JEE Advanced (2024)

PAPER 2

General Instructions:

SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –1 in all other cases.

1. Considering only the principal values of the inverse trigonometric functions, the value of

$$\tan\left[\sin^{-1}\left(\frac{3}{5}\right)-2\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)\right] \text{ is:}$$

(A) $\frac{7}{24}$ **(B)** $\frac{-7}{24}$ **(C)** $\frac{-5}{24}$ **(D)** $\frac{5}{24}$

2. Let $S = \{(x, y) \in R \times R : x \geq 0, y \geq 0, y^2 \leq 4x, y^2 \leq 12 - 2x$ and $3y + \sqrt{8}x \leq 5\sqrt{8}\}$. If the area of the region S is $\alpha\sqrt{2}$, then α is equal to:

(A) $\frac{17}{2}$ **(B)** $\frac{17}{3}$ **(C)** $\frac{17}{4}$ **(D)** $\frac{17}{5}$

3. Let $k \in R$. If $\lim_{x\to 0^+}(\sin(\sin kx)+\cos x + x)^{\frac{2}{x}} = e^6$, then the value of k is:

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

4. Let $f : R \to R$ be a function defined by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x^2}\right), & \text{if } x \neq 0, \\ 0, & \text{if } x = 0 \end{cases}$$

Then, which of the following statements is TRUE?

(A) $f(x) = 0$ has infinitely many solutions in the interval $\left[\frac{1}{10^{10}}, \infty\right)$.

(B) $f(x) = 0$ has no solutions in the interval $\left[\frac{1}{\pi}, \infty\right)$.

(C) The set of solutions of $f(x) = 0$ in the interval $\left(0, \frac{1}{10^{10}}\right)$ is finite.

(D) $f(x) = 0$ has more than 25 solutions in the interval $\left(\frac{1}{\pi^2}, \frac{1}{\pi}\right)$.

General Instructions:

SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY OR MORE THAN ONE** of these four options is the correct answer.
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered);
Negative Marks : –2 in all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option(s) (i.e., the question is unanswered) will get 0 marks; and
choosing any other combination of options will get –2 marks.

5. Let S be the set of all $(\alpha, \beta) \in R \times R$ such that

$$\lim_{x\to\infty} \frac{\sin(x^2)(\log_e x)^{\alpha} \sin\left(\frac{1}{x^2}\right)}{x^{\alpha\beta}(\log_e(1+x))^{\beta}} = 0.$$

Then, which of the following is (are) correct?

(A) $(-1, 3) \in S$ **(B)** $(-1, 1) \in S$

(C) $(1, -1) \in S$ **(D)** $(1, -2) \in S$

6. A straight line drawn from the point P(1, 3, 2), parallel to the line $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-6}{1}$, intersects the plane L_1: $x - y, + 3z = 6$, at the point Q. Another straight line which passes through Q and is perpendicular to the plane L_1 intersects the plane L_2: $2x - y + z = -4$ at the point R. Then, which of the following statements is (are) TRUE?

(A) The length of the line segment PQ is $\sqrt{6}$

(B) The coordinates of R are (1, 6, 3)

(C) The centroid of the triangle PQR is $\left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$

(D) The perimeter of the triangle PQR is $\sqrt{2} + \sqrt{6} + \sqrt{11}$

7. Let A_1, B_1, C_1, be three points in the xy-plane. Suppose that the lines A_1C_1 and B_1C_1 are tangents to the curve $y^2 = 8x$ at A_1 and B_1 respectively. If O = (0,0) and C_1 = (–4, 0), then which of the following statements is (are) TRUE?

(A) The length of the line segment OA_1 is $4\sqrt{3}$

(B) The length of the line segment A_1B_1 is 16

(C) The orthocentre of the triangle $A_1B_1C_1$ is (0,0)

(D) The orthocentre of the triangle $A_1B_1C_1$ is (1,0)

***General Instructions*:**

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 If **ONLY** the correct option is chosen;

  Zero Marks : 0 in all other cases.

8. Let $f : R \to R$ be a function such that $f(x + y) = f(x) + f(y)$ for all $x, y \in R$, and $g : R \to (0, \infty)$ be a function such that $g(x + y) = g(x)g(y)$ for all $x, y \in R$. If $f\left(\frac{-3}{5}\right) = 12$ and $g\left(\frac{-1}{3}\right) = 2$, then the value of $\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right)g(0)$ is __________ .

