JEE (Main) QUESTION PAPER 2024
27th January Shift 1

Time : 3 Hours **Total Marks : 300**

General Instructions :

1. *There are three subjects in the question paper consisting of Mathematics Q. no. 1 to 30.*
2. *This Paper is divided into two sections:*
   • Section A Consists of 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.
   *• Section B consist of 10 questions, **Numerical Value Type Questions** - In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.*
3. *There will be only one correct choice in the given four choices in Section A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice for Section A questions and zero mark will be awarded for not attempted question.*
4. *For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.*

Mathematics

Section A

Q. 1. If (a, b) be the orthocentre of the triangle whose vertices are (1, 2), (2, 3) and (3, 1), and $I_1 = \int_a^b x\sin(4x - x^2)dx$, $I_2 = \int_a^b \sin(4x - x^2)dx$, then $36\frac{I_1}{I_2}$ is equal to:

(1) 80 **(2)** 66

(3) 72 **(4)** 88

Q. 2. Let $a_1, a_2 .... a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_k = 50$ and $\sum_{\forall k<j} a_k . aj = 1100$. Then the standard deviation of $a_1, a_2, ... a_{10}$ is equal to:

(1) 10 **(2)** $\sqrt{5}$

(3) $\sqrt{115}$ **(4)** 5

Q. 3. The distance, of the point (7, –2, 11) from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-1}{-3} = z - \frac{5}{6}$, is

(1) 12 **(2)** 18

(3) 21 **(4)** 14

Q. 4. Let $x = x(t)$ and $y = y(t)$ be solutions of the differential equations $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ respectively, $a, b \in$ R. Given that $x(0) = 2$; $y(0) = 1$ and $3y(1) = 2x(1)$, the value of t, for which $x(t) = y(t)$, is :

(1) $\log_{\frac{2}{3}} 2$ **(2)** $\log_4 3$

(3) $\log_{\frac{4}{3}} 2$ **(4)** $\log_3 4$

Q. 5. Four distinct points $(2k, 3k)$, (1, 0), (0, 1) and (0, 0) lie on a circle for k equal to:

(1) $\frac{1}{13}$ **(2)** $\frac{2}{13}$

(3) $\frac{5}{13}$ **(4)** $\frac{3}{13}$

Q. 6. The portion of the line $4x + 5y = 20$ in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is :

(1) $\frac{8}{5}$ **(2)** $\frac{25}{41}$

(3) $\frac{2}{5}$ **(4)** $\frac{30}{41}$

Q. 7. If $a = \lim_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$ and $b = \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$, then the value of ab^3 is :

(1) 30 **(2)** 36
(3) 25 **(4)** 32

Q. 8. If the shortest distance between the lines $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$ and $\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is :

(1) 10 **(2)** 8
(3) 5 **(4)** 7

Q. 9. If $S = \{z \in C : |z - i| = |z + i| = |z - 1|\}$, then, $n(S)$ is :

(1) 2 **(2)** 0
(3) 3 **(4)** 1

Q. 10. Consider the function.

$$f(x) = \begin{cases} \frac{a(7x-12-x^2)}{b|x^2-7x+12|} & , x < 3 \\ 2^{\frac{\sin(x-3)}{x-[x]}} & , x > 3 \\ b & , x = 3 \end{cases}$$

where $[x]$ denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a, b) such that $f(x)$ is continuous at $x = 3$, then the number of elements in S is :

(1) 1 **(2)** 4
(3) 2 **(4)** Infinitely many

Q. 11. The length of the chord of the ellipse $\frac{x^2}{25}+\frac{y^2}{16} = 1$, whose mid point is $\left(1, \frac{2}{5}\right)$, is equal to :

(1) $\frac{\sqrt{1541}}{5}$ **(2)** $\frac{\sqrt{1691}}{5}$
(3) $\frac{\sqrt{1741}}{5}$ **(4)** $\frac{\sqrt{2009}}{5}$

Q. 12. If A denotes the sum of all the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ and B denotes the sum of all the coefficients in the expansion of $(1 + x^2)^n$, then :

(1) $A = B^3$ **(2)** 3A = B
(3) A = 3 B **(4)** $B = A^3$

Q. 13. The number of common terms in the progressions 4,9,14,19,.......up to 25th term and 3, 6, 9, 12,........, up to 37th term is:

(1) 5 **(2)** 8
(3) 7 **(4)** 9

Q. 14. If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is d, then d^2 is equal to

(1) 16 **(2)** 20
(3) 24 **(4)** 36

Q. 15. Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Given below are two statements:

Statement I: $f(-x)$ is the inverse of the matrix $f(x)$.

Statement II: $f(x) f(y) = f(x + y)$.

