CBSE EXAMINATION PAPER 2025
Mathematics
Class-12th
(Solved)
(Delhi & Outside Delhi Sets)

Time : 3 Hours **Max. Marks : 80**

General Instructions :

Read the following instructions very carefully and strictly follow them:

*(i) This Question Paper contains **38** questions. **All** questions are **compulsory**.*

*(ii) Question paper is divided into **FIVE** Sections.–Section **A, B, C, D** and **E**.*

*(iii) In **Section A:** Question no. **1** to **18** are Multiple Choice Questions (MCQs) and Questions no. **19 & 20** are Assertion-Reason based questions of 1 mark each.*

*(iv) In **Section B:** Question no. **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

*(v) In **Section C:** Question no. **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

*(vi) In **Section D:** Question no. **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

*(vii) In **Section E:** Question no. **36** to **38** are case study based questions, carrying **4** marks each.*

*(viii) There is no overall choice. However, an internal choice has been provided in **2** questions in Section-**B**, **3** questions in Section-**C**, **2** questions in Section-**D** and **2** questions in Section-**E**.*

(ix) Use of calculator is NOT allowed.

Delhi Set-1 **65/1/1**

SECTION A

This section comprises of 20 Multiple Choice Questions (MCQs) of 1 mark each. $20 \times 1 = 20$

1. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is :

(A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **(D)** $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. If vector $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and vector $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, then which of the following is correct?

(A) $\vec{a} \parallel \vec{b}$ **(B)** $\vec{a} \perp \vec{b}$

(C) $|\vec{b}| > |\vec{a}|$ **(D)** $|\vec{a}| = |\vec{b}|$

3. $\int_{-1}^{1} \frac{|x|}{x} dx, x \neq 0$ is equal to:

(A) –1 **(B)** 0

(C) 1 **(D)** 2

4. Which of the following is <u>not</u> a homogeneous function of x and y?

(A) $y^2 - xy$ **(B)** $x - 3y$

(C) $\sin^2 \frac{y}{x} + \frac{y}{x}$ **(D)** $\tan x - \sec y$

5. If $f(x) = |x| + |x - 1|$, then which of the following is correct?

(A) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$.

(B) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$.

(C) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.

(D) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.

6. If A is a square matrix of order 2 such that det $(A) = 4$, then det (4 adj A) is equal to:

(A) 16 **(B)** 64

(C) 256 **(D)** 512

7. If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P(E/\bar{F})$ is equal to:

(A) $\frac{1}{6}$ **(B)** $\frac{1}{2}$

(C) $\frac{2}{3}$ **(D)** $\frac{7}{9}$

8. The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is:

(A) 0 **(B)** 2

(C) 4 **(D)** 5

9. Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$, $C = [9 \;\; 8 \;\; 7]$, which of the following is defined?

(A) Only AB **(B)** Only AC

(C) Only BA **(D)** All AB, AC and BA

10. If $\int \frac{2^{\frac{1}{x}}}{x^2} dx = k \cdot 2^{\frac{1}{x}} + C$, then k is equal to:

(A) $\frac{-1}{\log 2}$ **(B)** $-\log 2$

(C) -1 **(D)** $\frac{1}{2}$

11. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = \sqrt{37}$, $|\vec{b}| = 3$ and $|\vec{c}| = 4$, then angle between $\vec{b}$ and $\vec{c}$ is:

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ **(D)** $\frac{\pi}{2}$

12. The integrating factor of differential equation $(x + 2y^3)\frac{dy}{dx} = 2y$ is:

(A) $e^{\frac{y^2}{2}}$ **(B)** $\frac{1}{\sqrt{y}}$

(C) $\frac{1}{y^2}$ **(D)** $e^{-\frac{1}{y^2}}$

13. If $A = \begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$ is a scalar matrix, then y^x is equal to:

(A) 0 **(B)** 1

(C) 7 **(D)** ± 7

14. The corner points of the feasible region in graphical representation of a L.P.P. are (2, 72), (15, 20) and (40, 15). If $Z = 18x + 9y$ be the objective function, then

(A) Z is maximum at (2, 72), minimum at (15, 20)

(B) Z is maximum at (15, 20) minimum at (40, 15)

(C) Z is maximum at (40, 15), minimum at (15, 20)

(D) Z is maximum at (40, 15), minimum at (2, 72)

15. If A and B are invertible matrices, then which of the following is not correct?

(A) $(A + B)^{-1} = B^{-1} + A^{-1}$

(B) $(AB)^{-1} = B^{-1}A^{-1}$

(C) adj $(A) = |A| A^{-1}$

(D) $|A|^{-1} = |A^{-1}|$

16. If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct?

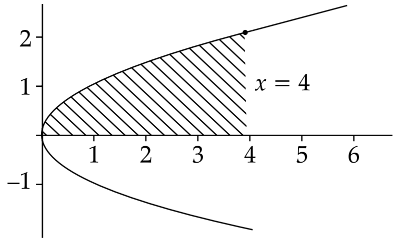
(A) It will only have a maximum value.

(B) It will only have a minimum value.

(C) It will have both maximum and minimum values.

(D) It will have neither maximum nor minimum value.

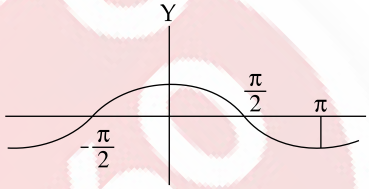
17. The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x-axis is given by



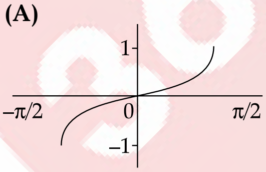
(A) $\int_0^4 x \, dx$ **(B)** $\int_0^2 y^2 \, dy$

(C) $2\int_0^4 \sqrt{x} \, dx$ **(D)** $\int_0^4 \sqrt{x} \, dx$

18. The graph of a trigonometric function is as shown. Which of the following will represent graph of the inverse?



(A)



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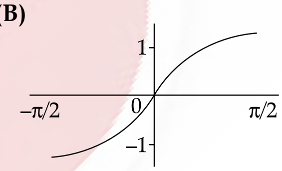

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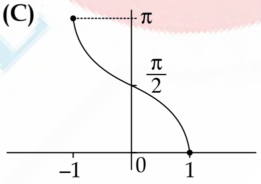
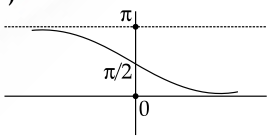


(D)



Assertion – Reason Based Questions

Direction : Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A) : Let $\mathbb{Z}$ be the set of integers. A function $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x) = 3x - 5$, $\forall x \in Z$ is a bijective.

Reason (R) : A funcion is a bijective if it is both surjective and injective.

20. Assertion (A) : $f(x)=\begin{cases}3x-8, & x\le 5\\ 2k, & x<5\end{cases}$

is continuous at $x = 5$ for $k = \frac{5}{2}$

Reason (R) : For a function f to be continuous at $x = n$.

$\lim\limits_{x\to n^-} f(x) = \lim\limits_{x\to n^+} f(x) = f(a)$.

SECTION B

This section comprises of 5 Very Short Answers (VSA) type questions of 2 marks each. $5 \times 2 = 10$

21. (a) Differentiate $2^{\cos^2 x}$ w.r.t. $\cos^2 x$.

OR

(b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.

22. Evaluate : $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

23. The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$. Find the area of the parallelogram.

24. Find the intervals in which function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$ is (i) increasing (ii) decreasing.

25. (a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings.

OR

(b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points $A(2, 1, 3)$ and $B(8, -1, 0)$ respectively.

SECTION C

This section comprises of 6 Short Answers (SA) of 3 marks each. $6 \times 3 = 18$

26. The side of an equilateral triangle is increasing at the rate of 3 cm/s. At what rate its area increasing when the side of the triangle is 15 cm?

27. Solve the following linear programming problem graphically :

Maximise $Z = x + 2y$

Subject to the constraints :

$$x - y \ge 0$$
$$x - 2y \ge -2$$
$$x \ge 0, y \ge 0$$

28. (a) Find : $\int \frac{x+\sin x}{1+\cos x}\,dx$

OR

(b) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

29. (a) Verify that lines given by $\vec{r} = (1-\lambda)\hat{i} + (\lambda-2)\hat{j} + (3-2\lambda)\hat{k}$ and $\vec{r} = (\mu+1)\hat{i} + (2\mu-1)\hat{j} - (2\mu+1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines.

OR

(b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}, \vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.

30. (a) The probability distribution for the number of students being absent in a class on a Saturday is as follows:

X	0	2	4	5
$P(X)$	p	$2p$	$3p$	p

where X is the number of students absent.

(i) Calculate p. **1**

(ii) Calculate the mean of the number of absent students on Saturday. **2**

OR

(b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.

31. Sketch the graph of $y = |x + 3|$ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.

SECTION D

This section comprises of 4 Long Answers (LA) type questions of 5 mark each. $4 \times 5 = 20$

32. (a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

(b) If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = \sin\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

33. Find the absolute maximum and absolute minimum of function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$.

34. (a) Find the image A' of the point $A(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A'.

OR

(b) Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point $Q(2, 4, -1)$ is 7 units. Also, find the equation of line joining P and Q.

35. A school wants to allocate students into three clubs: Sports, Music and Drama, under following conditions:

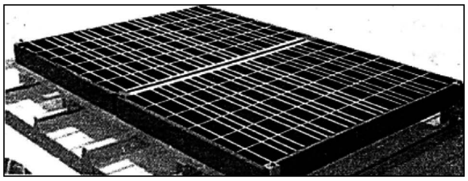
- The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club.
- The number of students in Music club should be 20 more than half the number of students in Sports club.
- The total number of students to be allocated in all three clubs are 180.

Find the number of students allocated to different clubs, using matrix method.

SECTION E

This section comprises of 3 case study based questions of 4 marks each. $3 \times 4 = 12$

36.



A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.

Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres.

Based on this information, answer the following questions:

(i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y. **1**

(ii) Write the area of the solar panel as a function of x. **1**

(iii) **(a)** Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area. **2**

OR

(iii) **(b)** Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition. **2**

37. A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set A = {1, 2, 3} :

$R_1 = \{(2, 3), (3, 2)\}$

$R_2 = \{(1, 2), (1, 3), (3, 2)\}$

$R_3 = \{(1, 2), (2, 1), (1, 1)\}$

$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$

$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$

The students are asked to answer the following questions about the above relations :

(i) Identify the relation which is reflexive, transitive but not symmetric.

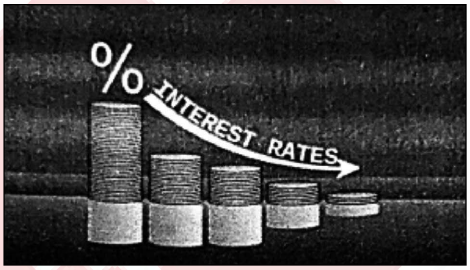
(ii) Identify the relation which is reflexive and symmetric but not transitive,

(iii) **(a)** Identify the relations which are symmetric but neither reflexive nor transitive.

OR

(b) What pairs should be added to the relation R_2 to make it an equivalence relation?

38.



A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively.

Based on the above information, answer the following :

(i) What is the probability that a customer after availing the loan will default on the loan repayment? **2**

(ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest? **2**

Delhi Set-2 **65/1/2**

Note: Except these, all other questions have been given in Delhi Set-1

SECTION A

2. If $\vec{\alpha} = \hat{i} - 4\hat{j} + 9\hat{k}$ and $\vec{\beta} = 2\hat{i} - 8\hat{j} + \lambda\hat{k}$ are two mutually parallel vectors, then λ is equal to:

(A) –18 **(B)** 18

(C) $\frac{-34}{9}$ **(D)** $\frac{34}{9}$

3. $\int \frac{1 - 2\sin x}{\cos^2 x} dx$ is equal to:

(A) $\tan x - 2\sec x + C$

(B) $-\tan x + 2\sec x + C$

(C) $-\tan x - 2\sec x + C$

(D) $\tan x + 2\sec x + C$

6. If A is a square matrix of order 3 such that $\det(A) = 9$, then $\det(9A^{-1})$ is equal to:

(A) 9 (B) 9^2
(C) 9^3 (D) 9^4

9. Let A be a matrix of order $m \times n$ and B is a matrix such that A^TB and BA^T are defined. Then, the order of B is:

(A) $m \times m$ (B) $n \times n$
(C) $m \times n$ (D) $n \times m$

12. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

(A) xe^x (B) $\frac{e^x}{x}$
(C) $\frac{x}{e^x}$ (D) $xe^{\frac{1}{x}}$

13. Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = \hat{j} - 2\hat{i}$. Then which of the following is true?

(A) $a_{12} > 0$ (B) all $a_{ij} < 0$
(C) $a_{13} + a_{31} = -6$ (D) $a_{23} > a_{32}$

SECTION B

22. Find the values of 'a' for which $f(x) = x^2 - 2ax + b$ is an increasing function for $x > 0$.

24. Evaluate : $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

SECTION C

27. Solve the following linear programming problem graphically:

Maximise $Z = 20x + 30y$

Subject to the constraints :

$$x + y \le 80$$
$$2x + 3y \ge 100$$
$$x \ge 14$$
$$y \ge 14$$

28. The area of an expanding rectangle is increasing at the rate of 48 cm^2/s. The length of the rectangle is always square of its breadth. At what rate the length of rectangle increasing at an instant, when breadth = 4.5 cm?

30. (a) Find : $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$

OR

(b) Evaluate : $\int_0^{\frac{\pi}{2}} \frac{5\sin x + 3\cos x}{\sin x + \cos x} dx$

SECTION D

32. (a) Differentiate $\tan^{-1}\frac{\sqrt{1-x^2}}{x}$ w.r.t. $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$, $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$

OR

(b) Find $\frac{dy}{dx}$, if $y = x^{\tan x} + \frac{\sqrt{x^2+1}}{2}$.

Delhi Set-3 **65/1/3**

Note: Except these, all other questions have been given in Delhi Set-1 and Set-2

SECTION A

This section comprises 20 Multiple Choice Questions (MCQs) of 1 mark each.

2. The unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ is

(A) $\hat{k}$ (B) $-\hat{k} + \hat{j}$
(C) $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

3. If $\int_0^1 \frac{e^x}{1+x} dx = \alpha$, then $\int_0^1 \frac{e^x}{(1+x)^2} dx$ is equal to

(A) $\alpha - 1 + \frac{e}{2}$ (B) $\alpha + 1 - \frac{e}{2}$
(C) $\alpha - 1 - \frac{e}{2}$ (D) $\alpha + 1 + \frac{e}{2}$

6. If $\begin{bmatrix} x+y & 3y \\ 3x & x+3 \end{bmatrix} = \begin{bmatrix} 9 & 4x+y \\ x+6 & y \end{bmatrix}$, then $(x - y) = ?$

(A) -7 (B) -3
(C) 3 (D) 7

7. Let M and N be two events such that $P(M) = 0.6$, $P(N) = 0.2$ and $P(M \cap N) = 0.5$, then $P(M'/N')$ is

(A) $\frac{7}{8}$ (B) $\frac{2}{5}$
(C) $\frac{1}{2}$ (D) $\frac{2}{3}$

11. A system of linear equations is represented as $AX = B$, where A is coefficient matrix, X is variable matrix and B is the constant matrix. Then the system of equations is

(A) Consistent, if $|A| \ne 0$, solution is given by $X = BA^{-1}$.
(B) Inconsistent if $|A| = 0$ and (adj A) $B = 0$
(C) Inconsistent if $|A| \ne 0$
(D) May or may not be consistent if $|A| = 0$ and (adj A) $B = 0$

13. The order and degree of differential function $\left[1+\left(\frac{dy}{dx}\right)^2\right]^5 = \frac{d^2y}{dx^2}$ are

(A) order 1, degree 1 **(B)** order 1, degree 2

(C) order 2, degree 1 **(D)** order 2, degree 2

SECTION B

22. Find the values of 'a' for which $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ is decreasing on $\mathbb{R}$.

24. Solve for x,

$$2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 4\sqrt{3}$$

SECTION C

26. Solve the following linear programming problem graphically:

Maximise $Z = 8x + 9y$

Subject to the constraints:

$$2x + 3y \le 6$$
$$3x - 2y \le 6$$
$$y \le 1$$
$$x \ge 0, y \ge 0$$

27. **(a)** Find : $\int \frac{2x-1}{(x-1)(x+2)(x-3)}dx$

OR

(b) Evaluate : $\int_0^5 (|x-1|+|x-2|+|x-5|)\,dx$

28. A spherical medicine ball when dropped in water dissolves in such a way that the rate of decrease of volume at any instant is proportional to its surface area. Calculate that rate of decrease of its radius.

SECTION D

33. Find: $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx.$

Outside Delhi Set-1 **65/2/1**

SECTION A

(This section comprises of 20 multiple choice questions (MCQs) of 1 mark each.) (20 × 1 = 20)

1. The projection vector of vector $\vec{a}$ on vector $\vec{b}$ is:

(A) $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$ **(B)** $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

(C) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$ **(D)** $\left(\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right)\vec{b}$

2. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval:

(A) (0, 2) **(B)** $(-\infty, 2]$

(C) [1, 2] **(D)** $[2, \infty)$

3. If $f(2a - x) = f(x)$, then $\int_0^{2a} f(x)\,dx$ is:

(A) $\int_0^{2a} f\left(\frac{x}{2}\right)dx$ **(B)** $\int_0^a f(x)\,dx$

(C) $2\int_a^0 f(x)\,dx$ **(D)** $2\int_0^a f(x)\,dx$

4. If $A = \begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{bmatrix}$ is a symmetric matrix, then $(2x + y)$ is:

(A) – 8 **(B)** 0

(C) 6 **(D)** 8

5. If $y = \sin^{-1}x, -1 \le x \le 0$, then the range of y is:

(A) $\left(\frac{-\pi}{2}, 0\right)$ **(B)** $\left[\frac{-\pi}{2}, 0\right]$

(C) $\left[\frac{-\pi}{2}, 0\right)$ **(D)** $\left(\frac{-\pi}{2}, 0\right]$

6. If a line makes angles of $\frac{3\pi}{4}, \frac{\pi}{3}$ and θ with the positive directions of x, y and z-axis respectively, then θ is:

(A) $\frac{-\pi}{3}$ only **(B)** $\frac{\pi}{3}$ only

(C) $\frac{\pi}{6}$ **(D)** $\pm\frac{\pi}{3}$

7. If E and F are two events such that P(E) > 0 and P(F) ≠ 1, then $P(\bar{E}|\bar{F})$ is:

(A) $\frac{P(\bar{E})}{P(\bar{F})}$ **(B)** $1 - P(\bar{E}|F)$

(C) $1 - P(E|F)$ **(D)** $\frac{1-P(E \cup F)}{P(\bar{F})}$

8. Which of the following can be both a symmetric and skew-symmetric matrix?

(A) Unit Matrix **(B)** Diagonal Matrix

(C) Null Matrix **(D)** Row Matrix

9. The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point (4, –3, 7) is:

(A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$

(B) $x = 3t + 4, y = t + 3, z = 2t + 7$

(C) $x = 3t + 4, y = t - 3, z = 2t + 7$

(D) $x = 3t + 4, y = -t + 3, z = 2t + 7$

10. Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify 4 AB +3(AB + BA) – 4 BA, where A and B are both matrices of order 2 × 2. It is known that $A \ne B \ne I$ and $A^{-1} \ne B$.

Their answers are given as:

Abhay : 6 AB

Bina : 7 AB – BA

Chhaya : 8 AB

Devesh : 7 BA – AB

Who answered it correctly?

(A) Abhay **(B)** Bina

(C) Chhaya **(D)** Devesh

11. A cylindrical tank of radius 10 cm is being filled with sugar at the rate of 100 π cm^3/s. The rate, at which the height of the sugar inside the tank is increasing, is:

(A) 0.1 cm/s **(B)** 0.5 cm/s

(C) 1 cm/s **(D)** 1.1 cm/s

12. Let $\vec{p}$ and $\vec{q}$ be two unit vectors and α be the angle between them. Then $(\vec{p}+\vec{q})$ will be a unit vector for what value of α?

