JEE Advanced (2024)

PAPER 1

General Instructions:

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);
  Negative Marks : −1 in all other cases.

1. Let $f(x)$be a continuously differentiable function on the interval $(0, \infty)$ such that $f(1) = 2$ and for each $x > 0$. Then, for all $x > 0$, $f(x)$ is equal to $\lim_{t\to x}\frac{t^{10}f(x)-x^{10}f(t)}{t^9-x^9}=1$

(A) $\frac{31}{11x}-\frac{9}{11}x^{10}$ **(B)** $\frac{9}{11x}+\frac{13}{11}x^{10}$

(C) $\frac{-9}{11x}+\frac{13}{11}x^{10}$ **(D)** $\frac{13}{11x}+\frac{9}{11}x^{10}$

2. A student appears for a quiz consisting of only true-false type questions and answers all the questions. The student knows the answers of some questions and guess the answers for the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guessed it is $\frac{1}{2}$. Also, assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is $\frac{1}{6}$. Then the probability that the student knows the answer of a randomly chosen question is:

(A) $\frac{1}{12}$ **(B)** $\frac{1}{7}$ **(C)** $\frac{5}{7}$ **(D)** $\frac{5}{12}$

3. Let $\frac{\pi}{2}<x<\pi$ be such that $\cot x = \frac{-5}{\sqrt{11}}$.

Then, $\left(\sin\frac{11x}{2}\right)(\sin 6x-\cos 6x)+\left(\cos\frac{11x}{2}\right)(\sin 6x+\cos 6x)$ is equal to:

(A) $\frac{\sqrt{11}-1}{2\sqrt{3}}$ **(B)** $\frac{\sqrt{11}+1}{2\sqrt{3}}$ **(C)** $\frac{\sqrt{11}+1}{3\sqrt{2}}$ **(D)** $\frac{\sqrt{11}-1}{3\sqrt{2}}$

4. Consider the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let $S(p, q)$ be a point in the first quadrant such that $\frac{p^2}{9}+\frac{q^2}{4}>1$. Two tangents are drawn from S to the ellipse, of which one meets the ellipse at one end point of the minor axis and the other meets the ellipse at a point T in the fourth quadrant. Let R be the vertex of the ellipse with positive x-coordinate and O be the centre of the ellipse. If the area of the triangle ΔORT $\frac{3}{2}$, then which of the following options is correct?

(A) $q=2, p=3\sqrt{3}$ **(B)** $q=2, p=4\sqrt{3}$

(C) $q=1, p=5\sqrt{3}$ **(D)** $q=1, p=6\sqrt{3}$

General Instructions:

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;

choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e., the question is unanswered) will get 0 marks and
choosing any other option(s) will get −2 marks.

5. Let $S = \{a+b\sqrt{2} : a, b \in \mathbb{Z}\}$, $T_1 = \{(-1+\sqrt{2})^n : n \in \mathbb{N}\}$, and $T_2 = \{(1+\sqrt{2})^n : n \in \mathbb{N}\}$. Then, which of the following statements is (are) TRUE?

(A) $\mathbb{Z} \cup T_1 \cup T_2 \subset S$

(B) $T_1 \cap \left(0, \frac{1}{2024}\right) = \phi$, where ϕ denotes the empty set.

(C) $T_2 \cap (2024, \infty) \neq \phi$

(D) For any given $a, b \in \mathbb{Z}$, $\cos\left(\pi\left(a+b\sqrt{2}\right)\right) + i\sin\left(\pi\left(a+b\sqrt{2}\right)\right) \in \mathbb{Z}$ if and only if $b = 0$, where $i = \sqrt{-1}$.

6. Let $\mathbb{R}^2$ denote $\mathbb{R} \times \mathbb{R}$. Let $S = \{(a, b, c) : a, b, c \in \mathbb{R}$ and $ax^2 + 2bxy + cy^2 > 0$ for all $(x, y) \in \mathbb{R}^2 - \{(0,0)\}\}$. Then, which of the following statements is (are) TRUE?