9. A bag contains N balls out of which 3 balls are white, 6 balls are green, and the remaining balls are blue. Assume that the balls are identical otherwise. Three balls are drawn randomly one after the other without replacement. For $i = 1, 2, 3$, let and W_i,G_i and B_i denote the events that the ball drawn in the i^{th} draw is a white ball, green ball, and blue ball, respectively. If the probability $P(W_1 \cap G_2 \cap B_3) = \frac{2}{5N}$ and the conditional probability $P(B_3 \cap W_1 \cap G_2) = \frac{2}{9}$, then N equals ....

10. Let the function $f : R \to R$ be defined by

$$f(x) = \frac{\sin x}{e^{\pi x}} \frac{(x^{2023} + 2024x + 2025)}{(x^2 - x + 3)} + \frac{2}{e^{\pi x}} \frac{(x^{2023} + 2024x + 2025)}{(x^2 - x + 3)}$$

Then, the number of solutions of $f(x) = 0$ in R is ....

11. Let $\vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{q} = \hat{i} - \hat{j} + \hat{k}$. If for some real number α, β and γ, we have $15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha(2\vec{p} + \vec{q}) + \beta(\vec{p} - 2\vec{q}) + \gamma(\vec{p} \times \vec{q})$, then the value of γ is ....

12. A normal with slope $\frac{1}{\sqrt{6}}$ is drawn from the point $(0, -\alpha)$to the parabola $x^2 = -4ay$, where $a > 0$. Let L be the line passing through $(0, -\alpha)$ and parallel to the directrix of the parabola. Suppose that L intersects the parabola at two points A and B. Let r denote the length of the latus rectum and s denote the square of the length of the line segment AB. If $r : s = 1 : 16$, then the value of 24α is ....

13. Let the function $f : [1, \infty) \to R$ be defined by

$$f(t) = \begin{cases} (-1)^{n+1}2, & \text{if } t = 2n-1, n \in N \\ \frac{(2n+1-t)}{2} f(2n-1) + \frac{(t-(2n-1))}{2} f(2n+1), & \text{if } 2n-1 < t < 2n+1, n \in N \end{cases}$$

Define $g(x) = \int_1^x f(t)dt$, $x \in (1, \infty)$. Let α denote the number of solutions of the equation $g(x) = 0$ in the interval (1, 8] and $\beta = \lim_{x\to 1+} \frac{g(x)}{x-1}$. Then, the value of $\alpha + \beta$ is equal to ....

General Instructions:

SECTION 4 (Maximum Marks : 12)

- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 in all other cases.

PARAGRAPH I

Let $S = 1, 2, 3, 4, 5, 6$ and X be the set of all relations R from S to S that satisfy both the following properties:

i. R has exactly 6 elements.

ii. For each $(a, b) \in$ R, we have $|a - b| \geq 2$.

Let $Y = \{R \in X$: The range of has exactly one element$\}$ and $Z = \{R \in X : R$ is a function from to S to S$\}$.

Let n(A) denote the number of elements in a set A.

(There are two questions based on PARAGRAPH I, the question given below is one of them).

14. If $n(X) = {}^mC_6$, then the value of m is ....

15. If the value of n(Y) + n(Z) is k^2, then $|k|$ is ....

PARAGRAPH II

Let $f : \left[0, \frac{\pi}{2}\right] \to [0, 1]$ be the function defined by $f(x) = \sin^2 x$ and the let $g : \left[0, \frac{\pi}{2}\right] \to [0, \infty)$ be the function defined by $g(x) = \sqrt{\frac{\pi x}{2} - x^2}$

(There are two questions based on PARAGRAPH II, the question given below is one of them).