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{3}$

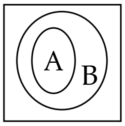
(C) $\frac{\pi}{2}$ **(D)** $\frac{2\pi}{3}$

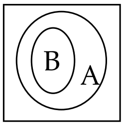
13. The line $x = 1 + 5\mu$, $y = -5 + \mu$, $z = -6 - 3\mu$ passes through which of the following point?

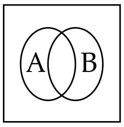
(A) (1, –5, 6) **(B)** (1, 5, 6)

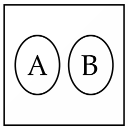
(C) (1, –5, –6) **(D)** (–1, –5, 6)

14. If A denotes the set of continuous functions and B denotes the set of differentiable functions, then which of the following depicts the correct relation between set A and B?

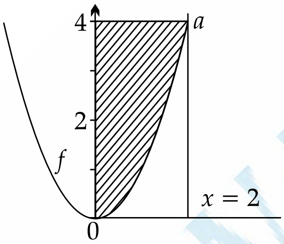
(A) 

(B) 

(C) 

(D) 

15. The area of the shaded region (figure) represented by the curves $y = x^2$, $0 \le x \le 2$ and y-axis is given by:



(A) $\int_0^2 x^2\,dx$ **(B)** $\int_0^2 \sqrt{y}\,dy$

(C) $\int_0^4 x^2\,dx$ **(D)** $\int_0^4 \sqrt{y}\,dy$

16. A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct?

(A) The objective function maximises the difference of the profit earned from products X and Y.

(B) The objective function measures the total production of products X and Y.

(C) The objective function maxmises the combined profit earned from selling X and Y.

(D) The objective function ensures the company produces more of product X than product Y.

17. If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct?

(A) A = B **(B)** AB = BA

(C) A = 0 or B = 0 **(C)** A = I or B = I

18. If p and q are respectively the order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$, then $(p - q)$ is

(A) 0 **(B)** 1

(C) 2 **(D)** 3

ASSERTION – REASON BASED QUESTIONS

Direction: Question number 19 and 20 are Assertion (A) and Reason (R) based questions. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below:

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true but Reason (R) is false.

(D) Assertion (A) is false but Reason (R) is true.

19. Assertion (A) : A = diag [3 5 2] is a scalar matrix of order 3 × 3.

Reason (R) : If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.

20. Assertion (A) : Every point of the feasible region of a Linear Programming Problem is an optimal solution.

Reason (R) : The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region.

SECTION – B

(This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.) (5 × 2 = 10)

21. (a) A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, find $\vec{a}$.

OR

(b) If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $QR = \frac{3}{2}QP$.

22. Evaluate: $\int_0^{\frac{\pi}{4}} \sqrt{1+\sin 2x}\,dx$

23. Find the values of 'a' for which $f(x) = \sin x - ax + b$ is increasing on R.

24. If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors, then find x, such that $\alpha = (x-2)\vec{a} + \vec{b}$ and $\vec{\beta} = (3+2x)\vec{a} - 2\vec{b}$ are collinear.

25. (a) If $x = e^{\frac{x}{y}}$, then prove that $\dfrac{dy}{dx} = \dfrac{x-y}{x\log x}$.

OR

(b) If $f(x) = \begin{cases} 2x-3, -3 \le x \le -2 \\ x+1, -2 < x \le 0 \end{cases}$

Check the differentiability of f(x) at $x = -2$

SECTION – C

(This section comprises of 6 Short Answer (SA) type questions of 3 marks each.) (6 × 3 = 18)

26. (a) Solve the differential equation $2(y+3) - xy\dfrac{dy}{dx} = 0$; given $y(1) = -2$.

OR

(b) Solve the following differential equation:

$$\left(1+x^2\right)\frac{dy}{dx} + 2xy = 4x^2.$$

27. Let R be a relation defined over N, where N is set of natural numbers, defined as "mRn if and only if m is a multiple of n, $m, n \in N$." Find whether R is reflexive, symmetric and transitive or not.

28. Solve the following linear programming problem graphically:

Minimise Z = $x - 5y$

subject to the constraints:

$x - y \ge 0$

$-x + 2y \ge 2$

$x \ge 3, y \le 4, \ge 0$

29. (a) If $y = \log\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)^2$, then show that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.

OR

(b) If $x\sqrt{1+y} + y\sqrt{1+x} = 0, -1 < x < 1, x \ne y$, then prove that $\dfrac{dy}{dx} = \dfrac{-1}{(1+x)^2}$.

30. (a) A die with number 1 to 6 is biased such that $P(2) = \dfrac{3}{10}$ and probability of other numbers is equal. Find the mean of the number of times the number 2 appears on the dice, if the dice is thrown twice.

OR

(b) Two dice are thrown. Defined are the following two events A and B. A = $\{(x, y) : x + y = 9\}$, B = $\{(x, y) : x \ne 3\}$, where (x, y) denote a point in the sample space.

Check if events A and B are independent or mutually exclusive.

31. Find: $\displaystyle\int \frac{1}{x}\sqrt{\frac{x+a}{x-a}}\,dx$.

SECTION – D

(This section comprises of 4 Long Answer (LA) type questions of 5 marks each.) (6 × 3 = 18)

32. Using integration, find the area of the region bounded by the line $y = 5x + 2$, the x – axis and the ordinates $x = -2$ and $x = 2$.

33. Find: $\displaystyle\int \frac{x^2+x+1}{(x+2)\left(x^2+1\right)}dx$.

34. (a) Find the shortest distance between the lines:

$$\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3} \text{ and}$$

$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}.$$

OR

(b) Find the image A′ of the point A(2, 1, 2) in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of the line joining AA′. Find the foot of perpendicular from point A on the line l.

35. (a) Given $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB. Hence, solve the system of linear equation :

$x - y + z = 4$

$x - 2y + 2z = 9$

$2x + y + 3z = 1$

OR

(b) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find A^{-1}.

Hence, solve the system of linear equations:

$x - 2y = 10$

$2x - y - z = 8$

$-2y + z = 7$

SECTION – E

(This section comprises of 3 case study based questions of 4 marks each.) (3 × 4 = 12)

36. A school is organising a debate competition with participants as speakers S = $\{S_1, S_2, S_3, S_4\}$ and these are judged by judges J = $\{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as R = $\{(x, y)$: speaker x is judged by judge y, $x \in S, y \in J\}$.

Based on the above, answer the following:

(i) How many relations can be there from S to J? **1**

(ii) A student identifies a function from S to J as f = $\{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$ Check if it is bijective. **1**

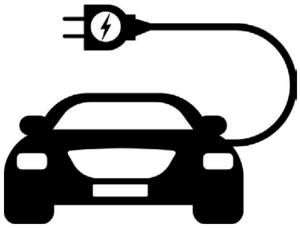
(iii) (a) How many one-one functions can be there from set S to set J? **2**

OR

(iii) (b) Another student considers a relation $R_1 = \{(S_1, S_2), \{S_2, S_4)\}$ in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. **2**

37. Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated).

Based on the above, answer the following:



(i) (a) What is the probability that a randomly selected car is an electric car? **2**

OR

(i) (b) What is the probability that a randomly selected car is a petrol car? **2**

(ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet? **1**

(iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi? **1**

38.

A small town is analysing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres.

Based on the above, answer the following:

(i) Find the intervals on which the $f(x)$ is increasing or decreasing, $x = [0, \pi]$. **2**

(ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion. **2**

Outside Delhi Set-2 65/2/2

Note: Except these, all other questions have been given in Outside Delhi Set-1

SECTION A

This section comprises of 20 multiple choice questions (MCQs) of 1 mark each.

1. The values of x for which the angle between the vectors $\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$ is obtuse, is:

(A) 0 or $\frac{1}{2}$ **(B)** $x > \frac{1}{2}$

(C) $\left(0, \frac{1}{2}\right)$ **(D)** $\left[0, \frac{1}{2}\right]$

3. $\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to:

(A) $\tan x + \cot x + C$

(B) $(\tan x + \cot x)^2 + C$

(C) $\tan x - \cot x + C$

(D) $(\tan x - \cot x)^2 + C$

4. Let P be a skew-symmetric matrix of order 3. If det(P) = α, then $(2025)^\alpha$ is:

(A) 0 **(B)** 1

(C) 2025 **(D)** $(2025)^3$

5. The principal value of $\sin^{-1}\left(\cos\frac{43\pi}{5}\right)$ is:

(A) $\frac{-7\pi}{5}$ **(B)** $\frac{-\pi}{10}$

(C) $\frac{\pi}{10}$ **(D)** $\frac{3\pi}{5}$

11. In the following probability distribution, the value of p is:

X	0	1	2	3
P(X)	p	p	0.3	$2p$

(A) $\frac{7}{40}$ **(B)** $\frac{1}{10}$

(C) $\frac{9}{35}$ **(D)** $\frac{1}{4}$

12. If $\overrightarrow{PQ} \times \overrightarrow{PR} = 4\hat{i} + 8\hat{j} - 8\hat{k}$, then the area ($\Delta$PQR) is:

(A) 2 sq units **(B)** 4 sq units

(C) 6 sq units **(D)** 12 sq units

SECTION B

22. Evaluate: $\int_0^\pi \frac{\sin 2px}{\sin x}\,dx, p \in N.$

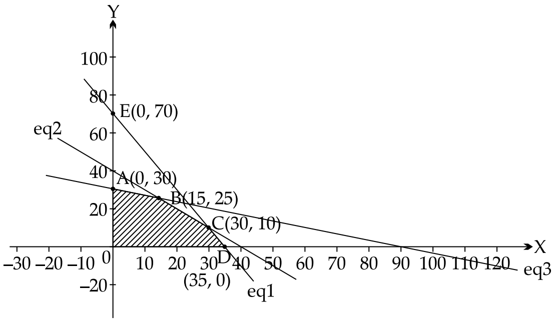
24. Let $\vec{p} = 2\hat{i} - 3\hat{j} - \hat{k}$, $\vec{q} = -3\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + 2\hat{k}$. Express $\vec{r}$ in the form of $\vec{r} = \lambda\vec{p} + \mu\vec{q}$ and hence, find the values of λ and μ.

SECTION C

This section comprises of 6 Short Answer (SA) type questions of 3 marks each.

27. Prove that $f: N \to N$ defined as $f(x) = ax + b$ $(a, b \in N)$ is one-one but not onto

28.



The feasible region along with the corner points for a linear programming problem are shown in the graph. Write all the constraints for the given linear programing problem.

31. f and g are continuos functions on interval $[a, b]$. Given that $f(a - x) = f(x)$ and $g(x) + g(a - x) = a$, show that $\int_0^a f(x)g(x)dx = \frac{a}{2}\int_0^a f(x)dx$.

SECTION D

33. Find: $\int \frac{5x}{(x+1)(x^2+9)}dx$

Outside Delhi Set-3 **65/2/3**

Note: Except these, all other questions have been given in Outside Delhi Set-1 and 2

SECTION A

1. If $\vec{p}$ and $\vec{q}$ are unit vectors, then which of the following values of $\vec{p}\cdot\vec{q}$ is not possible?

(A) $\frac{-1}{2}$ **(B)** $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}}{2}$ **(D)** $\sqrt{3}$

3. If $\int_0^a x\,dx \le \frac{a}{2} + 6$, then:

(A) $-4 \le a \le 3$

(B) $a \le 4$, $a \le -3$

(C) $-3 \le a \le 4$

(D) $-3 \le a \le 0$

4. If A and B are square matrices of same order, then $(AB^T - BA^T)$ is a:

(A) symmetric matrix

(B) skew-symmetric matrix

(C) null matrix

(D) unit matrix

5. The value of $\cos\left(\frac{\pi}{6} + \cot^{-1}\left(-\sqrt{3}\right)\right)$ is:

(A) -1 **(B)** $\frac{-\sqrt{3}}{2}$

(C) 0 **(D)** 1

11. The probability distribution of a random variable X is given by:

X	–4	–3	–2	–1	0
P(X)	0.1	0.2	0.3	0.2	0.2

Then E(X) of distribution is

(A) –1.8 **(B)** –1

(C) 1 **(D)** 1.8

12. If projection of $\vec{a} = \alpha\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units, then α is:

(A) –13 **(B)** –5

(C) 13 **(D)** 5

SECTION B

22. Find: $\int 2x^3 e^{x^2}dx$

24. If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a}\cdot\vec{b} = 4$, then evaluate $|\vec{a} + 2\vec{b}|$.

SECTION C

27. Let $\mathbb{R}$ be a relation on set of real numbers $\mathbb{R}$ defined as $\{(x, y) : x - y + \sqrt{3}$ is an irrational number, $x, y \in \mathbb{R}\}$ Verify $\mathbb{R}$ for reflexivity, symmetry and transitivity.

28. Solve the following linear programming problem graphically:

Minimise $Z = 2x + y$

subject to the constraints:

$3x + y \ge 9$

$x + y \ge 7$

$x + 2y \ge 8$

$x, y \ge 0$

31. If $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$, then find the values of a and b.

SECTION D

33. Find: $\int\left(\sqrt{\tan x} + \sqrt{\cot x}\right)dx$

ANSWERS

Delhi Set-1 **65/1/1**

SECTION A

1. **Option (D) is correct.**

Explanation:

Given, $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

A matrix is invertible if its determinant is non-zero. The determinant of A is:

$\det(A) = (-1)\times(1\times1) = -1 \neq 0$

Since $\det(A) \neq 0$, the matrix is **invertible**.

For a **diagonal matrix** of the form:

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

The inverse is given by:

$$A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$$

For the given matrix:

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{-1} & 0 & 0 \\ 0 & \frac{1}{1} & 0 \\ 0 & 0 & \frac{1}{1} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. **Option (B) is correct.**

Explanation: Given vectors,

$\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$

$\vec{b} = \hat{i} - \hat{j} + \hat{k}$

Two vectors are parallel if one is a scalar multiple of the other, i.e.,

$$\vec{a} = \lambda\vec{b}$$

for some scalar λ. Ratio of given vectors,

$$\frac{3}{1} = 3, \frac{2}{-1} = -2, \frac{-1}{1} = -1$$

Since these ratios are not equal, $\vec{a}$ and $\vec{b}$ are not parallel.

Two vectors are perpendicular if their dot product is zero:

$$\vec{a}\cdot\vec{b} = (3\times1) + (2\times-1) + (-1\times1)$$
$$= 3 - 2 - 1 = 0$$

Since the dot product is zero, $\vec{a}$ and $\vec{b}$ are perpendicular.

3. **Option (B) is correct.**

Explanation: Let

$$I = \int_{-1}^{1} \frac{|x|}{x} dx$$

For $x > 0, |x| = x,$ so $\frac{|x|}{x} = 1.$

For $x < 0, |x| = -x,$ so $\frac{|x|}{x} = -1.$.

Thus,

$$I = \int_{-1}^{0} (-1)dx + \int_{0}^{1} (1)dx$$

On solving each integral, we get

$$\int_{-1}^{0} (-1)dx = -\int_{-1}^{0} dx = -[x]_{-1}^{0} = -(0-(-1)) = -1$$

$$\int_{0}^{1} 1\,dx = [x]_0^1 = 1 - 0 = 1$$

$\therefore \quad I = (-1) + (1) = 0$

4. **Option (D) is correct.**

Explanation: A function $f(x, y)$ is homogeneous of degree n if:

$f(tx, ty) = t^n f(x, y)$

for some integer n.

(A) $y^2 - xy$

Substituting $x \to tx$ and $y \to ty$:

$(ty)^2 - (tx)(ty) = t^2y^2 - t^2xy = t^2(y^2 - xy)$

So, this is homogeneous of degree **2**.

(B) $x - 3y$

Substituting $x \to tx, y \to ty$:

$tx - 3(ty) = t(x - 3y)$

So, this is homogeneous of degree **1**.

(C) $\sin^2\left(\frac{y}{x}\right) + \frac{y}{x}$

Here, $\frac{y}{x}$ is a ratio and does not change with scaling.

Thus, the function remains the same under scaling, making it homogeneous of degree 0.

(D) $\tan x - \sec y$

Here, x and y appear inside trigonometric functions, and scaling x and y does not give a common power of t.

Thus, this is not a homogeneous function.

5. Option (C) is correct.

Explanation: The given function:

$f(x) = |x| + |x - 1|$

We need to analyse its continuity and differentiability at $x = 0$ and $x = 1$.

For Continuity

A function is continuous at $x = a$ if:

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$$

when $x = 0$

For $x > 0$:

$|x| = x$ and $|x - 1| = 1 - x$. So:

$f(x) = x + (1 - x) = 1$

For $x < 0$:

$|x| = -x$ and $|x - 1| = 1 - x$. So:

$f(x) = -x + (1 - x) = 1 - 2x$

Function value at $x = 0$ is 1.

$f(0) = |0| + |0 - 1| = 0 + 1 = 1$

Limits:

$$\lim_{x\to 0^-} f(x) = 1 - 2(0) = 1,\ \lim_{x\to 0^+} f(x) = 1$$

Since both limits match $f(0)$, $f(x)$ **is continuous at** x=0.

When $x = 1$

For $x > 1$:

$|x| = x$ and $|x - 1| = x - 1$. so,

$f(x) = x + (x - 1) = 2x - 1$

For $x < 1$:

$|x| = x$ and $|x - 1| = 1 - x$. so,

$f(x) = x + (1 - x) = 1$

Function value at $x = 1$:

$f(1) = |1| + |1 - 1| = 1 + 0 = 1$

Limits:

$$\lim_{x\to 1^-} f(x) = 1,\ \lim_{x\to 1^+} f(x) = 2(1) - 1 = 1$$

Since both limits match $f(1)$, $f(x)$ **is continuous at** $x = 1$.

For Differentiability

A function is differentiable at $x = a$ if:

$$\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

exists and is finite.

Check at $x = 0$

For $x > 0$:

$f(x) = 1 \Rightarrow f'(x) = 0$

For $x < 0$:

$f(x) = 1 - 2x \Rightarrow f'(x) = -2$

Left and Right Derivatives:

$f_-'(0) = -2,\ f_+'(0) = 0$

Since $f_-'(0) \neq f_+'(0)$, $f(x)$ **is not differentiable at** $x = 0$.

Check at $x = 1$

For $x > 1$:

$f(x) = 2x - 1 \Rightarrow f'(x) = 2$

For $x < 1$:

$f(x) = 1 \Rightarrow f'(x) = 0$

Left and Right Derivatives:

$f(x) = 2x - 1 \Rightarrow f'(x) = 2$

Since $f_-'(1) \neq f_+'(1)$, $f(x)$ **is not differentiable at** x=1.

$f(x)$ **is continuous at both** $x = 0$ **and** $x = 1$.

$f(x)$ **is not differentiable at** $x = 0$ **and** $x = 1$.

6. Option (B) is correct.

Explanation: We are given that is a square matrix of order 2 with:

$$\det(A)=4$$

We know that

For a square matrix A of order n, the determinant of the adjugate (adjoint) matrix is given by:

$$\det(\text{adj } A) = (\det A)^{n-1}$$

For a **2 × 2 matrix** ($n = 2$):

$$\det(\text{adj } A) = (\det A)^{2-1} = (\det A)^1 = 4$$

Using the property:

$$\det(kB) = k^n \det(B)$$

where k is a scalar and B is an $n{\times}n$ matrix.