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is true but Statement II is false
(2) Both Statement I and Statement II are false
(3) Both Statement I and Statement II are true
(4) Statement I is false but Statement II is true

Q. 16. Let $\vec{a} = \hat{i}+2\hat{j}+\hat{k}$, , $\vec{b} = 3\left(\hat{i}-\hat{j}+\hat{k}\right)$. Let $\vec{c}$ be the vector such that $\vec{a}\times\vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$ Then $\vec{a}.\left(\left(\vec{c}\times\vec{b}\right)-\vec{b}-\vec{c}\right)$ is equal to :

(1) 20 **(2)** 24
(3) 36 **(4)** 32

Q. 17. The function $f : N - \{1\} \to N$; defined by $f(n)$ = the highest prime factor of n, is :

(1) neither one-one nor onto
(2) one-one only
(3) both one-one and onto
(4) onto only

Q. 18. Let S = {1, 2, 3,...., 10}. Suppose M is the set of all the subsets of S, then the relation R = $\{(A, B) : A \cap B = \phi : A, B \in M\}$ is :

(1) reflexive only
(2) symmetric and reflexive only
(3) symmetric and transitive only
(4) symmetric only

Q. 19. If $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx = a+b\sqrt{2}+c\sqrt{3}$ where a, b, c are rational numbers, then $2a + 3b - 4c$ is equal to:

(1) 4 **(2)** 7
(3) 10 **(4)** 8

Q. 20. ${}^{n-1}C_r = (k^2 - 8)\ {}^nC_{r+1}$ if and only if:

(1) $2\sqrt{3} < k \le 3\sqrt{2}$ **(2)** $2\sqrt{2} < k \le 3$
(3) $2\sqrt{3} < k < 3\sqrt{3}$ **(4)** $2\sqrt{2} < k < 2\sqrt{3}$

Section B

Q. 21. The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i}-2\hat{j}+2\hat{k}$ and $\alpha\hat{i}+2\alpha\hat{j}-2\hat{k}$ is acute, is

Q. 22. A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let a = P(X = 3), b = P(X ≥ 3) and C = P(X > 6| X > 3). Then $\frac{b+c}{a}$ is equal to

Q. 23. If the solution of the differential equation $(2x + 3y - 2)dx + (4x + 6y - 7)dy = 0$, $y(0) = 3$, is $ax + by + 3\log_e|2x + 3y - \gamma| = 6$, then $\alpha + 2\beta + 3\gamma$ is equal to

Q. 24. If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, A, B, C ≥ 0, then 5(3A – 2B – C) is equal to ....

Q. 25. If $8 = 3+\frac{1}{4}(3+p)+\frac{1}{4^2}(3+2p)+\frac{1}{4^3}(3+3p) + \ldots\ldots\ldots\ \infty$ then the value of p is

Q. 26. Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $B = [B_1, B_2, B_3]$, where B_1, B_2, B_3 are column matrics, and $AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$AB_1 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

If $\alpha = |B|$ and β is the sum of all the diagonal elements of B, then $\alpha^3 + \beta^3$ is equal to

Q. 27. Let $f(x) = x^3 + x^2 f'(1) + xf''(2) + f''(3)$. $x \in$ R. Then $f'(10)$ is equal to

Q. 28. Let the area of the region $\{(x, y): x - 2y + 4 \ge 0, x + 2y^2 \ge 0, x + 4y^2 \le 8, y \ge 0\}$ be $\frac{m}{n}$, where m and n are coprime numbers. Then $m + n$ is equal to

Q. 29. Let for a differentiable function $f: (0, \infty) \to$ R,

$f(x) - f(y) > \log_e\left(\frac{x}{y}\right) + x - y, \forall x, y \in (0, \infty)$

Then $\sum_{n=1}^{20} f'\left(\frac{1}{n^2}\right)$ is equal to

Q. 30. Let the set of all $a \in$ R such that the equation $\cos 2x + a\sin x = 2a - 7$ has a solution be $[p, q]$ and $r = \tan 9° - \tan 27° - \frac{1}{\cot 63°} + \tan 81°$, then pqr is equal to ....

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Answer Key

Mathematics			
Q. No.	**Answer**	**Topic Name**	**Chapter Name**
1	(3)	Orthocentre	Staright Lines
2	(4)	Standard Deviation	Statistics
3	(4)	Dstance	Three Dimensional Geometry
4	(3)	Solution	Differential Equation
5	(3)	Equation of circla	Circles
6	(4)	Angle between line	Straight lines
7	(4)	limits	Limit and continuity
8	(2)	Shirtest distance	Three Dimensional Geometry
9	(2)	Complex number	complex number
10	(1)	Continuity	Limit and continuity
11	(2)	Ellipse	Conin section
12	(1)	General term	Binomial theorem
13	(3)	General term	Sequence and Series
14	(2)	Normal to a circle	Conin section
15	(3)	Matrix	Matrices and Determinants
16	(2)	Dot product	Vectors
17	(1)	One-One Onto	Functions
18	(4)	Types of Relations	Relations and Functions
19	(4)	Definite Integral	Integral Calculus
20	(2)	Combinations	permutation and Combinations
21	[5]	Angle between vectors	Vectors
22	[12]	Probability	Probability
23	[29]	Homogenous Differential equation	Differential Equation
24	[5]	Cube root of unity	Quadraic Equations
25	[9]	Sum	Sequence and Series
26	[27]	Matrix	Matrices and Determinants
27	[202]	Differentiation	Continuity and Differentiation
28	[119]	Area under the curve	Definite Integral
29	[2890]	Functions	Relations and Functions
30	[48]	Trigonometric values	Trigonometry

JEE (Main) SOLVED PAPER 2024

27th January Shift 1

ANSWERS WITH EXPLANATIONS

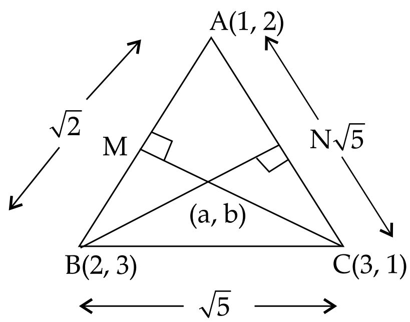
1. Option (3) is correct.

