JEE (Main) QUESTION PAPER 2024
29^{th} January Shift 2

Time : 3 Hours **Total Marks : 300**

General Instructions :

1. *There are three subjects in the question paper consisting of Mathematics Q. no. 1 to 30.*
2. *This Paper is divided into two sections:*
   - *Section A Consists of 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.*
   - *Section B consist of 10 questions, **Numerical Value Type Questions** - In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.*
3. *There will be only one correct choice in the given four choices in Section A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice for Section A questions and zero mark will be awarded for not attempted question.*
4. *For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.*

Mathematics

Section A

Q. 1. The function $f(x) = 2x+3(x)^{\frac{2}{3}}$, $x \in$ R, has

(1) exactly one point of local minima and no point of local maxima

(2) exactly one point of local maxima and no point of local minima

(3) exactly one point of local maxima and exactly one point of local minima

(4) exactly two points of local maxima and exactly one point of local minima

Q. 2. Let A be the point of intersection of the lines $3x + 2y = 14$, $5x - y = 6$ and B be the point of intersection of the lines $4x + 3y = 8$, $6x + y = 5$. The distance of the point P(5, –2) from the line AB is

(1) $\frac{13}{2}$ **(2)** 8

(3) $\frac{5}{2}$ **(4)** 6

Q. 3. If the mean and variance of five observations are $\frac{24}{5}$ and $\frac{194}{25}$ respectively and the mean of the first four observations is $\frac{7}{2}$, then the variance of the first four observations in equal to

(1) $\frac{4}{5}$ **(2)** $\frac{77}{12}$

(3) $\frac{5}{4}$ **(4)** $\frac{105}{4}$

Q. 4. The distance of the point (2, 3) from the line $2x - 3y + 28 = 0$, measured parallel to the line $\sqrt{3x} - y + 1 = 0$, is equal to

(1) $4\sqrt{2}$ **(2)** $6\sqrt{3}$

(3) $3+4\sqrt{2}$ **(4)** $4+6\sqrt{3}$

Q. 5. If $\log_e a$, $\log_e b$, $\log_e c$ are in an A.P. and $\log_e a - \log_e 2b$, $\log_e 2b - \log_e 3c$, $\log_e 3c - \log_e a$ are also in an A.P, then $a : b : c$ is equal to

(1) 9 : 6 : 4 **(2)** 16 : 4 : 1

(3) 25 : 10 : 4 **(4)** 6 : 3 : 2

Q. 6. Let A = $\begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$ and P = $\begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$.

The sum of the prime factors of $1|P^{-1}AP - 2I|$ is equal to

(1) 26 **(2)** 27

(3) 66 **(4)** 23

Q. 7. If each term of a geometric progression $a_1, a_2, a_3, \ldots$ with $a_1 = \frac{1}{8}$ and $a_2 \neq a_1$, is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \ldots + a_n$, then $S_{20} - S_{18}$ is equal to

(1) 2^{15} **(2)** -2^{18}

(3) 2^{18} **(4)** -2^{15}

Q. 8. An integer is chosen at random from the integers 1, 2, 3, …, 50. The probability that the chosen integer is a multiple of atleast one of 4, 6 and 7 is

(1) $\frac{8}{25}$ **(2)** $\frac{21}{50}$

(3) $\frac{9}{50}$ **(4)** $\frac{14}{25}$

Q. 9. Let P(3, 2, 3), Q(4, 6, 2) and R(7, 3, 2) be the vertices of ΔPQR. Then, the angle $\angle QPR$ is

(1) $\frac{\pi}{6}$ **(2)** $\cos^{-1}\left(\frac{7}{18}\right)$

(3) $\cos^{-1}\left(\frac{1}{18}\right)$ **(4)** $\frac{\pi}{3}$

Q. 10. Let r and θ respectively be the modulus and amplitude of the complex number $z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$, then (r, θ) is equal to

(1) $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$ **(2)** $\left(2\sec\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

(3) $\left(2\sec\frac{5\pi}{8}, \frac{3\pi}{8}\right)$ **(4)** $\left(2\sec\frac{11\pi}{8}, \frac{11\pi}{8}\right)$

Q. 11. If $\sin\left(\frac{y}{x}\right) = \log|x| + \frac{\alpha}{2}$ is the solution of the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ and $y(1) = \frac{\pi}{3}$, then α^2 is equal to

(1) 3 **(2)** 12

(3) 4 **(4)** 9

Q. 12. Let $y = \log_e\left(\frac{1-x^2}{1+x^2}\right)$, $-1 < x < 1$. Then at $x = \frac{1}{2}$, the value of $225(y' - y'')$ is equal to

(1) 732 **(2)** 746

(3) 742 **(4)** 736

Q. 13. If $\int \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sqrt{\sin^3 x \cos^3 x \sin(x-\theta)}} dx = A\sqrt{\cos\theta\tan x - \sin\theta} + B\sqrt{\cos\theta - \sin\theta\cot x} + C$ where C is the integration constant, then AB is equal to

(1) $4\text{cosec}(2\theta)$ **(2)** $4\sec\theta$

(3) $2\sec\theta$ **(4)** $8\text{cosec}(2\theta)$

Q. 14. Let $x = \frac{m}{n}$ (m, n are co-prime natural numbers) be a solution of the equation $\cos(2\sin^{-1}x) = \frac{1}{9}$ and let $\alpha, \beta(\alpha > \beta)$ be the roots of the equation $mx^2 - nx - m + n = 0$. Then the point (α, β) lies on the line

(1) $3x + 2y = 2$ **(2)** $5x - 8y = -9$

(3) $3x - 2y = -2$ **(4)** $5x + 8y = 9$

Q. 15. Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to

(1) 18 **(2)** 16

(3) 12 **(4)** 15

Q. 16. If R is the smallest equivalence relation on the set {1, 2, 3, 4} such that {(1, 2), (1, 3)} $\subset$ R, then the number of elements in R is ....