(A) $\left(2, \frac{7}{2}, 6\right) \in S$ **(B)** If $\left(3, b, \frac{1}{12}\right) \in S$, then $|2b| < 1$.

(C) For any given $(a, b, c) \in S$, the system of linear equations $ax + by = 1$, $bx + cy = -1$ has a unique solution.

(D) For any given $(a, b, c) \in S$, the system of linear equations $(a + 1)x + by = 0$, $bx + (c + 1)y = 0$ has a unique solution.

7. Let $\mathbb{R}^3$ denote the three-dimensional space. Take two points $P = (1, 2, 3)$ and $Q = (4, 2, 7)$. Let *dist* (X, Y) denote the distance between two points X and Y in $\mathbb{R}^3$. Let
$S = \{X \in \mathbb{R}^3 : (dist(X, P))^2 - (dist(X, Q))^2 = 50\}$ and
$T = \{Y \in \mathbb{R}^3 : (dist(Y, Q))^2 - (dist(Y, P))^2 = 50\}$.
Then, which of the following statements is (are) TRUE?

(A) There is a triangle whose area is 1 and all of whose vertices are from S.

(B) There are two distinct points L and M in T such that each point on the line segment LM is also in T.

(C) There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

(D) There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

***General Instructions*:**

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the correct integer is entered;
  Zero Marks : 0 in all other cases.

8. Let $a = 3\sqrt{2}$ and $b = \frac{1}{5^{1/6}\sqrt{6}}$. If $x, y, \in \mathbb{R}$ are such that $3x + 2y = \log_a (18)^{\frac{5}{4}}$ and $2x - y = \log_b\left(\sqrt{1080}\right)$, then $4x + 5y$ is equal to ________.

9. Let $f(x) = x^4 + ax^3 + bx^2 + c$ be a polynomial with real coefficients such that $f(1) = -9$. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3 + 3ax^2 + 2bx = 0$, where $i = \sqrt{-1}$. If $\alpha_1, \alpha_2, \alpha_3$, and α_4 are all the roots of the equation $f(x) = 0$, then $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$ is equal to ______.

10. Let

$$S = \left\{ A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a, b, c, d, e \in \{0,1\} \text{ and } |A| \in \{-1,1\} \right\},$$

where $|A|$ denotes the determinant of A. Then, number of elements in S is ______.

11. A group of 9 students $s_1, s_2, \ldots, s_9$, is to be divided to form three teams X, Y, and Z of sizes 2, 3, and 4, respectively. Suppose that s_1 cannot be selected for the team X, and s_2 cannot be selected for the team Y. Then, the number of ways to form such teams, is _______.

12. Let $\overrightarrow{OP} = \frac{\alpha-1}{\alpha}\hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{OQ} = \hat{i} + \frac{\beta-1}{\beta}\hat{j} + \hat{k}$ and $\overrightarrow{OR} = \hat{i} + \hat{j} + \frac{1}{2}\hat{k}$ be three vectors, where $\alpha, \beta \in \mathbb{R} - \{0\}$ and O denotes the origin. If $(\overrightarrow{OP} \times \overrightarrow{OQ}) \cdot \overrightarrow{OR} = 0$ and the point $(\alpha, \beta, 2)$ lies on the plane $3x + 3y - z + l = 0$, then the value of l is ________.

13. Let X be a random variable, and let $P(X = x)$ denote the probability that X takes the value x. Suppose that the points $(x, P(X = x))$, $x = 0, 1, 2, 3, 4$, lie on a fixed straight line in the xy-plane, and $P(X = x) = 0$ for all $x \in \mathbb{R} - \{0, 1, 2, 3, 4\}$. If the mean of X is $\frac{5}{2}$, and the variance of X is a, then the value of 24α is _______.

***General Instructions*:**

SECTION 4 (Maximum Marks: 12)

- This section contains **FOUR (04)** paragraphs.
- Each set has **ONE** Multiple Choice Question.