16. The value of $2\int_0^{\frac{\pi}{2}} f(x)g(x)dx - \int_0^{\frac{\pi}{2}} g(x)dx$ is ....

17. The value of $\frac{16}{\pi^3}\int_0^{\frac{\pi}{2}} f(x)g(x)dx$ is ....

ANSWER KEY

Q.No.	Answer key	Topic's name	Chapter's name
1	B	Properties of ITF	Inverse Trigonometric Function
2	B	Area Between two Curves	Area Under the Curves
3	B	Limit Using L'Hospital's	Limits
4	D	Trigonometry Equation	Trigonometry
5	B, C	Limit of a Function	Limits
6	A, C	Line and Plane	3 D
7	A, C	Line and Parabola	Parabola
8	[51]	Functional Rule	Function
9	[11]	Conditional Probability	Probability
10	[1]	No of Solution of Equation	Application of Derivatives
11	[2]	Product of Vectors	Vector
12	[12]	Normal of Parabola	Parabola
13	[5]	Definite Integration	Definite Integration
14	[20]	Number of Relation	Relation and Function
15	[36]	Number of Functions	Relation and Function
16	[0]	Definite Integration Using Properties	Definite Integration
17	[0.25]	Definite Integration Using Properties	Definite Integration

ANSWERS WITH EXPLANATIONS

1. Correct option is (B).

Let $A = \sin^{-1}\dfrac{3}{5}$ and $B = 2\cos^{-1}\dfrac{2}{\sqrt{5}}$

$\sin A = \dfrac{3}{5}$ $\qquad \dfrac{B}{2} = \cos^{-1}\dfrac{2}{\sqrt{5}}$

$\tan A = \dfrac{3}{4}$ $\qquad \cos\dfrac{B}{2} = \dfrac{2}{\sqrt{5}}$

$$\tan\frac{B}{2} = \frac{1}{2}$$

$$\therefore \quad \tan B = \frac{2\tan\frac{B}{2}}{1-\tan^2\frac{B}{2}} = \frac{2\times\frac{1}{2}}{1-\frac{1}{4}}$$

$$= \frac{4}{3}$$

Now ATQ

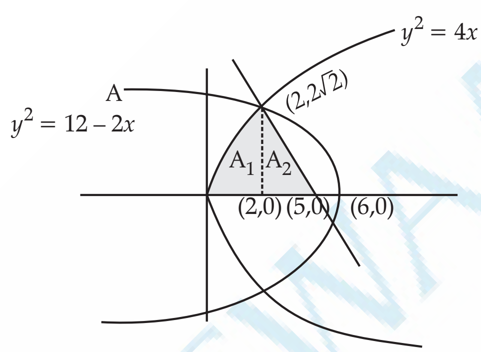
$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$$

$$= \frac{\frac{3}{4}-\frac{4}{3}}{1+\frac{3}{4}\times\frac{4}{3}} = \frac{-\frac{7}{12}}{2} = -\frac{7}{24}$$

2. Correct option is (B).

$x \ge 0, y \ge 0, y^2 \le 4x, y^2 \le 12-2x,$

and $3y+\sqrt{8}x \le 5\sqrt{8}$



Solving $\quad y^2 = 4x \;\&\; y^2 = 12-2x$

$\Rightarrow \quad 4x = 12-2x$

$\quad x = 2$

$\Rightarrow \quad y^2 = 8$

$\Rightarrow \quad y = \pm 2\sqrt{2}$

Point of intersections are $(2, 2\sqrt{2})$ and $(2, -2\sqrt{2})$

$$3y+\sqrt{8}x = 5\sqrt{8}$$

$\Rightarrow (2, 2\sqrt{2})$ satisfies the equation

Shaded Area $= A_1 + A_2$

$$= \int_0^2 \sqrt{4x}\,dx + \frac{1}{2}\times 3\times\sqrt{8}$$

$$= 2.\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2 + 3\sqrt{2} = \frac{2}{3}(2)^{\frac{3}{2}} + 3\sqrt{2}$$

$$= \frac{2}{3}\times 4\sqrt{2} + 3\sqrt{2} = \frac{8\sqrt{2}}{3} + 3\sqrt{2}$$

$$= \frac{17\sqrt{2}}{3}$$

$$\therefore \quad \alpha = \frac{17}{3}$$

3. Correct option is (B).

$$\lim_{x\to 0+}(\sin(\sin kx)+\cos x + x)^{\frac{2}{x}} = e^6$$

(1^∞ form)

$$\Rightarrow \quad e^{\lim\limits_{x\to 0+}(\sin(\sin kx)+\cos x + x - 1).\frac{2}{x}} = e^6$$

$$2\lim_{x\to 0+}\frac{(\sin(\sin kx)+\cos x + x - 1)}{x} = 6$$

Using L'Hospital's

$$\lim_{x\to 0+}\frac{\cos(\sin kx).\cos kx.k - \sin x + 1}{1} = 3$$

$\Rightarrow \quad k - 0 + 1 = 3$

$\quad k = 2$

4. Correct option is (D).

$$f(x) = \begin{cases} x^2\sin\left(\dfrac{\pi}{x^2}\right) & \text{if } x \ne 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Option (A)