(1) 10 **(2)** 12

(3) 8 **(4)** 15

Q. 17. Let a unit vector $\hat{u} = x\hat{i} + y\hat{j} + z\hat{k}$ make angles, $\frac{\pi}{2}, \frac{\pi}{3}$ and $\frac{2\pi}{3}$ with the vectors $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}, \frac{1}{\sqrt{2}}\hat{j}+\frac{1}{\sqrt{2}}\hat{k}$ and $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}$ respectively. If $\vec{v} = \frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}+\frac{1}{\sqrt{2}}\hat{k}$, then $|\hat{u}-|\vec{v}|^2$ is equal to

(1) $\frac{11}{2}$ **(2)** $\frac{5}{2}$

(3) 9 **(4)** 7

Q. 18. Let $\overrightarrow{OA} = a$, $\overrightarrow{OB} = 12\vec{a}+4\vec{b}$ and $\overrightarrow{OC} = \vec{b}$, where O is the origin. If S is the parallelogram with adjacent sides OA and OC, then $\frac{\text{area of the quadrilateral OABC}}{\text{area of S}}$ is equal to ....

(1) 6 **(2)** 10

(3) 7 **(4)** 8

Q. 19. The function $f(x) = \frac{x}{x^2-6x-16}$, $x \in R-\{-2, 8\}$

(1) decreases in $(-2, 8)$ and increases in $(-\infty, -2) \cup (8, \infty)$

(2) decreases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

(3) decreases in $(-\infty, -2)$ and increases in $(8, \infty)$

(4) increases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

Q. 20. The sum of the solutions $x \in R$ of the equation $\frac{3\cos 2x+\cos^3 2x}{\cos^6 x-\sin^6 x} = x^3-x^2+6$ is

(1) 0 **(2)** 1

(3) –1 **(4)** 3

Section B

Q. 21. Let α, β be the roots of the equation $x^2-\sqrt{6}x+3 = 0$ such that Im (α) > Im (β). Let a, b be integers not divisible by 3 and n be a natural number such that $\frac{\alpha^{99}}{\beta}+\alpha^{98} = 3^n(a+ib)$, $i = \sqrt{-1}$. Then $n + a + b$ is equal to ....

Q. 22. Let the slope of the line $45x + 5y + 3 = 0$ be $27r_1+\frac{9r_2}{2}$ for some $r_1, r_2 \in R$. Then

$$\lim_{x\to 3}\left(\int_3^x \frac{8t^2}{\frac{3r_2x}{2}-r_2x^2-r_1x^3-3x}\right)$$ is equal to ....

Q. 23. Let P(α, β) be a point on the parabola $y^2 = 4x$. If P also lies on the chord of the parabola $x^2 = 8y$ whose mid point is $\left(1\frac{5}{4}\right)$, then $(\alpha - 28)(\beta - 8)$ is equal to ....

Q. 24. Let O be the origin, and M and N be the points on the lines $\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$ and $\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\overrightarrow{OM}.\overrightarrow{ON}$ is equal to ....

Q. 25. Let for any three distinct consecutive terms a, b, c of an A.P, the lines $ax + by + c = 0$ be concurrent at the point P and Q(α, β) be a point such that the system of equations $x + y + z = 6$, $2x + 5y + \alpha z = \beta$ and $x + 2y + 3z = 4$, has infinitely many solutions. Then $(PQ)^2$ is equal to ....

Q. 26. Remainder when 64^{32} is divided by 9 is equal to ....

Q. 27. Let $f(x) = \sqrt{\lim_{r\to x}\left\{\frac{2r^2[f(r)]^2-f(x)f(r)}{r^2-x^2}-r^3 e^{\frac{f(r)}{r}}\right\}}$ be differentiable in $(-\infty, 0) \cup (0, \infty)$ and $f(1) = 1$. Then the value of ea, such that $f(a) = 0$, is equal to ....

Q. 28. Let the set C = $\{(x, y) | x^2 - 2^y = 2023, x, y \in N\}$. Then $\sum_{(x,y)\in C} (x+y)$ is equal to ....

Q. 29. If $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1-\sin 2x}\,dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$, where α, β and γ are rational numbers, then $3\alpha + 4\beta - \gamma$ is equal to ....

Q. 30. Let the area of the region $\{(x, y): 0 \le x \le 3, 0 \le y \le \min\{x^2 + 2, 2x + 2\}\}$ be A. Then 12A is equal to ....

■■■

Mathematics			
Q. No.	**Answer**	**Topic Name**	**Chapter Name**
1	(3)	Maxima and Minima	Application of Derivatives
2	(4)	Point and Line	Straight Line
3	(3)	Mean and Variance	Statistics
4	(4)	Line and Line	Straight Line
5	(1)	AP	Sequence and Series
6	(1)	Value of Determinant	Matrix and Determinant
7	(4)	GP	Sequence and Series
8	(2)	Probability	Probability
9	(4)	Dot product	Vector
10	(1)	Modulus and Argument	Complex number
11	(1)	Solution of Differential Equation	Differential Equation
12	(4)	Value of Derivatives	Differentiation
13	(4)	Method of Substitution	Indefinite Integral
14	(4)	Trigonometric Equation	ITF
15	(4)	Distribution	Permutation and Combination
16	(1)	Equivalence Relation	Relation and Function
17	(2)	Dot Product	Vector
18	(4)	Cross Product	Vector
19	(2)	Increasing and Decreasing Function	Application of Derivatives
20	(2)	Trigonometry Equation	Trigonometry
21	[49]	Algebra of Complex Number	Complex Number
22	[12]	Leibnitz Theorem	Limit
23	[–192]	Chord	Parabola
24	[9]	Shortest Line	3D
25	[113]	System of Linear Equations	Determinants
26	[1]	Divisibility Problem	Binomial Theorem
27	[2]	Limi and Differential Equation	Limit and Differential Equation
28	[46]	Solution of Equation	Basic Mathematics
29	[6]	Definite Integration	Definite Integral
30	[164]	Area Between 2 Curves	Area Under the Curves

JEE (Main) SOLVED PAPER 2024

29th January Shift 2

ANSWERS WITH EXPLANATIONS

Section A

1. Option (3) is correct.

$$f(x) = 2x+3x^{\frac{2}{3}}$$

Diff. wrfx. $\quad f'(x) = 2+2x^{\frac{-1}{3}} = 2+\dfrac{2}{x^{\frac{1}{3}}}$

$$= 2\left(\frac{x^{\frac{1}{3}}+1}{x^{\frac{1}{3}}}\right)$$

$$f^1(x) = 0$$

$$x^{\frac{1}{3}}+1 = 0$$

$$x = -1$$

Critical point $\quad x = 0, -1$

using first order derivative test

$\oplus$ $\quad -1 \quad\quad 0 \quad +$

$-1 \downarrow$ Point of local maxima

$0 \downarrow$ Point of Local minima.