- Each set has **TWO** lists: **List-I** and **List-II.**
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **FIVE** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen.
Zero Marks : 0 If none of the options is chosen (i.e the question is unanswered);
Negative Marks : –1 in all other cases.

14. Let a and b be the distinct roots of the equation $x^2 + x - 1 = 0$. Consider the set $T = \{1, \alpha, \beta\}$. For a 3×3 matrix $M = \{a_{ij}\}_{3\times 3}$, define $R_i = a_{i1} + a_{i2} + a_{i3}$ and $C_j = a_{1j} + a_{2j} + a_{3j}$ for $i = 1, 2, 3$ and $j = 1, 2, 3$.
Match each entry in **List-I** to the correct entry in **List-II**.

List-I	List-II
(P) The number of matrices $M = (a_{ij})_{3\times 3}$ with all entries in T such that $R_i = C_j = 0$ for all i, j, is	(1) 1
(Q) The number of symmetric matrices $M = (a_{ij})_{3\times 3}$ with all entries in T such that $C_j = 0$ for all j, is	(2) 12
(R) Let $M = (a_{ij})_{3\times 3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for $i > j$. Then, the number of elements in the set $\left\{\begin{pmatrix}x\\y\\z\end{pmatrix} : x, y, z \in \mathbb{R}, M\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}a_{12}\\0\\-a_{23}\end{pmatrix}\right\}$ is	(3) infinite
(S) Let $M = (a_{ij})_{3\times 3}$ be a matrix with all entries in T such that $R_i = 0$ for all i. Then, the absolute value of the determinant of M is	(4) 6
	(5) 0

The correct option is:
(A) (P) → (4) (Q) → (2) (R) → (5) (S) → (1)
(B) (P) → (2) (Q) → (4) (R) → (1) (S) → (5)
(C) (P) → (2) (Q) → (4) (R) → (3) (S) → (5)
(D) (P) → (1) (Q) → (5) (R) → (3) (S) → (4)

15. Let the straight line $y = 2x$ touch a circle with centre $(0, \alpha)$, $\alpha > 0$, and radius r at a point A_1. Let B_1 be the point on the circle such that the line segment A_1B_1 is a diameter of the circle. Let $\alpha + r = 5 + \sqrt{5}$.
Match each entry in **List-I** to the correct entry in **List-II**.

List-I	List-II
(P) α equals	(1) (–2, 4)
(Q) r equals	(2) $\sqrt{5}$
(R) A_1 equals	(3) (–2, 6)
(S) B_1 equals	(4) 5
	(5) (2, 4)

The correct option is:
(A) (P) → (4) (Q) → (2) (R) → (1) (S) → (3)
(B) (P) → (2) (Q) → (4) (R) → (1) (S) → (3)
(C) (P) → (4) (Q) → (2) (R) → (5) (S) → (3)
(D) (P) → (2) (Q) → (4) (R) → (3) (S) → (5)

16. Let $\gamma \in \mathbb{R}$ be such that the lines $L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3}$ and $L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma}$ intersect. Let R_1 be the point of intersection of L_1 and L_2. Let $O = (0, 0, 0)$ and $\hat{n}$ denote a unit normal vector to the plane containing both the lines L_1 and L_2.
Match each entry in **List-I** to the correct entry in **List-II**.

List-I	List-II
(P) γ equals	(1) $-\hat{j} - \hat{j} + \hat{k}$
(Q) A possible choice for $\hat{n}$ is	(2) $\sqrt{\frac{3}{2}}$
(R) $\overrightarrow{OR_1}$ equals	(3) 1
(S) A possible value of $\overrightarrow{OR_1} \cdot \hat{n}$ is	(4) $\frac{1}{\sqrt{6}}\hat{i} - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$
	(5) $\sqrt{\frac{2}{3}}$