Equation of AB is

$$(y-3)=\frac{2-3}{1-2}(x-2)$$

$\Rightarrow y - 3 = x - 2$

$\Rightarrow y - x = 1$ ...(i)



Also, equation of CM is

$(y - 1) = -1\,(x - 3) \Rightarrow y - 1 = -x + 3 \Rightarrow x + y = 4$

and equation of BN is

$(y - 3) = 2\,(x - 2) \Rightarrow y - 3 = 2x - 4 \Rightarrow 2x - y = 1$

On solving, we get

$$x=\frac{5}{3}, y=\frac{7}{3}$$

$$I_1 = \int_a^b x\sin(4x-x^2)dx \Rightarrow 2I = 4I_2$$

Using properties of definite Integration

$$\Rightarrow I_1 = 2I_2 \Rightarrow \frac{I_1}{I_2}=2$$

$$\therefore \frac{36\,I_1}{I_2}=72$$

2. Option (4) is correct.

Since, $\sum_{k=1}^{10} a_k = 50$

$$\Rightarrow \text{Mean (X)} = \frac{\sum_{k=1}^{10} a_k}{10}=\frac{50}{10}=5$$

Also, $\left(\sum_{i=1}^{10} a_i\right)^2 = \sum_{i=1}^{10} a_i^2 + 2(\Sigma a_i a_j)$

$$\Rightarrow 2500 = \sum_{i=1}^{10} a_i^2 + 2\times 1100$$

$$\Rightarrow \sum_{i=1}^{10} a_i^2 = 300$$

$$\therefore \sigma^2 = \frac{300}{10} - 5 = 25$$

So, $\sigma = \sqrt{25} = 5$

3. Option (4) is correct.

Let A ≡ (7, –2, 11) and B be any point on the line

$$\frac{x-6}{1}=\frac{y-4}{0}=\frac{z-8}{3} \quad \text{...(i)}$$

Now, let $\frac{x-7}{2}=\frac{y+2}{-3}=\frac{z-11}{6}=\lambda$ (say)

$x = 2\lambda + 7, y = -3\lambda - 2, z = 6\lambda + 11$

$\therefore$ B ≡ $(2\lambda + 7, -3\lambda - 2, 6\lambda\ x\ 11)$

As point B lies on (i), so,

$$\frac{2\lambda+7-6}{1}=\frac{-3\lambda-2-4}{0}=\frac{6\lambda+11-8}{3}$$

$\Rightarrow -3\lambda - 6 = 0 \Rightarrow -3\lambda = 6 \Rightarrow \lambda = -2$

$\therefore$ B ≡ (3, 4, –1)

Hence, $AB = \sqrt{(7-3)^2+(-2-4)^2+(11+1)^2}$

$= \sqrt{16+36+144} = \sqrt{196} = 14$ units

4. Option (3) is correct.

We have, $\frac{dx}{dt}+ax=0 \Rightarrow \frac{dx}{dt}=-ax$

$$\Rightarrow \int\frac{dx}{x}=-a\int dt$$

$\Rightarrow \ln x = -at + C_1$

$\Rightarrow x = Ce^{-at}$

As, $x(0) = 2$

So, $2 = Ce^0 \Rightarrow C = 2$

$\Rightarrow x = 2e^{-at}$

Also, $\frac{dy}{dt}+by=0 \Rightarrow \frac{dy}{dt}=-by$

$$\Rightarrow \int\frac{dy}{y}=-b\int dt$$

$\Rightarrow \ln y = -b + C_2$

$\Rightarrow y = Ke^{-bt}$

As, $y(0) = 1$

So, $1 = Ke^0 \Rightarrow k = 1 \Rightarrow y = e^{-bt}$

Given, $3y(1) = 2x(1)$

$\Rightarrow 3e^{-b} = 2(2e^{-a})$

$$\Rightarrow e^{a-b} = \frac{4}{3}$$

If $x(t) = y(t)$

$\Rightarrow 2e^{-at} = e^{-bt}$

$$\Rightarrow 2\frac{e^{-at}}{e^{-bt}} = 1$$

$\Rightarrow 2e^{-(a-b)t} = 1$

$$\Rightarrow 2\left(\frac{4}{3}\right)^{-t} = 1 \Rightarrow \left(\frac{4}{3}\right)^{t} = 2$$

$$\Rightarrow t = \log_{\frac{4}{3}} 2$$

5. Option (3) is correct.

Given four points lie on circle.