$\Rightarrow$ Exactly are point local maxima & exactly one point local minima.

2. Option (4) is correct.

Point of intersection of the lines

$$3x + 2y = 14$$

and $\quad 5x - y = 6$ is A(2, 4)

Point of intersection of lines

$$4x + 3y = 8$$

$$\& \ 6x + y = 5 \text{ is } B\left(\frac{1}{2}, 2\right)$$

Eq^n of line AB

$$\Rightarrow \quad y-4 = \frac{4-2}{2-\frac{1}{2}}(x-2)$$

$$\Rightarrow \quad y-4 = \frac{4}{3}(x-2)$$

$$3y - 12 = 4x - 8$$

$$4x - 3y + 4 = 0$$

Distance of line

$$4x - 3y + 4 = 0$$

from P(5, 2) is

$$\left|\frac{4(5)-3(-2)+4}{5}\right| = \frac{30}{5}$$

$$= 6$$

3. Option (3) is correct.

$$\bar{X}_{o/d} = \frac{24}{5}$$

$$\Rightarrow \frac{a_1+a_2+a_3+a_4+a_5}{5} = \frac{24}{5}$$

$$= \sum_{i=1}^{5} ai$$

$$= 24 \quad \text{...(i)}$$

$$(\text{Var})_{o/d} = \frac{194}{25}$$

$$n = 5$$

$$\bar{X}_{\text{New}} = \frac{7}{2}$$

$$\frac{\sum_{i=1}^{4} ai}{4} = \frac{7}{2}$$

$$\Rightarrow \quad \sum_{i=1}^{4} ac = 14 \quad \text{...(ii)}$$

by (i) & (ii) $\quad a_5 = 10$

$$(\text{Var})_{o/d} = \frac{194}{25}$$

$$\frac{\sum xc^2}{5} - (\bar{X})^2 = \frac{194}{25}$$

$$\frac{\sum_{i=1}^{4} ai^2 + 100}{5} - \left(\frac{24}{5}\right)^2 = \frac{194}{25}$$

$$\frac{\sum_{i=1}^{4} ac^2 + 100}{5} = \frac{194}{25} + \frac{576}{26}$$

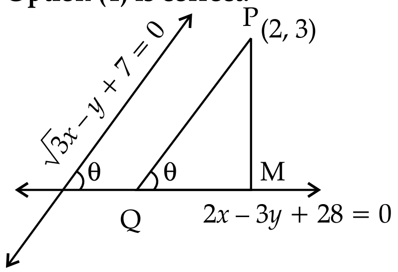
$$\sum_{i=1}^{4} ac^2 = \frac{720}{5} - 100 = \frac{270}{5}$$

$$= 54$$

New Variance

$$\frac{\sum_{i=1}^{4} ac^2}{4} - (\bar{X})^2 = \frac{54}{4} - \left(\frac{7}{2}\right)^2 = \frac{5}{4}$$

4. Option (4) is correct.



Let 'Q' be the angle *b/w* two given lines

$$\tan\theta = \left|\frac{\frac{2}{3} - \sqrt{3}}{1 + \frac{2}{3}\sqrt{3}}\right|$$

$$\tan\theta = \left|\frac{2 - 3\sqrt{3}}{3 + 2\sqrt{3}}\right|$$

$$\tan\theta = \frac{3\sqrt{3} - 2}{2\sqrt{3} + 3}$$

Now

$$\text{Pm} = \left|\frac{2(2) - 3(3) + 28}{\sqrt{4+9}}\right|$$

$$= \frac{23}{\sqrt{13}}$$

$$\because \text{PQ} = \frac{\text{Pm}}{\sin\theta}$$

$$= \text{Pm cosec}\theta$$

$$= \frac{23}{\sqrt{13}}\left(\frac{\sqrt{\left(2\sqrt{3}+3\right)^2 + \left(3\sqrt{3}-2\right)^2}}{3\sqrt{3}-2}\right)$$

$$= \frac{23}{\sqrt{13}}\left(\frac{\sqrt{12 + 9 + 12\sqrt{3} + 27 + 4 - 12\sqrt{3}}}{3\sqrt{3}-2}\right)$$

$$= \frac{23}{\sqrt{13}} \times \frac{\sqrt{52}}{\left(3\sqrt{3}-2\right)}$$

$$= \frac{46}{3\sqrt{3}-2} \times \frac{3\sqrt{3}+2}{3\sqrt{3}+2}$$

$$= \frac{46\left(3\sqrt{3}+2\right)}{23}$$

$$= 4 + 6\sqrt{3}$$

5. Option (1) is correct.

$\log_e a, \log_e b, \log_e c$ are in AP

$$\Rightarrow \quad 2\log_e b = \log_e a + \log_e c$$

$$\Rightarrow \quad b^2 = ac$$

$\because \log_e a - \log_e 2b, \log_e 2b - \log_e 3c,$

$\log_e 3c - \log_e a$ are in AP

$\Rightarrow \log_e \frac{a}{2b}, \log_e \frac{2b}{3c}, \log_e \frac{3c}{a}$ are in AP

$$\Rightarrow \quad 2\log_e \frac{2b}{3c} = \log_e \frac{a}{2b} + \log_e \frac{3c}{a}$$

$$\Rightarrow \quad \left(\frac{2b}{3c}\right)^2 = \frac{a}{2b} \times \frac{3c}{a}$$

$$\Rightarrow \quad \frac{4b^2}{9c^2} = \frac{3c}{2b}$$

$$8b^3 = 27c^3$$

$$2b = 3c$$

$$\because \quad b^2 = ac$$

$$\left(\frac{3c}{2}\right)^2 = ac$$

$$\Rightarrow \quad 9c = 4a$$

$$\therefore \quad a : b : c$$

$$\frac{9c}{4} : \frac{3c}{2} : c = 9 : 6 : 4$$

6. Option (1) is correct.

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$$

$$\text{let B} = \text{P}^{-1}\text{AP} - 2\text{I}$$

Post multiply by P

$$\text{PB} = \text{AP} - 2\text{P}$$

$$\{\because \text{PP}^{-1} = \text{I} \ \&\ \text{PI} = \text{P}\}$$

Post multiply by P^{-1}

$$\text{PBP}^{-1} = \text{A} - 2\text{I}$$

$$\text{Now } |\text{PBP}^{-1}| = |\text{A} - 2\text{I}|$$

$$|\text{P}|.|\text{B}|.|\text{P}^{-1}| = \begin{bmatrix} 2-2 & 1 & 2 \\ 6 & 2-2 & 11 \\ 3 & 3 & 2-2 \end{bmatrix}$$

$$|P|.|B|.\frac{1}{|P|} = \begin{bmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{bmatrix}$$