The correct option is:
(A) (P) → (3) (Q) → (4) (R) → (1) (S) → (2)
(B) (P) → (5) (Q) → (4) (R) → (1) (S) → (2)
(C) (P) → (3) (Q) → (4) (R) → (1) (S) → (5)
(D) (P) → (3) (Q) → (1) (R) → (4) (S) → (5)

17. Let $f : \mathbb{R} - \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be functions defined by

$$f(x) = \begin{cases} x|x|\sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0, \end{cases} \text{ and } g(x) = \begin{cases} 1 - 2x, & 0 \le x \le \frac{1}{2}, \\ 0, & \text{otherwise.} \end{cases}$$

Let $a, b, c, d, \in \mathbb{R}$. Define the function $h : \mathbb{R} \to \mathbb{R}$ by

$$h(x) = af(x) + b\left(g(x) + g\left(\frac{1}{2} - x\right)\right) + c(x - g(x)) + dg(x), x \in \mathbb{R}.$$

Match each entry in **List-I** to the correct entry in **List-II**.

List-I	List-II
(P) If $a = 0, b = 1, c = 0,$ and $d = 0$, then	(1) h is one-one.
(Q) If $a = 1, b = 0, c = 0,$ and $d = 0$, then	(2) h is onto.
(R) If $a = 0, b = 0, c = 1,$ and $d = 0$, then	(3) h is differentiable on $\mathbb{R}$.
(S) If $a = 0, b = 0, c = 0,$ and $d = 1$, then	(4) the range of h is [0, 1].
	(5) the range of h is {0, 1}.

The correct option is:
(A) (P) → (4) (Q) → (3) (R) → (1) (S) → (2)

(B) (P) → (5) (Q) → (2) (R) → (4) (S) → (3)

(C) (P) → (5) (Q) → (3) (R) → (2) (S) → (4)

(D) (P) → (4) (Q) → (2) (R) → (1) (S) → (3)

Answer Key

Q.No.	Answer key	Topic's name	Chapter's name
1	(B)	Linear Differential Equation	Differential Equation
2	(C)	Bayes Theorem	Probability
3	(B)	Sum or Difference	Trigonometric Ratio and Identities
4	(A)	Tangent	Ellipse
5	(A, C, D)	Operations on Sets	Sets
6	(B, C, D)	Cramer Rule	Determinant
7	(A, B, C, D)	Plane	Three Dimensional
8	8	Log	Essential Math
9	20	Biquadratic	Quadratic Equation
10	16	Miscellaneous	Matrices
11	665	Division into Groups	Permutation and Combination
12	5	Scalar Product	Vector
13	42	Variance	Stat
14	(C)	Miscellaneous	Matrices
15	(C)	Tangent	Circle
16	(C)	Intersecting Lines	Three Dimensional
17	(C)	Miscellaneous	Function

ANSWERS WITH EXPLANATIONS

1. Correct option is (B).

$$\lim_{t\to x}\frac{t^{10}f(x)-x^{10}f(t)}{t^9-x^9}=1$$

Form is $\frac{0}{0}$ hence using L'Hospital's

$$\lim_{t\to x}\frac{10t^9f(x)-x^{10}f'(t)}{9t^8}=1$$

$$\Rightarrow \frac{10x^9f(x)-x^{10}f'(x)}{9x^8}=1$$

$$\Rightarrow f'(x)-\frac{10}{x}f(x)=\frac{-9}{x^2}$$

$$\textbf{IF} = e^{\int\frac{-10}{x}dx}=e^{-10\ln x}=x^{-10}$$

$$\Rightarrow yx^{-10}=\int\frac{-9}{x^{12}}dx$$

$$\Rightarrow yx^{-10}=\frac{9}{11}x^{-11}+c$$

given $f(1) = 2$ $\Rightarrow c=\frac{13}{11}$

$$\Rightarrow y=\frac{9}{11x}+\frac{13}{11}x^{10}$$

2. Correct option is (C).

Let P (guess) = λ

P (knows the answer) = $1-\lambda$

P (Correct answer/guess) = $\frac{1}{2}$, P

(Correct answer/known the answer) v

$$P\left(\frac{\text{guess}}{\text{correct answer}}\right)=\frac{\lambda\times\frac{1}{2}}{\lambda\times\frac{1}{2}+(1-\lambda)\times 1}=\frac{1}{6}$$