$x \in \left[\dfrac{1}{10^{10}}, \infty\right)$

$\Rightarrow \dfrac{1}{10^{10}} \le x < \infty$

$0 < \dfrac{1}{x} \le 10^{10}$

$0 < \dfrac{\pi}{x^2} \le 10^{20}\pi$

$\Rightarrow \quad f(x) = 0$

$\Rightarrow \quad x^2\sin\dfrac{\pi}{x^2} = 0$ has finite solution

(B) $x \in \left[\dfrac{1}{\pi}, \infty\right)$

$\Rightarrow \dfrac{1}{\pi} \le x < \infty$

$0 < \dfrac{1}{x} \le \pi$

$0 < \dfrac{\pi}{x^2} \le \pi^3$

$\quad f(x) = 0$

$\Rightarrow \quad x^2\sin\dfrac{\pi}{x^2} = 0$ has finite solution.

(C) $x \in \left(0, \frac{1}{10^{10}}\right)$

$0 < x < \frac{1}{10^{10}}$

$10^{20} < \frac{1}{x^2} < \infty$

$10^{20}\pi < \frac{\pi}{x^2} < \infty$

$$f(x) = 0$$

$$x^2 \sin\frac{\pi}{x^2} = 0 \text{ has infinite no. of solutions.}$$

(D) $x \in \left(\frac{1}{\pi^2}, \frac{1}{\pi}\right)$

$\frac{1}{\pi^2} < x < \frac{1}{\pi}$

$\pi < \frac{1}{x} < \pi^2$

$\pi^2 < \frac{1}{x^2} < \pi^4$

$\pi^3 < \frac{\pi}{x^2} < \pi^5$

$$\pi^3 \simeq 30.9$$
$$\pi^5 \simeq 305.24$$

$\because$ $f(x) = 0$

$\Rightarrow$ $x \sin\frac{\pi}{x^2} = 0$

$\Rightarrow$ $\sin\theta = 0$

$\Rightarrow$ $\theta = n\pi$

Equation has more than 25 solutions.

5. Correct options are (B, C).

$$\lim_{x\to\infty} \frac{\sin x^2 (\ln x)^{\alpha} \sin\frac{1}{x^2}}{x^{\alpha\beta}.[\ln(1+x)]^{\beta}} = 0$$

$$\lim_{x\to\infty} \frac{\sin x^2.(\ln x)^{\alpha}.\left(\frac{\sin\frac{1}{x^2}}{\frac{1}{x^2}}\right).\frac{1}{x^2}}{x^{\alpha\beta}.[\ln(1+x)]^{\beta}} = 0$$

$\because$ $\sin x^2 \in [-1, 1]$

$\Rightarrow$ $\lim_{x\to\infty} \frac{\sin x^2.(\ln x)^{\alpha}}{x^{\alpha\beta+2}.[\ln(1+x)]^{\beta}} = 0$

$\Rightarrow \lim_{x\to\infty} \frac{(\sin x^2)(\ln x)^{\alpha-\beta}}{x^{\alpha\beta+2}}$

$\Rightarrow$ At $x \to \infty$, $1 + x \simeq x$

(A) $(-1, 3) \Rightarrow \alpha = -1, \beta = 3$

$\lim_{x\to\infty} \frac{\sin x^2.(\ln x)^{-4}}{x^{-1}}$

$\Rightarrow$ $\lim_{x\to\infty} \frac{\sin x^2 (\ln x)^{-4}}{x^{-1}} = \infty$ (wrong)

(B) $(-1, 1) \Rightarrow \alpha = -1, \beta = 1$

$$\lim_{x\to\infty} \frac{\sin x^2 (\ln x)^{-2}}{x} = \frac{\sin x^2}{x(\ln x)^2} = 0 \text{ correct}$$

(C) $(-1, 1) \Rightarrow \alpha = 1, \beta = -1$

$$\lim_{x\to\infty} \frac{\sin x^2 (\ln x)^2}{x} \to \infty \text{ correct}$$

(D) $(1, -2) \Rightarrow \alpha = 1, \beta = -2$

$$\lim_{x\to\infty} \sin x^2 \frac{(\ln x)^3}{x^0} \to 0 \text{ wrong}$$

6. Correct options are (A, C).

Line passing through (1, 3, 2) & parallel to

$$\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-6}{1} \text{ is}$$

Line (l): $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z-2}{1} = \lambda$