(expand wrf 12,) $|B| = -1(-33) + 2(18)$

$\Rightarrow \quad |P^{-1}AP - 2I| = 33 + 36 = 69$

Prime factors of 69 is 3 & 23

$$\text{sum} = 3 + 23 = 26$$

7. Option (4) is correct.

$\because a_1, a_2, a_3$............ are in G.P.

$$a_1 = \frac{1}{8}$$

$$a_2 = \frac{a_3 + a_4}{2} \quad ...(i)$$

$\therefore a_1 = a$ &'r' be the common ratio

$\therefore eq^n$ (i) becomes

$$ar = \frac{ar^2 + ar^3}{2}$$

$$2 = r + r^2$$

$$r^2 + r - 2 = 0$$

$$r^2 + 2r - r - 2 = 0$$

$$(r + 2)(r - 1) = 0$$

$$r = -2, 1$$

$$a_2 \neq a_1$$

$\Rightarrow \quad r \neq 1$

$\therefore \quad r = -2$

Now $\quad S_{20} - S_{18} = a_{19} + a_{20}$

$$= ar^{18} + ar^{19}$$

$$= ar^{18}(1 + r)$$

$$= \frac{1}{8}(-2)^{18}(1-2)$$

$$= \frac{-2^{18}}{23}$$

$$= -2^{15}$$

8. Option (2) is correct.

$$U = \{1, 2, 3.......50\}$$

A = Set of integers which is multiple of 4

B = Set of integers which is multiple of 6

C = Set of integers which is multiple of 7

$$n(A) = 12$$

$$n(B) = 8$$

$$n(C) = 7$$

$$n(A \cap B) = 4$$

$$n(B \cap C) = 1$$

$$n(C \cap A) = 1$$

$$n(A \cap B \cap C) = 0$$

$\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$$= 12 + 8 + 7 - 4 - 1 - 1 + 0$$

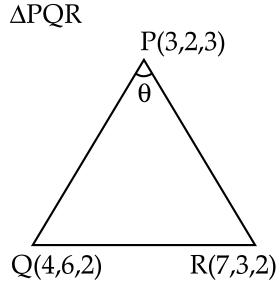
$$= 21$$

$\therefore$ required probability

$$= \frac{21}{50}$$

9. Option (4) is correct.

P(3, 2, 3), Q(4, 6, 2) & R(7, 3, 2) be the vertices of ΔPQR



let $\quad \angle QPR = \theta$

$$\overrightarrow{OP} = -\hat{i} - 4\hat{j} + \hat{k}$$

$$\overrightarrow{RP} = -4\hat{i} - \hat{j} + \hat{k}$$

$\therefore$

$$\cos\theta = \left|\frac{\overrightarrow{QP}.\overrightarrow{RP}}{|\overrightarrow{QP}||\overrightarrow{RP}|}\right|$$

$$= \left|\frac{4+4+1}{\sqrt{18}\sqrt{18}}\right|$$

$$= \frac{9}{18} = \frac{1}{2}$$

$$Q = \frac{\pi}{3}$$

10. Option (1) is correct.

$$z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$$

$$= 2 - 2i\frac{\sin\frac{5\pi}{8}}{\cos\frac{5\pi}{8}}$$

$$= \frac{2}{\cos\frac{5\pi}{8}}\left(\cos\frac{5\pi}{8} - i\sin\frac{5\pi}{8}\right)$$

$$= \frac{2}{\cos\left(\pi - \frac{3\pi}{8}\right)}\left(\cos\left(\pi - \frac{3\pi}{8}\right) - i\sin\left(\pi - \frac{3\pi}{8}\right)\right)$$

$$= \frac{-2}{\cos\frac{3\pi}{8}}\left(-\cos\frac{3\pi}{8} - i\sin\frac{3\pi}{8}\right)$$

$$= 2\sec\frac{3\pi}{8}\left(\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}\right)$$

In polar form any complex no. z can be written

$$z = |z|(\cos\theta + i\sin\theta)$$

$$\therefore \quad |z| = r$$

$$= 2\sec\frac{3\neq}{8}$$

$$\text{Arg}(z) = \theta$$

$$= \frac{3\neq}{8}$$

11. Option (1) is correct.

$$\sin\frac{y}{x} = \ln x+\frac{\alpha}{2}$$

$$\because \quad y(1) = \frac{\pi}{3}$$

$$\therefore \quad \sin\frac{\pi}{3} = \ln(1)+\frac{\alpha}{2}$$

$$\frac{\sqrt{3}}{2} = 0+\frac{\alpha}{2}$$

$$\alpha = \sqrt{3}$$

$$\alpha^2 = 3$$

12. Option (4) is correct.

$$y = \log_e\left(\frac{1-x^2}{1+x^2}\right) -1<x<1$$

$$y' = \frac{1}{\left(\frac{1-x^2}{1+x^2}\right)}.\frac{(1+x^2)(-2x)-(1-x^2)(2x)}{(1+x^2)^2}$$

$$= \frac{2x(-1-x^2-1+x^2)}{(1-x^2)(1+x^2)}$$

$$= \frac{-4x}{1-x^4}$$

$$y'' = \frac{(1-x^4)(-4)-(-4x)(-4x^3)}{(1-x^4)^2}$$

$$= \frac{4[-1+x^4-4x^4]}{(1-x^4)^2}$$

$$= 4\frac{(-3x^4-1)}{(1-x^4)^2} = -4\left[\frac{3x^4+1}{(1-x^4)^2}\right]$$

$$\text{At} \quad x = \frac{1}{2}$$

$$\Rightarrow \quad y' = \frac{-4\times\frac{1}{2}}{1-\frac{1}{16}}$$

$$= \frac{-2\times 16}{15}$$

$$= -\frac{32}{15}$$

$$y'' = -4\frac{\left(\frac{3}{10}+1\right)}{\left(1-\frac{1}{10}\right)^2}$$

$$= \frac{-4\times 19}{16}\times\frac{16\times 16}{15\times 15} = \frac{-64\times 19}{225}$$