$$3\lambda=1-\frac{\lambda}{2}$$

$$\Rightarrow \lambda=\frac{2}{7}$$

Probability

$$=1-\lambda=\frac{5}{7}$$

3. Correct option is (B).

Let $\lambda = \sin\left(\frac{11x}{2}\right)\sin 6x-\sin\left(\frac{11x}{2}\right)\cos 6x$

$+\cos\left(\frac{11x}{2}\right)\sin 6x+\cos\left(\frac{11x}{2}\right)\cos 6x$

$\sin\left(6x-\frac{11x}{2}\right)+\cos\left(6x-\frac{11x}{2}\right)$

$$\Rightarrow \lambda=\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right) \quad ...(i)$$

$$\tan x=-\frac{\sqrt{11}}{5}=\frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$$

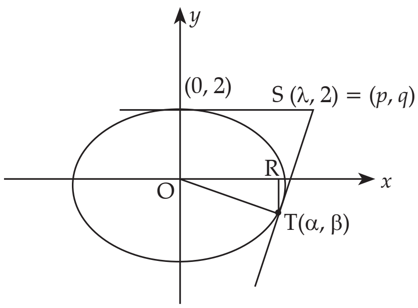
$$\Rightarrow -1+\tan^2\frac{x}{2} = \frac{10}{\sqrt{11}}\tan\frac{x}{2}$$

$$\Rightarrow \tan\frac{x}{2} = \frac{\frac{10}{\sqrt{11}}+\sqrt{\frac{100}{11}+4}}{2}$$

$$= \frac{10+12}{2\sqrt{11}} = \sqrt{11}$$

$$\therefore \quad \lambda = \frac{\sqrt{11}+1}{2\sqrt{3}}$$

4. Correct option is (A).



Area of ΔORT = $\left|\frac{3\times\beta}{2}\right| = \frac{3}{2}$

$\therefore \quad \beta < 0$

$\therefore \quad \beta = -1$

(α, β) lie on ellipse $\Rightarrow \frac{\alpha^2}{9}+\frac{1}{4}=1$

α Equation of tangent at 'T' $> 0 \Rightarrow \alpha = \frac{3\sqrt{3}}{2}$

$$\frac{3\sqrt{3}\cdot x}{18}-\frac{y}{4}=1$$

$\because (\lambda, 2)$ lie on tangent

$\Rightarrow \quad \lambda = \quad 3\sqrt{3} = p$ and $q = 2$

5. Correct option is (A, C, D).

If $b = 0$ then $a \in \mathbb{Z} \Rightarrow \mathbb{Z} \subset S$

$$\left(\sqrt{2}-1\right)^n = {}^nC_0\left(\sqrt{2}\right)^n - {}^nC_1\left(\sqrt{2}\right)^{n-1} + \ldots$$

$= \quad a_1+b_1\sqrt{2}$ where a_1, $b_1, \in \mathbb{Z}$

Similarly $\left(\sqrt{2}+1\right)^n = a_2+b_2\sqrt{2}$ where $a_2, b_2, \in \mathbb{Z}$

$\therefore \mathbb{Z} \cup T_1 \cup T_2 \subset S$ (true)

Option (B)

$0 < \sqrt{2}-1 < 1$

$\therefore \left(\sqrt{2}-1\right)^n \to 0$ as $n \to \infty$

Hence $T_1 \cap \left(0, \frac{1}{2024}\right) \neq \phi$

if n is large

($\therefore$ Option (B) is wrong)

Option (C)

$\sqrt{2}+1 > 1$

$\therefore \left(\sqrt{2}+1\right)^n \to \infty$ as $n \to \infty$

Hence $T_2 \cap (2024, \infty) \neq \phi$

(Option (C) is correct)