Therefore,

$$\det(4 \text{ adj } A) = 16 \times 4 = 64$$

7. Option (C) is correct.

Explanation: Given,

$$P(E) = \frac{2}{3},\ P(F) = \frac{3}{7}$$

Since $\overline{F}$ is the complement of F:

$$P(\overline{F}) = 1 - P(F) = 1 - \frac{3}{7} = \frac{4}{7}$$

Since E and F are independent:

$$P(E \cap \overline{F}) = P(E) \times P(\overline{F})$$

$$= \frac{2}{3} \times \frac{4}{7} = \frac{8}{21}$$

Apply Conditional Probability Formula

$$P(E \mid \overline{F}) = \frac{P(E \cap \overline{F})}{P(\overline{F})}$$

$$= \frac{\frac{8}{21}}{\frac{4}{7}}$$

$$= \frac{8}{21} \times \frac{7}{4} = \frac{2}{3}$$

8. Option (C) is correct.

Explanation: Given,

$f(x) = x^3 - 3x + 2$

On differentiating, we get

$f'(x) = 3x^2 - 3$

For critical points, put $f'(x) = 0$:

$$3x^2 - 3 = 0$$
$$x^2 = 1$$
$$x = \pm 1$$

Since $x = -1$ is outside the given interval [0, 2], we consider $x = 1$.

Now, value of $f(x)$ at critical points and endpoints:

$$f(0) = 0^3 - 3(0) + 2 = 2$$

$$f(1) = 1^3 - 3(1) + 2 = 1 - 3 + 2 = 0$$

$$f(2) = 2^3 - 3(2) + 2 = 8 - 6 + 2 = 4$$

The maximum value is **4**.

9. Option (A) is correct.

Explanation: Given matrices:

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$$

$$C = \begin{bmatrix} 9 & 8 & 7 \end{bmatrix}$$

The order of matrix A is 3×3, and the order of matrix B is 3×1.

Since the number of columns of A (which is 3) matches the number of rows of B (which is also 3), multiplication AB is **defined**.

The order of matrix C is 1×3.

Since the number of columns of A (3) does not match the number of rows of C (1), multiplication AC is **not defined**.

Also, the number of columns of B (which is 1) does not match the number of rows of A (which is 3), multiplication BA is **not defined**.

10. Option (A) is correct.

Explanation: Let,

$$I = \int \frac{2^{\frac{1}{x}}}{x^2} dx$$

Put

$$t = \frac{1}{x}$$

Differentiating both sides, we get

$$dt = -\frac{dx}{x^2}$$

Rewriting the integral in terms of t, i.e.,

$$I = \int 2^t \cdot (-dt)$$

$$\left[\because \int a^x dx = \frac{a^x}{\log a}\right]$$

$$I = -\frac{2^t}{\log 2} + C$$

Substituting back $t = \frac{1}{x}$, we get

$$I = -\frac{2^{\frac{1}{x}}}{\log 2} + C$$

Comparing with:

$$I = k \cdot 2^{\frac{1}{x}} + C$$

we get:

$$k = -\frac{1}{\log 2}$$

11. Option (C) is correct.

Explanation: Given,

$$\vec{a} + \vec{b} + \vec{c} = 0$$

$$|\vec{a}| = \sqrt{37}, |\vec{b}| = 3, |\vec{c}| = 4$$

From the given equation, we have

$$\vec{a} = -(\vec{b} + \vec{c})$$

Taking the square of both sides:

$$|\vec{a}|^2 = |\vec{b} + \vec{c}|^2$$

Using the vector identity:

$$|\vec{b} + \vec{c}|^2 = |\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{b}||\vec{c}|\cos\theta$$

Substituting values:

$$37 = 3^2 + 4^2 + 2(3)(4) \cos \theta$$
$$37 = 9 + 16 + 24 \cos \theta$$
$$37 - 25 = 24 \cos \theta$$
$$12 = 24 \cos \theta$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$$

12. Option (B) is correct.

Explanation: Given the differential equation:

$$\left(x + 2y^3\right)\frac{dy}{dx} = 2y$$

We rewrite it in the standard form:

$$\frac{dy}{dx} - \frac{2y}{x + 2y^3} = 0$$

Taking the reciprocal of both sides:

$$\frac{dx}{dy} = \frac{x + 2y^3}{2y}$$

$$\frac{dx}{dy} - \frac{x}{2y} = y^2$$

This is now in the standard linear form:

$$\frac{dx}{dy} + P(y)x = Q(y)$$

where:

$$P(y) = -\frac{1}{2y} \text{ and } Q(y) = y^2$$

The integrating factor (IF) is given by:

$$\text{IF} = e^{\int P(y)dy}$$

$$\text{IF} = e^{\int -\frac{1}{2y}dy}$$
$$= e^{-\frac{1}{2}\ln y}$$
$$= y^{-1/2}$$

Thus, the integrating factor is: $\frac{1}{\sqrt{y}}$

13. Option (B) is correct.

Explanation:

$$A = \begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$$

It is given that A is a scalar matrix. A scalar matrix is a diagonal matrix where all diagonal elements are equal.

For A to be a scalar matrix:

It must be a diagonal matrix, meaning all non-diagonal elements should be zero.

Here, x is a non-diagonal element. So, for A to be a scalar matrix, $x = 0$.

All diagonal elements must be equal.

The diagonal elements of A are 7, 7, y.

For A to be a scalar matrix, $y = 7$.

Now,

$y^x = 7^0 = 1$

14. Option (C) is correct.

Explanation: The corner points of the feasible region are:

(2, 72) (15, 20) (40, 15)

Given, $Z = 18x + 9y$

At (2, 72):

$Z = 18(2) + 9(72) = 36 + 648 = 684$

At (15, 20):

$Z = 18(15) + 9(20) = 270 + 180 = 450$

At (40, 15):

$Z = 18(40) + 9(15) = 720 + 135 = 855$

Maximum value of Z is at (40, 15).

Minimum value of Z is at (15, 20).

15. Option (A) is correct.

Explanation: We analyse the given matrix properties:

(A) Incorrect because $(A + B)^{-1} \neq A^{-1} + B^{-1}$ in general.

(B) Correct, as the inverse of a product follows the rule $(AB)^{-1} = B^{-1}A^{-1}$.

(C) Correct, because adjugate property states $\text{adj}(A) = |A|A^{-1}$.

(D) Correct, since determinant property states $|A^{-1}| = \frac{1}{|A|}$.

16. Option (C) is correct.

Explanation: If the feasible region of a Linear Programming Problem (LPP) is bounded, the objective function $Z = ax + by$ will always attain both maximum and minimum values within the feasible region.

17. Option (D) is correct.

Explanation: We need to find the area of the shaded region in the first quadrant, which is bounded by the parabola $y^2 = x$ and the vertical line $x = 4$.

From $y^2 = x$, get

$$y = \sqrt{x}$$

The required area is given by:

$$\int_0^4 (\text{Upper function} - \text{Lower function})\,dx$$

Here, the upper function is $y = \sqrt{x}$ and the lower function is the x-axis $y = 0$. Thus, the area is: $\int_0^4 \sqrt{x}\,dx$

18. Option (C) is correct.

Explanation: The figure is the graph of cos x. Option C represents the graph of $\cos^{-1}x$.

19. Option (D) is correct.

Explanation: $f(x) = 3x - 5$ is bijective if it is one-to-one and onto.

For one-one

A function is one-one if:

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

For
$$f(x) = 3x - 5,$$
$$3x_1 - 5 = 3x_2 - 5$$
$$3x_1 = 3x_2$$
$$x_1 = x_2$$

Since this holds for all integers, $f(x)$ is one-one.

For onto

A function is onto if for every integer $y \in \mathbb{Z}$, there exists an integer $x \in \mathbb{Z}$ such that:

$f(x) = y$

Solving for x:

$$y = 3x - 5$$
$$x = \frac{y+5}{3}$$

For x to be an integer, $y+5$ must be divisible by 3. However, not all integers y satisfy this condition (e.g., $y = 1$ gives $x = \frac{6}{3} = 2$, which is fine, but $y = 2$ gives $x = \frac{7}{3}$, which is not an integer).

Since not all integers are mapped to by some $x \in \mathbb{Z}$ the function is not Onto.

The assertion (A) is false, but the reason (R) is true because a function is bijective if and only if it is both one-one and onto.

20. Option (D) is correct.

Explanation: To check the continuity of the function $f(x)$ at $x = 5$, we need to verify:

$$\lim_{x\to5^-} f(x) = \lim_{x\to5^+} f(x) = f(5)$$

Left-Hand Limit (LHL)

For $x \leq 5$, the function is given by:

$f(x) = 3x - 8$

Taking the left-hand limit:

$$\lim_{x\to5^-} f(x) = 3(5) - 8 = 15 - 8 = 7$$

Right-Hand Limit (RHL)

For $x > 5$, the function is given by:
$f(x) = 2k$
Taking the right-hand limit:

$$\lim_{x\to 5^+} f(x) = 2k$$

Function 's value at $x = 5$
$f(5) = 3(5) - 8 = 7$
For $f(x)$ to be continuous at $x = 5$:

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x) = f(5)$$

$$7 = 2k = 7$$

$$k = \frac{7}{2}$$

Thus, the given assertion (A) is false but Reason (R) is true by the definition of continuity.

SECTION B

21. **(a)** Let $y = 2\cos^2 x$
Taking the natural logarithm on both sides:
$\ln y = \cos^2 x \ln 2$
Differentiating with respect to $\cos^2 x$:

$$\frac{1}{y}\frac{dy}{d(\cos^2 x)} = \ln 2$$

Multiplying both sides by y:

$$\frac{dy}{d(\cos^2 x)} = y\ln 2$$

Substitute $y = 2\cos^2 x$

$$\frac{dy}{d(\cos^2 x)} = 2^{\cos^2 x}\ln 2$$

Thus,

$$\frac{d}{d(\cos^2 x)}\left(2^{\cos^2 x}\right) = 2^{\cos^2 x}\ln 2$$

OR

(b) Rewriting the given equation as:
$x^2 + y^2 = \tan(a^2)$
Differentiate both sides w.r.t. x
Using implicit differentiation:

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(\tan(a^2))$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

22. We need to evaluate:

$$\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$$

$$= \tan^{-1}\left[2\sin\left(2\cos^{-1}\cos\frac{\pi}{6}\right)\right] \qquad \left[\because \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}\right]$$

$$= \tan^{-1}\left[2\sin\left(\frac{\pi}{3}\right)\right]$$

$$= \tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right] \qquad \left[\because \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}\right]$$

$$= \tan^{-1}\sqrt{3}$$

$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) = \frac{\pi}{3} \qquad \left[\because \sqrt{3} = \tan\frac{\pi}{3}\right]$$

23. We are given the diagonals of a parallelogram as:
$a = 2\hat{i} - \hat{j} + \hat{k} \quad b = \hat{i} + 3\hat{j} - \hat{k}$

The area of a parallelogram when its diagonals are given as:

$$\text{Area} = \frac{1}{2}|\vec{a}\times\vec{b}|$$

Now,

$$a\times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix}$$

Expanding along the first row:

$$\vec{a}\times\vec{b} = \hat{i}\begin{vmatrix} -1 & 1 \\ 3 & -1 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix}$$

Thus, the cross-product vector is:

$$\vec{a}\times\vec{b} = \left(-2\hat{i} + 3\hat{j} + 7\hat{k}\right)$$

$$|\vec{a}\times\vec{b}| = \sqrt{(-2)^2 + 3^2 + 7^2} = \sqrt{4+9+49} = \sqrt{62}$$

Thus, the area of the parallelogram $= \frac{1}{2}\times\sqrt{62}$ sq. units

24. The given function is:

$$f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$$

To find the intervals where the function is increasing or decreasing, we first differentiate $f(x)$:

$$f'(x) = \frac{d}{dx}\left(5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}\right)$$

$$f'(x) = 5\cdot\frac{3}{2}x^{\frac{3}{2}-1} - 3\cdot\frac{5}{2}x^{\frac{5}{2}-1}$$

$$f'(x) = \frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}}$$

$$f'(x) = \frac{15}{2}x^{\frac{1}{2}}(1-x)$$

To find critical points, put $f'(x) = 0$:

$$\frac{15}{2}x^{\frac{1}{2}}(1-x) = 0$$

$$x^{\frac{1}{2}} = 0 \Rightarrow x = 0$$

and $1 - x = 0 \Rightarrow x = 1$
Thus, the critical points are $x = 0$ and $x = 1$.

For $x < 0$ (not in the domain since $x^{1/2}$ is undefined for negative x)

For $0 < x < 1$:

$x^{\frac{1}{2}}$ is positive.

$1 - x$ is positive.

$f'(x) > 0 \rightarrow$ Function is **increasing**.

For $x > 1$:

$x^{\frac{1}{2}}$ is positive.

$1 - x$ is negative.

$f'(x) < 0 \rightarrow$ Function is **decreasing**.

Therefore, $f(x)$ is:

Increasing in $(0, 1)$

Decreasing in $(1, \infty)$

25. (a) The given vectors are:

$$\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$$

The angle θ between two vectors a and b is given by:

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\ |\vec{b}|}$$

The dot product is:

$$\vec{a}\cdot\vec{b} = (3)(2) + (1)(-2) + (2)(4)$$

$$= 6 - 2 + 8 = 12$$

Now,

$$|\vec{a}| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{9+1+4} = \sqrt{14}$$

and

$$|\vec{b}| = \sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{4+4+16} = \sqrt{24} = 2\sqrt{6}$$

Hence,

$$\cos\theta = \frac{12}{\sqrt{14}\times 2\sqrt{6}}$$

$$= \frac{12}{2\sqrt{84}} = \frac{6}{\sqrt{84}}$$

$$= \frac{6}{2\sqrt{21}} = \frac{3}{\sqrt{21}}$$

Thus,

$$\theta = \cos^{-1}\left(\frac{3}{\sqrt{21}}\right)$$

OR

(b) Here,

$$\vec{AB} = (8-2)\hat{i} + (-1-1)\hat{j} + (0-3)\hat{k}$$

$$= 6\hat{i} - 2\hat{j} - 3\hat{k}$$

So,

$$|\vec{AB}| = \sqrt{6^2 + (-2)^2 + (-3)^2}$$

$$= \sqrt{36+4+9} = \sqrt{49} = 7$$

The unit vector opposite to $\vec{AB} = \frac{-\vec{AB}}{|\vec{AB}|}$

$$= -\frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{3}{7}\hat{k}$$

Hence,

A vector of magnitude 21 in this direction is:

$$21\times\left(-\frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{3}{7}\hat{k}\right)$$

$$= -18\hat{i} + 6\hat{j} + 9\hat{k}$$

SECTION C

26. The area A of an equilateral triangle with side length s is given by:

$$A = \frac{\sqrt{3}}{4}s^2$$

Differentiating both sides with respect to time t:

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4}\cdot 2s\cdot\frac{ds}{dt}$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2}s\frac{ds}{dt}$$

Substitute given values

$s = 15$ cm and $\frac{ds}{dt} = 3$ cm/s

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2}\times 15\times 3$$

$$= \frac{\sqrt{3}\times 45}{2}$$

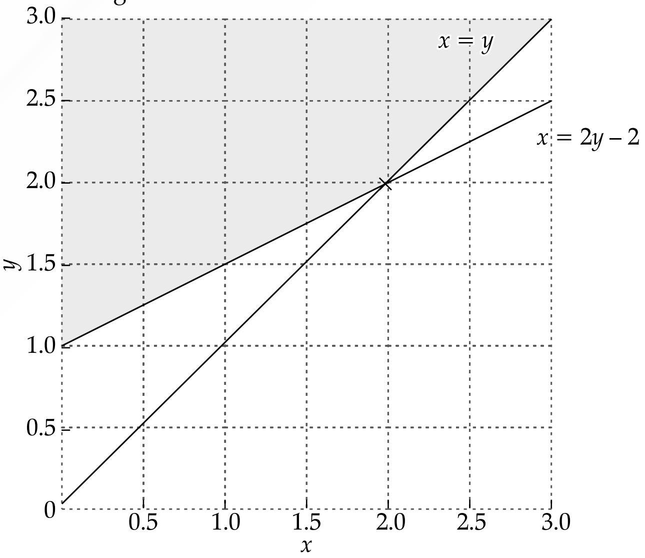
$$= \frac{45\sqrt{3}}{2}\ \text{cm}^2/\text{s}$$

27. Maximise $Z = x + 2y$

Subject to the constraints:

$$x - y \geq 0$$
$$x - 2y \geq -2$$
$$x \geq 0, y \geq 0$$

Converting the given inequalities into equations and plotting on graph we get the following feasible region as shown below.



Here, the feasible region lies in the first quadrant and is bounded on the left by the line $x=2y-2$ and below by $x=y$. However, the region extends indefinitely towards the right and upwards, meaning it is not enclosed within a finite boundary.

Corner points of feasible region are: (0,1) and (2,2)

Value of Objective Function, $Z=x+2y$ at corner points

$Z(0,1) = 0 + 2(1) = 2$

$Z(2,2) = 2 + 2(2) = 6$

Thus, Maximum Z occurs at (2,2) with $Z_{max} = 6$

28. **(a)** Let

$$I=\int \frac{x+\sin x}{1+\cos x}dx$$

we know that,

$$1+\cos x=2\cos^2\frac{x}{2}$$

$$\sin x=2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$\therefore \quad I=\int \frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}dx$$

$$I=\int \frac{x}{2\cos^2\frac{x}{2}}+\int \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$I=\frac{1}{2}\int x\sec^2\frac{x}{2}dx+\int \tan\frac{x}{2}dx$$

Using integration by parts

$$=\frac{1}{2}x\frac{\tan\frac{x}{2}}{\frac{1}{2}}-\int \frac{1}{2}\times\frac{\tan\frac{x}{2}}{\frac{1}{2}}dx+\int \tan\frac{x}{2}dx+C$$

$$I=x\tan\frac{x}{2}+C$$

OR

(b) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

$$I=\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x\sqrt{2\times 2\sin x\cos x}}$$

$$I=\frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x\sqrt{\frac{\sin x}{\cos x}\times\cos x\times\cos x}}$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x\sqrt{\tan x\times\cos^2 x}}$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^4 x\sqrt{\tan x}}=\frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\sec^4 x}{\sqrt{\tan x}}dx$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{(1+\tan^2 x)\sec^2 x\,dx}{\sqrt{\tan x}}$$

Let $t=\sqrt{\tan x}\Rightarrow t^2=\tan x\Rightarrow 2tdt=\sec^2 dx$

When $x=0$

$$t=\sqrt{\tan 0}=0$$

and $x=\frac{\pi}{4}$

$$t=\sqrt{\tan\frac{\pi}{4}}=1$$

Now

$$=\frac{1}{2}\int_0^1 \frac{(1+t^4)}{t}2t\,dt$$

$$=\int_0^1 (1+t^4)dt=\left[t+\frac{t^5}{5}\right]_0^1$$

$$=1+\frac{1}{5}=\frac{6}{5}$$

29. **(a)** The given lines are:

$$\vec{r}=(1-\lambda)\hat{i}+(\lambda-2)\hat{j}+(3-2\lambda)\hat{k}$$

$$\vec{r}=(\mu+1)\hat{i}+(2\mu-1)\hat{j}-(2\mu+1)\hat{k}$$

Comparing with $r=\vec{a}+\lambda\vec{b}$:

First Line:

Point A:(1, −2, 3)

Direction Vector $\vec{b}_1$: (−1, 1, −2)

Second Line:

Point B:(1, −1, −1)

Direction Vector $\vec{b}_2$: (1, 2, −2)

Two lines are skew if:

They are neither parallel nor intersecting.

For Parallelism

If two lines are parallel, then one direction vector should be a scalar multiple of the other. Here:

$(-1, 1, -2) \neq k(1, 2, -2)$ for any k

Since the vectors are not proportional, the lines are not parallel.

For Intersection

Equating the parametric coordinates:

$$1-\lambda=\mu+1$$

$$\lambda-2=2\mu-1$$

$$3-2\lambda=-(2\mu+1)$$

Solving these, we find no common solution for λ and μ, proving that the lines do not intersect.

Since the lines are neither parallel nor intersecting, they are **skew lines**.