Now $225(y'-y'')$

$$= 225\left(\frac{-32}{15}+\frac{64\times 19}{225}\right)$$

$$= 32(-15+38)$$

$$= 32\times 23$$

$$= 736$$

13. Option (4) is correct.

$$\int\frac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sqrt{\sin^3 x.\cos^3 x.\sin(x-\theta)}}dx$$

$$= A\sqrt{\cos\theta.\tan x-\sin\theta}+13\sqrt{\cos\theta-\sin\theta\cot x}+c$$

$$\text{let} \quad I = \int\frac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sqrt{\sin^3 x\cos^3 x\sin(x-\theta)}}dx$$

$$= \int\frac{dx}{\sqrt{\cos^3 x(\sin x\cos\theta-\cos x\sin\theta)}}$$

$$+ \int\frac{dx}{\sqrt{\sin^3 x(\sin x\cos\theta-\cos x\sin\theta)}}$$

$$= \int\frac{\sec^2 x dy}{\sqrt{\tan x\cos\theta-\sin\theta}}+\int\frac{\cos ec^2 x dx}{\sqrt{\cos\theta-\cot x\sin\theta}}$$

$$\downarrow$$

$$\tan x = y \qquad -\cot x = z$$

$$\sec^2 x\, dx = dy \qquad \text{cosec}^2 x dx = dz$$

$$= \int\frac{dy}{\sqrt{y\cos\theta-\sin\theta}}+\int\frac{dz}{\sqrt{\cos\theta+\sin\theta 2}}$$

$$= \frac{2\sqrt{y\cos\theta-\sin\theta}}{\cos\theta}+\frac{2\sqrt{\cos\theta+\sin\theta 2}}{\sin\theta}$$

$$\therefore \quad A = 2\sec\theta$$

$$B = 2\,\text{cosec}\,\theta$$

$$AB = 2\sec\theta\,.\,2\,\text{cosec}\,\theta$$

$$= \frac{4}{\sin\theta\ \text{cosec}\theta} = \frac{8}{\sin 2\theta}$$

$$= 8\,\text{cosec}\,2\theta$$

14. Option (4) is correct.

$$\cos(2\sin^{-1}x) = \frac{1}{9}$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\therefore \quad \cos(2\sin^{-1}x) = 1 - 2[\sin(\sin^{-1}x)]^2$$

$$= \frac{1}{9}$$

$$1 - 2(x^2) = \frac{1}{9}$$

$$1 - \frac{1}{9} = 2x^2$$

$$\frac{8}{9} = 2x^2$$

$$x^2 = \frac{4}{9}$$

$$x = \pm\frac{2}{3}$$

$$\therefore \quad x = \frac{2}{3} = \frac{m}{n}$$

$$m = 2$$

$$n = 3$$

$\therefore$ eqn is $mx^2 - nx + m + n = 0$

$$\Rightarrow \quad 2x^2 - 3x - 2 + 3 = 0$$

$$\Rightarrow \quad 2x^2 - 3x + 1 = 0$$

$$\Rightarrow \quad 2x^2 - 2x - x + 1 = 0$$

$$\Rightarrow \quad 2x^2(x-1) - (x-1) = 0$$

$$(2x-1)(x-1) = 0$$

$$\Rightarrow \quad x = 1, \frac{1}{2}$$

$$\therefore \quad \alpha = 1,$$

$$\beta = 1$$

$$\Rightarrow \quad (\alpha, \beta) = \left(1, \frac{1}{2}\right)$$

(1) $\quad 5(1) - 8\left(\frac{1}{2}\right) = -9$

$$\Rightarrow \quad 1 = -9 \text{ false}$$

(2) $\quad 3(1) - 2\left(\frac{1}{2}\right) = -2$

$$\Rightarrow \quad 2 = -2 \text{ false}$$

(3) $\quad 3(1) + 2\left(\frac{1}{2}\right) = -2$

$$\Rightarrow \quad 4 = 2 \text{ false}$$

(1) $\quad 5(1) + 8\left(\frac{1}{2}\right) = 9$

$$\Rightarrow \quad 9 = 9 \text{ true}$$

15. Option (4) is correct.

8 identical objects into 4 identical shelves where any number of shelves may remain empty is given by

008, 0017, 0026, 0035, 0044 = 5

$$\left.\begin{matrix} 0 & 1 & 1 & 6 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & 3 & 4 \\ 0 & 2 & 2 & 4 \end{matrix}\right]$$

0224

$$\left.\begin{matrix} 1 & 1 & 1 & 5 \\ 1 & 1 & 2 & 4 \\ 1 & 1 & 3 & 3 \\ 1 & 2 & 2 & 3 \end{matrix}\right]$$

$\underline{2222 - 1}$

Total = 15

16. Option (1) is correct.

A = {1, 2, 3, 4}
R = {(1, 2) (1, 3)}

for reflexive
we need to have = (1, 1), (2, 2), (3, 3), (4, 4)
For symmetric
(1, 2) $\in$ R $\Rightarrow$ (2, 1) $\in$ R
(1, 3) $\in$ R $\Rightarrow$ (3, 1) $\in$ R
$\therefore$ R becomes
{(1, 1) (2, 2) (3, 3) (4, 4) (1, 2) (2, 1), (1, 3), (3, 1)}
Now (3, 1) & (1, 2) both belongs to R
So for transitive (3, 2) must be on R
$\therefore$ (3, 2) must be added
again for symmetric (2, 3) also be an element
$\therefore$ R = {(1, 1) (2, 2) (3, 3) (4, 4) (1, 2) (2, 1) (1, 3) (3, 1) (3, 2) (2, 3)}
$\therefore$ No. of elements in
R = 10