Option (D)

Answer = A, C, D

$a+b\sqrt{2}$ must be an Integer

$\therefore b = 0$

6. Correct options are (B, C, D).

$ax^2 + 2bxy + cy^2 > 0$

Let $\frac{y}{x} = t$

$ct^2 + 2bt + a > 0$

$\therefore c > 0$ and $b^2 - ac < 0$

$\left(2, \frac{7}{2}, 6\right)$ does not satisfy above condition

(option (A) wrong)

Option (B) $\Rightarrow \left(3, b, \frac{1}{12}\right) \in S$

$\therefore \ b^2 < \frac{1}{4}$

(option (B) correct)

Option (C) For unique solution

$$\begin{vmatrix} a & b \\ b & c \end{vmatrix} \neq 0$$

(option (C) correct)

Option (D) For unique solution

$(a + 1)(c + 1) - b^2 \neq 0$

$\{b^2 - ac < 0$ and $c >$ o

$\therefore a, c > 0$

(option (D) correct)

7. Correct options are (A, B, C, D).

Let X = (x, y, z)

$\therefore (x-1)^2 + (y-2)^2 + (z-3)^2 - \{(x-4)^2 + (y-2)^2 + (z-7)^2\} = 50$

$\Rightarrow S : 6x + 8z - 105 = 0$

Let $Y = (x, y, z)$

Similarly, we get $T : 6x + 8z - 5 = 0$

(A) S is a plane equation hence correct

(B) T is a plane equation hence correct

(C) distance between both planes

$$\Rightarrow \frac{100}{\sqrt{36+64}} = 10$$

Hence correct.

There are infinitely many rectangles of perimeter 48 because there are infinity many point on the plane. A square of perimeter 48 having side 12 if the framing square on the plane $S = T$.

So Option (D) is correct.

8. Correct answer is [8].

$a = \sqrt{18}$, $b = (1080)^{-\frac{1}{6}}$

$\Rightarrow 3x + 2y = \frac{5}{2}$

and $2x - y = -3$

$\therefore x = -\frac{1}{2}, y = 2$

hence $4x + 5y = 8$

9. Correct answer is [20].

$x(4x^2 + 3ax + 2b) = 0$ **has roots** $0, \pm i\sqrt{3}$

$\Rightarrow a = 0, \frac{2b}{4} = (\sqrt{3}\,i)(-\sqrt{3}\,i)$

$\Rightarrow b = 6$

$f(1) = -9 \Rightarrow c = -16$

$\therefore x^4 + 6x^2 - 16 = 0$

$(x^2 + 8)(x^2 - 2) = 0$

$\Rightarrow x = \sqrt{2}, 2\sqrt{2}\,i, -2\sqrt{2}\,i, -\sqrt{2}$

$|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 = 20$

10. Correct answer is [16].

$|A| = d - e + c(b - a) = \pm 1$

Case I $\Rightarrow c = 1$

$d + b - a - e = \pm 1$

Three of them one $= {}^4C_3$

Three of them zero $= {}^4C_3$

$\Rightarrow (4 + 4) \Rightarrow$ **8 ways**

Case II $\Rightarrow c = 0$

$d - e = \pm 1$

a, b **can take an given value**

Total ways $\Rightarrow 2 \times 4 \Rightarrow$ **8 ways.**

11. Correct answer is [665].

Total ways without any conditon

$= {}^9C_2 \times {}^7C_3 \times {}^4C_4 = \frac{9!}{2!\,3!\,4!}$

$S_1 \in X$ **then** $\Rightarrow {}^8C_1 \times {}^7C_3 \times {}^4C_4 = \frac{8!}{3!4!}$

$S_2 \in Y$ **then** $\Rightarrow {}^8C_2 \times {}^6C_2 \times {}^4C_4 = \frac{8!}{2!\,2!\,4!}$

$S_1 \in X$ **and** $S_2 \in Y \Rightarrow {}^7C_1\, {}^6C_2\, {}^4C_4 = \frac{7!}{2!\,4!}$