Any point $(\lambda + 1, 2\lambda + 3, \lambda + 2)$

Put in plane L_1:

$$x - y + 3z = 6$$

$\lambda + 1 - 2\lambda - 3 + 3\lambda + 6 = 6$

$2\lambda = 2$

$\lambda = 1$

$\because$ Q $\Rightarrow$ (2, 5, 3)

line L_2 is passing through Q and $\perp^r$ to

$x - y + 3z = 6$

$\therefore$ Equation of line L_2:

$$\frac{x-2}{1} = \frac{y-5}{-1} = \frac{z-3}{3} = \mu$$

Any point on $L_2 = (\mu + 2, -\mu + 5, 3\mu + 3)$

Put in plane L_2 : $2x - y + z = -4$

$2u + 4 + u - 5 + 3u + 3 = -4$

$\mu = -1$

$\therefore$ R (1, 6, 0)

(A) $PQ = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

(B) R = (1, 6, 0)

(C) Centroid of ΔPQR $= \left(\frac{1+2+1}{3}, \frac{3+5+6}{3}, \frac{2+3+0}{3}\right)$

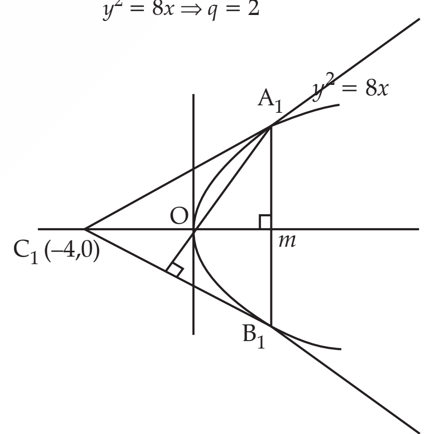
$= \left(\frac{4}{3}, \frac{14}{3}, \frac{5}{3}\right)$

(D) Perimeter of PQ = PQ + QR + PR

$= \sqrt{6} + \sqrt{11} + \sqrt{33}$

7. Correct options are (A, C).

$$y^2 = 8x \Rightarrow q = 2$$



Let $A_1 (at^2, 2at) = (2t^2, 4t)$

Equation of tangent at A_1

$$y^2 = 8x$$

$\Rightarrow \quad yy_1 = 8\left(\frac{x+x_1}{2}\right)$

$\Rightarrow \quad 4ty = 4(x + 2t^2)$

$\Rightarrow \quad ty = x + 2t^2$

$\therefore$ It passes through $C_1(-4, 0)$

$\Rightarrow \quad 0 = -4 + 2t^2$

$\Rightarrow \quad 2t^2 = 4$

$\Rightarrow \quad t = \pm\sqrt{2}$

$\therefore \quad A_1 = (4, 4\sqrt{2})$ & $B_1 = (4, -4\sqrt{2})$

(A) $\quad OA_1 = \sqrt{(4-0)^2 + (4\sqrt{2}-0)^2}$

$= \sqrt{16+32} = \sqrt{48} = 4\sqrt{3}$

(B) $\quad A_1B_1 = \sqrt{0+(8\sqrt{2})^2} = 8\sqrt{2}$

(C) Orthocentre of $A_1B_1C_1$

$\Rightarrow$ Point of intersection C_1m & line $\perp^r$ to B_1C_1

$C_1m = y = 0$ &

Equation of line $\perp^r$ B_1C_1 & passing through A_1

$$(y-4\sqrt{2}) = \frac{-1}{\frac{-4\sqrt{2}}{8}}(x-4)$$

$$y-4\sqrt{2} = \sqrt{2}(x-4)$$

$\Rightarrow \quad y = \sqrt{2}x$

$\therefore$ Orthocentre (0, 0).