The shortest distance between two skew lines is given by:

$$d=\frac{\left|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)\right|}{\left|(\vec{b}_1\times\vec{b}_2)\right|}$$

where:

$$\vec{a}_1=(1,-2,3),\ \vec{a}_2=(1,-1,-1)$$

$$\vec{b}_1=(-1,1,-2),\ \vec{b}_2=(1,2,-2)$$

Using the determinant formula:

$$\left(\vec{b}_1 \times \vec{b}_2\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= \hat{i}[(1)(-2)-(2)(-2)] - \hat{j}[(-1)(-2)-(1)(-2)] + \hat{k}[(-1)(2)-(1)(1)]$$

$$= \hat{i}(-2+4) - \hat{j}(2+2) + \hat{k}(-2-1)$$

$$= \hat{i}(2) - \hat{j}(4) - \hat{k}(3)$$

$= (2, -4, -3)$

and

$$\vec{a}_2 - \vec{a}_1 = (1-1, -1-(-2), -1-3) = (0, 1, -4)$$

Now, $\left(\vec{a}_2 - \vec{a}_1\right) \cdot \left(\vec{b}_1 \times \vec{b}_2\right) =$

$(0, 1, -4) \cdot (2, -4, -3) = (0)(2) + (1)(-4) + (-4)(-3)$
$= 0 - 4 + 12 = 8$

$$\left|\left(\vec{b}_1 \times \vec{b}_2\right)\right| = \sqrt{2^2 + (-4)^2 + (-3)^2}$$

$$= \sqrt{4+16+9} = \sqrt{29}$$

$$d = \frac{|8|}{\sqrt{29}}$$

$$= \frac{8}{\sqrt{29}}$$

$$= \frac{8\sqrt{29}}{29}$$

OR

(b) Given Position Vectors

Bowler's position (B):

$$\vec{B} = 2\hat{i} + 8\hat{j}$$

Wicketkeeper's position (W):

$$\vec{W} = 6\hat{i} + 12\hat{j}$$

Leg slip fielder's position (F):

$$\vec{F} = 12\hat{i} + 18\hat{j}$$

Assume that W divides the line segment BF in the ratio $k : 1$.

The position vector of a point $\vec{P}$ dividing the line segment joining two points with position vectors $\vec{A}$ and $\vec{B}$ in the ratio $k : 1$ is given by:

$$\vec{P} = \frac{k\vec{B} + \vec{A}}{k+1}$$

For case:

$$\vec{W} = \frac{k\vec{F} + \vec{B}}{k+1}$$

Substituting the given vectors:

$$6\hat{i} + 12\hat{j} = \frac{k\left(12\hat{i} + 18\hat{j}\right) + \left(2\hat{i} + 8\hat{j}\right)}{k+1}$$

On equating $\hat{i}$ -component:

$$6 = \frac{12k+2}{k+1}$$

$$6(k+1) = 12k+2$$
$$6k+6 = 12k+2$$
$$6k - 12k = 2 - 6$$
$$k = \frac{2}{3}$$

On equating $\hat{j}$ -component:

$$12 = \frac{18k+8}{k+1}$$

$$12(k+1) = 18k+8$$
$$12k+12 = 18k+8$$
$$12k - 18k = 8 - 12$$
$$k = \frac{2}{3}$$

Since both components give the same ratio, we conclude that the wicketkeeper W divides the segment BF in the ratio $2 : 3$

30. (a) Given Probability Distribution Table

X	0	2	4	5
$P(X)$	p	$2p$	$3p$	p

Since the sum of all probabilities must be equal to 1, Therefore,

$$p + 2p + 3p + p = 1$$
$$7p = 1$$
$$p = \frac{1}{7}$$

The mean $E(X)$ is given by:

$E(X) = \sum XP(X)$

Substituting the values from the table:

$$E(X) = 0 \cdot p + 2 \cdot (2p) + 4 \cdot (3p) + 5 \cdot p$$

$E(X) = 0 + 4p + 12p + 5p$

$E(X) = 21p$

Substituting $p = \frac{1}{7}$:

$$E(X) = 21 \times \frac{1}{7} = 3$$

OR

(b) Total candidates $= 3000$

Females $= \frac{2}{3} \times 3000 = 2000$

Males $= 3000 - 2000 = 1000$

Probability of selecting a male $= P(M) = \frac{1000}{3000} = \frac{1}{3}$

Probability of selecting a female $= P(F) = \frac{2000}{3000} = \frac{2}{3}$

Probability of distinction given male
$= P(D|M) = 0.4$

Probability of distinction given female
$= P(D|F) = 0.35$

The probability that a randomly selected candidate gets a distinction is:

$$P(D) = P(D|M)P(M) + P(D|F)P(F)$$

Substituting the values:

$$P(D) = \left(0.4 \times \frac{1}{3}\right) + \left(0.35 \times \frac{2}{3}\right)$$

$$P(D) = \frac{0.4}{3} + \frac{0.7}{3}$$

$$P(D) = \frac{1.1}{3} = 0.3667$$

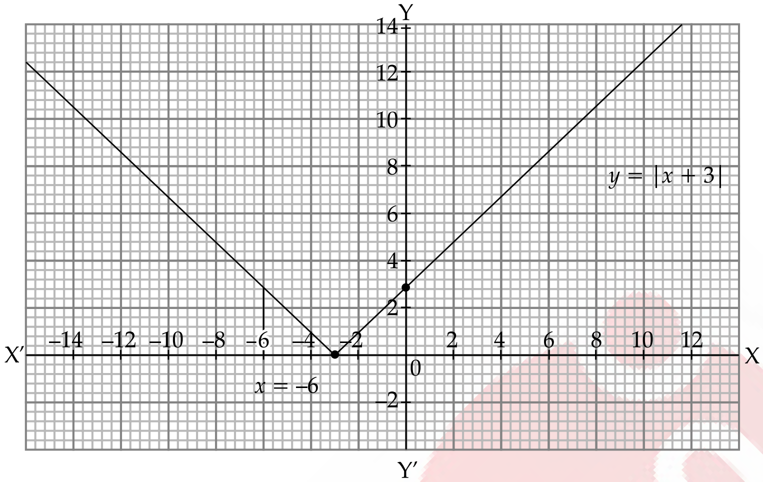
31. The function given is:

$y = |x + 3|$

The absolute value function can be rewritten as:

$$y = \begin{cases} x+3, & \text{if } x \geq -3 \\ -(x+3), & \text{if } x < -3 \end{cases}$$

The function has a **V-shape** with a vertex at $(-3, 0)$. The area enclosed between the curve and the x-axis in the interval $x = -6$ to $x = 0$



The area enclosed by the curve and the x-axis is given by:

$$A = \int_{-6}^{-3} -(x+3)dx + \int_{-3}^{0} (x+3)dx$$

First Integral:

$$\int_{-6}^{-3} -(x+3)dx = \int_{-6}^{-3} (-x-3)dx$$

$$= \int_{-6}^{-3} -x\,dx - \int_{-6}^{-3} 3dx$$

$$= \left[-\frac{x^2}{2}\right]_{-6}^{-3} - [3x]_{-6}^{-3}$$

$$= \left[-\frac{(-3)^2}{2} + \frac{(-6)^2}{2}\right] - [3(-3) - 3(-6)]$$

$$= \left[-\frac{9}{2} + \frac{36}{2}\right] - [-9 + 18]$$

$$= \left[\frac{27}{2}\right] - [9]$$

$$= \frac{27}{2} - \frac{18}{2} = \frac{9}{2}$$

Second Integral:

$$\int_{-3}^{0} (x+3)dx$$

$$= \int_{-3}^{0} x\,dx + \int_{-3}^{0} 3dx$$

$$= \left[\frac{x^2}{2}\right]_{-3}^{0} + [3x]_{-3}^{0}$$

$$= \left[\frac{(0)^2}{2} - \frac{(-3)^2}{2}\right] + [3(0) - 3(-3)]$$

$$= \left[0 - \frac{9}{2}\right] + [0 + 9]$$

$$= -\frac{9}{2} + 9$$

$$= -\frac{9}{2} + \frac{18}{2} = \frac{9}{2}$$

Total Area,

$$A = \frac{9}{2} + \frac{9}{2} = \frac{18}{2} = 9$$

Thus, the enclosed area is 9 square units.

SECTION D

32. **(a)** Given, $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

Let $x = \sin\theta \Rightarrow \theta = \sin^{-1} x$

$y = \sin\alpha \Rightarrow \alpha = \sin^{-1} y$

$$\sqrt{1-\sin^2\alpha} + \sqrt{1-\sin^2\alpha} = a(\sin\theta - \sin\alpha)$$

$$\Rightarrow \sqrt{\cos^2\theta} + \sqrt{\cos^2\alpha} = a(\sin\theta - \sin\alpha)$$

$$\cos\theta + \cos\alpha = a(\sin\theta - \sin\alpha)$$

$$\Rightarrow 2\cos\left(\frac{\theta+\alpha}{2}\right)\cos\left(\frac{\theta-\alpha}{2}\right) = a\left(2\cos\left(\frac{\theta+\alpha}{2}\right)\sin\left(\frac{\theta-\alpha}{2}\right)\right)$$

$$\Rightarrow a = \cot\left(\frac{\theta-\alpha}{2}\right)$$

$$\Rightarrow \frac{\theta-\alpha}{2} = \cot^{-1} a \Rightarrow \theta - \alpha = 2\cot^{-1} a$$

Thus, On differentiating w.r.t x both sides, we get

$$\sin^{-1}x - \sin^{-1}y = 2\cot^{-1}a$$

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

Hence proved

OR

(b) $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ $\qquad y = \sin\theta$

$$\frac{dx}{d\theta} = a\left(-\sin\theta + \frac{1}{\tan\frac{\theta}{2}} \times \frac{1}{2}\sec^2\frac{\theta}{2}\right)$$

$$= a\left(-\sin\theta + \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} \times \frac{1}{2}\frac{1}{\cos^2\frac{\theta}{2}}\right)$$

$$= a\left(-\sin\theta + \frac{1}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= a\left(-\sin\theta + \frac{1}{\sin\theta}\right)$$

$$\frac{dx}{d\theta} = a\left(\frac{1-\sin^2\theta}{\sin\theta}\right) = a\left(\frac{\cos^2\theta}{\sin\theta}\right) \qquad ...(1)$$

Now $y = \sin\theta$

$$\frac{dy}{d\theta} = \cos\theta \qquad ...(2)$$

Now divide eq (2) by (1)

$$\Rightarrow \frac{dy}{dx} = \frac{(\cos\theta)}{a\left(\frac{\cos^2\theta}{\sin\theta}\right)} = \frac{1}{a}\tan\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{a}\tan\theta$$

$$\frac{d^2y}{dx^2} = \frac{1}{a}\sec^2\theta\frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{1}{a}\sec^2\theta\frac{\sin\theta}{a\cos^2\theta} \qquad \left[\text{Subsitute } \frac{d\theta}{dx}\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{4}} = \frac{1}{a}\left(\sqrt{2}\right)^2 \times \frac{\frac{1}{\sqrt{2}}}{a\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \frac{1}{a^2}2\times\sqrt{2} = \frac{2\sqrt{2}}{a^2}$$

33. Given,

$f(x) = 2x^3 - 15x^2 + 36x + 1$

defined on the closed interval [1, 5]

Differentiate $f(x)$ with respect to x:

$$f'(x) = \frac{d}{dx}\left(2x^3 - 15x^2 + 36x + 1\right) = 6x^2 - 30x + 36$$

Put $f'(x) = 0$ to find critical points:

$6x^2 - 30x + 36 = 0$

or, $(x - 2)(x - 3) = 0$

So, the critical points are $x = 2$ and $x = 3$.

Now, evaluate $f(x)$ at Critical Points and Endpoints

At $x = 1$:

$$f(1) = 2(1)^3 - 15(1)^2 + 36(1) + 1$$

$$= 2 - 15 + 36 + 1 = 24$$

At $x = 2$:

$$f(2) = 2(2)^3 - 15(2)^2 + 36(2) + 1$$

$$= 2(8) - 15(4) + 36(2) + 1 = 16 - 60 + 72 + 1 = 29$$

At $x = 3$:

$$f(3) = 2(3)^3 - 15(3)^2 + 36(3) + 1$$

$$= 2(27) - 15(9) + 36(3) + 1 = 54 - 135 + 108 + 1 = 28$$

At $x = 5$:

$$f(5) = 2(5)^3 - 15(5)^2 + 36(5) + 1$$

$$= 2(125) - 15(25) + 36(5) + 1$$

$$= 250 - 375 + 180 + 1 = 56$$

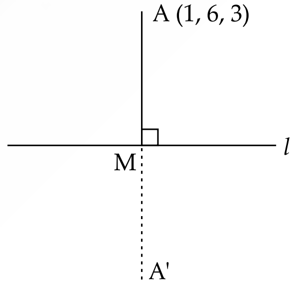
The highest value is 56 at $x = 5$, so absolute maximum is $f(5) = 56$.

The lowest value is 24 at $x = 1$, so absolute minimum is $f(1) = 24$.

Absolute Maximum: $f(5) = 56$

Absolute Minimum: $f(1) = 24$

34. **(a)** Given line (l), $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$



Direction ration of l is (1, 2, 3)

Direction ration of AM is $(\lambda - 1, 2\lambda - 5, 3\lambda - 1)$

Here, $AM \perp l$

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$$

$$\Rightarrow \lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$$

$$14\lambda = 14 \Rightarrow \lambda = 1$$

M is the mid point of AA'

$A = (1, 6, 3)$, $A' = (x_1, y_1, z_1)$, $M = (1, 3, 5)$

$$1=\frac{x_1+1}{2},\ \frac{y_1+6}{2}=3,\ \frac{z_1+3}{2}=5$$

$x_1 = 1, y_1 = 0, z_1 = 7$

Now eq. of line AA' is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-2_1}$$

$$\frac{x-1}{0}=\frac{y-0}{6}=\frac{z-7}{-4}$$

OR

(b) The given line equation is:

$$\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=t$$

From this, the parametric coordinates of any point P on the line are:

$x = -5 + t, y = -3 + 4t, z = 6 - 9t.$

The distance between $P(x, y, z)$ and $Q(2, 4, -1)$ is given by:

$$\sqrt{(x-2)^2+(y-4)^2+(z+1)^2}=7.$$

Substituting x,y,z:

$$\sqrt{(-5+t-2)^2+(-3+4t-4)^2+(6-9t+1)^2}=7.$$

$$\sqrt{(-7+t)^2+(-7+4t)^2+(7-9t)^2}=7.$$

$$\sqrt{(t-7)^2+(4t-7)^2+(7-9t)^2}=7.$$

Squaring both sides:

$$(t-7)^2+(4t-7)^2+(7-9t)^2=49.$$

$$(t^2-14t+49)+(16t^2-56t+49)+(81t^2-126t+49)=49.$$

$$t^2-14t+49+16t^2-56t+49+81t^2-126t+49=49.$$

$$(1+16+81)t^2+(-14-56-126)t+(49+49+49)=49.$$

$$\begin{aligned}98t^2-196t+147&=49.\\ 98t^2-196t+98&=0.\\ t^2-2t+1&=0\\ (t-1)^2&=0.\\ t&=1\end{aligned}$$

Substituting $t = 1$ in parametric equations:

$$x=-5+1=-4,\ y=-3+4(1)=1,\ z=6-9(1)=-3.$$

So, the required point is:

$P(-4, 1, -3)$.

Equation of Line Joining $P(-4, 1, -3)$ and $Q(2, 4, -1)$

Direction ratios:

$$(2-(-4),4-1,-1-(-3))=(6,3,2).$$

Equation of the required line:

$$\frac{x+4}{6}=\frac{y-1}{3}=\frac{z+3}{2}.$$

This is the required equation of the line joining P and Q.

35. Let,

x be the number of students in the Sports club.

y be the number of students in the Music club.

z be the number of students in the Drama club.

From the given conditions:

$$\begin{aligned}\text{Sports club} &= \text{Music} + \text{Drama}\\ x &= y+z\\ x-y-z&=0\\ \text{Music club} &= 20 + \text{Half of Sports club}\\ y&=\frac{x}{2}+20\\ x-2y &= -40\\ \text{Total students} &= 180\\ x+y+z &= 180\end{aligned}$$

Thus, the system of equations is:

$$\begin{aligned}x-y-z&=0\\ x-2y&=40\\ x+y+z&=180\end{aligned}$$

The system of equations can be written as:

$$\begin{bmatrix}1&-1&-1\\1&-2&0\\1&1&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\-40\\180\end{bmatrix}$$

We solve for X:

$$AX = B$$

where,

$$A=\begin{bmatrix}1&-1&-1\\1&-2&0\\1&1&1\end{bmatrix},\ X=\begin{bmatrix}x\\y\\z\end{bmatrix},\ B=\begin{bmatrix}0\\-40\\180\end{bmatrix}$$

Now, solving for X:

$$X = A^{-1}B$$

$$\det(A)=\begin{vmatrix}1&-1&-1\\1&-2&0\\1&1&1\end{vmatrix}$$

Expanding along the first row:

$$=1((-2)(1)-(0)(1))+1((1)(1)-(0)(1))-1((1)(1)-(-2)(1))$$

$= (-2) +1 - (1+2)$

$= -2 + 1 -3 = -4$

Since $\det(A) \neq 0$, the system has a unique solution.

The cofactor of an element a_{ij} is given by:

$C_{ij} = (-1)^{i+j} M_{ij}$

where M_{ij} is the minor

$$C_{11}=(-1)^{1+1}\begin{vmatrix}-2&0\\1&1\end{vmatrix}=(-1)^2(-2\cdot1-0\cdot1)=-2$$

$$C_{12}=(-1)^{1+2}\begin{vmatrix}1&0\\1&1\end{vmatrix}=(-1)^3(1\cdot1-0\cdot1)=-1$$

$$C_{13}=(-1)^{1+3}\begin{vmatrix}1&-2\\1&1\end{vmatrix}=(-1)^4(1\cdot1-(-2)\cdot1)=3$$

$$C_{21}=(-1)^{2+1}\begin{vmatrix}-1 & -1\\ 1 & 1\end{vmatrix}=(-1)^3(-1\cdot1-(-1)\cdot1)=0$$

$$C_{22}=(-1)^{2+2}\begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix}=(-1)^4(1\cdot1-(-1)\cdot1)=2$$

$$C_{23}=(-1)^{2+3}\begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix}=(-1)^5(1\cdot(1)-(-1)\cdot1)=-2$$

$$C_{31}=(-1)^{3+1}\begin{vmatrix}-1 & -1\\ -2 & 0\end{vmatrix}=(-1)^4(-1\cdot0-(-1)\cdot(-2))=-2$$

$$C_{32}=(-1)^{3+2}\begin{vmatrix}1 & -1\\ 1 & 0\end{vmatrix}=(-1)^5(1\cdot0-(-1)\cdot1)=-1$$

$$C_{33}=(-1)^{3+3}\begin{vmatrix}1 & -1\\ 1 & -2\end{vmatrix}=(-1)^6(1\cdot(-2)-(-1)\cdot(1))=-1$$

Form the Cofactor Matrix

$$C=\begin{bmatrix}-2 & -1 & 3\\ 0 & 2 & -2\\ -2 & -1 & -1\end{bmatrix}$$

Thus, the adjoint (adjugate) of A is:

$$\text{adj}(A)=C^T=\begin{bmatrix}-2 & 0 & -2\\ -1 & 2 & -1\\ 3 & -2 & -1\end{bmatrix}$$

The inverse of A is given by:

$$A^{-1}=\frac{1}{\det(A)}\cdot\text{Adj}(A)$$

Computing the adjugate matrix and inverse, then multiplying with B, we obtain:

$$X=\begin{bmatrix}90\\ 25\\ 65\end{bmatrix}$$

Thus, the number of students in each club is:

Sports Club: $x = 90$

Music Club: $y = 25$

Drama Club: $z = 665$

SECTION E

36. **(i)** The total boundary material consists of:

Two lengths $(2x)$

Three widths $(3y)$ (since the partition is parallel to the width)

Thus, the boundary equation is:

$2x + 3y = 300$

Solving for y in terms of x:

$$y=\frac{300-2x}{3}$$

(ii) The area of the rectangle is:

$A = x \cdot y$

Substituting y from the boundary equation:

$$A(x)=\frac{x(300-2x)}{3}$$

$$A(x)=\frac{300x-2x^2}{3}$$

$$A(x)=100x-\frac{2}{3}x^2$$

(iii) **(a)** To find the critical points, differentiate $A(x)$:

$$\frac{dA}{dx}=100-\frac{4}{3}x \quad ...(i)$$

Put $\frac{dA}{dx}=0$ to find critical points:

$$100-\frac{4}{3}x=0$$

$$\frac{4}{3}x=100$$

$$x=\frac{100\times3}{4}=75$$

Now, find y:

$$y=\frac{300-2(75)}{3}$$

$$y=\frac{300-150}{3}=\frac{150}{3}=50$$

On differentiating eq. (i), we get

$$\frac{d^2A}{dx^2}=-\frac{4}{3}$$

Since $\frac{d^2A}{dx^2}<0$, the function has a maximum at $x = 75$.