Required ways $\Rightarrow$ **Total ways** $- n(S_1 \cup S_2)$

$$\Rightarrow \frac{9!}{2!\,3!\,4!} - \frac{8!}{3!\,4!} - \frac{8!}{2!\,2!\,4!} + \frac{7!}{2!\,4!}$$

$\Rightarrow$ **665**

12. Correct answer is [5].

$(\overline{OP} \times \overline{OQ}) \cdot \overline{OR} = 0$

$$\begin{vmatrix} 1-\frac{1}{\alpha} & 1 & 1 \\ 1 & 1-\frac{1}{\beta} & 1 \\ 1 & 1 & 1-\frac{1}{2} \end{vmatrix} = 0$$

$C_1 \to C_1 \to C_2 \quad ; \quad C_2 \to C_2 \to C_3$

$$\begin{vmatrix} -\frac{1}{\alpha} & 0 & 1 \\ \frac{1}{\beta} & -\frac{1}{\beta} & 1 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{vmatrix} = 0$$

$$-\frac{1}{\alpha}\left(-\frac{1}{2\beta} - \frac{1}{2}\right) + \frac{1}{2\beta} = 0$$

$$\frac{1}{\alpha\beta} + \frac{1}{\alpha} + \frac{1}{\beta} = 0 \Rightarrow \alpha + \beta + 1 = 0$$

$\{\alpha, \beta, 2\}$ **lie on** $3x + 3y - z + l = 0$

$3(\alpha + \beta) - 2 + l = 0 \Rightarrow l = 5$

13. Correct answer is [42].

mean $= \sum_{x=0}^{4} x\,P(x) = \frac{5}{2}$ **...(1)**

$x, P(x)$ **lie on a line**

$\therefore P(1) - P(0) = P(2) - P(1) = P(3) - P(2) = P(4) - P(3) = m$

$\sum_{r=0}^{4} P(r) = 1 \Rightarrow 5P(0) + 10m = 1$ **...(2)**

We can also write $P(n) = n \cdot m + P(0)$ $n \in N$

From equation (1)

$\therefore \frac{5}{2} = 30m + 10\,P(0)$ **...(3)**

$\Rightarrow m = \frac{1}{20}, \; P(0) = \frac{1}{10}$

α **= variance** $= \sum_{x=0}^{4} x^2 P(x) - \left(\frac{5}{2}\right)^2$

$= \frac{1}{20}(3 \times 1 + 4 \times 2^2 + 5 \times 3^2 + 6 \times 4^2) - \frac{25}{4}$

$\alpha = \frac{7}{4}$

Hence, $24\alpha = 42$

14. Correct option is (C).

$x^2 + x - 1 = 0$ **has roots** α, β

$\Rightarrow\ \alpha + \beta = -1, \alpha\beta = -1$

(P) $R_i = C_i = 0$

$\begin{pmatrix} \alpha & \beta & 1 \\ & & \square \\ & & \end{pmatrix}$ $\rightarrow$ This can be arranged in six ways; $\rightarrow$ This can't be one hence two ways

Total ways $\Rightarrow 6 \times 2 \Rightarrow 12$

P $\rightarrow$ **2**

(Q) $A = \begin{pmatrix} 1 & \alpha & \beta \\ \alpha & & \\ \beta & \square & \end{pmatrix}$ $\rightarrow$ 6 ways and $A = A^T$; $\rightarrow$ If R_1 is filled C_1 is also fix.