8. Correct answer is [51].

$\because \quad f(x + y) = f(x) + f(y)$

$\therefore \quad f(x) = kx$

$\because \quad f\left(-\frac{3}{5}\right) = 12 = -k\left(\frac{3}{5}\right)$

$\Rightarrow \quad k = -20$

$f(x) = -20x$

$\because \quad g(x + y) = g(x).g(y)$

$g(x) = a^x$

$\because \quad g\left(-\frac{1}{3}\right) = 2 \Rightarrow 2 = a^{-\frac{1}{3}}$

$$a = \frac{1}{8}$$

$\therefore \quad g(x) = \left(\frac{1}{8}\right)^x$

Now $\left(f\left(\frac{1}{4}\right) + g(-2) - 8\right)g(0)$

$= (-5 + 64 - 8).(1) = 51$

9. Correct answer is [11].

3W 6G (N – 9) B

$P(W_1 \cap G_2 \cap B_3)$

$$= \frac{3}{N} \times \frac{6}{N-1} \times \frac{(N-9)}{N-2} = \frac{2}{5N}$$

$\Rightarrow \quad 45(N-9) = (N-1)(N-2)$

$\Rightarrow N^2 - 48N + 407 = 0$

$(N-11)(N-37) = 0$

$N = 11$ or $N = 37$

$\because \quad P\left(\frac{B_3}{W_1 \cap G_2}\right) = \frac{P(B_3 \cap W_1 \cap G_2)}{P(W_1 \cap G_2)}$

$$P = \frac{\frac{2}{5N}}{\frac{3}{N} \times \frac{6}{N-1}} = \frac{N-1}{45}$$

$\therefore \quad N = 11 \Rightarrow P = \frac{10}{45} = \frac{2}{9}$

& $\quad N = 37 \Rightarrow P = \frac{36}{45} = \frac{4}{5}$

$\therefore \quad N = 11$

10. Correct answer is [1].

$$f(x) = \frac{\sin x}{e^{\pi x}}\frac{(x^{2023}+2024x+2025)}{(x^2-x+3)} + \frac{2}{e^{\pi x}}\frac{x^{2023}+2029x+2025}{x^2-x+3}$$

$$= \frac{(x^{2023}+2024x+2025)(\sin x+2)}{(x^2-x+3)e^{\pi x}}$$

$\because \quad \sin x + 2 \neq 0, e^{\pi x} \neq 0, x^2 - x + 3 \neq 0$

let $\quad g(x) = x^{2023} + 2024x + 2025$

$g'(x) = 2023\, x^{2022} + 2024 > 0$

$\therefore$ $g(x)$ is increasing function

& $g(x)$ is odd degree polynomial

$\therefore \quad g(x) = 0$ has only one real root

$\therefore \quad f(x) = 0$ also has only one real root.

11. Correct answer is [2].

$$15\hat{i}+10\hat{j}+6\hat{k} = \alpha(2\vec{p}+\vec{q})+\beta(\vec{p}-2\vec{q})+\gamma(\vec{p}\times\vec{q})$$

$\because \quad \vec{p} = 2\hat{i}+\hat{j}+3\hat{k}$

$\vec{q} = \hat{i}-\hat{j}+\hat{k}$

$$\Rightarrow \quad \vec{p}\times\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 1 & -1 & 1 \end{vmatrix} = 4\hat{i}+\hat{j}-3\hat{k}$$

$\therefore \quad 15\hat{i}+10\hat{j}+6\hat{k} = \alpha(5\hat{i}+\hat{j}+7\hat{k})+\beta(3\hat{j}+\hat{k})+\gamma(4\hat{i}+\hat{j}-3\hat{k})$

$= \hat{i}(5\alpha+4\gamma)+\hat{j}(\alpha+3\beta+\gamma)+\hat{k}(7\alpha+\beta-3\gamma)$

$\therefore \quad 5\alpha + 4\gamma = 15$ ...(i)

$\alpha + 3\beta + \gamma = 10$ ...(ii)

$7\alpha + \beta - 3\gamma = 6$ ...(iii) × 3

$21\alpha + 3\beta - 9\gamma = 18$

$\alpha + 3\beta + \gamma = 10$

$-\quad -\quad -\quad -$

$20\alpha - 10\alpha = 8$

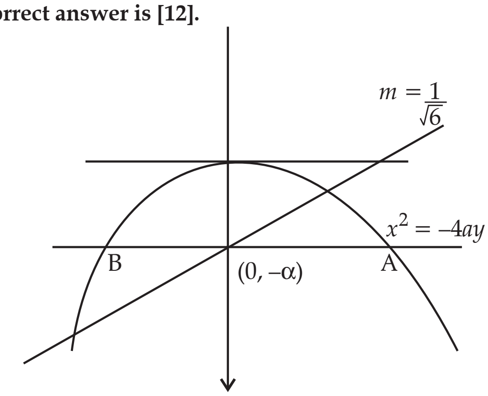
$10\alpha - 5\gamma = 4$ ...(iv)

by (i) $10\alpha + 8\beta = 30$ ...(v)

