CBSE EXAMINATION PAPER 2025
Applied Mathematics
Class-12th
(Solved)

Time : 3 Hours | **Max. Marks : 80**

General Instructions:
Read the following instructions carefully and strictly follow them:

*(i) This Question Paper contains **38** questions. **All** questions are **compulsory.***

*(ii) This Question paper is divided into **FIVE** sections Section - **A, B, C, D** and **E.***

*(iii) In **Section A,** Question no. **1** to **18** are Multiple Choice Questions (MCQs) and Question no. **19 & 20** are Assertion-Reason based questions of 1 mark each.*

*(iv) In **Section B,** Question no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

*(v) In **Section C,** Question no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

*(vi) In **Section D,** Question no. **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

*(vii) In **Section E,** Question no. **36** to **38** are case study based questions, carrying **4** marks each.*

*(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section-**B**, **2** questions in Section-**C**, **2** questions in Section-**D** and **3** questions in Section-**E**.*

(ix) Use of calculators is NOT allowed.

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SECTION A

This section comprises Multiple Choice Questions (MCQs) of 1 mark each. Select the correct option (Question 1 to Question 18) :

1. –41 mod 9 is

(A) 5 **(B)** 4
(C) 3 **(D)** 0

2. If $a > b$ and $c < 0$, then which of the following is true?

(A) $a + c < b + c$ **(B)** $a - c < b - c$
(C) $ac > bc$ **(D)** $a - c > b + c$

3. If A and B are symmetric matrices of the same order, then $(AB' - BA')$ is a

(A) symmetric matrix
(B) null matrix
(C) diagonal matrix
(D) skew symmetric matrix

4. The inverse of matrix A $= \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$ is

(A) $\frac{1}{6}\begin{bmatrix} -4 & 2 \\ -1 & -1 \end{bmatrix}$ **(B)** $\begin{bmatrix} \frac{1}{3} & \frac{1}{6} \\ \frac{2}{3} & -\frac{1}{6} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{1}{6} & \frac{1}{6} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$ **(D)** $\begin{bmatrix} -\frac{2}{3} & \frac{1}{6} \\ -\frac{1}{3} & -\frac{1}{6} \end{bmatrix}$

5. If $\begin{vmatrix} 2x & 5 \\ 4 & x \end{vmatrix} = \begin{vmatrix} 3 & 5 \\ 4 & 6 \end{vmatrix}$, then the value of x is

(A) $\frac{3}{2}$ **(B)** 6
(C) 3 **(D)** ± 3

6. The slope of the normal to the curve $y = \frac{x-3}{x-4}$ at $x = 6$ is

(A) 4 **(B)** $-\frac{1}{4}$
(C) –4 **(D)** $\frac{1}{4}$

7. The rate of change of population $P(t)$ with respect to time (t), where α, β are the constant birth and death rates, respectively, is

(A) $\frac{dP}{dt} = (\alpha + \beta)P$ **(B)** $\frac{dP}{dt} = (\alpha - \beta)P$

(C) $\frac{dP}{dt} = \frac{\alpha + \beta}{P}$ **(D)** $\frac{dP}{dt} = \frac{\alpha - \beta}{P}$

8. A pair of dice is thrown two times. It X represents the number of doublets obtained, then the expectation of X is

(A) $\frac{1}{6}$ **(B)** 1

(C) $\frac{1}{3}$ **(D)** $\frac{11}{36}$

9. The mean of t-distribution is

(A) 0 **(B)** 1
(C) 2 **(D)** not defined

10. The variations which occur due to change in climate, festivals or weather conditions are known as

(A) secular variations **(B)** cyclic variations
(C) seasonal variations **(D)** irregular variations

11. In an LPP, the maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \le 40$, $x + 2y \le 60$, $x, y \ge 0$ is

(A) 120 **(B)** 140
(C) 150 **(D)** 130

12. The present value of a sequence of payments of ₹ 100 made at the end of every year and continuing forever, if the money is worth 5% compounded annually, is

(A) ₹ 2,000 **(B)** ₹ 20,000

(C) ₹ 5,000 **(D)** ₹ 12,000

13. The demand function of a monopolist is given by $p = 30 + 5x - 3x^2$, where x is the number of units demanded and p is the price per unit. The marginal revenue when 2 units are sold, is

(A) ₹ 28 **(B)** ₹ 23

(C) ₹ 1 **(D)** ₹ 14

14. If the cost function and revenue function of x items are respectively given as $C(x) = 100 + 0.015\ x^2$, $R(x) = 3x$, then the value of x for maximum profit is

(A) 50 **(B)** 100

(C) 150 **(D)** 200

15. If a random variable X has the probability distribution

$$P(X=x)=\begin{cases} k, & \text{if} \quad x=0 \\ 2k, & \text{if} \quad x=1 \text{ or } 2 \\ 0, & \text{otherwise,} \end{cases}$$

then the value of k is

(A) $\frac{1}{3}$ **(B)** $\frac{1}{5}$

(C) $\frac{1}{6}$ **(D)** $\frac{1}{4}$

16. The test statistic t for testing the significance of differences between the means of two independent samples is given by

(A) $t=\dfrac{\bar{x}-\bar{y}}{\sqrt{s}}$ **(B)** $t=\dfrac{\bar{x}-\bar{y}}{s\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}}$

(C) $t=\dfrac{\bar{x}-\bar{y}}{\dfrac{s}{\sqrt{n-1}}}$ **(D)** $t=\dfrac{\bar{x}+\bar{y}}{s\sqrt{\dfrac{1}{n_1}-\dfrac{1}{n_2}}}$

17. The effective rate of interest equivalent to a nominal rate of 4% compounded semi-annually, is

(A) 4.12% **(B)** 4.04%

(C) 4.08% **(D)** 4.14%

18. The CAGR of an investment, whose starting value is ₹ 5,000 and it grows to ₹ 25,000 in 4 years, is :

[Given $(5)^{0.25} = 1.4953$]

(A) 49.53% **(B)** 14.95%

(C) 495.3% **(D)** 1.49%

Questions number 19 and 20 are Assertion - Reason based questions of 1 mark each. Two statements are given – one labelled Assertion (A) and other labelled Reason (R). Select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below :

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is correct explanation of Assertion (A).

(B) Both Assertion (A) and Reasion (R) are true, but Reason (R) is not correct explanation for Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. **Assertion (A) :** The area of the region bounded by the line $y - 1 = x$, the x-axis and the ordinates $x = -1$ and $x = 1$ is 2 square units.

Reason (R) : The area of the region bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = b$ is given by

$$\int_a^b f(x)dx$$

20 **Assertion (A) :** The differential equation representing the family of curves $y = mx$, m being an arbitrary constant, is

$$x\frac{dy}{dx}-y=0.$$

Reason (R) : For a family of curves, the differential equation is obtained by differentiating the equation of family of curves with respect to x and then eliminating the arbitrary constant, if any.

SECTION B

*Questions Number **21** to **25** are Very Short Answer (VSA) type questions of 2 marks each.*

21. **(a)** The cost of Type I sugar is ₹ 25 per kg and Type II sugar is ₹35 per kg. If both Type I sugar and Type II sugar are mixed in the ratio of 3:2, find the price per kg of the mixture.