$\downarrow$ This cannot be α and β

Total ways $\Rightarrow$ **6**

Q $\rightarrow$ **4**

(R) M is skew symmetric $\Rightarrow |M| = 0$

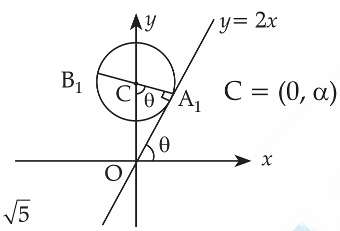
hence ∞ solution

R $\rightarrow$ 3

(S) If $R_i = 0 \Rightarrow |M| = 0$

S $\rightarrow$ 5

15. Correct option is (C).



tan θ = 2

$$\left|\frac{\alpha}{\sqrt{5}}\right| = r \Rightarrow \alpha = r\sqrt{5}$$

Given $\alpha + r = 5 + \sqrt{5}$

$\therefore\ \alpha = 5$ **and** $r = \sqrt{5}$

$OA_1 = \sqrt{\alpha^2 - r^2} = 2\sqrt{5}$

$A_1 = (2\sqrt{5}\cos\theta, 2\sqrt{5}\sin\theta) = (2, 4)$

$B_1 = (-2, 6)$

16. Correct option is (C).

L_1 and L_2 are intersecing lines; hence, the shortest distance = 0

$$\begin{vmatrix} 3 & 2 & \gamma \\ 1 & 2 & 3 \\ 5 & -10 & -25 \end{vmatrix} = 0 \Rightarrow \gamma = 1$$

$$\overline{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = -4\hat{i} + 8\hat{j} - 4\hat{k}$$

$$\Rightarrow \hat{n} = \frac{\hat{i}}{\sqrt{6}} - \frac{2\hat{j}}{\sqrt{6}} + \frac{\hat{k}}{\sqrt{6}}$$

Let $\dfrac{x+11}{1} = \dfrac{y+21}{2} = \dfrac{z+29}{3} = \lambda$

$\dfrac{x+16}{3} = \dfrac{y+11}{2} = \dfrac{z+4}{1} = \mu$

Both lines intersect at R_1

$\therefore\ \lambda - 11 = 3\mu - 16,\ 2\lambda - 21 = 2\mu - 11$

$\Rightarrow \lambda = 10$

$R_1 = -1, -1, 1$

$\overrightarrow{OR_1} = -\hat{i} - \hat{j} + \hat{k}$

$\overrightarrow{OR_1} \cdot \hat{n} = \sqrt{\dfrac{2}{3}}$

17. Correct option is (C).

$$g\left(\frac{1}{2} - x\right) = \begin{cases} 1 - 2\left(\frac{1}{2} - x\right), & 0 \le \frac{1}{2} - x \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} 2x, & 0 \le x \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$\therefore g(x) + g\left(\frac{1}{2} - x\right) = \begin{cases} 1, & 0 \le x \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow h(x) = \begin{cases} af(x) + b + c(3x-1) + d(1-2x), & 0 \le x \le \frac{1}{2} \\ af(x) + cx, & \text{otherwise} \end{cases}$$

$$(P) = h(x) = \begin{cases} 1, & 0 \le x \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

P $\rightarrow$ 5

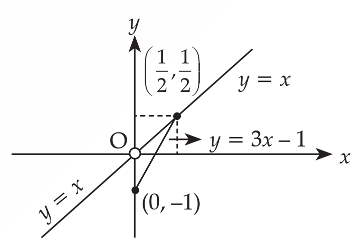
(Q) $\Rightarrow h(x) = f(x)$

$$\text{LHD} = \lim_{h \to 0^+} \frac{f(0) - f(0-h)}{h} = \lim_{h \to 0} h\sin(h^{-1}) = 0$$

$$\text{RHD} = \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} h\sin(h^{-1}) = 0$$

Q $\rightarrow$ 3

$$(R) \Rightarrow h(x) = \begin{cases} 3x - 1, & 0 \le x \le \frac{1}{2} \\ x, & \text{otherwise} \end{cases}$$



$h(x)$ is onto function.

(R) $\rightarrow$ 2

$$(S)\ h(x) = \begin{cases} 1 - 2x, & 0 \le x \le \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

$h(x)_{max} = 1 - 0 = 1$

$h(x)_{min} = 0$

$0 \le h(x) \le 1$

(S) $\rightarrow$ 4