Thus, the **maximum area** is:

$A_{max} = x.y = 75 \times 50$

$= 3750$ sq. m

OR

(b) Applying First Derivative Test:

$$A'(x)=\frac{300-4x}{3} \quad \text{[From (ii)]}$$

Put $A'(x)=0$, to get $x=75$

For $x<75$, $A'(x)>0$ (increasing).

For $x>75$, $A'(x)<0$ (decreasing).

Change in sign from positive to negative at $x=75$ confirms A is maximum.

Thus, the maximum area the company can enclose is:

$A=x \times y = 75 \times 50 = 3750$ square meters

[Since, $y=\frac{300-2x}{3}$]

37. A relation R on a set $A = \{1, 2, 3\}$ has the following properties:

Reflexive: R contains all pairs (a, a) for all $a \in A$.

Symmetric: If $(a, b) \in R$, then $(b, a) \in R$.

Transitive: If $(a, b) \in R$ and (b, c) then $(a, c) \in R$.

Relation $R_1 = \{(2, 3), (3, 2)\}$

Is not reflexive (does not contain (1, 1), (2, 2), or (3, 3))

Is symmetric (since (2, 3) and (3, 2) both exist)

Is not transitive (since (2, 3) and (3, 2) exist, but (2, 2) is missing)

Conclusion: Symmetric but neither reflexive nor transitive.

Relation R_2 = {(1, 2), (1, 3), (3, 2)}

This is not reflexive (missing (1, 1), (2, 2), (3, 3))

This is not symmetric (contains (1, 2) but not (2, 1), etc.)

This is not transitive (contains (1, 2) and (1, 3) but not (2, 3))

Conclusion: Not reflexive, not symmetric, not transitive.

Relation R_3 = {(1, 2), (2, 1), (1, 1)}

Not reflexive (as R missing (2, 2), (3, 3))

Symmetric (as R contains (1, 2) and (2, 1))

Not transitive (as R contains (1, 2) and (2, 1), but missing (2, 2)), i.e.,partial case, but does not complete transitivity for all elements.

Conclusion: Symmetric but neither reflexive nor transitive.

Relation R_4 = {(1, 1), (1, 2), (3, 3), (2, 2)}

Reflexive (as R contains (1, 1), (2, 2), (3, 3))

Not symmetric (as R contains (1, 2) but not (2, 1))

Transitive (since (1, 2) exists and no other required condition is violated)

Conclusion: Reflexive and transitive but not symmetric.

Relation R_5 = {(1, 1), (1, 2), (3, 3), (2, 2) (2, 1), (2, 3), (3, 2)}

Reflexive (as R contains (1, 1), (2, 2), (3, 3))

Symmetric (as R contains (1, 2) and (2, 1), (2, 3) and (3, 2))

Not transitive (as R contains (1, 2) and (2, 3) but not (1, 3))

Conclusion: Reflexive and symmetric but not transitive.

(i) R_4 is the relation reflexive, transitive but not symmetric.

(ii) R_5 is the relation which is reflexive and symmetric but not transitive.

(iii) **(a)** R_1 and R_3 are the relations which are symmetric but neither reflexive nor transitive.

OR

(b) To make R_2 an equivalence relation, it must be reflexive, symmetric, and transitive.

Reflexivity: Add (1, 1), (2, 2), (3, 3).

Symmetry: Add (2, 1), (3, 1) and (2, 3).

Transitivity: Ensure (1, 2), (2, 3) implies (1, 3) (which is already present).

Thus, the required pairs to be added are:

(1, 1), (2, 2), (3, 3), (2, 1), (3, 1) (2, 3)

38. Let:

A_1 be the event that a customer takes a **fixed rate loan**.

A_2 be the event that a customer takes a **floating rate loan**.

A_3 be the event that a customer takes a **variable rate loan**.

From the given data:

$P(A_1)=0.10, P(A_2)=0.20, P(A_3)=0.70$

Define D as the event that a customer **defaults** on loan repayment.

Given conditional probabilities:

$P(D|A_1)=0.05, P(D|A_2)=0.03, P(D|A_3)=0.01$

(i) Using the Total Probability Theorem:

Probability that a customer defaults on loan repayment,

$$P(D)=P(D|A_1)P(A_1)+P(D|A_2)P(A_2)$$
$$+P(D|A_3)P(A_3)$$

Substituting values:

$$P(D)=(0.05\times0.10)+(0.03\times0.20)+(0.01\times0.70)$$

$= 0.005 + 0.006 + 0.007 = 0.018$

Thus, the probability that a customer defaults on loan repayment is 0.018 or 1.8%.

(ii) Applying Bayes' Theorem:

Probability that a customer after availing the loan, defaults on loan repayment availed the loan at a variable rate of interest,

$$P(A_3|D)=\frac{P(D|A_3)P(A_3)}{P(D)}$$

Substituting values:

$$P(A_3|D)=\frac{(0.01\times0.70)}{0.018}$$

$$=\frac{0.007}{0.018}$$

$$\approx 0.0389$$

Thus, the probability that the customer availed the loan at a variable rate of interest given that he defaulted is 0.389 or 38.9%.

Delhi Set-2 **65/1/2**

SECTION A

2. Option (B) is correct.

Explanation: $\vec{\alpha} \parallel \vec{\beta} \Rightarrow \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

i.e., $\frac{1}{2}=\frac{-4}{-8}=\frac{9}{\lambda}$

$$\frac{9}{\lambda}=\frac{1}{2}$$

$\lambda = 18$

3. Option (A) is correct.

Explanation:

$$\int\frac{1-2\sin x}{\cos^2 x}dx=\int\frac{1}{\cos^2 x}dx-\int\frac{2\sin x}{\cos^2 x}dx$$

$$=\int\sec^2 x\,dx-\int 2\sec x\tan x\,dx$$

$= \tan x - 2\sec x + C$

6. Option (B) is correct.

Explanation:

Given $n = 3$, $|A| = 9$

$|9A^{-1}| = 9^3\,|A^{-1}| = 9^3 \times \frac{1}{|A|} = 9^3 \times \frac{1}{9} = 9^2$

[Since, det $(kA) = k^n$ det(A)]

9. Option (C) is correct.

Explanation:

Order of $A = m \times n$

Order of $A^T = n \times m$

A^TB and BA^T are defined

$\Rightarrow$ Order of $B = m \times n$

12. Option (B) is correct.

Explanation:

$$\frac{dy}{dx} + y = \frac{1+y}{x}$$

$$\frac{dy}{dx} + y = \frac{1}{x} + \frac{y}{x}$$

$$\frac{dy}{dx} + y - \frac{y}{x} = \frac{1}{x}$$

$$\frac{dy}{dx} + y\left(1 - \frac{1}{x}\right) = \frac{1}{x}$$

The given differential equation is lineas

I.F. $= e^{\int p\,dx}$

$$= e^{\int\left(1-\frac{1}{x}\right)dx}$$

$$= e^{x - \log x}$$

$$= \frac{e^x}{e^{\log x}}$$

$$= \frac{e^x}{x}$$

13. Option (D) is correct.

Explanation:

$$a_{ij} = \hat{j} - 2\hat{i}$$

$a_{12} = 2 - 2 = 0 \Rightarrow$ (A) and (B) are false

$a_{13} + a_{31} = (3 - 2) + (1 - 6) = 1 - 5 = -4$

$\Rightarrow$ (C) is false

$a_{23} = 3 - 4 = -1$

$a_{32} = 2 - 6 = -4$

$\Rightarrow a_{23} > a_{32} \Rightarrow$ (D) is true

SECTION B

22. $f(x) = x^2 - 2ax + b$

$f'(x) = 2x - 2a$

Since $f(x)$ is increasing in $(0, \infty)$, $f'(x) > 0$

$\Rightarrow \quad 2x - 2a > 0$

$\Rightarrow \quad -2a > -2x$

$\Rightarrow \quad a < x$

$\Rightarrow \quad a \in (-\infty, 0)$

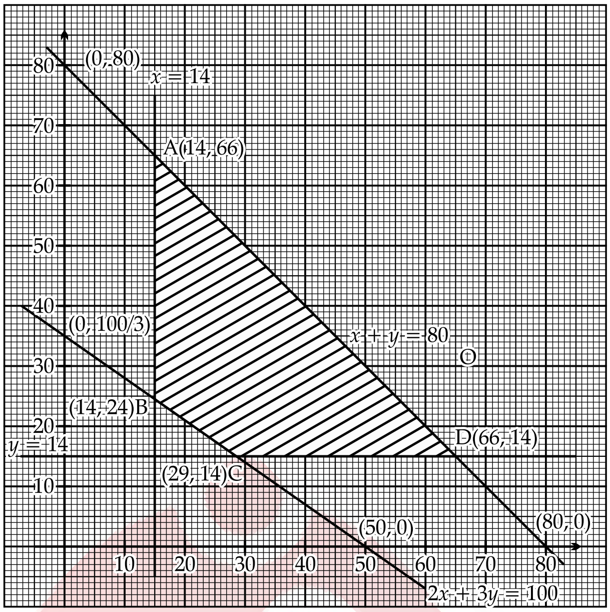
24. $$\sin^{-1}\left[\sin\frac{3\pi}{5}\right] = \sin^{-1}\left[\sin\left(\pi - \frac{2\pi}{5}\right)\right]$$

$$= \sin^{-1}\left[\sin\frac{2\pi}{5}\right]$$

$$= \frac{2\pi}{5}$$

SECTION C

27.



Corner points	$Z = 20x + 30y$
A (14, 66)	2260
B (14, 24)	1000
C (29, 14)	1000
D (66, 14)	1740

Maximum value of $Z = 2260$ at (14, 66)

28. Given, $\frac{dA}{dt} = 48\text{ cm}^2/\text{s}$

Let, l be length and b be breadth of rectangle.

ATQ, $l = b^2$

When $b = 4.5$ cm $\Rightarrow l = 4.5^2 = 20.25$ cm

Area, $A = lb = l \times l^{\frac{1}{2}} = l^{\frac{3}{2}}$

$$\frac{dA}{dt} = \frac{3}{2}\sqrt{l} \times \frac{dl}{dt}$$

$$48 = \frac{3}{2} \times \sqrt{20.25} \times \frac{dl}{dt}$$

$$48 = \frac{3}{2} \times 4.5 \times \frac{dl}{dt}$$

$$\frac{dl}{dt} = \frac{48 \times 2}{3 \times 4.5}$$

$$= \frac{64}{9}\text{ cm/s}$$

30. (a) $$\int \frac{\cos 2x}{(\sin x + \cos x)^2}\,dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{(\sin x + \cos x)^2}\,dx$$

$$= \int \frac{(\cos x + \sin x)(\cos x + \sin x)}{(\sin x + \cos x)^2}\,dx$$

$$= \int \frac{\cos x - \sin x}{\sin x + \cos x} dx$$

$= \log |\sin x + \cos x| + C$

$$\left[\because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C\right]$$

OR

(b) Let $I = \int_0^{\frac{\pi}{2}} \frac{5\sin x + 3\cos x}{\sin x + \cos x} dx$ ... (1)

Using the property $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

$$I = \int_0^{\frac{\pi}{2}} \frac{5\sin\left(\frac{\pi}{2} - x\right) + 3\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{5\cos x + 3\sin x}{\cos x + \sin x} dx \quad \text{... (2)}$$

$$(1) + (2) \Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{8\sin x + 8\cos x}{\sin x + \cos x} dx$$

$$= 8\int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$= 8\int_0^{\frac{\pi}{2}} 1\, dx$$

$$= 8[x]_0^{\frac{\pi}{2}}$$

$$= 8\left(\frac{\pi}{2} - 0\right)$$

$2I = 4\pi$

$\therefore \quad I = \pi$

SECTION D

32. (a) Let $u = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

$$= \tan^{-1}\left(\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}\right) \quad \text{Putting } x = \cos\theta$$

$$= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$\tan^{-1}(\tan\theta) = \theta = \cos^{-1}x$

$$\frac{du}{dx} = \frac{d}{dx}\left(\cos^{-1}x\right) = \frac{-1}{\sqrt{1-x^2}} \quad \text{...(1)}$$

Let $v = \cos^{-1}\left(2x\sqrt{1-x^2}\right)$ Putting $x = \cos\theta$

$$= \cos^{-1}\left(2\cos\theta\sqrt{1-\cos^2\theta}\right)$$

$= \cos^{-1}(2\cos\theta\sin\theta)$

$= \cos^{-1}(\sin 2\theta)$

$$= \cos^{-1}\left[\cos\left(\frac{\pi}{2} - 2\theta\right)\right]$$

$$= \frac{\pi}{2} - 2\theta$$

$$= \frac{\pi}{2} - 2\cos^{-1}x$$

$$\frac{dv}{dx} = \frac{-2}{\sqrt{1-x^2}} \quad \text{...(2)}$$

From (1) and (2), $\dfrac{du}{dv} = \dfrac{\frac{-1}{\sqrt{1-x^2}}}{\frac{-2}{\sqrt{1-x^2}}}$

$$= \frac{1}{2}$$

OR

(b) Given, $y = x^{\tan x} + \dfrac{\sqrt{x^2+1}}{2}$

Let $u = x^{\tan x}$ and $v = \dfrac{\sqrt{x^2+1}}{2}$

$\therefore y = u + v$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \text{... (1)}$$

Now $u = x^{\tan x}$

Taking log on both sides

$\log u = \log x^{\tan x}$

$\log u = \tan x \log x$

Differentiating both sides w.r .t x

$$\frac{1}{u}\frac{du}{dx} = \frac{\tan x}{x} + \sec^2 x \log x$$

$$\frac{du}{dx} = u\left(\frac{\tan x}{x} + \sec^2 x \log x\right)$$

$$\frac{du}{dx} = x^{\tan x}\left(\frac{\tan x}{x} + \sec^2 x \log x\right) \quad \text{... (2)}$$

and $v = \dfrac{\sqrt{x^2+1}}{2}$

$$\frac{dv}{dx} = \frac{2x}{2\times 2\sqrt{x^2+1}} = \frac{x}{2\sqrt{x^2+1}} \quad \text{... (3)}$$

Substituting (2) and (3) in (1)

$$\Rightarrow \frac{dy}{dx} = x^{\tan x}\left(\frac{\tan x}{x} + \sec^2 x \log x\right) + \frac{x}{2\sqrt{x^2+1}}$$

Delhi Set-3 **65/1/3**

SECTION A

2. Option (A) is correct.

Explanation: Required unit vector is :

$$\frac{(\hat{i}-\hat{j})\times(\hat{i}+\hat{j})}{|(\hat{i}-\hat{j})\times(\hat{i}+\hat{j})|}=\frac{2\hat{k}}{2}$$

$$= \hat{k}$$

3. Option (C) is correct.

Explanation: Apply integration by parts

Let $u = e^x$ and $dv = \frac{1}{(1+x)^2}$

Then, $du = e^x$ and $v = -\frac{1}{1+x}$

Applying the integration by parts formula:

$$\int_0^1 \frac{e^x}{(1+x)^2}dx = \left[-\frac{e^x}{1+x}\right]_0^1 - \int_0^1 -\frac{e^x}{1+x}dx$$

$$= \left(\frac{-e}{2}\right) - 1 + \alpha$$

$$= \alpha - \frac{e}{2} - 1$$

6. Option (B) is correct.

Explanation: Given,

$$\begin{bmatrix} x+y & 3y \\ 3x & x+3 \end{bmatrix} = \begin{bmatrix} 9 & 4x+y \\ x+6 & y \end{bmatrix}$$

On comparing corresponding elements of matrices, we get

$x + y = 9$ ……(i)

$3x = x + 6$

$x = 3$

And $x + 3 = y$

$y = 3 + 3 = 6$

The, $x - y = 3 - 6 = -3$

7. Option (A) is correct.

Explanation: $P(M) = 0.6$, $P(N) = 0.2$ and

$P(M \cap N) = 0.5,$

Now, $P(M \cup N) = P(M) + P(N) - P(M \cap N)$

$= 0.6 + 0.2 - 0.5 = 0.3$

$$[P(M'/N') = \frac{P(M' \cap N')}{P(N')}$$

$$= \frac{P((M \cup N)')}{1-P(N)}$$

$$= \frac{1-P(M \cup N)}{1-P(N)}$$

$$= \frac{1-0.3}{1-0.2} = \frac{0.7}{0.8} = \frac{7}{8}$$

11. Option (A & D) is correct.

Explanation: A is true.

If $|A| \neq 0$, then A is invertible.

The system has a unique solution given by:

$X = A^{-1}B$

The system is consistent (has a unique solution).

B is false.

Because if (adj A)B=0, the system is consistent (has infinitely many solutions).

C is false.

Because if $|A| \neq 0$, the system has a unique solution, meaning it is consistent.

D is true.

If $|A| = 0$, then A is not invertible.

The system may be:

Consistent with infinitely many solutions if (adj A) B=0.

Inconsistent (no solution) if (adj A) $B \neq 0$.

Thus, may or may not be consistent if $|A| = 0$ and (adj A)B=0.

13. Option (C) is correct.

Explanation: In the given differential equation, the highest order derivative is 2 and its degree is 1.

SECTION B

22. Given, $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$

Then, $f'(x) = \sqrt{3}\cos x + \sin x - 2a$

$$f'(x) = 2\left[\left(\frac{\sqrt{3}}{2}\right)\cos x + \left(\frac{1}{2}\right)\sin x\right] - 2a$$

$$f'(x) = 2\left[\left(\sin\frac{\pi}{3}\right)\cos x + \left(\cos\frac{\pi}{3}\right)\sin x\right] - 2a$$

$$f'(x) = 2\left[\sin\left(\frac{\pi}{3}+x\right) - a\right]$$

If $a > 1$, then for all real values of x, $f'(x) < 0$.