OR

(b) Pipe A can fill a tank in 1 hour and Pipe B can fill it in $\frac{11}{2}$ hours. If both the pipes are opened in the empty tank, how much time will they take to fill the tank ?

22. A boat goes 3.5 km upstream and then returns. Total time taken is 1 hour and 12 minutes. If the speed of the current is 1 km/h, then find the speed of the boat in still water.

23. A runs $\frac{3}{2}$ times as fast as B. If A gives a start of 40 m, how far must the winning post from the starting point be, so that A and B reach at the same time ?

24. Given $A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 4 & 5 \\ 0 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & -5 \\ -5 & 1 & -5 \\ 1 & -2 & 4 \end{bmatrix}$, find BA.

25. **(a)** If a fair coin is tossed 6 times, find the probability of getting atleast 4 heads.

OR

(b) Given that mean of a normal variate X is 9 and standard deviation is 3, then find:

(i) the z-score of the data point 15

(ii) the data point if its z-score is 4.

SECTION C

*Questions Number **26** to **31** are Short Answer (SA) type questions of 3 marks each.*

26. Find the units digit in 7^{295}.

27. Two numbers are selected at random (without replacement) from first six positive integers. Let X denotes the smaller of the two numbers obtained.

Calculate the mathematical expectation of X.

28. (a) If the mean and variance of a binomial distribution are $\frac{4}{3}$ and $\frac{8}{9}$, respectively, then find $P(x = 1)$.

OR

(b) The mortality rate for a certain disease is 0.007. Using Poisson distribution, calculate the probability for 2 deaths in a group of 400 people. [Use $e^{-2.8} = 0.0608$]

29. (a) There are two types of fertilisers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid. F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that he needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If F_1 costs ₹6 per kg and F_2 costs ₹5 per kg, how much of each type of fertiliser should be used so that the cost is minimum. Formulate a linear programming problem.

OR

(b) Solve the following linear programming problem graphically :

Maximise $z = 50x + 30y$

subject to $2x + y \le 18$

$3x + 2y \le 34$

$x, y \ge 0$

30. A machinist is making engine parts with axle diameter of 0.7 cm. A random sample of 10 parts shows mean diameter 0.742 cm with a standard deviation of 0.04 cm. On the basis of this sample, find if you would say that the work is inferior. (Given t_9 (0.05) = 2.262)

31. Calculate EMI under Flat-Rate System for a loan of ₹5,00,000 with 7.5% annual interest rate for 5 years.

SECTION D

*Questions Number **32** to **35** are Long Answer (LA) type questions of 5 marks each.*

32. (a) If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear equations :

$2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$

OR

(b) Using properties of determinants, prove that

$$\Delta = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

33. If the supply function is $p = 4 - 5x + x^2$, then find the producer's surplus when price is 18.

34. (a) Compute the seasonal indices by 4-year moving averages from the given data of production of paper (in thousand tons):

Year:	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Index number:	2450	1470	2150	1800	1210	1950	2300	2500	2480	2680

OR

(b) Fit a straight-line trend by method of least squares for the following data :

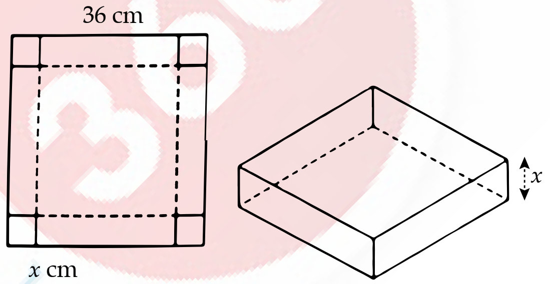
Year:	2011	2012	2013	2014	2015	2016
Production (in tons):	210	225	275	220	240	235

35. A machine costs ₹1,00,000 and its effective life is estimated to be 12 years. A sinking fund is created for replacing the machine by a new model at the end of its life time when its scrap realizes a sum of ₹5,000 only. Find what amount should be set a side at the end of each year, out of the profits for the sinking fund if it accumulates at 5% effective.
[Use $(1.05)^{12} = 1.7958$]

SECTION E

*Questions Number **36** to **38** are case-study based questions of 4 marks each.*

36. A man has an expensive square-shaped piece of golden board of side 36 cm. He wants to turn it into a box without top by cutting a square from each corner and folding the flaps. Let x cm be the side of square, which is cut from each corner.



Based on the above information, answer the following questions:

(i) Find the expression for the volume (V) of open box in terms of x.

(ii) Find $\frac{dV}{dx}$.

(iii) Find the value of x for which the volume (V) is maximum.

OR

(iii) Find the maximum volume of the open box.

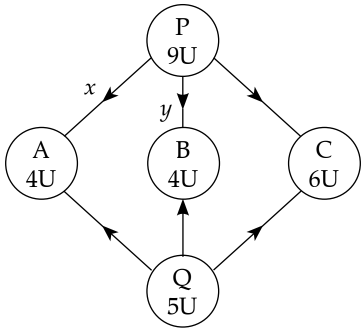
37. There are two factories located one at P and the other at Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 4, 4 and 6 units of the commodity while the production capacity of the factories at P and Q are 9 and 5 units respectively. The cost of transportation per unit is given as :

From / To	Cost (in ₹)		
	A	B	C
P	160	100	150
Q	100	120	100

Based on the above information, answer the following questions :

Let x units and y units of the commodity be transported from factory P to the depots at A and B respectively, then

Factory



Factory

(i) Find (in terms of x and y) how many units of commodity be transported from factory P to depot C.

(ii) Find how many units of commodity be transported from factory Q to A, B and C respectively.

(iii) Using (i) and (ii), find the total transportation cost Z.

OR

(iii) Using (i) and (ii), find the constraint inequalities for minimum cost Z.

38. Ramesh borrowed a home loan amount of ₹7,00,000 from a bank at an interest of 12% per annum for 30 years, to be paid in monthly installments.

Based on the above information, answer the following questions :

(i) Write the formula for calculating EMI by reducing balance method.

(ii) Write the values of P, i and n respectively.

(iii) Find the EMI. [Use $(1.01)^{-360} = 0.02781668$]

OR

(iii) If the loan is to be returned in 20 years, find EMI. [Use $(1.01)^{-240} = 0.09180584$]

ANSWERS

SECTION – A

1. Option (B) is correct.

Explanation: – 41 mod 9

On dividing – 41 by 9, we get remainder = 4

Hence, – 41 mod 9 = 4

2. Option (D) is correct.

Explanation: Here, $a - c = a + (-c)$, $b - c = b + (-c)$

Since, $-c$ is positive (because c is negative), adding the same positive number to both sides preserves the inequality.

$$a - c > b - c$$

Also, $b - c > b + c$ (because subtracting a negative number makes it larger), it follows that:

$$a - c > b + c$$

3. Option (D) is correct.

Explanation: Since, A and B are symmetric matrix.