So, the equation of circle is

$(x-1)(x-0) + (y-0)(y-1) = 0$

$\Rightarrow x^2 - x + y^2 - y = 0$ ...(i)

$\because (2k, 3k)$ lie on (i), we get

$\Rightarrow (2k)^2 - 2k + (3k)^2 - 3k = 0$

$\Rightarrow 13k^2 - 5k = 0 \Rightarrow k(13k - 5) = 0$

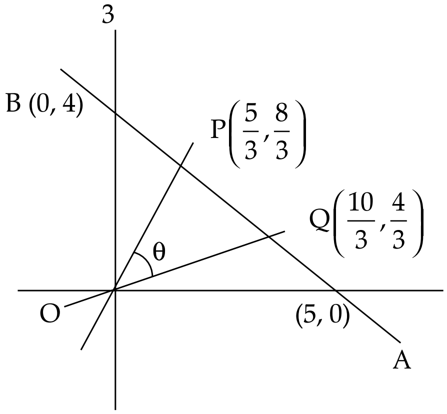
$$\Rightarrow k = 0 \text{ or } k = \frac{5}{13}$$

$$\therefore \text{Value of } k = \frac{5}{13}$$

6. Option (4) is correct.

The given line is $4x + 5y = 20$

$$m_1(\text{OP}) = \frac{\frac{8}{3}}{\frac{5}{3}} = \frac{8}{5}$$



$$\text{and } m_2(\text{QO}) = \frac{\frac{4}{3}}{\frac{10}{3}} = \frac{4}{10} = \frac{2}{5}$$

$$\therefore \tan\theta = \left|\frac{\frac{8}{5} - \frac{2}{5}}{1 + \frac{16}{25}}\right| = \left|\frac{\frac{6}{5}}{\frac{41}{25}}\right| = \frac{30}{41}$$

7. Option (4) is correct.

$$a = \lim_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$$

$$= \lim_{x\to 0} \frac{1+\sqrt{1+x^4} - 2}{x^4(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2})} \quad \text{[on rationalising]}$$

$$= \lim_{x\to 0} \frac{\sqrt{1+x^4} - 1}{x^4(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2})}$$

$$= \lim_{x\to 0} \frac{1+x^4 - 1}{x^4(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2})(\sqrt{1+x^4} + 1f)}$$

[on rationalising]

$$= \lim_{x\to 0} \frac{1}{(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2})(\sqrt{1+x^4} + 1)} = \frac{1}{(4\sqrt{2})}$$

$$b = \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$$

$$= \lim_{x\to 0} \frac{(1-\cos^2 x)(\sqrt{2} + \sqrt{1-\cos x})}{(\sqrt{2} - \sqrt{1+\cos x})(\sqrt{2} + \sqrt{1+\cos x})}$$

$$= \lim_{x\to 0} \frac{(1-\cos^2 x)(\sqrt{2} + \sqrt{1+\cos x})}{2 - 1 - \cos x}$$

$$= \lim_{x\to 0} (1+\cos x)(\sqrt{2} + \sqrt{1+\cos x})$$

$$= 2(2\sqrt{2}) = 4\sqrt{2}$$

$$\text{So, } ab^3 = \frac{1}{4\sqrt{2}} \times 64 \times 2\sqrt{2} = 32$$

8. Option (2) is correct.

Here, $\vec{a} = 4\hat{i} - \hat{j} + 0\hat{k}; \vec{b} = \lambda\hat{i} - \hat{j} + 2\hat{k}$

$\vec{p} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{q} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

So, $\vec{b} - \vec{a} = (\lambda - 4)\hat{i} + 2\hat{k}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = \hat{i}(-10+12) - \hat{j}(-5+6) + \hat{k}(4-4)$$

$$= 2\hat{i} - \hat{j} + 0\hat{k}$$

$|\vec{p}\times\vec{q}| = \sqrt{4+1} = \sqrt{5}$

So, shortest distance = $\dfrac{6}{\sqrt{5}}$

$= \left|\dfrac{[(\lambda-4)\hat{i}+2\hat{k}].[2\hat{i}-\hat{j}]}{\sqrt{5}}\right| = \dfrac{6}{\sqrt{5}}$

$\Rightarrow |2\lambda - 8| = 6 \Rightarrow |\lambda - 4| = 3$

$\Rightarrow \lambda = 7, 1$

So, required sum = 7 + 1 = 8

9. Option (2) is correct.

The number of solutions = 0

10. Option (1) is correct.

Since, $f(x)$ is continuous at $x = 3$.