24. $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 4\sqrt{3}$

Since,

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$$

Then, $2\tan^{-1}x + 2\tan^{-1}x = 4\sqrt{3}$

$4\tan^{-1}x = 4\sqrt{3}$

$\tan^{-1}x = \sqrt{3}$

$\tan^{-1}x = \tan^{-1}(60°)$

$x = 60°$

SECTION C

26. Given constraints are :

$2x + 3y \le 6$

$3x - 2y \le 6$

$y \le 1$

$x, y \ge 0$

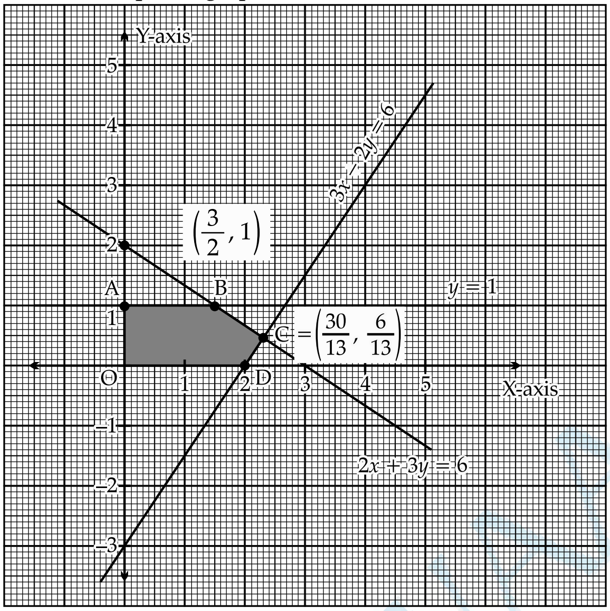
For the graph of $2x + 3y \le 6$, draw the graph of $2x + 3y = 6$

x	0	3
y	2	0

Putting origin in the equation, we get:
$2 \times 0 + 3 \times 0 \le 6$, so (0, 0) satisfy this constraint.
Hence, feasible region lie towards origin side of line.
For graph of $3x - 2y \le 6$.
We will draw the line graph of $3x - 2y = 6$

x	0	2
y	–3	0

Putting origin in the equation, we get:
$3 \times 0 - 2 \times 0 \le 6$, so (0, 0) satisfy this constraint.
Hence, feasible region lies towards origin side of line.
For graph of $y \le 1$
We draw the graph of line $y = 1$, which is parallel to x–axis and meet y–axis at 1. Putting origin in the equation, we get: $0 \le 1 \Rightarrow$ feasible region lies towards origin side of $y = 1$.
Also, $x \ge 0$, $y \ge 0$ says feasible region is in first quadrant.
The required graph is



Here, feasible region is bounded
Therefore, *OABCDO* is the required feasible region, having corner point $O(0, 0)$, $A(0, 1)$, $B\left(\frac{3}{2}, 1\right)$, $C\left(\frac{30}{13}, \frac{6}{13}\right)$, D (2, 0).

Corner points	$Z = 8x + 9y$
O (0, 0)	0
A (0, 1)	9
$B\left(\frac{3}{2}, 1\right)$	21
$C\left(\frac{30}{13}, \frac{6}{13}\right)$	22.6
D (2, 0)	16

Therefore, Z is maximum when $x = \frac{30}{13}$ and $y = \frac{6}{13}$. The maximum value of Z is 22.6

27. Let, $I = \int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$

Rewrite the fraction using partial–fraction decomposition
Then,

$$\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$$

Then, $2x - 1 = A(x + 2)(x - 3) + B(x - 1)(x - 3) + C(x - 1)(x + 2)$
Put $x = 3$, then $5 = 10C$

$$C = \frac{1}{2}$$

Put $x = -2$, then $-5 = 15B$

$$B = -\frac{1}{3}$$

Pur $x = 1$, then $1 = -6A$

$$A = -\frac{1}{6}$$

Therefore,

$$\frac{(2x-1)}{(x-1)(x+2)(x-3)} = \frac{-1}{6(x-1)} + \frac{-1}{3(x+2)} + \frac{1}{2(x-3)}$$

Now on integrating,

$$I = \int\left[-\frac{1}{6(x-1)} - \frac{1}{3(x+2)} + \frac{1}{2(x-3)}\right]dx$$

$$I = \frac{-1}{6}\cdot \ln(|x-1|) - \frac{1}{3}\cdot \ln(|x+2|) + \frac{1}{2}\cdot \ln(|x-3|)$$

OR

(b) Let $I = \int_0^5 (|x-1| + |x-2| + |x-5|)dx$

Let $f(x) = |x-1| + |x-2| + |x-5|$
We have three critical points $x = 1, 2, 5$
When $0 \le x < 1$
When $1 \le x < 2$
When $2 \le x \le 5$
$f(x) = -(x-1) - (x-2) - (x-5)$, if $0 \le x < 1$
$f(x) = (x-1) - (x-2) - (x-5)$, if $1 \le x < 2$
$f(x) = (x-1) + (x-2) - (x-5)$, if $2 \le x \le 5$
Therefore, $f(x) = -3x + 8$, if $0 \le x < 1$
$f(x) = -x + 6$, if $1 \le x < 2$
$f(x) = x + 2$, if $2 \le x \le 5$

$$I = \int_0^1 (8-3x)dx + \int_1^2 (6-x)dx + \int_2^5 (x+2)dx$$

$$= \left[8x - \frac{3}{2}x^2\right]_0^1 + \left[6x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} + 2x\right]_2^5$$

$$= \left(8 - \frac{3}{2}\right) + \left[(12-2) - \left(6 - \frac{1}{2}\right)\right] + \left[\left(\frac{25}{2} + 10\right) - (2+4)\right]$$

$$= \frac{13}{2} + \left(10 - \frac{11}{2}\right) + \frac{45}{2} - 6$$

$$= 4 + \frac{13}{2} - \frac{11}{2} + \frac{45}{2}$$

$$= 4 + \frac{47}{2}$$

= 4 + 23.5 = 27.5

28. Let V be the volume of the sphere and r be its radius at time t.

The volume of a sphere is given by: $V = \frac{4}{3}\pi r^3$ ...(i)

The surface area of a sphere is given by: $A = 4\pi r^2$ ...(ii)

According to the question, the rate of decrease of volume is proportional to the surface area.

$-\frac{dV}{dt} = kA$, where k is a positive constant of proportionality

On differentiating we get

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$$

Put the values of $\frac{dV}{dt}$ and A in Eq. (iii), we get

$$-4\pi r^2 \frac{dr}{dt} = k(4\pi r^2)$$

$$\frac{dr}{dt} = -k$$

This means the radius decreases at a constant rate k.

SECTION D

33.

Let $I = \int \sin^{-1}\sqrt{\frac{x}{a+x}}dx$

Putting $x = a\tan^2\theta \Rightarrow \tan\theta = \sqrt{\frac{x}{a}}$

$\Rightarrow dx = a(2\tan\theta)\sec^2\theta d\theta$

$$\therefore I = \int \sin^{-1}\sqrt{\frac{a\tan^2\theta}{a+a\tan^2\theta}}(2a\tan\theta\sec^2\theta)d\theta$$

$$= \int \sin^{-1}\sqrt{\frac{\tan^2\theta}{\sec^2\theta}}(2a\tan\theta\sec^2\theta)d\theta$$

$$= 2a\int \sin^{-1}(\sin\theta)\tan\theta\sec^2\theta d\theta$$

$$= 2a\int \theta\tan\theta\sec^2\theta d\theta$$

[Considering θ as first function and $\tan\theta\sec^2\theta$ as second function]

$$\therefore I = 2a\left[\frac{\theta\tan^2\theta}{2} - \int\frac{\tan^2\theta}{2}d\theta\right]$$

$$= a\left[\theta\tan^2\theta - \int(\sec^2\theta - 1)d\theta\right]$$

$$= a\left[\theta\tan^2\theta - \tan\theta + \theta\right] + C$$

$$= a\left[\theta(1+\tan^2\theta) - \tan\theta\right] + C$$

$$= a\left[\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)\left(1+\frac{x}{a}\right) - \frac{\sqrt{x}}{\sqrt{a}}\right] + C$$

$$= a\left[\frac{x+a}{a}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right) - \frac{\sqrt{x}}{\sqrt{a}}\right] + C$$

$$= (x+a)\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{a}}\right) - \sqrt{ax} + C$$

Outside Delhi Set-1 **65/2/1**

SECTION A

1. Option (B) is correct.

Explanation: Projection of vector $\vec{a}$ on vector $\vec{b}$ is $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$.

2. Option (D) is correct.

Explanation: Given, function is: $f(x) = x^2 - 4x + 6$

Then, $f'(x) = 2x - 4$

For function to be increasing, $f'(x) \geq 0$

$$2x - 4 \geq 0$$
$$2x \geq 4$$
$$x \geq 2$$

Then, function is increasing in the interval $[2, \infty)$

3. Option (D) is correct.

Explanation: Given, $f(2a - x) = f(x)$

Let, $I = \int_0^{2a} f(x) = 2\int_0^a f(x)dx$

Using additive property,

$$I = \int_0^a f(x)dx + \int_a^{2a} f(x)dx$$

Consider the integral $\int_a^{2a} f(x)dx$

Let $x = 2a - t$, then $dx = -dt$

When $x = a$, $t = a$ and $x = 2a$, $t = 0$

Therefore, $\int_0^{2a} f(x)dx = -\int_a^0 f(2a-t)dx$

$$= \int_0^a f(2a-t)dx$$

$$= \int_0^a f(2a-x)dx$$

We have $f(2a - x) = f(x)$

Therefore, $I = \int_0^a f(x)dx + \int_0^a f(x)dx$

$$= 2\int_0^a f(x)dx$$

4. Option (D) is correct.

Explanation: $A = \begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{bmatrix}$

Matrix A is symmetric, if

A = A′

$\therefore A' = \begin{bmatrix} 1 & 6x & 8x \\ 12 & 5 & 4 \\ 4y & 2x & 6 \end{bmatrix}$

Then, $\begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 6x & 8x \\ 12 & 5 & 4 \\ 4y & 2x & 6 \end{bmatrix}$

Then, $6x = 12$

$\Rightarrow$ $x = 2$

And $8x = 4y$

$y = \dfrac{8 \times 2}{4} = 4$

$\therefore 2x + y = 2 \times 2 + 4 = 8$

5. Option (B) is correct.

Explanation: The range of $y = \sin^{-1}x$ is $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ when domain is $[-1, 1]$.

If $-1 \le x \le 0$, then the range of $y = \sin^{-1}x$ is: $\left[-\dfrac{\pi}{2}, 0\right]$

6. Option (B) is correct.

Explanation: The angle with x-axis, $\alpha = \dfrac{3\pi}{4}$

The angle with y-axis, $\beta = \dfrac{\pi}{3}$

The angle with z-axis, $\gamma = \theta$

Now, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\cos^2\left(\frac{3\pi}{4}\right) + \cos^2\left(\frac{\pi}{3}\right) + \cos^2\theta = 1$$

$$\left(\frac{-1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \quad \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$$

$$\Rightarrow \quad \frac{3}{4} + \cos^2\theta = 1$$

$$\Rightarrow \quad \cos\theta = \pm\frac{1}{2}$$

Then, $\theta = \dfrac{\pi}{3}$

7. Option (D) is correct.

Explanation: Given, $P(E) > 0$ and $P(F) \ne 1$

$$P(\bar{E} \mid \bar{F}) = \frac{P(\bar{E} \cap \bar{F})}{P(\bar{F})}$$

$$= \frac{1 - P(E \cup F)}{P(\bar{F})}$$

8. Option (C) is correct.

Explanation: A matrix that is both symmetric and skew, symmetric is a zero matrix or null matrix.

9. Option (C) is correct.

Explanation: The Cartesian equation of the line passing through the point $(x, y, z,)$ and parallel to the vector $a\hat{i} + b\hat{j} + c\hat{k}$ is:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$x_1 = 4, y_1 = -3, z_1 = 7$

$a = 3, b = 1, c = 2$

Then, $\dfrac{x-4}{3} = \dfrac{y+3}{1} = \dfrac{z-7}{2} = t$

$x = 3t + 4$

$y = t - 3$

$z = 2t + 7$

10. Option (B) is correct.

Explanation: Given, $4AB + 3(AB + BA) - 4BA$

$= 4AB + 3AB + 3BA - 4BA$

$= 7AB - BA$

11. Option (C) is correct.

Explanation: Radius of tank, $r = 10$ cm

$\dfrac{dv}{dt} = 100\pi \text{ cm}^3/\text{s}$

Let, the height of tank be 'h' cm

$v = \pi r^2 h$

$\dfrac{dv}{dt} = \pi r^2 \dfrac{dh}{dt}$

$100\pi = \pi \times 10^2 \times \dfrac{dh}{dt}$

$\dfrac{dh}{dt} = 1$ cm/s

12. Option (D) is correct.

Explanation: $|\vec{p}| = |\vec{q}| = 1$

α is the angle between $\vec{p}$ and $\vec{q}$.

$$|\vec{p} + \vec{q}|^2 = 1^2$$

$$|\vec{p}|^2 + |\vec{q}|^2 + 2\vec{p} \cdot \vec{q} = 1$$

$$2 + 2\cos\alpha = 1$$

$$\cos\alpha = \frac{-1}{2}$$

$$\alpha = \frac{2\pi}{3}$$

13. Option (C) is correct.

Explanation: Given, line is $x = 1 + 5\mu$, $y = -5 + \mu$, $z = -6 - 3\mu$

Then, $\mu = \dfrac{x-1}{5}$, $\mu = \dfrac{y+5}{1}$, $\mu = \dfrac{z+6}{-3}$

Then, line passes through the point $(1, -5, -6)$

14. Option (B) is correct.

Explanation: Set A is continuous function and set B is a differentiable function.

If a function is differentiable at a point, then it must also be continuous at point. A continuous function does not necessarily have to be differentiable.

15. Option (D) is correct.

Explanation: Curve is, $y = x^2$
Then, area bounded by the curve
$\int_0^4 x\,dy = \int_0^4 \sqrt{y}\,dy$

16. Option (C) is correct.
Explanation: By property of objective function

17. Option (B) is correct.
Explanation: RHS $= (A - B)(A + B)$
$= A^2 + AB - BA - B^2$
This will become equal to LHS only when
$AB = BA$

18. Option (B) is correct.
Explanation: Given, $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$
$$3\left(\frac{dy}{dx}\right)^2 \frac{d^2y}{dx^2} = 0$$
Then, order, $p = 2$ and degree $q = 1$
Sum of order and degree
Then, $p - q = 2 - 1 = 1$

19. Option (D) is correct.
Explanation: In assertion, $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is a diagonal matrix, but not a scalar matrix because its diagonal elements are not equal.
The definition of scalar matrix given in reason is correct.

20. Option (D) is correct.
Explanation: No, not every point in the feasible region of an LPP is called an optimal solution.
The reason is infact true.

SECTION B

21. (a) If a vector $\vec{a}$ makes equal angles with all three axes, i.e., $\alpha = \beta = \gamma$
$3\cos^2\alpha = 1$
$\cos\alpha = \pm\frac{1}{\sqrt{3}}$
Now, components in each direction are equal.
Let $\vec{r} = a\hat{i} + a\hat{j} + a\hat{k}$
$\therefore |\vec{r}| = \sqrt{a^2 + a^2 + a^2} = \sqrt{3}a$
$|\sqrt{3}a| = 5\sqrt{3}$
$\Rightarrow a = \pm 5$
$\therefore \quad \vec{r} = 5\hat{i} + 5\hat{j} + 5\hat{k}$
or $-5\hat{i} - 5\hat{j} - 5\hat{k}$

OR

(b) Let, $\overrightarrow{OP} = \alpha$, $\overrightarrow{OQ} = \beta$ and $\overrightarrow{OR} = r$, when O is the origin.
Then, $\overrightarrow{QP} = \alpha - \beta$ and $\overrightarrow{QR} = r - \beta$
Give, QR $= \frac{3}{2}QP$
$$r - \beta = \frac{3}{2}(\alpha - \beta)$$
$$r = \beta + \frac{3}{2}(\alpha - \beta)$$
$$= \frac{3}{2}\alpha - \frac{1}{2}\beta$$
Hence , the position vector of point R is $\vec{r} = \frac{3}{2}\alpha - \frac{1}{2}\beta$

22. Let $I = \int_0^{\pi/4} \sqrt{1 + \sin 2x}\,dx$
$$= \int_0^{\pi/4} \sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x}.dx$$
$$= \int_0^{\pi/4} \sqrt{(\sin x + \cos x)^2}.dx$$
$$= \int_0^{\pi/4} (\sin x + \cos x).dx$$
$$= [-\cos x + \sin x]_0^{\pi/4}$$
$$= \left(\sin\frac{\pi}{4} - \cos\frac{\pi}{4}\right) - (\sin 0 - \cos 0)$$
$$= \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - (0 - 1)$$
$$= 1$$

23. We have, $f(x) = \sin x - ax + b$,
$f'(x) = \cos x - a$
For function to be increasing,
$f'(x) \geq 0$
$\cos x - a \geq 0$
Since, $\cos x \in [-1, 1]$
$\therefore a \leq -1$
$\Rightarrow a \in (-\infty, -1]$
Hence, the value of a is $(-\infty, -1)$.

24. $\vec{\alpha} = (x-2)\vec{a} + \vec{b}$ and $\vec{\beta} = (3+2x)\vec{a} - 2\vec{b}$ vector α and β collinear, if there exists a scalar λ such that :
$\alpha = \lambda\beta$
$\therefore (x-2)\vec{a} + \vec{b} = \lambda\left[(3+2x)\vec{a} - 2\vec{b}\right]$
$\Rightarrow (x-2)\vec{a} + \vec{b} = \lambda(3+2x)\vec{a} - 2\lambda\vec{b}$
For $\vec{a}$, $\quad x-2 = \lambda(3 + 2x) \quad$ ....(i)
For $\vec{b}$ $\quad 1 = -2\lambda$
$$\lambda = -\frac{1}{2}$$
Put $\lambda = -\frac{1}{2}$ in Eq. (i)
$$(x-2) = -\frac{1}{2}(2x+3)$$
$\Rightarrow \quad 2x - 4 = -2x - 3$
$\Rightarrow \quad 4x = 4 - 3$
$\Rightarrow \quad 4x = 1$
$\Rightarrow \quad x = \frac{1}{4}$
Hence, the value of x is $\frac{1}{4}$

25. (a) Given, $x = e^{x/y}$
Then, $\quad \log x = \frac{x}{y}$
$\Rightarrow \quad y\log x = x$
On differentiating both sides, we get

$\Rightarrow \frac{y}{x} + \log x \times \frac{dy}{dx} = 1$

$\Rightarrow \quad \log x \frac{dy}{dx} = 1 - \frac{y}{x}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{x-y}{x \log x}$

OR

(b) Given $f(x) = \begin{cases} 2x-3 & -3 \le x \le -2 \\ x+1 & -2 < x \le 0 \end{cases}$

Since, polynomial functions are continuous and differentiable at everywhere . So, $f(x)$ is differentiable on

$x \in [-3, -2]$ and $x \in (-2, 0]$

We will check the differentiability at $x = -2$

$$(\text{LHD at } x = -2) = \lim_{x\to-2^-} \frac{f(x)-f(-2)}{x-(-2)}$$

$$= \lim_{x\to-2^-} \frac{(2x-3)-(-1)}{x+2}$$

$$= \lim_{x\to-2^-} \frac{2x+4}{x+2} = 2$$

$$(\text{RHD at } x = -2) = \lim_{x\to-2^+} \frac{(x+1)-(-1)}{x+2} = \lim_{x\to-2^+} \frac{x+2}{x+2} = 1$$

Since, LHD at $x = -2 \neq$ RHD at $x = -2$

So, $f(x)$ is not differentiable at $x = -2$

SECTION C

26. (a) Given, differential equation is :

$$2(y+3) - xy\frac{dy}{dx} = 0$$

$\Rightarrow \quad 2(y+3) = xy\frac{dy}{dx}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2(y+3)}{xy}$

$\Rightarrow \quad \frac{y}{y+3}dy = \frac{2dx}{x}$

$\Rightarrow \quad \frac{(y+3-3)}{y+3}dy = \frac{2dx}{x}$

$\Rightarrow \quad \left[1 - \frac{3}{y+3}\right]dy = \frac{2dx}{x}$

On integrating both sides, we get

$\Rightarrow \int 1 \cdot dy - \int \frac{3}{y+3} \cdot dy = 2\int \frac{dx}{x}$

$\Rightarrow$ $y - 3[\log(y+3)] = 2\log x + C$

Put $\quad y(1) = -2$

$\Rightarrow \quad -2 - 3\log(1) = 2\log 1 + C$

$\Rightarrow \quad -2 = C$

$\Rightarrow \quad C = -2$

$\Rightarrow \quad y - 3\log(y+3) = 2\log x - 2$

is the particular solution of the differential equation

OR

(b) $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$

$\Rightarrow \frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{4x^2}{1+x^2}$

This is a linear d.e. of first order of the form :

$\frac{dy}{dx} + py = Q$

These, $\quad$ I.F $= e^{\int P\,dx}$

$= e^{\int\left(\frac{2x}{1+x^2}\right)dx}$

$= e^{\log(1+x^2)} = 1 + x^2$

The required solution is,

$y\,(\text{I.F.}) = \int Q(I \cdot F \cdot)dx + C$

$y\,(1+x^2) = \int\left(\frac{4x^2}{1+x^2}\right)(1+x^2) + C$

$= \int 4x^2 + C$

$\Rightarrow \quad y(1+x^2) = \frac{4x^3}{3} + C$

27. We know that a number is a factor as well as multiple of itself .

Since, 'n' will be a multiple of n for any natural number, hence (n, n) will be a subset of the relation . This makes the relation reflexive.