Therefore, $A' = A$ and $B' = B$

$$\begin{aligned}(AB' - BA')' &= (AB')' - (BA')' \\ &= (B')'A' - (A')'B' \\ &= BA' - AB' \\ &\qquad [\text{We know that } (A')' = A] \\ &= -(AB' - BA')\end{aligned}$$

Hence, $AB' - BA'$ is a skew symmetric matrix.

4. Option (C) is correct.

Explanation: Given, $A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

Cofactors:

$C_{11} = 1$ $\quad C_{12} = -2$

$C_{21} = 1$ $\quad C_{22} = 4$

$$\text{Adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}' = \begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$$

$$\begin{aligned}|A| &= (4)(1) - (-2)(1) \\ &= 4 + 2 \\ &= 6\end{aligned}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj } A$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$$

Therefore, $A^{-1} = \begin{bmatrix} \frac{1}{6} & \frac{1}{6} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix}$

5. Option (D) is correct.

Explanation: $\begin{vmatrix} 2x & 5 \\ 4 & x \end{vmatrix} = \begin{vmatrix} 3 & 5 \\ 4 & 6 \end{vmatrix}$

$$\begin{aligned}2x^2 - 20 &= 18 - 20 \\ 2x^2 &= 18 \\ x^2 &= \frac{18}{2} \\ x^2 &= 9 \\ x &= \pm 3\end{aligned}$$

6. Option (A) is correct.

Explanation: Given curve, $y = \dfrac{x-3}{x-4}$

Differentiation with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{(x-4)\frac{d}{dx}(x-3) - (x-3)\frac{d}{dx}(x-4)}{(x-4)^2} \\ &= \frac{(x-4)-(x-3)}{(x-4)^2} \\ &= \frac{x-4-x+3}{(x-4)^2} \\ \frac{dy}{dx} &= \frac{-1}{(x-4)^2}\end{aligned}$$

Slope of the normal (at $x = 6$)

$$= \frac{-1}{\left.\frac{dy}{dx}\right\}_{x=6}} = \frac{-1}{\frac{-1}{4}} = 4$$

7. Option (B) is correct.

Explanation: Births increase the population at a rate proportional to P, i.e., birth rate contribution: αP

Deaths decrease the population at a rate proportional to P, i.e., death rate contribution: βP

Thus, the net rate of population change is given by:

$$\frac{dP}{dt} = \text{(births per unit time)} - \text{(deaths per unit time)}$$

$$\frac{dP}{dt} = \alpha P - \beta P$$

$$\frac{dP}{dt} = (\alpha - \beta)P$$

8. Option (C) is correct.

Explanation: $X = 0, 1, 2$

X	0	1	2
$P(X)$	$\frac{30}{36} \times \frac{30}{36} = \frac{25}{36}$	$2 \times \frac{30}{36} \times \frac{6}{36} = \frac{10}{36}$	$\frac{6}{36} \times \frac{6}{36} = \frac{1}{36}$

$$P(X) = 0\times\frac{25}{36}+1\times\frac{10}{36}+2\times\frac{1}{36}$$
$$= 0+\frac{10}{36}+\frac{2}{36}$$
$$= \frac{12}{36}$$
$$= \frac{1}{3}$$

9. Option (A) is correct.

Explanation: Because of its symmetry, the mean of t-distribution is always located at the centre of distribution which is 0.

10. Option (C) is correct.

Explanation: Seasonal variations refer to fluctuations that occur due to changes in climate, weather conditions, or festivals. These variations repeat at regular intervals, typically within a year.

11. Option (B) is correct.

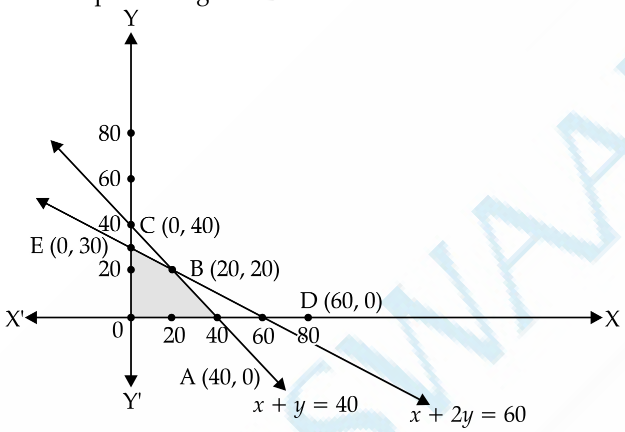
Explanation: $x + y = 40$

x	0	40	20
y	40	0	20

$x + 2y = 60$

x	0	60	20
y	30	0	20

Graph of the given LPP is shown as follows:



The corner points of feasible region are $O(0, 0)$, $A(40, 0)$, $B(20, 20)$ and $E(0, 30)$.

Corner points	$Z = 3x + 4y$
$O(0, 0)$	$3(0) + 4(0) = 0$
$A(40, 0)$	$3(40) + 4(0) = 120 + 0 = 120$
$B(20, 20)$	$3(20) + 4(20) = 60 + 80 = 140$
$E(0, 30)$	$3(0) + 4(30) = 0 + 120 = 120$

Maximum value of Z is 140 at point (20, 20).

12. Option (A) is correct.

Explanation: The given annuity is a perpetuity.

$$\text{Present value of perpetuity} = \frac{\text{Cash flow}}{\text{Interest rate}}$$
$$= \frac{100}{\frac{5}{100}}$$
$$= \frac{100\times 100}{5}$$
$$= ₹\,2000$$

13. Option (D) is correct.

Explanation: Revenue function,

$$R(x) = (30 + 5x - 3x^2)(x)$$
$$R(x) = 30x + 5x^2 - 3x^3$$
$$\text{Marginal revenue, } R'(x) = \frac{dR}{dx}$$
$$= 30 + 10x - 9x^2$$
$$\text{Marginal revenue at } x = 15 \text{ is } \left.\frac{dR}{dx}\right\}_{x=2}$$
$$= 30 + 10(2) - 9(2)^2$$
$$= 30 + 20 - 36$$
$$= ₹\,14$$

14. Option (B) is correct.

Explanation: Profit function,

$$P(x) = R(x) - C(x)$$
$$P(x) = 100 + 0.015x^2 - 3x$$
$$P'(x) = 0.03x - 3$$
i.e.,
$$P'(x) = 0$$
$$0.03x - 3 = 0$$
$$0.03x = 3$$
$$x = \frac{3}{0.03}$$
$$x = 100$$

Now,
$$\frac{d^2P}{dx^2} = \frac{d}{dx}(-0.03x + 3) = -0.03$$

Since, $\frac{d^2P}{dx^2} < 0$, the function has a maximum at $x = 100$.