$\therefore \lim_{x\to3^-} f(x) = f(3) = \lim_{x\to3^+} f(x)$

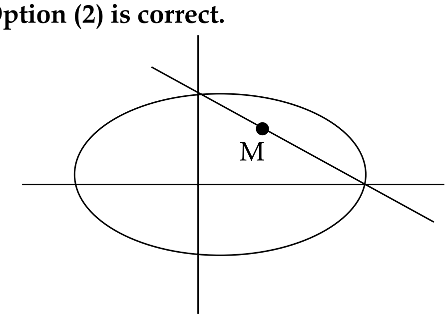
$\Rightarrow \lim_{x\to3^-}\dfrac{-a(x-4)(x-3)}{b|(x-3)(x-4)|} = b = \lim_{x\to3} 2^{\frac{\sin(x-3)}{x-3^+}}$

$\Rightarrow \dfrac{-a}{b} = b = 2^1 = 2$

$\Rightarrow b = 2$ and $\dfrac{a}{2} = 2 \Rightarrow a = -4$

$\therefore$ S = {(–4, 2)} i.e. only one pair

11. Option (2) is correct.



We have, T = S_1

$\Rightarrow \dfrac{x}{25} + \dfrac{y\times\frac{2}{5}}{16} = \dfrac{1}{25} + \dfrac{4}{16\times25}$

$\Rightarrow \dfrac{x}{25}+\dfrac{y}{40} = \dfrac{1}{25}+\dfrac{1}{100} = \dfrac{4+1}{100}$

$\Rightarrow \dfrac{x}{25}+\dfrac{y}{40} = \dfrac{1}{20} \Rightarrow \dfrac{8x+5y}{200} = \dfrac{1}{20}$

$\Rightarrow 8x + 5y - 10 = 0$

$\because \dfrac{x^2}{25}+\dfrac{y^2}{16} = 1$

$\Rightarrow \dfrac{16x^2+25y^2}{400} = 1 \Rightarrow 16x^2+25y^2 = 400$

$\Rightarrow 16x^2 + 100 + 64x^2 - 160x = 400$

$\Rightarrow 80x^2 - 160x - 300 = 0$

$\Rightarrow 2x^2 - 40x - 75 = 0$

$\Rightarrow 4x^2 - 8x - 15 = 0$

Now, chord length = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(x_2-x)^2+\dfrac{64}{25}(x_2-x_1)^2} = \sqrt{\dfrac{89}{25}}\sqrt{(x_2-x_1)^2}$

$= \dfrac{\sqrt{89}}{5}(x_1-x_2)$

$\therefore$ Required distance = $\dfrac{\sqrt{89}}{5}\times\sqrt{19} = \dfrac{\sqrt{1691}}{5}$

12. Option (1) is correct.

Put $x = 1$, we get

A = $(1 - 3 + 10)^n$ = B^n and B = $(1 + 1)^n = 2^n$

Clearly A = 8^n = $(2^n)^3$ = B^3

13. Option (3) is correct.

First A.P., 4, 9, 14, 19, ..., 48 to 25th term

Here, $a = 4$, $d = 5$

So, a_{25} = 4 + 24 (5) = 124

Second A.P., 3, 6, 9, 12, ... 48 to 37th term

Here, $a = 3$ $d = 3$

So, a_{37} = 3 + 36 (3) = 111

As, LCM (5, 3) = 15

So, we have, 9 + $(x - 1)$ 15 $\le$ 111

($\because$ 9 is the first common term)

$\Rightarrow 9 + 15n - 15 \le 111$

$\Rightarrow 15n - 6 \Rightarrow 111 \Rightarrow 15n \le 117$

$\Rightarrow n \le 7.4$. So, $n = 7$

14. Option (2) is correct.

Equation of circle is

$x^2 + y^2 - 4x - 16y + 64 = 0$

$\therefore$ Centre = (2, 8) and radius = 2

Normal, $y + tx = 2t + t^2$

The normal will pass through (2, 8)

$\Rightarrow 8 + 2t = 2t + t^3$

$\Rightarrow t^3 = 8 \Rightarrow t = 2$

$\therefore p(t) = p(t^2, 2t) = p(4, 4)$

$\therefore$ Shortest distance = $\sqrt{(2-4)^2+(8-4)^2}$

$= \sqrt{4+16} = \sqrt{20} \Rightarrow d^2 = 20$

15. Option (3) is correct.

Given $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$f(x)=\begin{bmatrix}\cos(-x) & -\sin(-x) & 0\\ \sin(x) & \cos(-x) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos x & \sin x & 0\\ -\sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$f(x)\,f(-x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos x & \sin x & 0\\ -\sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2 x+\sin^2 x+0 & \begin{matrix}\sin x\cos x-\\ \sin x\cos x+0\end{matrix} & 0+0+1\\ \begin{matrix}\sin x\cos x-\\ \sin x\cos x+0\end{matrix} & \sin^2 x+\cos^2 x+0 & 0+0+1\\ 0+0+0 & 0+0+0 & 0+0+1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

$\therefore$ Statement - I is true

$$f(x)f(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos x\cos y-\sin x\sin y & \begin{matrix}-\cos x\sin y\\ -\sin x\sin y\end{matrix} & 0+0+0\\ \sin x\sin y+\cos x\sin y & \begin{matrix}-\sin x\sin y\\ +\cos x\cos y\end{matrix} & 0+0+0\\ 0+0+0 & 0+0+0 & 0+0+1\end{bmatrix}$$

$$=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$= f(x + y)$

$\therefore$ Statement - II is true

16. Option (2) is correct.

Given, $\vec{a}=\hat{i}+2\hat{j}+\hat{k};\vec{b}=3(\hat{i}-\hat{j}+\hat{k})$

$$\vec{a}\left((\vec{c}\times\vec{b})-\vec{b}-\vec{c}\right)=\vec{a}.(\vec{c}\times\vec{b})-\vec{a}.\vec{b}-\vec{a}.\vec{c}$$

$$=(\vec{a}\times\vec{c}).\vec{b}-((1)(3)+(2)(3)+(1)(3))-(3)$$

$$=\vec{b}.\vec{b}-0-3(9+9+9)-23=24$$

17. Option (1) is correct.

$f:\mathrm{N}-\{1\}\to\mathrm{N}$

$f(n)$ = the highest prime factor of n.