$\Rightarrow \quad -13\alpha = -26$

$\alpha = 2$

12. Correct answer is [12].



Equation of normal

$$y = mx - 2a - \frac{a}{m^2}$$

$(0, -\alpha) \Rightarrow \quad -\alpha = 0 - 2a - 9(6)$

$$m = \frac{1}{\sqrt{6}}$$

$$-\alpha = -8a \Rightarrow \alpha = 8a$$

solving $\quad y = -8a$ with $x^2 = -4ay$

$$x^2 = 32a^2 \Rightarrow x = \pm 4\sqrt{2}a$$

$\therefore \quad A = (4\sqrt{2}a, -8a)\ \&\ B(-4\sqrt{2}a, -8a)$

$\because \quad r = L.R = 4a$

$$S = AB^2 = 128a^2$$

$\therefore \quad \frac{r}{5} = \frac{1}{16} \Rightarrow \frac{4a}{128a^2} = \frac{1}{16}$

$$a = \frac{1}{2}$$

$$24a = 12$$

13. Correct answer is (5).

$$f(t) = \begin{cases} (-1)^{n+1}\,2 & \text{if } t = 2n-1,\ n \in \mathbb{N} \\ \left(\frac{2n+1-t}{2}\right) f(2n-1) + \left(\frac{t-(2n-1)}{2}\right) f(2n+1) & \text{if } 2n-1 < t < 2n+1,\ n \in \mathbb{N} \end{cases}$$

$$f(t) = \begin{cases} 2 & t = 1 \\ -2 & t = 3 \\ 4 - 2t & 1 < t < 3 \\ 2 & t = 5 \\ -2 & t = 7 \\ \left(\frac{5-t}{2}\right) f(3) + \left(\frac{t-3}{2}\right) f(5) & 3 < t < 5\ (n = 2) \\ \left(\frac{7-t}{2}\right) f(3) + \left(\frac{t-5}{2}\right) f(5) & 5 < t < 7 \\ \left(\frac{9-t}{2}\right) f(7) + \left(\frac{t-7}{2}\right) f(9) & 7 < t < 9 \end{cases}$$

$$f(t) = \begin{cases} 2 & t = 1 \\ -2 & t = 3 \\ 4 - 2t & 1 < t < 3 \\ 2 & t = 5 \\ -2 & t = 7 \\ 2t - 3 & 3 < t < 5 \\ 12 - 2t & 5 < t < 7 \\ 2t - 16 & 7 < t < 9 \end{cases}$$

Now $g(x) = \int_1^x f(t)\,dt \qquad x \in (1, 8]$

$\Rightarrow \int_1^x 2(2-t)\,dt \quad 1 < x < 3 \Rightarrow 4x - x^2 - 3$

$\Rightarrow \int_1^3 2(2-t)\,dt + \int_3^x (2t-8)\,dt \qquad 3 < x \le 5$

$\Rightarrow 8x^2 - 8x + 15$

$\Rightarrow \int_1^3 2(2-t)\,dt + \int_3^5 (2t-8)\,dt + \int_5^x 2(6-t)\,dt$

$\Rightarrow 12x - x^2 - 35 \qquad 5 < x \le 7$

$\Rightarrow \int_1^3 2(2-t)\,dt + \int_3^5 (2t-8)\,dt + \int_5^7 2(6-t)\,dt + \int_7^x (2t-6)\,dt$

$\Rightarrow x^2 - 6x + 63 \qquad 7 < x < 8$

$$g(x) = \begin{cases} 4x - x^2 - 3 & 1 < x \le 3 \\ x^2 - 8x^2 + 15 & 3 < x \le 5 \\ 12x - x^2 - 35 & 5 < x \le 7 \\ x^2 - 6x + 63 & 7 < x < 8 \end{cases}$$