15. Option (B) is correct.

Explanation:

X	0	1	2
$P(x)$	k	$2k$	$2k$

$$k + 2k + 2k = 1$$
$$5k = 1$$
$$k = \frac{1}{5}$$

16. Option (B) is correct.

Explanation: $t = \dfrac{\bar{x}-\bar{y}}{s\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}}$

17. Option (B) is correct.

Explanation: We have, $r = \frac{4}{100} = 0.04$, $n = 2$

(compounded semi-annually)

$$\text{Effective rate of return} = \left(1+\frac{r}{n}\right)^{n} - 1$$

$$= \left(1+\frac{0.04}{2}\right)^{2} - 1$$

$$= (1.02)^2 - 1$$

$$= 0.0404$$

Hence, the effective rate $= 0.0404 \times 100\%$

$$= 4.04\%$$

18. Option (A) is correct.

Explanation: Final value = ₹ 25,000, Initial value = ₹ 5000, n = 4 years

The CAGR of the revenues over the 4 years period

$$= \left(\frac{\text{Final value}}{\text{Initial value}}\right)^{\frac{1}{n}} - 1$$

$$= \left(\frac{25000}{5000}\right)^{\frac{1}{4}} - 1$$

$$= (5)^{\frac{1}{4}} - 1$$

$$= (5)^{0.25} - 1$$

$$= 1.4953 - 1$$

$$= 0.4953$$

$$= 49.53\%$$

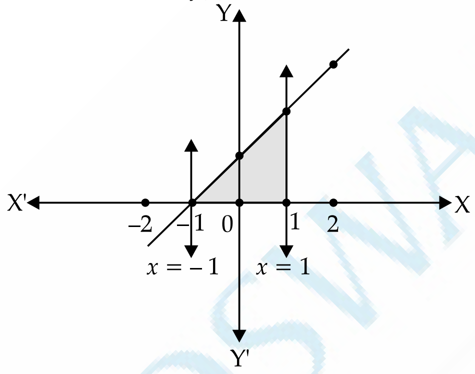
19. Option (A) is correct.

Explanation: Here, $a = -1$ and $b = 1$

$$y - 1 = x$$

$$y = x + 1$$

$$f(x) = x + 1$$



$$\text{Required area} = \int_a^b f(x)dx$$

$$= \int_{-1}^{1} (x+1)dx$$

$$= \left[\frac{x^2}{2} + x\right]_{-1}^{1}$$

$$= \left[\frac{1}{2}+1\right] - \left[\frac{1}{2}-1\right]$$

$$= \frac{3}{2} + \frac{1}{2}$$

$$= \frac{4}{2}$$

$= 2$ sq. units

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

20. Option (A) is correct.

Explanation: Given, $y = mx$ ...(i)

On differentiating both sides with respect to x

$$\frac{dy}{dx} = m \times 1$$

$$\frac{dy}{dx} = m \quad \text{...(ii)}$$

From Eq. (ii) value of m put in Eq. (i)

$$y = \frac{dy}{dx}x$$

$$x\frac{dy}{dx} - y = 0$$

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

SECTION – B

21. (a) Given, CP of two types of sugar Type-I and Type-II is ₹ 25 and ₹ 35, respectively and they mixed in the ratio of 3 : 2.

Let Type-I and Type-II sugar quantity be $3x$ and $2x$, respectively.

The price per kg of mixed variety of sugar

$$= \frac{(25)(3x)+(35)(2x)}{3x+2x}$$

$$= \frac{75x+70x}{5x}$$

$$= \frac{145x}{5x}$$

= ₹ 29

OR

(b) Given, Pipe A can fill a tank in 1 hour

Then part of tank filled in 1 hour by Pipe A $= \frac{1}{1}$ tank

Pipe B can fill a tank in $1\frac{1}{2}$ or $\frac{3}{2}$ hour

Then part of tank filled in 1 hour by Pipe B $= \frac{2}{3}$ tank

Tank filled by Pipe A and Pipe B together in 1 hour

$= \left(\frac{1}{1}+\frac{2}{3}\right)$ tank

$= \frac{5}{3}$ tank

So, tank gets completely filled in $\frac{3}{5}$ hours

$= \frac{3}{5}$ hours

$= \frac{3}{5} \times 60$ minutes

= 36 minutes

22. Let the speed of the boat in still water be v km/h.

Given, speed of current = 1 km/h

The speed of the boat in upstream is $(v - 1)$ km/h and in downstream is $(v + 1)$ km/h. The distance covered by boat in upstream and in downstream is 3.5 km respectively. The total time taken by boat is 1 hour and 12 minutes, or $\frac{72}{60}$ hours in going upstream and downstream.

Time taken by the boat in upstream $= \frac{3.5}{v-1}$ hour

Time taken by the boat in downstream $= \frac{3.5}{v+1}$ hour

As per the question,

$$\frac{3.5}{v-1} + \frac{3.5}{v+1} = \frac{72}{60}$$

$$\frac{3.5(v+1)+3.5(v-1)}{(v-1)(v+1)} = \frac{72}{60}$$

$$\frac{7v}{v^2-1} = \frac{6}{5}$$

$$7v = \frac{6}{5}(v^2-1)$$

$$7v = \frac{6v^2}{5} - \frac{6}{5}$$

$$35v = 6v^2 - 6$$

$$6v^2 - 35v - 6 = 0$$

$$6v^2 - 36v + 1v - 6 = 0$$

$$6v(v - 6) + 1(v - 6) = 0$$

$$(v - 6)(6v + 1) = 0$$

$$v = 6 \text{ km/h}$$

[$v = -\frac{1}{6}$ is not possible, as speed cannot be negative]

Hence, the speed of the boat in still water is 6 km/h.

23. Let the speed of B be v m/s, then the speed of A is $\frac{3}{2}v$ m/s. Since A gives B a start of 40 m,

Let d is the distance from the starting point to the winning post.

Here, Time taken by A = Time taken by B

$$\frac{d}{\frac{3}{2}v} = \frac{d-40}{v}$$

$$\frac{2d}{3v} = \frac{d-40}{v}$$

$$2d = 3(d - 40)$$

$$2d = 3d - 120$$

$$d = 120 \text{ m}$$

Hence, the required distance is 120 m.

24. Given, $A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 4 & 5 \\ 0 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & -5 \\ -5 & 1 & -5 \\ 1 & -2 & 4 \end{bmatrix}$

$$BA = \begin{bmatrix} 1 & 1 & -5 \\ -5 & 1 & -5 \\ 1 & -2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 3 & 4 & 5 \\ 0 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(2)+(1)(3)+(-5)(0) & (1)(0)+(1)(4)+(-5)(2) & (1)(1)+(1)(5)+(-5)(3) \\ (-5)(2)+(1)(3)+(-5)(0) & (-5)(0)+(1)(4)+(-5)(2) & (-5)(1)+(1)(5)+(-5)(3) \\ (1)(2)+(-2)(3)+(4)(0) & (1)(0)+(-2)(4)+(4)(2) & (1)(1)+(-2)(5)+(4)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 2+3+0 & 0+4-10 & 1+5-15 \\ -10+3-0 & 0+4-10 & -5+5-15 \\ 2-6+0 & 0-8+8 & 1-10+12 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -6 & -9 \\ -7 & -6 & -15 \\ -4 & 0 & 3 \end{bmatrix}$$