$\because f(2) = f(n) = 2$

But $2 \neq 4$

$\Rightarrow f$ is not one-one

There is no such n in the codomain such that $f(x) = 1$

So, f is not onto.

18. Option (4) is correct.

Given : S {1, 2, 3, ..., 10}

Also, M = P(S) = power set of S.

(i) Let A, B, $\in$ M

R = {(A, B) : A $\cap$ B $\neq \phi$, A, B, $\in$ M} [Given]

If A = B = $\phi \Rightarrow$ A $\cap$ B = ϕ

Hence, R is not reflexive.

(ii) As, if (A, B) 6M $\Rightarrow$ (B, A) $\in$ M

So, R is symmetric

(iii) Let (A, B) $\in$ M and (B, C) $\in$ M

$\Rightarrow$ A $\cap$ B = ϕ and B $\cap$ C = ϕ

But A $\cap$ C not necessarily empty

So, (A, C) $\notin$ M

So, R is not transitive.

19. Option (4) is correct.

$$\text{Let I} = \int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx$$

$$=\int_0^1 \frac{\sqrt{3+x}-\sqrt{1+x}}{(3+x)-(1+x)}dx \quad \text{[on rationalising]}$$

$$=\int_0^1\left[\frac{\sqrt{3+x}-\sqrt{1+x}}{2}\right]dx$$

$$=\frac{1}{2}\left\{\frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{(1+x)^{\frac{3}{2}}}{\frac{3}{2}}\right\}\Bigg|_0^1$$

$$=\frac{1}{3}\left\{\left[(1+3)^{\frac{3}{2}}-(1+1)^{\frac{3}{2}}\right]-\left[(0+3)^{\frac{3}{2}}-(1+0)^{\frac{3}{2}}\right]\right\}$$

$$=\frac{1}{3}\left\{8-2\sqrt{2}-3\sqrt{3}+1\right\}$$

$$=\frac{1}{3}\left\{9-2\sqrt{2}-3\sqrt{3}\right\}=3-\frac{2}{\sqrt{3}}\sqrt{2}-\sqrt{3}$$

$$=a+b\sqrt{2}+c\sqrt{3}$$

$\Rightarrow a = \frac{3}{0}, b = -\frac{2}{3}, C = -1$

So, $2a + 3b - 4c = -2 + 4 = 8$

20. Option (2) is correct.

Given, ${}^{n-1}C_r = (k^2 - 8)\, {}^nC_{r+1}$

$\Rightarrow \frac{(n-1)!}{r!(n-r-1)!} = (k^2-8)\frac{n(n-1)!}{(n-r-1)!(r+1)r!}$

$\Rightarrow \frac{r+1}{n} = k^2 - 8$

$\Rightarrow 0 < k^2 - 8 \leq 1 \Rightarrow 8 \leq k^2 \leq 9$

$\Rightarrow k \in (2\sqrt{2}, 3]$

21. Correct answer is [5].

Since, the angle is acute

$\therefore \cos\theta \geq 0$

$\Rightarrow (\alpha^2 - 4\alpha - 41 \geq 0)$

$\Rightarrow (\alpha - 2)^2 \geq 8$

$\Rightarrow |(\alpha - 2)| \geq 2\sqrt{2}$

$\therefore \alpha \in (-\infty, -2\sqrt{2}+2) \cup [2+2\sqrt{2}; \infty]$

$\therefore$ Least value of $\alpha = 5$

22. Correct answer is [12].

$P(A) = P$ (getting a six) $= \frac{1}{6}$

$P(X = 3) = \frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} = \frac{25}{216} = a$

$P(X \geq 3) = \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \ldots$

$= \frac{\frac{25}{216}}{1-\frac{5}{6}} = \frac{25}{36}$

$\{\because$ First term $= \frac{25}{216}$ and $r = \frac{5}{6}\}$

$P\left(\frac{X \geq 6}{X > 3}\right) = \frac{P(X \geq 6)}{P(X > 3)}$

$= \frac{\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^6\left(\frac{1}{6}\right) + \ldots.}{\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \ldots}$

$= \frac{25}{36} = C$

So, $\frac{b+c}{a} = \frac{\frac{25}{36}+\frac{25}{36}}{\frac{25}{216}} = \frac{50}{36}\times\frac{216}{25} = 12$

23. Correct answer is [29].

Given: D.E. is

$(2x + 3y - 2)\, dx + (4x + 6y - 7)\, dy = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-2x+3y-2}{4x+6y-7}$ ...(i)