If m is a multiple of n, then n is not a multiple of m. $m\ R\ n \to n\ R\ m$ is not true . Thus, the relation is not symmetric.

Now, m is a multiple of n, n is a multiple of p. Then m is a multiple of p.

Thus, the relation is transitive .

Hence, the given relation is reflexive and transitive but not symmetric.

28. Minimise, $Z = x - 5y$

Subject to constraints

$x - y \ge 0$

$-x + 2y \ge 2$

$x \ge 3, y \le 4, y \ge 0$

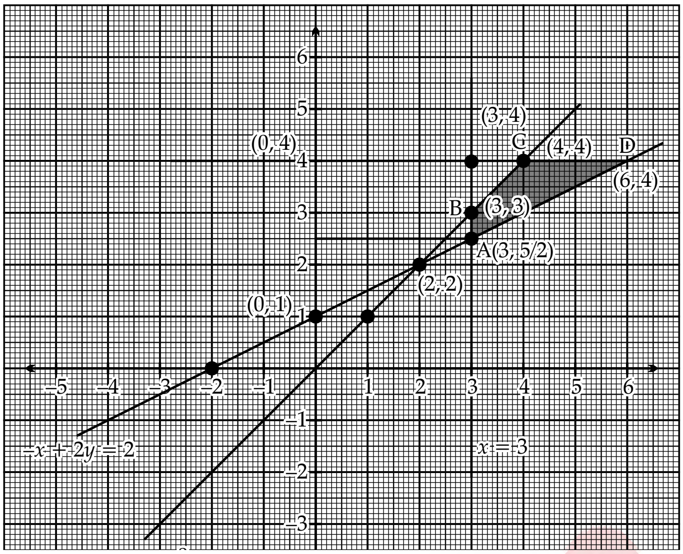
convert the in equations to equations then plot them on graph.

$x - y = 0$

x	1	2	3
y	1	2	3

$-x + 2y = 2$

x	0	-2	3
y	1	0	5/2



The corner points of flexible region are

A $\left(3,\frac{5}{2}\right)$, B (3, 3), C (4,4) and D (6,4)

corner points	$Z = x - 5y$
A $\left(3,\frac{5}{2}\right)$	$-\frac{19}{2} \simeq -9.5$
B (3, 3)	−12
C (4, 4)	−16
D (6, 4)	−14

Therefore, the minimum value of Z is –16 at point C (4, 4).

29. **(a)** Given, $y = \log\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$

$$y_1 = 2\log\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$$

On differentiating both sides, we get

$$y_1 = 2\times\frac{1}{\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)}\times\left(\frac{1}{2\sqrt{x}}-\frac{1}{2x^{3/2}}\right)$$

$$= \frac{2\sqrt{x}}{(x+1)}\times\frac{1}{2\sqrt{x}}\left(1-\frac{1}{x}\right)$$

$$= \frac{1}{x+1}\times\frac{(x-1)}{x}$$

$$\Rightarrow \quad xy_1 = \frac{x-1}{x+1}$$

On differentiating both sides, we get

$$y_1 + xy_2 = \frac{(x+1)(1)-(x-1)\cdot 1}{(x+1)^2}$$

$$\Rightarrow \quad y_1 + xy_2 = \frac{x+1-x+1}{(x+1)^2}$$

$\Rightarrow x\,(x+1)^2 y_2 + (x+1)^2 y_1 = 2$

Hence, Proved

OR

(b) Given $x\sqrt{1+y}+y\sqrt{1+x}=0$

$$\Rightarrow \quad x\sqrt{1+y} = -y\sqrt{1+x}$$

Squaring on both sides, we get

$$\Rightarrow \quad x^2(1+y) = y^2(1+x)$$

$$\Rightarrow \quad x^2 + x^2y = y^2 + xy^2$$

$$\Rightarrow \quad x^2 - y^2 = xy^2 - x^2y$$

$$\Rightarrow \quad (x-y)(x+y) = xy(y-x)$$

$$\Rightarrow \quad (x+y) = -xy$$

$$\Rightarrow \quad (1+x)y = -x$$

$$y = \frac{-x}{1+x}$$

Differentiating on both sides w.r.t. x,we get

$$\frac{dy}{dx} = -\frac{(1+x)\frac{d}{dx}x - x\frac{d}{dx}(1+x)}{(1+x)^2}$$

$$= -\frac{(1+x)-x}{(1+x)^2} = \frac{-1}{(1+x)^2}$$

$$\frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

Hence, Proved

30. **(a)** Let $p(1) = p(3) = p(4) = p(5) = p(6) = x$

We are given that $p(2) = \frac{3}{10}$

$$\therefore\ 5x + \frac{3}{10} = 1 \Rightarrow x = \frac{1}{5}\left(\frac{7}{10}\right) = \frac{7}{50}$$

Let x be the r.v representing the number of times, the number 2 appears when the die is thrown twice, x can take value 0, 1, 2.

Here, $p(X) = \frac{3}{10}$

$$P\left(\overline{X}\right) = 1 - \frac{3}{10} = \frac{7}{10}$$

X	0	1	2
$P(x)$	$\frac{7}{10}\times\frac{7}{10}=\frac{49}{100}$	$\frac{3}{10}\times\frac{7}{10}+\frac{7}{10}\times\frac{3}{10}=\frac{42}{100}$	$\frac{3}{10}\times\frac{3}{10}=\frac{9}{100}$

$$E(X) = \Sigma x_i\, p(x = x_i)$$

$$= 0\times\frac{49}{100}+1\times\frac{42}{100}+2\times\frac{9}{100}$$

$$= \frac{42}{100}+\frac{18}{100}=\frac{60}{100}=\frac{3}{5} = 0.6$$

OR

(b) Event A = $\{(x, y): x + y = 9\}$

Event B = $\{(x, y): x \neq 3\}$

When two dice are thrown, there is a total of 36 outcomes.

$\therefore$ A = {(3, 6), (4, 5), (5, 4), (6, 3)}

n(A) = 4

$\therefore$ B = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6) (4, 1) (4, 2) (4, 3) (4, 5) (2, 6) (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)}

n(B) = 30

$n(A\cap B) = 3$

$P(A) = \frac{4}{36}=\frac{1}{9}$, $P(B) = \frac{30}{36}=\frac{10}{12}=\frac{5}{6}$

$P(A\cap B)= \frac{3}{36}=\frac{1}{12}$

$P(A)\,.\,P(B) = \frac{1}{9}\cdot\frac{5}{6}=\frac{5}{54}$

$P(A\cap B) = \frac{1}{12} \neq P(A)\cdot P(B)$

So, A and B are not independent.

31. Let

$$I = \int\frac{1}{x}\sqrt{\frac{x+a}{x-a}}dx$$

$$= \int\frac{1}{x}\sqrt{\frac{(x+a)(x+a)}{(x-a)(x+a)}}\cdot dx$$

$$= \int\frac{1}{x}\times\frac{x+a}{\sqrt{x^2-a^2}}dx$$

$$= \int\frac{1}{x}\times\frac{x}{\sqrt{x^2-a^2}}dx+\int\frac{1}{x}\times\frac{a}{\sqrt{x^2-a^2}}dx$$

$$= \int\frac{1}{\sqrt{x^2-a^2}}dx+a\int\frac{1}{x\sqrt{x^2-a^2}}dx$$

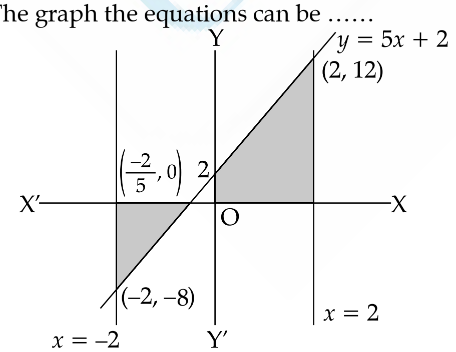
$$= \log\left|x+\sqrt{x^2-a^2}\right|+\frac{a}{a}\sec^{-1}\left(\frac{x}{a}\right)+C$$

$$= \log\left|x+\sqrt{x^2-a^2}\right|+\sec^{-1}\left(\frac{x}{a}\right)+C$$

SECTION D

32. Given, line is $y = 5x + 2$, x-axis and ordinates $x = -2$ and $x = 2$

The graph the equations can be ......



$$\text{Required area} = \int_{-2}^{-2/5}|5x+2|dx+\int_{2/5}^{2}|5x+2|dx+$$

$$I_1 = \int_{-2}^{-2/5}(5x+2)dx$$

$$I_1 = \left[\frac{5x^2}{2}+2x\right]_{-2}^{-2/5}$$

$$= \left(\frac{5(-2/5)^2}{2}+2(-2/5)\right)-\left(\frac{5(-2)^2}{2}+2(-2)\right)$$

$$= \left(\frac{5(4/25)}{2}-\frac{4}{5}\right)-\left(\frac{5(4)}{2}+4\right)$$

$$= \left(\frac{20}{50}-\frac{4}{5}\right)-\left(\frac{20}{2}-4\right)$$

$$= \left(\frac{2}{5}-\frac{4}{5}\right)-(10-4)$$

$$= \left(-\frac{2}{5}\right)-6$$

$$= -\frac{2}{5}-6=-\frac{32}{5}$$

$$I_2 = \int_{-2/5}^{-2}(5x+2)dx$$

$$I_2 = \left[\frac{5x^2}{2}+2x\right]_{-2/5}^{2}$$

$$= \left(\frac{5(2)^2}{2}+2(2)\right)-\left(\frac{5(-2/5)^2}{2}+2(-2/5)\right)$$

$$= \left(\frac{5(4)}{2}+4\right)-\left(\frac{5(4/25)}{2}-\frac{4}{5}\right)$$

$$= \left(\frac{20}{2}+4\right)-\left(\frac{20}{50}-\frac{4}{5}\right)$$

$$= (10+4)-\left(\frac{2}{5}-\frac{4}{5}\right)$$

$$= 14-\left(-\frac{2}{5}\right)$$

$$= = 14+\frac{2}{5}=\frac{70}{5}+\frac{2}{5}=\frac{72}{5}$$

$$A = |I_1|+|I_2|$$

$$= \left|-\frac{32}{5}\right|+\left|\frac{72}{5}\right|$$

$$= \frac{32}{5}+\frac{72}{5}$$

$$= \frac{104}{5}$$

$$=20.8 \text{ sq.units}$$

33. Let $I = \int\frac{x^2+x+1}{(x+2)(x^2+1)}dx$

We will apply partial fraction decomposition

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1} \quad \text{....(i)}$$

Then, $x^2 + x + 1 = A(x^2 + 1) + (Bx + C)(x + 2)$

$\Rightarrow \quad x^2 + x + 1 = A(x^2 + 1) + Bx^2 + 2Bx + Cx + 2C$

$= (A + B)x^2 + (2B + C)x + (A + 2C)$

$\therefore \quad A + B = 1 \quad \text{...(i)}$

$2B + C = 1 \quad \text{... (ii)}$

and $\quad A + 2C = 1 \quad \text{... (iii)}$

From Eq. (i), $\quad B = 1 - A$

Put the value of B in Eq. (ii), we get

$\Rightarrow \quad 2(1 - A) + C = 1$

$\Rightarrow \quad 2 - 2A + C = 1$

$\Rightarrow \quad -2A + C = -1$

Solving Eq. (iii) and (iv), we get $A = \frac{3}{5}$ and $C = \frac{1}{5}$

$\therefore B = \frac{2}{5}$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{3/5}{x+2} + \frac{\left(2/5\right)x + \left(1/5\right)}{x^2+1}$$

$$\therefore \quad I = \int \frac{3/5}{(x+2)} + \frac{\left(2/5\right)x + \left(1/5\right)}{x^2+1}dx$$

$$= \frac{3}{5}\int \frac{1}{x+2}dx + \frac{2}{5}\int \frac{x}{x^2+1}dx + \frac{1}{5}\int \frac{1}{x^2+1}dx$$

$$= \frac{3}{5}\log(x+2) + \frac{1}{5}\log(x^2+1) + \frac{1}{5}\tan^{-1}x + C$$

34. **(a)** Equation of line 1:

$$\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$$

Equation of line 2:

$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$$

Shortest distance between the lines is

Here $\vec{a}_1 = -\hat{i} + \hat{j} + 9\hat{k}$

$\vec{a}_2 = 3\hat{i} - 15\hat{j} + 9\hat{k}$

$\therefore \vec{a}_2 - \vec{a}_1 = \left(3\hat{i} - 15\hat{j} + 9\hat{k}\right) - \left(-\hat{i} + \hat{j} + 9\hat{k}\right)$

$= 4\hat{i} - 16\hat{j}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 2 & -7 & 5 \end{vmatrix} = (5-21)\hat{j} - (10+6)\hat{j} + (-14-2)\hat{k}$$

$= -16\hat{i} - 16\hat{j} - 16\hat{k}$

$\left(\vec{a}_2 - \vec{a}_1\right)\cdot\left(\vec{b}_1 \times \vec{b}_2\right) = \left(4\hat{i} - 16\hat{j}\right)\cdot\left(-16\hat{i} - 16\hat{j} - 16\hat{k}\right)$

$= -64 + 256$

$= 192$

$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(-16)^2 + (-16)^2 + (-16)^2} = 16\sqrt{3}$

$d = \frac{|192|}{16\sqrt{3}} = \frac{192}{16\sqrt{3}} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ units

OR

(b) Line $\ell : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\left(\hat{i} - \hat{j} - \hat{k}\right)$

Coordinates of point A (2,1,2)

Cartesian equation of the line is :

$$\frac{x-4}{1} = \frac{y-2}{-1} = \frac{z-2}{-1} = \lambda \text{ (Say)} \quad \text{...(i)}$$

Then Point B is $(\lambda+4, -\lambda+2, -\lambda+2)$

DRS of AB are $(\lambda + 4 - 2, -\lambda + 2 - 1, -\lambda + 2 - 2)$

i.e., $(\lambda + 2, -\lambda + 1, -\lambda)$

As given equation of line is perpendicular to A B

$\therefore 1.(\lambda+2) + (-1)(-\lambda+1) + (-1)(-\lambda) = 0$

$\Rightarrow \quad \lambda + 2 - (-\lambda+1) - (-\lambda) = 0$

$\Rightarrow \quad \lambda + 2 + \lambda - 1 + \lambda = 0$

$\Rightarrow \quad 3\lambda + 1 = 0$

$\lambda = -\frac{1}{3}$

DRS of AB is $\left(\frac{5}{3}, \frac{4}{3}, \frac{1}{3}\right)$

$\therefore$ Equation of line AA′ is

$$\frac{x-2}{5/3} = \frac{y-1}{4} = \frac{x-2}{1/3}$$

$$\Rightarrow \frac{3(x-2)}{5} = \frac{3(y-1)}{4} = 3(x-2) \quad \text{... (ii)}$$

Coordinate of B $= (\lambda+4, -\lambda+2, -\lambda+2)$

$$= \left(-\frac{1}{3}+4, -\left(-\frac{1}{3}\right)+2, -\left(-\frac{1}{3}\right)+2\right)$$

$$= \left(\frac{11}{3}, \frac{7}{3}, \frac{7}{3}\right)$$

If A′ (α, β, γ) is the image of A and B is the mid point of line AA′

Now,

$$\frac{2+\alpha}{2} = \frac{11}{3}, \frac{1+\beta}{2} = \frac{7}{3}, \frac{2+\gamma}{2} = \frac{7}{3}$$

$$\Rightarrow \alpha = \frac{16}{3}, \beta = \frac{11}{3}, \gamma = \frac{8}{3}$$

$$\text{A}'(\alpha, \beta, \gamma) = \left(\frac{16}{3}, \frac{11}{3}, \frac{8}{3}\right)$$

Equation af line AA'

$$\vec{r} = \left(2\hat{i} + \hat{j} + 2\hat{k}\right) + \left(\left(\frac{16}{3} - 2\right)\hat{i} + \left(\frac{11}{3} - 1\right)\hat{j} + \left(\frac{8}{3} - 2\right)\hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} + \hat{j} + 2\hat{k}\right) + \left(\frac{10}{3}\hat{i} + \frac{8}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$$

35. **(a)** Given, $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

Then, $AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$AB = 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 8I$$

Given equation is

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

Here, $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$

$\therefore$ $BX = C$

$$X = B^{-1}C$$

$$X = \frac{1}{8}AC \text{(from (i))}$$

$$= \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -16 & +36 & +4 \\ -28 & +9 & +3 \\ 20 & -27 & -1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$\therefore x = 3, y = -2, \text{ and } z = -1$

OR

(b) Given, $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$

$$|A| = 1(-1 - 2) - 2(-2 - 0) + 0$$
$$= 1(-3) - 2(-2)$$
$$= -3 + 4 = 1 \neq 0$$

So, A^{-1} will exist

Now, $A^{-1} = \dfrac{adj|A|}{|A|}$

Let a_{ij} be the co-factors of the elements a_{ij} in $A = [a_{ij}]$ then,

$$C_{11} = (-1)^{1+1}(-1 - 2) = -3$$
$$C_{12} = (-1)^{1+2}(-2 - 0) = 2$$
$$C_{13} = (-1)^{1+3}(2 - 0) = 2$$
$$C_{21} = (-1)^{2+1}(2 - 0) = -2$$
$$C_{22} = (-1)^{2+2}(1 - 0) = 1$$
$$C_{23} = (-1)^{2+3}(-1 - 0) = 1$$
$$C_{31} = (-1)^{3+1}(-4 - 0) = -4$$
$$C_{32} = (-1)^{3+2}(-2 - 0) = 2$$
$$C_{33} = (-1)^{3+3}(-1 + 4) = 3$$

$\therefore$ $\text{adj. } A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}^T$

$$= \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

Given, equations are :

$x - 2y = 10$

$2x - y - z = 8$

$-2y + z = 7$ This is of form $CX = D$

Then, $C = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, D = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

Here, $C = A^T$

Now, $CX = D$

$$C^{-1}CX = C^{-1}D$$
$$X = C^{-1}D$$
$$X = [A^T]^{-1}D$$
$$= [A^{-1}]^T D$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} -30 & +16 & +14 \\ -20 & +8 & +7 \\ -40 & +16 & +21 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

Hence, $x = 0, y = -5$ and $z = -3$

SECTION E

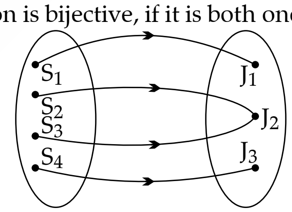
36. **(i)** $S = \{S_1, S_2, S_3, S_4\}$

$J = \{J_1, J_2, J_3\}$

Here, $n(S) = 4$, $n(J) = 3$

$$\text{No . of relations} = 2\, n^{(S) \times n(J)}$$
$$= 2^{\,4\times3}$$
$$= 2^{12}$$

(ii) A function is bijective, if it is both one-one and onto



Here, S_2 and S_3 have same images as J_2 . So, function is not one-one

But function is onto.