25. **(a)** The probability of getting at least 4 heads when a fair coin is tossed 6 times can be found by adding the probabilities of getting 4, 5 or 6 heads. Since, the coin is fair, the probability of heads (H) or tails (T) is 0.5 each. The probability of getting exactly k heads in n tosses follows a binomial distribution:

$$P(X = k) = {}^nC_k p^k (1 - p)^{n-k}$$

For this problem, $p = 0.5$ and $n = 6$, so

$P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6)$

$= {}^6C_4 0.5^4(1 - 0.5)^{6-4} + {}^6C_5 0.5^5(1 - 0.5)^{6-5} + {}^6C_6 0.5^6(1 - 0.5)^{6-6}$

$= {}^6C_4 0.5^4(0.5)^2 + {}^6C_5 0.5^5(0.5)^1 + {}^6C_6 0.5^6(0.5)^0$

$= {}^6C_4(0.5)^6 + {}^6C_5(0.5)^6 + {}^6C_6(0.5)^6$
$= (15)(0.5)^6 + (6)(0.5)^6 + (1)(0.5)^6$
$= (15 + 6 + 1)(0.5)^6$
$= 22\,(0.5)^6$

$$22'\left(\frac{1}{2}\right)^6 = \frac{22}{64}$$

$$= \frac{11}{32}$$

Hence, the probability of getting at least 4 heads is $\frac{11}{32}$.

OR

(b) (i) The z-score is given by the formula:

$$z = \frac{X-\mu}{\sigma}$$

where μ is the mean, σ is the standard deviation and X is the data point. Given $\mu = 9$, $\sigma = 3$ and $X = 15$:

$$z = \frac{15-9}{3}$$

$$= \frac{6}{3}$$

$$= 2$$

(ii) To find data point X, given, $z = 4$,

$\mu = 9, \sigma = 3,$

$$z = \frac{X-\mu}{\sigma}$$

$$4 = \frac{X-9}{3}$$

$$12 = X - 9$$
$$X = 12 + 9$$
$$X = 21$$

SECTION – C

26. To find the units digit of 7^{295}, we follow below steps:

$7^1 \equiv 7 \pmod{10}$
$7^2 \equiv 9 \pmod{10}$
$7^3 \equiv 3 \pmod{10}$
$7^4 \equiv 1 \pmod{10}$
$7^5 \equiv 7 \pmod{10}$

The pattern in the units digit repeats after every 4 powers,

Since, $295 \equiv 3 \pmod 4$

The units digit of 7^{295} will be the same as the units digit of 7^3, which is 3.

27. To calculate the mathematical expectation of X, the smaller of two numbers selected without replacement from the first six positive integers, we need to consider all possible selections of two numbers and their corresponding probabilities.

The total number of ways to select 2 numbers from 6 is ${}^6C_2 = 15$. The possible values of X are 1 to 5 (since 6 cannot be the smaller number when two different numbers are chosen). We calculate the probability of each value of X and multiply it by the value of X to find the expectation.

For $X = 1$, there are 5 choices for the second number, so the probability is $\frac{5}{15}$.

For $X = 2$, there are 4 choices for the second number, so the probability is $\frac{4}{15}$.

For $X = 3$, there are 3 choices for the second number, so the probability is $\frac{3}{15}$.

For $X = 4$, there are 2 choices for the second number, so the probability is $\frac{2}{15}$.

For $X = 5$, there is 1 choice for the second number, so the probability is $\frac{1}{15}$.

X	1	2	3	4	5
$P(X)$	$\frac{5}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$	$\frac{1}{15}$

Now we calculate the expectation:

$$E(X) = (1)\left(\frac{5}{15}\right) + (2)\left(\frac{4}{15}\right) + (3)\left(\frac{3}{15}\right) + (4)\left(\frac{2}{15}\right) + (5)\left(\frac{1}{15}\right)$$

$$= \frac{5+8+9+8+5}{15}$$

$$= \frac{35}{15}$$

$$= \frac{7}{3}$$

28. (a) For a binomial distribution, if the mean μ is $\frac{4}{3}$ and the variance σ^2 is $\frac{8}{9}$, we can find the probability $P(x = 1)$ using the binomial probability formula:

$$P(x = r) = {}^nC_r p^r (q)^{n-r}$$
$$q = 1 - p$$

The mean and variance of a binomial distribution are given $\mu = np$ and $\sigma^2 = np(1 - p)$, respectively. We have:

$$\mu = np = \frac{4}{3}$$

$$\sigma^2 = np(1-p) = \frac{8}{9}$$

From $\mu = np$, we can solve for p:

$$p = \frac{\mu}{n} = \frac{\frac{4}{3}}{n}$$

Substituting this into the variance formula:

$$\sigma^2 = n\left(\frac{\frac{4}{3}}{n}\right)\left(1-\frac{\frac{4}{3}}{n}\right) = \frac{8}{9}$$

$$\frac{4}{3}\left(1-\frac{\frac{4}{3}}{n}\right) = \frac{8}{9}$$

$$4\left(1-\frac{4}{3n}\right) = \frac{8}{3}$$

$$4-\frac{16}{3n} = \frac{8}{3}$$

$$4-\frac{8}{3} = \frac{16}{3n}$$

$$\frac{12-8}{3} = \frac{16}{3n}$$

$$\frac{4}{3} = \frac{16}{3n}$$

$$n = 4$$

$$p = \frac{\frac{4}{3}}{n} = \frac{\frac{4}{3}}{4} = \frac{1}{3}$$

$$q = 1-p = 1-\frac{1}{3} = \frac{2}{3}$$

Using the binomial probabilities formula:

$$P(x=r) = {}^nC_r p^r (q)^{n-r}$$

$$P(x=1) = {}^4C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{4-1}$$

$$P(x=1) = {}^4C_1\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^3$$

$$= 4\times\frac{1}{3}\times\frac{8}{27}$$

$$= \frac{32}{81}$$

OR

(b) For a Poisson distribution, the probability of observing exactly k events is given by:

$$P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!}$$

where λ is the average number of events, which can be calculated as the product of the number of trials and the probability of an event in each trial. In this case, the mortality rate is 0.007 and there are 400 people, so:

$$\lambda = 400 \times 0.007 = 2.8$$

Substituting $\lambda = 2.8$ and $k = 2$ into the Poisson formula:

$$P(X=2) = \frac{e^{-2.8}\times 2.8^2}{2!}$$

Given $e^{-2.8} = 0.0608$,

$$\therefore \quad P(X=2) = \frac{0.0608\times 2.8^2}{2}$$

$$P(X=2) = \frac{0.0608\times 7.84}{2}$$

$$P(X=2) = 0.0608 \times 3.92$$

$$P(X=2) = 0.238336$$

29. (a) Let x kg of fertiliser F_1, y kg of fertiliser F_2 be required and Z be the total cost of the fertilisers used.