Let $2x + 3y = \mu$

$\Rightarrow 2 + 3\frac{dy}{dx} = \frac{du}{dx}$

So, (i) becomes

$\Rightarrow \frac{1}{3}\left(\frac{du}{dx} - 2\right) = -\frac{u-2}{24-7}$

$\Rightarrow \frac{du}{dx} = \frac{-3u+6}{24-7} + 2 = \frac{-3u+6+4u-14}{2u-7}$

$\Rightarrow \frac{du}{dx} = \frac{4-8}{24-7}$

$\Rightarrow \int \frac{24-7}{4-8}du = \int dx$

$\Rightarrow \int\left[1 + \frac{4+1}{4-8}\right]du = \int dx$

$\Rightarrow 2u + \ln|4-8| = x + C$

$\Rightarrow 2(2x + 3y) + 9\ln|2x + 3y - 8| = x + C$

Given: $y(0) = 3$

$\Rightarrow 2(2(0) + 9 + 9\ln|2(0) + 4 - 8| = 0 + C$

$\Rightarrow 18 + 9(0) = C \Rightarrow C = 18$

$\therefore$ Solution is :

$4x + 6y + 19\ln|2x + 3y - 8| = x + 18$

$\Rightarrow 3x + 6y + 9\ln|2x + 3y - 8| = 18$

$\Rightarrow x + 2y + 3\ln|2x + 3y - 8| = 6$

On comparing, we get $\alpha = 1, \beta = 2, \gamma = 8$

$\therefore \alpha + 2\beta + 3\gamma = 1 + 4 + 24 = 29$

24. Correct answer is [5].

Given: $1 + x + x^2 = 0$

$\therefore \alpha = \omega, \omega^2$

Now, $(1 + \alpha)^7 = a + b\alpha + c\alpha^2$

$\Rightarrow (1 + \omega)^7 = a + b\omega + c\alpha^2$

$\Rightarrow -\omega^{14} = a + b\omega + c\omega^2$

$\Rightarrow -\omega^2 = a + b\omega + c\omega^2$

$\Rightarrow a + b\omega + (c + 1)\,\omega^2 = 0$

$\Rightarrow a + b\left(\frac{-1}{2} + \frac{\sqrt{3}}{2}i\right) + (c+1)\left(\frac{-1}{2} - \frac{\sqrt{3}}{2}i\right) = 0$

$\Rightarrow a - \frac{b}{2} - \frac{c+1}{2} = 0 \Rightarrow 2a - b - c - 1 = 0$

& $\frac{\sqrt{3}}{2}b - \frac{\sqrt{3}}{2}(c+1) = 0$

$\Rightarrow b - c - 1 = 0$

So, $a = b$

$\therefore 5(3a - 2b - c) = 5(b - c) = 5$

Correct answer is [9].

Given:

$$3+\frac{1}{4}(3+p)+\frac{1}{4^2}(3+2p)+\frac{1}{4^3}(3+3o)+\ldots\infty=8$$

$$\Rightarrow\left(3+\frac{3}{4}+\frac{3}{4^2}+\ldots\right)+\left(\frac{p}{4}+\frac{2p}{4^2}+\frac{3p}{4^3}+\ldots.\right)=8$$

$$\Rightarrow\left[\frac{3}{1-\frac{1}{4}}\right]+\left[\frac{p}{4}+\frac{2p}{4^2}+\frac{3p}{4^3}+..\right]=8$$

$$\Rightarrow\frac{p}{4}+\frac{2p}{4^2}+\frac{3p}{4^3}+\ldots=4$$

Let $x=\frac{p}{4}+\frac{2p}{4^2}+\frac{3p}{4^3}+\ldots$

On dividing by 4

$$\frac{x}{4}=\frac{p}{4^2}+\frac{2p}{4^3}+\frac{3p}{4^4}+\ldots$$

$$\therefore\left(x-\frac{x}{4}\right)=\left(\frac{p}{4}\right)+\left(\frac{2p}{4^2}-\frac{p}{4^2}\right)+\left(\frac{3p}{4^3}-\frac{2p}{4^3}\right)+\ldots$$

$$\Rightarrow\frac{3x}{4}=\frac{p}{4}+\frac{p}{4^2}+\frac{p}{4^3}+\ldots.=\left[\frac{\frac{p}{4}}{1-\frac{1}{4}}\right]=\frac{p}{3}$$

$$\Rightarrow\frac{p}{9}=4\Rightarrow p=q$$

26. Correct answer is [27].

$$A=\begin{bmatrix}2&0&1\\1&1&0\\1&0&1\end{bmatrix}$$

Let $B_1=\begin{bmatrix}a\\b\\c\end{bmatrix} B_2=\begin{bmatrix}d\\e\\f\end{bmatrix} B_3=\begin{bmatrix}g\\h\\i\end{bmatrix}$

$$AB_1=\begin{bmatrix}2a+c\\a+b\\a+c\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}\Rightarrow a=1,b=-1,c=-1$$

$$AB_2=\begin{bmatrix}2d+b\\d+e\\d+f\end{bmatrix}=\begin{bmatrix}2\\3\\0\end{bmatrix}\Rightarrow d=2,e=1,f=-2$$

$$AB_3=\begin{bmatrix}2g+h\\g+h\\g+i\end{bmatrix}=\begin{bmatrix}3\\2\\1\end{bmatrix}\Rightarrow g=2,h=0,i=-1$$

$$B=\begin{bmatrix}1&2&2\\-1&1&0\\-1&-2&-1\end{bmatrix}$$

$\Rightarrow |B| = 1(-1+0)-2(1+0)+2(2+1)$

$= -1-2+6 = 3$

$\therefore \alpha = 3, \beta = 1+1-1 = 1$

$\therefore \alpha^3+\beta^3 = 3^3+1^3 = 27$

27. Correct answer is [202].

Given: $f(x) = x^3 + x^2 f'(1) + xf''(2) + f''(3)$

$\Rightarrow f'(x) = 3x^2 + 2xf'(1) + f'(2)$ ...(A)