17. Option (2) is correct.

unit vector $\hat{u} = x\hat{i} + y\hat{j} + z\hat{k}$ makes $\frac{\pi}{2}, \frac{\pi}{3}$ & $\frac{2\pi}{3}$

with the vectors $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}$, $\frac{1}{\sqrt{2}}\hat{j}+\frac{1}{\sqrt{2}}\hat{k}$ & $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}$ respectively.

$$\therefore \quad \frac{x}{\sqrt{2}}+\frac{z}{\sqrt{2}} = |\hat{u}|\sqrt{\frac{1}{2}+\frac{1}{2}}.\cos\sqrt{\frac{\pi}{2}}$$

$$= 0$$

$$\Rightarrow \quad x + z = 0 \quad ...(i)$$

$$\& \quad \sqrt{\frac{y}{2}+\frac{z}{\sqrt{2}}} = |\vec{u}|\sqrt{\frac{1}{2}+\frac{1}{2}}\cos\frac{\pi}{3}$$

$$\frac{y+z}{\sqrt{2}} = \frac{1}{2}$$

$$\Rightarrow \quad y + z = \frac{1}{\sqrt{2}} \text{ ....(ii)}$$

$$\& \quad \frac{x}{\sqrt{2}}+\frac{y}{\sqrt{2}} = |\hat{u}|\sqrt{\frac{1}{2}+\frac{1}{2}}\cos\frac{2\pi}{3}$$

$$\frac{x+y}{\sqrt{2}} = 1\times1\times-\frac{1}{2}$$

$$\Rightarrow \quad x + y = -\frac{1}{\sqrt{2}} \text{ ...(iii)}$$

add (i), (ii) & (iii)

$$7x + 2y + 2z = 0$$

$$x + y + z = 0$$

$$y = 0$$

$$x = -\frac{1}{\sqrt{2}}$$

$$z = \frac{1}{\sqrt{2}}$$

$$\hat{u} = -\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}$$

$$|\hat{u}-\mathrm{J}|^2 = \left(\frac{2}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+0^2$$

$$= \frac{4}{2}+\frac{1}{2} = \frac{5}{2}$$

18. Option (4) is correct.

$$\overrightarrow{OA} = \vec{a}$$

$$\overrightarrow{OB} = 12\vec{a}+4\vec{b}$$

$$OC = b$$

$\because$ S is parallelogram with adjacent sides OA & OC.

Now

$$\text{Area of quad.} \quad OABC = \frac{1}{2}|\vec{d}_1\times\vec{d}_2|$$

(Where $\vec{d}_1 \varnothing \vec{d}_2$ are diagram)

$$= \frac{1}{2}|\overrightarrow{OB}\times\overrightarrow{AC}|$$

$$= \frac{1}{2}|(12\vec{a}\times4\vec{b})\times(\vec{b}-\vec{a})|$$

$$= \frac{1}{2}|12(\vec{a}\times\vec{b})-4(\vec{b}\times\vec{a})|$$

$$= \frac{1}{2}|16(\vec{a}\times\vec{b})|$$

$$= 8|\vec{a}\times\vec{b}|$$

$$\text{Area of} \quad S = |\vec{a}\times\vec{b}|$$

$$\therefore \frac{\text{Area of quad.}\varnothing\text{ABC}}{\text{Area of S}} = 8$$

19. Option (2) is correct.

$$f(x) = \frac{x}{x^2-6x-16} \quad x\in \text{R } \{-2, 8\}$$

$$f'(x) = \frac{(x^2-6x-16)(1)-x(2x-6)}{(x^2-6x-16)^2}$$

$$= \frac{x^2-6x-16-2x^2+6x}{(x^2-6x-16)^2}$$

$$= \frac{-x^2-16}{(x^2-6x-16)^2}$$

$$= \frac{-(x^2-16)}{(x^2-6x-16)^2} < 0$$

$\because$ f^1(x) < 0 for $x \in$ domain

$\therefore$ It is a decreasing f4^n in its domain

$x \in (-\infty, -2) \cup (2, 8) \cup (8, \infty)$

20. Option (2) is correct.

$$\frac{3\cos 2x+\cos^3 2x}{\cos^6 x-\sin 6x} = x^3 - x^2 + 6$$

L.H.S

$$\frac{\cos 2x(3+\cos^2 2x)}{(\cos^2 x-\sin^2 x)(\cos^4 x+\sin^4 x+\cos^2 x\sin^2 x)}$$

$\{\because \quad \cos2x = \cos^2 x - \sin^2 2x\}$

$$\frac{\cos 2x(3+\cos^2 2x)}{\cos 2x((\cos^2 x+\sin^2 x)^2-\cos^2 x\sin^2 x)}$$

$$\frac{3+\cos^2 24}{1-\frac{1}{4}\sin^2 2x}$$

$\{\because \sin 2x = 2\sin x \cos x\}$

$$= \frac{4(3+1-\sin^2 2x)}{4-\sin^2 2x}$$

$$= 4\frac{(4-\sin^2 2x)}{4-\sin^2 2x}$$

$$= 4$$

Now $x^3 - x^2 + 6 = 4$

$x^3 - x^2 + 2 = 0$

$\therefore$ sum of solution $= 1$

Section B

21. Correct answer is [49].

α & β are roots of eq^n

$$x^2 - \sqrt{6}x + 3 = 0$$

$$x = \frac{\sqrt{6} \pm \sqrt{6-12}}{2}$$

$$= \frac{\sqrt{6} \pm \sqrt{6}i}{2}$$

$$= \frac{\sqrt{6}}{2}(1 \pm i)$$

$$\text{Im}(\alpha) > \text{Im}(\beta)$$

$$\alpha = \frac{\sqrt{6}}{2}(1+i), ,$$

$$\beta = \frac{\sqrt{6}}{2}(1-i),$$

$$\alpha = re^{i\theta}$$

$$= \sqrt{3}e^{i\frac{\neq}{4}}$$

$$\alpha^{98} = \left(\sqrt{3}\right)^{98}\left(e^{i98\times\frac{\pi}{4}}\right)$$

$$= 3^{49}\left[e^{i49\frac{\pi}{2}}\right]$$

$$= 3^{49}\left(0 + ^{i\sin 49\frac{\pi}{2}}\right)$$

$$= i(3^{49})$$

$$\therefore \quad \frac{\alpha^{99}}{\beta} + a^{98} = \alpha^{98}\left(\frac{\alpha}{\beta} + 1\right)$$