	Nitrogen	Phosphoric acid	Cost
Fertiliser F_1	10%	6%	₹ 6 kg
Fertiliser F_2	5%	10%	₹ 5 kg
Minimum Requirement	14 kg	14 kg	

Thus, the mathematical formulation of the given LPP is:

Minimise $Z = 6x + 5y$

Subject to constraints

$$x\times\frac{10}{100}+y\times\frac{5}{100} \geq 14$$

$$\Rightarrow \quad \frac{x}{10}+\frac{y}{20} \geq 14$$

$$\Rightarrow \quad 2x + y \geq 280$$

$$x\times\frac{6}{100}+y\times\frac{10}{100} \geq 14$$

$$\Rightarrow \quad \frac{3x}{50}+\frac{y}{10} \geq 14$$

$$\Rightarrow \quad 3x + 5y \geq 700$$

$$x, y \geq 0$$

Thus, required LPP is

Minimize $Z = 6x + 5y$

Subject to:

$$2x + y \geq 280$$
$$3x + 5y \geq 700$$
$$x, y \geq 0$$

OR

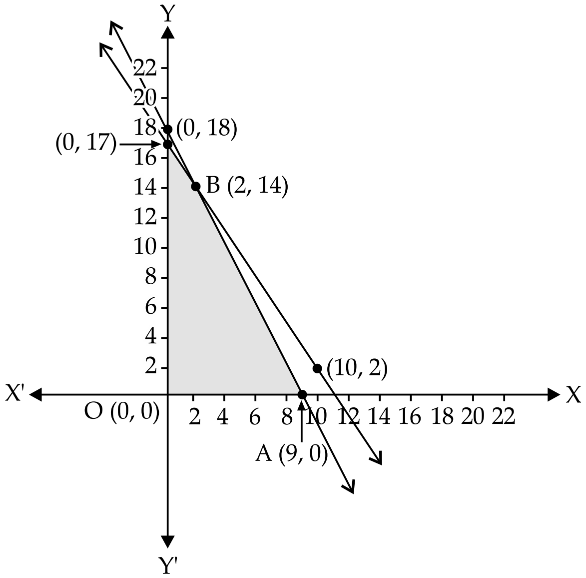
(b) $2x + y = 18$

x	9	0	2
y	0	18	14

$3x + 2y = 34$

x	0	10	2
y	17	2	14

Graph of the given LPP is shown as follows:



The corner points of feasible region are (0, 0), (9, 0), (2, 14) and E(0, 17).

Corner points	$Z = 50x + 30y$
(0, 0)	50(0) + 30(0) = 0
(9, 0)	50(9) + 30(0) = 450 + 0 = 450
(2, 14)	50(2) + 30(14) = 100 + 420 = 520
(0, 17)	50(0) + 30(17) = 0 + 510 = 510

Maximum value of Z is 520 at points (2, 14).

30. We define,

Null Hypothesis (H_0): The mean diameter is 0.7 cm (i.e., the work is not inferior).

$$H_0 : \mu = 0.7$$

Alternative Hypothesis (H_1): The mean diameter is greater than 0.7 cm (i.e, the work is inferior).

$$H_1 : \mu > 0.7$$

Given, $\bar{x} = 0.742$, $\mu = 0.7$, $s = 0.04$ and $n = 10$

where $\bar{x}$ is the sample mean, μ is the hypothesized population mean (0.7 cm), s is the sample standard deviation, and n is the sample size.

The t-statistic is given by:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Substitute the given values:

$$t = \frac{0.742 - 0.7}{0.04 / \sqrt{10}}$$

$$t = \frac{0.042}{0.04/\sqrt{10}}$$

$$t = \frac{0.042}{0.01265}$$

$$t = 3.3202$$

The critical value from the t-distribution with $n - 1 = 10 - 1 = 9$ degrees of freedom and given $t_9(0.05) = 2.62$

Since 3.3202 > 2.262, we reject the null hypothesis.

Hence work is inferior, as the mean diameter of the engine parts is significantly different from the expected diameter of 0.7 cm.

31. Given, the principal is ₹ 5,00,000, the annual interest rate is 7.5%, the time period is 5 years and the total number of monthly installments over 5 years is $5 \times 12 = 60$

To calculate the EMI under the flat-rate system, we use the formula:

$$\text{EMI} = \frac{\text{Principal} + (\text{Principal} \times \text{Rate of Interest} \times \text{Time})}{\text{Total number of installments}}$$

$$\text{EMI} = \frac{500000 + \left(500000 \times \frac{7.5}{100} \times 5\right)}{60}$$

$$\text{EMI} = \frac{500000 + (500000 \times 0.075 \times 5)}{60}$$

$$\text{EMI} = \frac{500000 + (37500 \times 5)}{60}$$

$$\text{EMI} = \frac{500000 + 187500}{60}$$

$$\text{EMI} = \frac{687500}{60}$$

EMI = 11458.33

Therefore, the EMI under the flat-rate system is ₹ 11458.33

32. **(a)** Given, $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$$|A| = (2)\begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix} - (-3)\begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix} + (5)\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}$$

$= (2)(-4 + 4) + (3)(-6 + 4) + (5)(3 - 2)$

$= (2)(0) + (3)(-2) + (5)(1)$

$= 0 - 6 + 5$

$= -1$, which is not equal to 0.

So, given matrix is non – singular and its inverse exists.

Cofactors:

$C_{11} = 0$ $\quad C_{12} = +2$ $\quad C_{13} = 1$

$C_{21} = -1$ $\quad C_{22} = -9$ $\quad C_{23} = -5$

$C_{31} = 2$ $\quad C_{32} = 23$ $\quad C_{33} = 13$

$$\text{Adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$= \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

To solve the following system of linear equations, we first write in matrix form:

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\; B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$AX = B$$
$$X = A^{-1}B$$

$$X = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} (0)(11)+(1)(-5)+(-2)(-3) \\ (-2)(11)+(9)(-5)+(-23)(-3) \\ (-1)(11)+(5)(-5)+(-13)(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x = 1, y = 2$ and $z = 3$

OR

(b)

$$\text{LHS} = \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} (b+c)^2 & a^2-(b+c)^2 & a^2-(b+c)^2 \\ b^2 & (c+a)^2-b^2 & b^2-b^2 \\ c^2 & c^2-c^2 & (a+b)^2-c^2 \end{vmatrix}$$

$$= \begin{bmatrix} (b+c)^2 & (a+b+c)(a-b-c) & (a+b+c)(a-b-c) \\ b^2 & (c+a+b)(c+a-b) & 0 \\ c^2 & 0 & (a+b+c)(a+b-c) \end{bmatrix}$$

Taking $(a + b + c)$ common from C_3 and C_2.

$$= (a+b+c)^2\begin{vmatrix} (b+c)^2 & a-b-c & a-b-c \\ b^2 & a+c-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

Applying $R_1 \to R_1 - (R_2 + R_3)$

$$= (a+b+c)^2 \begin{vmatrix} b^2+c^2+2bc-b^2-c^2 & a-b-c-a-c+b-0 & a-b-c-0-a-b+c \\ b^2 & a+c-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

$$= (a+b+c)^2 \begin{vmatrix} 2bc & -2c & -2b \\ b^2 & a+c-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

On taking 2 common from R_1

$$= 2(a+b+c)^2 \begin{vmatrix} bc & -c & -b \\ b^2 & a+c-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