$\Rightarrow f'(1) = 3 + 2f'(1) + f''(2)$

$\Rightarrow f'(1) + f''(2) = -3$ ...(B)

Now, $f'(x) = 6x + 2f'(1)$

$\Rightarrow f'(2) = 12 + 2f'(1)$ ...(C)

$\therefore f'(1) + 12 + 2f'(1) = -3$

$\Rightarrow 3f'(1) = -15 \Rightarrow f'(1) = -5$

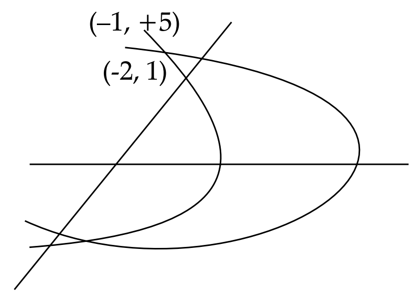
$\Rightarrow f''(2) = 12 - 10 = 2$

Putting $x = 10$ in (A),

$f'(10) = 3(100) + 2(10)(-5) + 2$

$= 300 - 100 + 2 = 202$

28. Option (119) is correct.



Required area

$$=\int_{-2}^{-1}\frac{x+4}{2}dx+\int_{-1}^{8}\frac{\sqrt{8-x}}{2}dx-\frac{1}{\sqrt{2}}\int_{-2}^{0}\sqrt{x}dx$$

$$=\frac{1}{2}\left(\frac{x^2}{2}+4x\right)\Bigg|_{2}^{-1}+\frac{1}{2}\left[\frac{-2}{3}(8-x)^{\frac{3}{2}}\right]_{-1}^{8}+\frac{1}{\sqrt{2}}\times\frac{2}{3}(-x)^{\frac{3}{2}}\Bigg]_{-2}^{0}$$

$$=\frac{1}{2}\left[\frac{1}{2}-4-2+8\right]-\frac{1}{3}\left[(0)-(9)^{\frac{3}{2}}\right]+\frac{\sqrt{2}}{3}\left[0-2\sqrt{2}\right]$$

$$=\frac{107}{12}=\frac{m}{n}$$

$\Rightarrow m+n = 107+12 = 119$

29. Correct answer is [2890].

$$f(x)-f(y)\geq \log_e\left(\frac{x}{y}\right)+x-y$$

$\Rightarrow f(x) - f(y) \geq \log x - \log x + x - y$

$\Rightarrow f(x) - \log x - x \geq f(y) - \log y - y$

Interchanging x and y

$f(y) - \log y - y \geq f(x) - \log x - x$

So, $f(x) = \log x + x + \lambda$

$\Rightarrow f'(x) = \frac{1}{x} + 1$ ...(i)

$\therefore \sum_{n=1}^{20} f'\left(\frac{1}{x^2}\right)$ [Using (i)]

$= \sum_{n=1}^{20} x^2 + = \sum_{n=1}^{20} 1$

$= \frac{20 \times 21 \times 4}{6} = 20$

$= 2870 + 20 = 2890$

30. Correct answer is [48].

Given $\cos^2 x + a \sin x = 2a - 7$

$\Rightarrow 1 - 2\sin^2 x + 7 = -a \sin 2x + 2a$

$\Rightarrow 2(4 - \sin^2 x) = a(2 - \sin x)$

$\Rightarrow 2(2 + \sin x) = a$

$\Rightarrow a = 4 + 2\sin x - 1 \leq \sin x \leq 1$

$\Rightarrow -2 \leq 2\sin \leq 2$

$\Rightarrow 2 \leq a \leq 6$

$\therefore p = 2 \;\&\; q = 6$

$r = \tan 9° - \tan 27° - \frac{1}{\cos 63°} + \tan 81°$

$= \tan 9° + \cot 9° - \tan 27° - \cot 27°$

$= \frac{\sin 9°}{\cos 9°} + \frac{\cos 9°}{\sin° 9} - \left[\frac{\sin 27°}{\cos 27°} + \frac{\cos 27°}{\sin 27°}\right]$

$= \frac{\sin^2 9° + \cos^2 9°}{\sin 9° \cos 9°} - \left[\frac{\sin^2 27° + \cos^2 27°}{\sin 27° \cos 27°}\right]$

$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°}$

$= 2\left\{\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1}\right\} = 8\left\{\frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4}\right\}$

$= 2\{2\} = 4$

$\therefore pqr = 2 \times 6 \times 4 = 48$

■■■