$$\Rightarrow i\left(3^{49}\left(\frac{1+c}{1-c} + 1\right)\right.$$

$$\Rightarrow 3^{49}\left(\frac{i-1}{1-c} + i\right)$$

$$= 3^{49}(-1+c)$$

$\Rightarrow$ $a = -1,$

$b = 1$

$n = 49$

$n + a + b = 49$

22. Correct answer is [12].

$$45x + 5y + 3 = 0$$

$$\text{Slope} = -\frac{45}{5}$$

$$= 27r_i + \frac{9r_2}{2}$$

$$-9 = 27r_1 + \frac{9r_2}{2}$$

$$-2 = 6r_1 + r_2$$

$$6r_1 + r_2 + 2 = 0$$

$$\lim_{x\to 3}\left(\int_3^x \frac{3t^2}{\frac{3r_2 x}{2} - r_2x^2 - r_1x^3 - 3x}\,dt\right)$$

$$\lim_{x\to 3}\frac{\frac{8}{3}(x^3-27)}{\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x}$$

L1 Hospital rule

$$\lim_{x\to 3}\frac{\frac{8}{3}.3x^2}{\frac{3r_2}{2} - 2r_2x - 3r_1x^2 - 3}$$

$$\frac{8(9)}{\frac{3r_2}{2} - 6r_2 - 27r_1 - 3} = \frac{8\times 9\times 2}{3r_2 - 12r_2 - 54r_1 - 6}$$

$$= \frac{144}{-9(r_2 + 6r_1) - 6}$$

$$= \frac{144}{12}$$

$$= 12$$

23. Correct answer is [–192].

eq^n of of chord bisected at point $\left(1, \frac{5}{4}\right)$ to Parabola

$$x^2 = 8y$$

$$T = S_1$$

$$xx_1 - 4(y + y_1) = x_1^2 - 8y_1$$

$$\therefore \quad (x_1\, y_1) = \left(1, \frac{5}{4}\right)$$

$$\Rightarrow \quad x - 4y - 5 = -9$$

$$\Rightarrow \quad x - 4y + 4 = 0$$

$\because$ (α, α) lies on

$$x - 4y + 4 = 0$$

& $y^2 = 4x$

$$\alpha - 4\beta + 4 = 0 \;\&$$

$$\alpha^2 = 4\alpha$$

$$\Rightarrow \quad \beta^1 = 4(4\alpha - 4)$$

$\Rightarrow \quad \beta^1 - 16\alpha + 16 = 0$

$\Rightarrow \quad \beta = \dfrac{16 \pm \sqrt{(16)^2 - 4(1)(16)}}{2}$

$\beta = 8 \pm 4\sqrt{3}$

$8 - \beta = \mp 4\sqrt{3}$

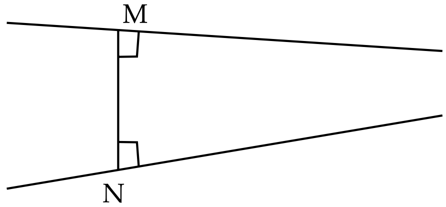
Now $\quad \alpha = 4\beta - 4$

$= 28 \pm 16\sqrt{3}$

$\therefore \quad (\alpha - 28)(8 - \beta) = \left(\pm 16\sqrt{3}\right)\left(\mp 4\sqrt{3}\right)$

$= -192$

24. Correct answer is [9].



$$\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$$

$$\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$$

let m $(4\lambda + 5, \lambda + 4, 3\lambda + 5)$ & N$(12\mu - 8, 5\mu - 2, 9\mu - 11)$ are two point on given lines.

$\therefore \ \overrightarrow{MN} \perp \overrightarrow{b_1}$,

& $\overrightarrow{MN} \perp \overrightarrow{b_2}$

$\overrightarrow{MN}.\overrightarrow{b_1} = 0$

$4(12\mu - 4\lambda - 13) + 1(5\mu - \lambda - b) + 3(9\mu - 3\lambda - 16) = 0$

$\Rightarrow 48\mu - 16\lambda - 52 + 5\mu - \lambda - 6 + 27\mu - 9\lambda - 48 = 0$

$\Rightarrow \quad 80\mu - 26\lambda = 106$

$40\mu - 13\lambda = 53 \quad$ ...(i)

$\overrightarrow{MN}.\overrightarrow{b_2} = 0$

$12(12\mu - 4\lambda - 13) + 5(5\mu - \lambda - 6) + 9(9\mu - 3\lambda - 16) = 0$

$\Rightarrow 144\mu - 48\lambda - 156 + 25\mu - 5\lambda - 30 + 81\mu - 27\lambda - 144 = 0$

$\Rightarrow \quad 250\mu - 80\lambda = 330$

$25\mu - 8\lambda = 33 \quad$ ...(ii)

Solving (i) & (ii) we get

$\mu = 1,$

$\lambda = -1$

$\therefore \quad M = (1, 3, 2)$

& $\quad N = (4, 3, -2)$

$\overrightarrow{OM} = \hat{i} + 3\hat{j} + 2\hat{k}$

$\overrightarrow{ON} = 4\hat{i} + 3\hat{j} - 2\hat{k}$

$\overrightarrow{OM}.\overrightarrow{ON} = 4 + 9 - 4$

$= 9$

25. Correct answer is [113].

$\because$ a, b, c are in AP

$2b = a + c$

$a - 2b + c = 0$

$\because \quad ax + by + c = 0$

three lines are concurrent point of intersection is

$$\frac{x}{1} = \frac{y}{-2} = \frac{1}{1}$$

$x = 1$

$y = -2$

$\therefore$ P(1, – 2)

&

Given system $x + y + z = 6$

$2x + 5y + \alpha z = \beta$

$x + 2y + 32 = 4$ has

Infinite solve

$\therefore \quad \Delta = \Delta_n = \Delta_y$

$= \Delta_2$

$= 0$

$\Delta = 0$

$$\Rightarrow \quad \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{bmatrix} = 0$$