Applying $C_3 \to cC_3$ and $C_2 \to bC_2$

$$= \frac{2(a+b+c)^2}{bc} \begin{vmatrix} bc & -bc & -bc \\ b^2 & ab+bc-b^2 & 0 \\ c^2 & 0 & ac+bc-c^2 \end{vmatrix}$$

Applying $C_3 \to C_3 + C_1$ and $C_2 \to C_2 + C_1$

$$= \frac{2(a+b+c)^2}{bc} \begin{vmatrix} bc & 0 & 0 \\ b^2 & ab+bc & b^2 \\ c^2 & c^2 & bc+ca \end{vmatrix}$$

On expanding along R_1

$$= \frac{2(a+b+c)^2}{bc} \{(bc)\ [(ab+bc)(bc+ca) - b^2c^2] - 0 + 0\}$$

$$= \frac{2(a+b+c)^2}{bc} \{b^2c^2[(a+c)(a+b) - bc]\}$$

$$= \frac{2(a+b+c)^2}{bc} \times b^2c^2[(a+c)(a+b) - bc]$$

$= 2bc(a+b+c)^2\,[a^2 + ab + ac + bc - bc]$

$= 2bc(a+b+c)^2\,[a^2 + ab + ac]$

$= 2bc(a+b+c)^2\,a(a+b+c)$

$= 2abc\,(a+b+c)^3$

= R.H.S. **Hence Proved.**

33. The producer's surplus is given by:

$$PS = \int_0^{x_0} p(x)dx - x_0 p_0$$

where,

$p(x) = 4 - 5x + x^2$ is the supply function

$p_0 = 18$ is the given equilibrium price

x_0 is the quantity supplied when $p = 18$, found by solving $4 - 5x + x^2 = 18$.

Thus,

$$4 - 5x + x^2 = 18$$
$$x^2 - 5x + 4 - 18 = 0$$
$$x^2 - 5x - 14 = 0$$
$$x^2 - 7x + 2x - 14 = 0$$
$$(x-7)(x+2) = 0$$
$$x = 7, -2$$

Since, quantity cannot be negative, we take $x_0 = 7$.

Now, $PS = \int_0^7 (4 - 5x + x^2)dx - (7 \times 18)$

$$= \int_0^7 (4 - 5x + x^2)dx - 126$$

$$= \left[4x - \frac{5x^2}{2} + \frac{x^3}{3}\right]_0^7 - 126$$

$$= \left[4(7) - \frac{5(7)^2}{2} + \frac{(7)^3}{3}\right] - \left[4(0) - \frac{5(0)^2}{2} + \frac{(0)^3}{3}\right] - 126$$

$$= \left[28 - \frac{5(49)}{2} + \frac{343}{3}\right] - 126$$

$$= \left[28 - \frac{245}{2} + \frac{343}{3}\right] - 126$$

$$= \left[\frac{168}{6} - \frac{735}{6} + \frac{686}{6}\right] - 126$$

$$= \frac{168 - 735 + 686 - 756}{6} = \frac{854 - 1491}{6}$$

$$= \frac{-637}{6} = -106.17$$

Producer's surplus = 106.17 units (absolute value).

34. **(a)** Computation of four-yearly moving averages:

Year	Index number	4-yearly centred moving total	4-yearly moving average
2001	2450	–	
2002	1470	–	
		7870	1967.5
2003	2150	–	
		6630	1657.5
2004	1800	–	
		7110	1777.5
2005	1210	–	
		7260	1815
2006	1950	–	
		7960	1990
2007	2300	–	
		9230	2307.5
2008	2500	–	
		9960	2490
2009	2480	–	
2010	2680	–	

OR

(b) Computation of trend values by the method of least squares.

In case of EVEN number of years, let us consider

$$X = \frac{(x - \text{Arithimetic mean of two middle years})}{0.5}$$

Year	Production (in tons)				Trend value
(x)	(Y)	$X = \dfrac{x - 2013.5}{0.5}$	XY	X^2	Y_t
2011	210	–5	–1050	25	225.9525
2012	225	–3	–675	9	229.2383
2013	275	–1	–275	1	232.5241
2014	220	1	220	1	235.8099
2015	240	3	720	9	239.0957
2016	235	5	1175	25	242.3815
$n = 8$	$\sum Y = 1405$	$\sum X = 0$	$\sum XY = 115$	$\sum X^2 = 70$	

$$a = \frac{\Sigma Y}{n} = \frac{1405}{6}$$

$$= 234.167$$

$$b = \frac{\Sigma XY}{\Sigma X^2} = \frac{115}{70}$$

$$= 1.6429$$

Therefore, the required equation of the straight line trend is given by

$$Y = a + bX;$$

$$Y = 234.167 + 1.6429X$$

Trend values($Y_t = 234.167 + 1.6429X$)

Year 2011, $X = -5$, $Y_t = 225.9525$

Year 2012, $X = -3$, $Y_t = 229.2383$

Year 2013, $X = -1$, $Y_t = 232.5241$

Year 2014, $X = 1$, $Y_t = 235.8099$

Year 2015, $X = 3$, $Y_t = 239.0957$

Year 2016, $X = 5$, $Y_t = 242.3815$

35. To find the amount that should be set aside at the end of each year for the sinking fund, we use the formula for the future value of an annuity:

$$FV = P\frac{(1+r)^n - 1}{r}$$

where FV is the future value, P is the payment per period, r is the interest rate per period and n is the number of periods.

Machine cost = ₹ 1,00,000, Scrap value = ₹ 5,000, Amount to be accumulated = 1,00,000 – 5000 = ₹ 95,000, $r = 5\%$ and $n = 12$

$$95{,}000 = \left[P\frac{(1.05)^{12} - 1}{0.05}\right]$$

$$95{,}000 = \left[P\frac{1.7958 - 1}{0.05}\right]$$

$$95{,}000 = \left(P\frac{0.7958}{0.05}\right)$$

$$P = \frac{95000}{0.7958 / 0.05}$$

$$P = \frac{95000 \times 0.05}{0.7958}$$

$$P = ₹\ 5968.84$$

SECTION – E

36. (i) After cutting out squares of side x from each corner, the new length and width of the base of the box are $36 - 2x$ each, and the height of the box is x. The volume V is given by

$$V = \text{length} \times \text{width} \times \text{height}$$
$$= (36 - 2x)(36 - 2x)x.$$

Expanding this, we get

$$V = (36 - 2x)^2 x$$
$$= (1296 + 4x^2 - 144x)x$$
$$V = 4x^3 - 144x^2 + 1296x$$

(ii) To find $\frac{dV}{dx}$, we differentiate V with respect to x:

$$\frac{dV}{dx} = \frac{d}{dx}(4x^3 - 144x^2 + 1296x)$$

$$\frac{dV}{dx} = 12x^2 - 288x + 1296$$

(iii) To find the value of x for which V is maximum,

Put $\frac{dV}{dx} = 0$

$$12x^2 - 288x + 1296 = 0$$
$$12(x^2 - 24x + 108) = 0$$
$$x^2 - 18x - 6x + 108 = 0$$
$$x(x - 18) - 6(x - 18) = 0$$
$$(x - 18)(x - 6) = 0$$
$$x = 18 \text{ or } x = 6$$

Since, x cannot be 18, so we have $x = 6$

$$\frac{dV}{dx} = 12x^2 - 288x + 1296$$

Again differentiating w.r.t. to x, we get

$$\frac{d^2V}{dx^2} = 24x - 288$$

$$\left.\frac{d^2V}{dx^2}\right]_{x=6} = 24(6) - 288$$
$$= 144 - 288$$
$$= -144 < 0$$

Therefore, at $x = 6$, V is maximum.