$9(x) = 0$

$\Rightarrow x = 3, 5, 7$

$\alpha = 3$

$$\beta = \lim_{x \to 1+} \frac{9(x)}{x-1} = \frac{-(x-1)(x-3)}{x-1} = 2$$

$\therefore \alpha + \beta = 3 + 2 = 5$

14. Correct answer is (20).

$\because \quad |a - b| \ge 2$

$a = 1 \quad b = 3, 4, 5, 6 \Rightarrow 4$ elements
$a = 2 \quad b = 4, 5, 6 \Rightarrow 3$ elements
$a = 3 \quad b = 1, 5, 6 \Rightarrow 3$ elements
$a = 4 \quad b = 1, 2, 6 \Rightarrow 3$ elements
$a = 5 \quad b = 1, 2, 3 \Rightarrow 3$ elements
$a = 6 \quad b = 1, 2, 3, 4 \Rightarrow 4$ elements
Total = 20 elements

$\because$ X has exactly 6 elements

$\therefore$ No. of ways $^{20}C_6 = n(X)$

$\Rightarrow \quad m = 20$

15. Correct answer is [36].

$|a - b| \ge 2 \Rightarrow 20$ elements

Y are such relations belonging to X whose range has only 1 element.

$\therefore$ No such relation is possible

$\Rightarrow \quad n(Y) = 0$

Z is a function

$\therefore$ No. of ways $= {}^4C_1\,{}^3C_1\,{}^3C_1\,{}^3C_1\,{}^3C_1\,{}^4C_1$

$= 4 \times 3 \times 3 \times 3 \times 3 \times 4$

$= (36)^2$

$\therefore \quad k^2 = 36^2$

$k = 36$

16. Correct answer is [0].

$$f(x) = \sin^2 x$$

$$g(x) = \sqrt{\frac{\pi x}{2} - x^2}$$

$$I_1 = 2\int_0^{\frac{\pi}{2}} f(x)g(x)dx \text{ and } I_2 = \int_0^{\frac{\pi}{2}} g(x)dx$$

$$I_1 = 2\int_0^{\frac{\pi}{2}} f(x)g(x)dx$$

$$= I_1 = 2\int_0^{\frac{\pi}{2}}\left(\sin^2 x.\sqrt{\frac{\pi x}{2} - x^2}\right)dx \quad \text{...(i)}$$

Using properly $x \to a + b - x$

$$I_1 = 2\int_0^{\frac{\pi}{2}} \sin^2\left(\frac{\pi}{2} - x\right).\sqrt{\frac{\pi}{2} - \left(\frac{\pi}{2} - x\right) - \left(\frac{\pi}{2} - x\right)^2}\, dx$$

$$I_1 = 2\int_0^{\frac{\pi}{2}} \cos^2(x).\sqrt{\frac{\pi x}{2} - x^2}dx \quad \text{...(ii)}$$

add equation (i) and (ii) to get,

$$2I_1 = 2\int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^2 x).\sqrt{\frac{\pi}{2}x - x^2}\, dx$$

$$I_1 = \int_0^{\frac{\pi}{2}} \sqrt{\frac{\pi}{2}x - x^2}\, dx = I_2$$

$\therefore \quad I_1 - I_2 = 0$

17. Correct answer is [0.25].

$$\frac{16}{\pi^3}\int_0^{\frac{\pi}{2}} f(x)g(x)dx$$

$$\frac{16}{\pi^3}.\frac{I_1}{2} \quad \text{(from previous question)}$$

$$= \frac{8}{\pi^3}\int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{\pi}{4}\right)^2 - \left(x - \frac{\pi}{4}\right)^2}$$

$$= \frac{8}{\pi^3}\left[\frac{x - \frac{\pi}{4}}{2}\sqrt{\left(\frac{\pi}{4}\right)^2 - \left(x - \frac{\pi}{4}\right)^2} + \frac{\left(\frac{\pi}{4}\right)^2}{2}\sin^{-1}\left(\frac{x - \frac{\pi}{4}}{\frac{\pi}{4}}\right)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{8}{\pi^3}\left[\left(0 + \frac{\pi^2}{32}\sin^{-1}1\right) - \left(0 + \frac{\pi^2}{32}\sin^{-1}(-1)\right)\right]$$

$$= \frac{8}{\pi^3}\left[\frac{\pi^2}{32} \times \frac{\pi}{2} + \frac{\pi^2}{32}\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{8}{\pi^3}\left(\frac{\pi^3}{32}\right)$$

$$= \frac{1}{4} = 0.25$$

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