$1(15 - 2\alpha) - 1(6 - \alpha) + 1(4 - 5) = 0$

$= 15 - 2\alpha - 6 + \alpha - 1$

$= 0$

$\alpha = 8$

$\Delta_2 = 0$

$$\begin{bmatrix} 1 & 1 & 6 \\ 2 & 5 & \beta \\ 1 & 2 & 4 \end{bmatrix} = 0$$

$1(20 - 2\beta) - 1(8 - \beta) + 6(4 - 5) = 0$

$\Rightarrow \quad 20 - 2\beta - 8 + \beta - 6 = 0$

$\beta = 6$

$\therefore$ Q (8, 6) $\quad PQ^2 = 49 + 64$

$= 113$

26. Correct answer is [1].

$(64)32^{32}$ is divided by 9 then for remainder$\rightarrow$

$(64)^K = (1 + 63)K$

when $\quad K = 32^{32}$

$= 1 + K(63) + \dfrac{K(K-1)}{2!}(63)^2 + \ldots$

$= 1 + 63\lambda$
$= 1 + 9\mu$
$\therefore$ remainder = I

27. Correct answer is [2].

$$f(x) = \sqrt{\lim_{r\to x}\frac{2r^2[(f(x))^2 - f(x).f(x)]}{r^2 - x^2} - r^3 e^{\frac{f(z)}{r}}}$$

$$= \sqrt{\lim_{r\to x}\frac{2r^2 f(x)[f(x) - f(x)]}{(r-x)-(r+x)} - \lim_{x\to\infty} r^3 e^{\frac{r(x)}{r}}}$$

$$= \sqrt{\lim_{x\to\infty}\frac{2r^2 - f(x)}{r+x}.\left(\frac{f(r)-f(x)}{r-x}\right) - x^3 e^{\frac{f(x)}{x}}}$$

$$= \sqrt{\frac{2x^2.f(x)}{2x}.f^1(x) - x^3 e\frac{f(x)}{x}}$$

Squaring both sides

$$f^2(x) = x.f(x).f^1(x) - x^3 e^{\frac{f(x)}{x}}$$

$$y^2 = xy\frac{dy}{dx} - x^3 e^{\frac{y}{x}}$$

$$x^3 e^{\frac{y}{x}} dx = xydy - y^2 dx$$

$$x^3 dx = y\frac{(xdy - ydx)}{e^{\frac{y}{x}}}$$

$$x^3 dx = y.(xdy - ydx).e^{-\frac{y}{x}}$$

$$\frac{x}{y}dx = \frac{(xdy - ydx)}{xz}.e - \frac{y}{x} \quad \text{....(i)}$$

$$e^{-\frac{y}{x}} = t$$

$$\Rightarrow \quad -\frac{y}{x} = lnt$$

$$e^{-\frac{y}{x}}(-1)\left(\frac{xdy - yx}{x^2}\right) = dt$$

$$e^{-\frac{y}{x}}\left(\frac{xdy - ydx}{x^2}\right) = -dt$$

Now eqn (i) become

$$-\frac{dx}{\ln t} = dt$$

$$dx = \text{lnt}.dt$$

$$\int dx = \int \text{lnt}\ dt$$

$$x = t[\text{lnt} - 1] + C$$

$$x = e^{-\frac{y}{x}}\left(-\frac{y}{x} - 1\right) + C$$

$$x + \left(1 + \frac{y}{x}\right)e^{-\frac{y}{x}} = C \quad \text{...(i)}$$

$f(1) = 1$
$\Rightarrow \quad x = 1$
$\Delta \quad y = 1$
Satisfies eqn (i)
$1 + (1 + 1)e^{-1} = C$
$\Rightarrow 1 + 2e^{-1}$
Now $\quad f(a) = 0$
$x = a$
& $\quad y = 0$
$a + (1 + 0)\, e^{-0} = C$
$a + 1 = C$
$a = C - 1$
$a = 1 + 2e^{-1} - 1$
$a = 2e^{-1}$
$ae = 2$

28. Correct answer is [46].

$x^2 - 2y = 2023$
$x^2 = 2023 + 2y$
let $\quad y = 1$
$\Rightarrow \quad x^2 = 2023 + 2$
$= 2025$
$x = 45$
$\therefore \quad x + y = 46$

29. Correct answer is [6].

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{5}} \sqrt{1 - \sin 2x}\ dx = a + \beta\sqrt{2} + \gamma\sqrt{3}$$

let
$$\text{I} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1 - \sin 2x}\ dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} |\sin x - \cos x| dx$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\cos x - \sin x)dy + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sin x - \cos x)dx$$

$$= [\sin x + \cos x]_{\frac{\pi}{6}}^{\frac{\pi}{4}} - [\cos x + \sin x]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

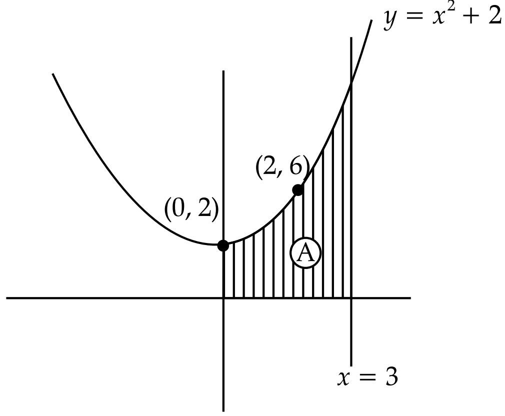
$$= \left(\sqrt{2}-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{2}+\frac{\sqrt{3}}{2}-\sqrt{2}\right)$$

$$= 2\sqrt{2}-1-\sqrt{3}$$

$$\alpha = -1$$
$$\beta = 2$$
$$\gamma = -1$$
$$3\alpha + 4\beta - \gamma = -3 + 8 + 1$$
$$= 6$$

30. Correct answer is [164].

Region = $\{(x, y): 0 \le x \le 3, 0 < y < \min(x^2 + 2, 2x + 2)\}$



$$A = \int_0^2 x^2 + 2 + \int_2^3 2x + 2$$

$$= \left[\frac{x^3}{3}+2x\right]_0^2 + [x^2+2x]_2^3$$

$$= \frac{8}{3}+4+[9+6-4-4]$$

$$= \frac{8}{3}+4+7$$

$$= 11+\frac{8}{3}$$

$$12\,A = 12 \times 11 + 32$$
$$= 132 + 32$$
$$= 164$$

■■■