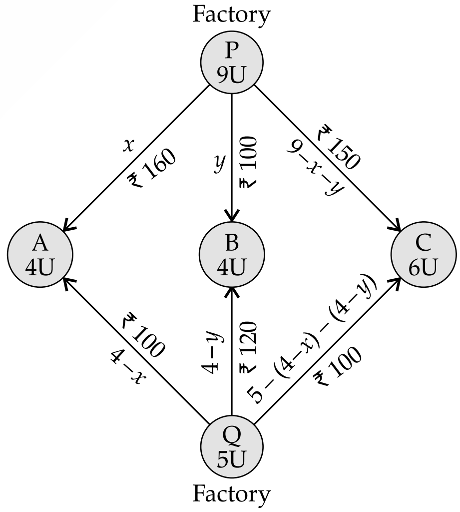
OR

(iii)
$$V_{max} = 4(6)^3 - 144(6)^2 + 1296(6)$$
$$= 4(216) - 144(36) + 7776$$
$$= 864 - 5184 + 7776$$
$$= 3456 \text{ cm}^3$$

37. Given cost of transportation per unit is

From/To	Cost (in ₹)		
	A	B	C
P	160	100	150
Q	100	120	100

Let x units and y units of the commodity be transported from the factory at P to the depots at A and B, respectively. Then, $(9 - (x + y))$ units will be transported to the depot at C.



Hence, we have $x, y \geq 0$ and $9 - (x + y) \geq 0$

i.e., $x + y \leq 9$ and $y, x \geq 0$

Now, the weekly requirement of the depot at A is 4 units of the commodity, x units is supplied from factory P to depot A. So, $(4 - x)$ units need to be transported to the depot A from the factory at Q. Obviously, $4 - x \geq 0$ or $x \leq 4$

Similarly, y units is supplied from factory Q to depot B, so $(4 - y)$ units and $\{5 - (4 - x + 4 - y)\} = (x + y - 3)$ units are to be transported from the factory at Q to the depots at B and C respectively. Thus, $4 - y \geq 0$, $x + y - 3 \geq 0$ or $y \leq 4$, $x + y \geq 3$

Let Z be the total transportation cost. Then, $Z = 160x + 100y + 150[9 - (x + y)] + 100(4 - x) + 120(4 - y) + 100(x + y - 3) = 10x - 70y + 1930$

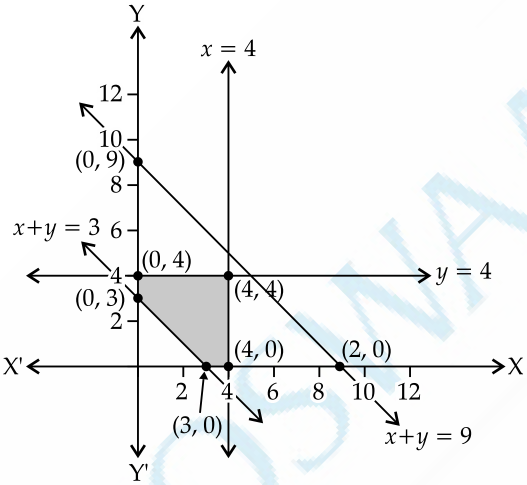
Thus, the required LPP is

minimise $Z = 10(x - 7y + 193)$

subject to constraints

$$x + y \leq 9$$
$$x + y \geq 3$$
$$x \leq 4$$
$$y \leq 4$$
$$x, y \geq 0$$

Let us draw the graph for the system of inequalities representing constraints.



The feasible region is shaded in the graph, which is bounded.

Let us evaluate the objective function $Z = 10(x - 7y + 193)$ at the corner points.

Corner points	$Z = 10x - 70y + 1930$
(3, 0)	$10(3) - 70(0) + 1930$ $= 30 - 0 + 1930 = 1960$
(0, 3)	$10(0) - 70(3) + 1930$ $= 0 - 210 + 1930 = 1720$
(4, 4)	$10(4) - 70(4) + 1930$ $= 40 - 280 + 1930 = 1690$
(0, 4)	$10(0) - 70(4) + 1930$ $= 0 - 280 + 1930 = 1650$
(4, 0)	$10(4) - 70(0) + 1930$ $= 40 - 0 + 1930 = 1970$

Z is minimum at $B(0, 4)$ and minimum cost is ₹ 1650. So, the minimum transportation cost is ₹ 1650 when 0, 4 and 5 units are transported from factory at P and 4, 0 and 1 units are transported from factory at Q to the depots A, B and C, respectively.

(i) 5 units

(ii) 4, 0 and 1 units are transported from factory at Q to the depots A, B and C, respectively.

(iii) Minimum transportation cost is ₹ 1650.

Or

(iii) Subject to constraints

$$x + y \leq 9$$
$$x + y \geq 3$$
$$x \leq 4$$
$$y \leq 4$$
$$x, y \geq 0$$

LPP is to minimise $Z = 10(x - 7y + 193)$

38. **(i)** The formula for the Equated Monthly Installment (EMI) is given by:

$$\text{EMI} = \frac{P \times r \times (1+r)^n}{(1+r)^n - 1}$$

where:

P is the principal loan amount.

r is the monthly interest rate.

n is the number of monthly installments.

(ii) We have, P = ₹ 7,00,000 (Principal),

Annual interest rate = 12% per annum,

So monthly interest rate = 1% = 0.01

For 30 years, $n = 30 \times 12 = 360$ months

(iii)

$$\text{EMI} = \frac{P \times r \times (1+r)^n}{(1+r)^n - 1}$$

$$\text{EMI} = \frac{P \times r}{1-(1+r)^{-n}}$$

$$= \frac{700000 \times 0.01}{1-(1+0.01)^{-360}}$$

$$= \frac{7000}{1-(1.01)^{-360}}$$

$$= \frac{7000}{1-0.02781668}$$

$$= \frac{7000}{0.97218332}$$

$$= 7200.28$$

$$\approx ₹\,7200$$

OR

(iii) Finding the EMI for 20 years:

$$n = 20 \times 12 = 240 \text{ months}$$

$$EMI = \frac{P \times r}{1-(1+r)^{-n}}$$

$$= \frac{700000 \times 0.01}{1-(1+0.01)^{-240}}$$

$$= \frac{7000}{1-(1.01)^{-240}}$$

$$= \frac{7000}{1-0.09180584}$$

$$= \frac{7000}{0.97218332}$$

$$= 7{,}707.60$$

$$\approx ₹\,7708$$