So, the given function is not bijective.

(iii) **(a)** $n(S) = 4$, $n(J) = 3$

Since $n(S) > n(J)$,

no; of one-one functions from S to $J = 0$

OR

(b) For function to be reflexive ordered pairs to be added are : $\{(S_1, S_1)\ (S_2, S_2), (S_3, S_3)\ (S_4, S_4)\}$

So, the relation $R_1 = \{(S_1, S_1), (S_1, S_2), (S_2, S_2), (S_2, S_4) (S_3, S_3) (S_4, S_4)\}$

So, the relation is reflexive but not symmetric

Minimum number of ordered pairs to be added = 4

37. (i) (a) Amber = 60% market share, 20% electric cars

Bonzi = 30% market share, 10% electric cars

Comet = 10% market share, 5% electric cars

$$P(\text{electric car}) = \frac{60}{100}\times\frac{20}{100}+\frac{30}{100}\times\frac{10}{100}+\frac{10}{100}\times\frac{5}{100}$$

$$= 0.12 + 0.03 + 0.005$$

$$= 0.155 = \frac{155}{1000} = \frac{31}{200}$$

OR

(b) Amber = 60% market share, 80% petrol cars

Bonzi = 30% market share, 90% petrol cars

Comet = 10% market share, 95% petrol cars

P(petrol car)

$$= (0.60 \times 0.80) + (0.30 \times 0.90) + (0.10 \times 0.95)$$

$$= 0.48 + 0.27 + 0.095$$

$$= 0.845 = \frac{845}{1000} = \frac{169}{200}$$

(ii) $$p\left(\frac{\text{Comet}}{\text{Electric}}\right) = \frac{p(\text{Comet} \cap \text{Electric})}{p(\text{Electric})}$$

$$= \frac{0.05\times 0.10}{0.155} = \frac{5}{155} = \frac{1}{31}$$

(iii) p(Amber or Bonzi|Electric) = [p(Electric|Amber or Bonzi + p(Amber or Bozi|p(Electric)]

p(Amber or Bozi) = 0.60 + 0.30

= 0.90

p(Electric|Amber) = 0.20

p(Electric|Boniz)= 0.10

p [Electric|Amber or Bonzi]

$$= \frac{[(0.60\times 0.20)+(0.30\times 0.10)]}{0.90}$$

$$\frac{0.12+0.03}{0.90} = \frac{0.15}{0.90}$$

$$= \frac{1}{6}$$

$$\text{Required probability} = \frac{\left[\left(\frac{1}{6}\right)\times 0.90\right]}{0.155}$$

$$= \frac{0.15}{0.155} = \frac{150}{155}$$

$$= \frac{30}{31}$$

38. (i) Given $f(x) = e^x \sin x$

$f'(x) = e^x \sin x + e^x \cos x$

$= e^x(\sin x + \cos x)$

For finding critical points, put $f'(x) = 0$

$e^x(\sin x + \cos x) = 0$

$\tan x = -1$

Then, in the interval $[0,\pi]$, the solution is $x = \frac{3\pi}{4}$.

For $x \in \left[0, \frac{3\pi}{4}\right)$, $e^x(\sin x + \cos x) > 0$, so $f'(x) > 0$ and $f(x)$ is increasing.

For $x \in \left(\frac{3\pi}{4}, \pi\right]$ $e^x(\sin x + \cos x) < 0$, $f'(x) < 0$ and $f(x)$ is decreasing.

(ii) For finding the nature of the critical points at $x = \frac{3\pi}{4}$, calculate $f''(x)$.

$f''(x) = e^x(\sin x + \cos x) + e^x(\cos x - \sin x)$

$= 2\, e^x \cos x$

$$\text{Now, } f''\left(\frac{3\pi}{4}\right) = 2e^{3\pi/4}\cos\left(\frac{3\pi}{4}\right)$$

$$= 2e^{3\pi/4}\left(-\frac{\sqrt{2}}{2}\right)$$

$$= -\sqrt{2}\, e^{3\pi/4}$$

Since, $f''\left(\frac{3\pi}{4}\right) < 0$, the critical point at

$x = \frac{3\pi}{4}$ is a local maximum.

At $x = 0$, $f(0) = 0$

At $x = \pi$, $f(\pi) = 0$

Hence, local maximum is at $x = \frac{3\pi}{4}$

Point of inflexion are at $x = 0$ and $x = \pi$.

Outside Delhi Set-2 **65/2/2**

SECTION A

1. Option (C) is correct

Explanation:

Given that,

$$\vec{a} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$$

$$\vec{b} = 7\hat{i} - 2\hat{j} + x\hat{k}$$

Let the angle between the vectors be θ

As the angle between the vectors is obtuse,

So, $\cos\theta < 0$

$$\frac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|} < 0 \quad \left(\cos\theta = \frac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|}\right)$$

$\Rightarrow \quad \vec{a}.\vec{b} < 0$

$\Rightarrow \quad 14x^2 - 8x + x < 0$

$\Rightarrow \quad 14x^2 - 7x < 0$

$\Rightarrow \quad 7x(2x - 1) < 0$

$\Rightarrow \quad x < \frac{1}{2}$

(On simplifying the inequality)

$\Rightarrow \quad x \in (0, \frac{1}{2})$

The values of x for which the angle between the given vectors is obtuse is $\left(0, \frac{1}{2}\right)$.

3. Option (C) is correct

Explanation:

$$\int \frac{dx}{\sin^2 x \cdot \cos^2 x}$$

$$= \int \left(\frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x}\right) dx \quad \{\text{using } \sin^2 x + \cos^2 x = 1\}$$

$$\int \left(\frac{\sin^2 x}{\sin^2 x \cdot \cos^2 x}\right) dx + \int \left(\frac{\cos^2 x}{\sin^2 x \cdot \cos^2 x}\right) dx$$

$$= \tan x - \cot x + C$$

4. Option (B) is correct

Explanation:

Given that,

P is skew-symmetric matrix of order 3

$$\det(P) = \alpha$$

We know that,

For a skew-symmetric matrix of **odd order**, such as a 3×3 matrix, the determinant of P is **always zero**.

$\Rightarrow \quad \det(P) = \alpha = 0$

So, $\quad = (2025)^0$

$\quad = 1$

5. Option (B) is correct

Explanation: Principal value of

$$\sin^{-1}\left(\cos\frac{43\pi}{5}\right)$$

$$= \sin^{-1}\left(\cos\left(8\pi + \frac{3\pi}{5}\right)\right)$$

$$= \sin^{-1}\left(\cos\frac{3\pi}{5}\right)$$

$$= \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)\right\} \quad \left[\therefore \cos\frac{3\pi}{5} = \sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)\right]$$

$$= \sin^{-1}\left\{\sin\left(\frac{-\pi}{10}\right)\right\}$$

$$= \frac{-\pi}{10}$$

11. Option (A) is correct

Explanation:

For given probability distribution,

X	0	1	2	3
$P(X)$	p	p	0.3	$2p$

$\sum P(X) = 1$

$\Rightarrow P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1$

$\Rightarrow \quad p + p + 0.3 + 2p = 1$

$\Rightarrow \quad 4p + 0.3 = 1$

$\Rightarrow \quad 4p = 0.7$

$\Rightarrow \quad 4p = \frac{7}{10}$

$\Rightarrow \quad p = \frac{7}{40}$

So, the value of p is $\frac{7}{40}$.

12. Option (B) is correct

Explanation:

Given that,

$\overrightarrow{PQ} \times \overrightarrow{PR} = 4\hat{i} + 8\hat{j} - 8\hat{k}$

Area $(\Delta PQR) = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}|$

Now,

$|\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{(4)^2 + (8)^2 + (-8)^2}$

$|\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{16 + 64 + 64}$

$|\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{144}$

$|\overrightarrow{PQ} \times \overrightarrow{PR}| = 12$

So,

Area $(\Delta PQR) = \frac{1}{2} \times 12 = 6$ sq units

SECTION – B

22. Let $I = \int_0^\pi \frac{\sin 2px}{\sin x} dx$, $P \in N$

Using the property of definite integrals,

$\int_0^a f(x)dx = \int_0^a f(a-x)dx$

So, substituting $x \to \pi - x$ in given integral.

$$I = \int_0^\pi \frac{\sin(2p(\pi - x))}{\sin(\pi - x)} dx$$

Using the identities:

$\sin(\pi - x) = \sin x$

$\sin(2p(\pi - x)) = -\sin(2px)$

$$I = \int_0^\pi \frac{-\sin 2px}{\sin x} dx$$

$I = -I$

$2I = 0$

$I = 0$

Therefore, $\int_0^\pi \frac{\sin 2px}{\sin x} dx = 0$

24. Given that,

$\vec{p} = 2\hat{i} - 3\hat{j} - \hat{k}$

$\vec{q} = -3\hat{i} + 4\hat{j} + \hat{k}$

$\vec{r} = \hat{i} + \hat{j} + 2\hat{k}$

Expressing $\vec{r}$ in the form of,

$\vec{r} = \lambda\vec{p} + \mu\vec{q}$

$\vec{r} = \lambda\left(2\hat{i} - 3\hat{j} - \hat{k}\right) + \mu\left(-3\hat{i} + 4\hat{j} + \hat{k}\right)$

$\hat{i} + \hat{j} + 2\hat{k} = \lambda\left(2\hat{i} - 3\hat{j} - \hat{k}\right) + \mu\left(-3\hat{i} + 4\hat{j} + \hat{k}\right)$

$$\hat{i}+\hat{j}+2\hat{k}=(2\lambda-3\mu)\hat{i}+(-3\lambda+4\mu)\hat{j}+(-\lambda+\mu)\hat{k}$$

On comparing the coefficients of both sides,

$2\lambda - 3\mu = 1$ ....(i)

$-3\lambda + 4\mu = 1$ ....(ii)

$-\lambda + \mu = 2$ ....(iii)

On simplifying Eqs. (i), (ii) & (iii),

We get,

$\lambda = -7$ and $\mu = -5$

SECTION C

27. To prove: f: N⇒N defined as $f(x) = ax+b$ (a, $b \in$ N) is one–one but not onto.

Proof:

Given that,

$f(x) = ax+b \quad (a, b \in \text{N})$

Case 1: f is injective(one–one)

A function is one–one (or injective) if for every pair of distinct elements $x_1, x_2 \in$ N, we have:

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

For the given function, $f(x) = ax + b$, assume that $f(x_1) = f(x_2)$, so:

$ax_1 + b = ax_2 + b$

By subtracting b from both sides, we get:

$ax_1 = ax_2$

$\Rightarrow x_1 = x_2$

Therefore, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$, which shows that f is one–one(injective).

Case2: f is not onto (surjective)

A function is **onto** (or surjective) if for every element $y \in$ N, there exists an $x \in$ N such that:

$f(x) = y$

Now, for the given function $f(x) = ax + b$,

Suppose for some $y \in$ N, we want to find an x such that:

$ax + b = y$

On rearranging the equation:

$ax = y - b$

$x = (y - b)/a$

Now, for x to be a natural number (i.e., $x \in$ N), the expression $(y - b)$/a must be a positive integer.

However, there are values of $y \in$ N for which $(y - b)$/a is not a natural number.

For example,

For $y = 4$, $a = 2$ and $b = 1$, (1, 2, 4 ∈ N)

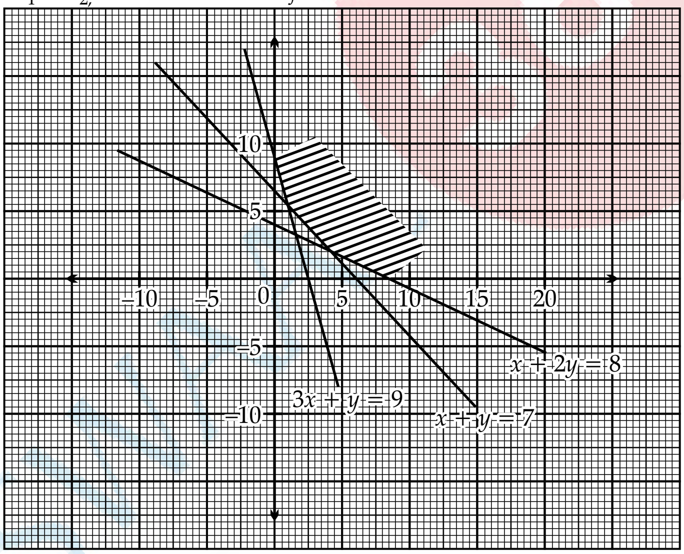
$$\Rightarrow x = \frac{(4-1)}{2} = \frac{3}{2} \notin \text{N}$$

This shows that there are some values of y for which no $x \in$ N satisfies $f(x) = y$.

Therefore, $f(x)=ax+b$ is **not onto**.

Hence proved.

28. From the given graph:



The corner points of the feasible region are labelled as:

A(0, 30), B(15, 25), C(30, 10), D(35, 0),

E(0, 70) (not a part of the feasible region but present on the graph)

Observing the graph, the feasible region is constrained by the following lines:

Equation 1 (eq1): Passing through points (30,10) and (35,0).

Equation 2 (eq2): Passing through points (0,30) and (30,10).

Equation 3 (eq3): Passing through points (0,30) and (15,25).

Additionally, there are **non**–negativity **constraints**:

$x \geq 0$ and $y \geq 0$

Let the variables be x and y

Constraint from (eq1):

Equation of line passing through (30, 10) and (35, 0)

$\Rightarrow y = -2x + 70$

Constraint is $y \leq -2x + 70$ ....(i)

Constraint from (eq2):

Equation of line passing through (0,30) and (30,10)

$$\Rightarrow y = \left(\frac{-2}{3}\right)x + 30$$

Constraint is $y \leq \left(\frac{-2}{3}\right)x + 30$ ....(ii)

Constraint from (eq3):

Equation of line passing through (0,30) and (15,25)

$\Rightarrow y = \left(\frac{-1}{3}\right)x + 30$

Constraint is $y \le \left(\frac{-1}{3}\right)x + 30$ ....(iii)

So, from Eqs. (i), (ii), (iii) and including non–negativity all constraints are as follows:

$y \le -2x + 70,$

$y \le \left(\frac{-2}{3}\right)x + 30,$

$y \le \left(\frac{-1}{3}\right)x + 30,$

$x \ge 0$ and $y \ge 0.$

31. Given that,

f and g are continuous functions on interval $[a, b]$

$f(a - x) = f(x)$ and $g(x) + g(a - x) = a$

To prove: $\int_0^a f(x)g(x)dx = \frac{a}{2}\int_0^a f(x)dx$

Taking LHS,

$I = \int_0^a f(x)\cdot g(x)dx$ ....(i)

$I = \int_0^a f(a-x)\cdot g(a-x)dx$

$\because \int_0^m f(x)dx = \int_0^m f(m-x)dx$

$I = \int_0^a f(x)\cdot(a - g(x))dx$

$\because g(a-x) = a - g(x)$ & $f(x) = f(a-x)$

$I = \int_0^a af(x)dx - \int_0^a f(x)\cdot g(x)dx$

$I = \int_0^a af(x)dx - I$ $\therefore$ from (1)

$2I = \int_0^a af(x)dx$

$I = \frac{a}{2}\int_0^a f(x)dx$

LHS = RHS

Hence Proved

SECTION D

33. Let $I = \int \frac{5x}{(x+1)(x^2+9)}dx$ ....(1)

Using the partial fraction decomposition

$$\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$$

$$5x = A(x^2+9) + (Bx+C)(x+1)$$

On simplifying and coefficients, we get

$$A = -\frac{1}{2},\quad B = \frac{1}{2},\quad C = \frac{9}{2}$$

Now , rewrinting the integral,

$$I = -\frac{1}{2}\int\frac{dx}{x+1} + \frac{1}{2}\int\frac{x}{x^2+9}dx + \frac{9}{2}\int\frac{dx}{x^2+9}$$

$I = I_1 + I_2 + I_3$ ....(ii)

Calculating each integral,

$I_1 = -\frac{1}{2}\int\frac{dx}{x+1} = -\frac{1}{2}\log(x+1) + C_1$ ....(iii)

$I_2 = \frac{1}{2}\int\frac{x}{x^2+9}dx$

Using $v = x^2 + 9$ so $dv = 2xdx$

$I_2 = \frac{1}{4}\int\frac{dv}{v} = \frac{1}{4}\log v = \frac{1}{4}\log(x^2+9) + C_2$ ....(iv)

$\frac{9}{2}\int\frac{dx}{x^2+9}, \because \int\frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

$= \frac{3}{2}\tan^{-1}\left(\frac{x}{3}\right) + C_3$ ....(v)

From Eqs. (iii) and (iv) writing the final integral (I),

$$I = -\frac{1}{2}\log(x+1) + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\left(\frac{x}{3}\right) + C$$

Outside Delhi Set-3 **65/2/3**

SECTION A

1. Option (D) is correct.

Explanation: Since $\vec{p}$ and $\vec{q}$ are unit vectors,

$\Rightarrow |\vec{p}| = |\vec{q}| = 1$

Now, $\vec{p}\cdot\vec{q} = |\vec{p}||\vec{q}|\cos\theta$

$\vec{p}\cdot\vec{q} = \cos\theta$

We know that maximum value of $\cos\theta = 1$, so $\sqrt{3}$ is not possible.

3. Option (C) is correct.

Explanation: Here, $\int_0^a x dx \le \frac{a}{2} + 6$

$$\left[\frac{x^2}{2}\right]_0^a \le \frac{a}{2} + 6$$

$$\frac{a^2}{2} \le \frac{a}{2} + 6$$

$$a^2 - a - 12 \le 0$$

$$a^2 - 4a + 3a - 12 \le 0$$

$$a(a-4) + 3(a-4) \le 0$$

$$(a-4)(a+3) \le 0$$

$(a - 4) \le 0$ or $(a + 3) \ge 0$

$a \le 4$ or $a \ge -3$

Thus, $-3 \le a \le 4$

4. Option (B) is correct.

Explanation: A matrix M is skew-symmetric if $M^T = -M$. Let's check:

$(AB^T - BA^T)^T$

Using the property $(XY)^T = Y^T X^T$:

$(AB^T - BA^T)^T = (AB^T)^T - (BA^T)^T$

$= (B^T)^T A^T - (A^T)^T B^T$

$= BA^T - AB^T$

$= -(AB^T - BA^T)$

Since $(AB^T - BA^T)^T = -(AB^T - BA^T)$, the matrix is skew-symmetric.

5. Option (A) is correct.

Explanation: $\cos\left[\frac{\pi}{6} + \cot^{-1}\left(-\sqrt{3}\right)\right]$

$$= \cos\left[\frac{\pi}{6} + \pi - \frac{\pi}{6}\right]$$

(Since, $\cot^{-1}\left(\sqrt{3}\right) = \frac{\pi}{6}$, so, $\cot^{-1}\left(-\sqrt{3}\right) = \pi - \frac{\pi}{6}$)

$= \cos \pi$

$= -1$

11. Option (A) is correct.

Explanation:

X_i	$P(X_i)$	$X_i.P(X_i)$
–4	0.1	–0.4
–3	0.2	–0.6
–2	0.3	–0.6
–1	0.2	–0.2
0	0.2	0
		$\sum X_iP(X_i) = -1.8$

The expected value $E(X)$ of a discrete random variable X with probability distribution $P(X)$ is given by:

$$E(X) = \sum X_iP(X_i), \text{ thus } E(X) = -1.8$$

12. Option (D) is correct.

Explanation: Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$$4 = \frac{(\alpha i + j + 4k)(2i + 6j + 3k)}{\sqrt{(2)^2 + (6)^2 + (3)^2}}$$

$$4 = \frac{2\alpha + 6 + 12}{\sqrt{4 + 36 + 9}}$$

$$4 = \frac{2\alpha + 18}{\sqrt{49}}$$

$$4 = \frac{2\alpha + 18}{7}$$

$28 = 2\alpha + 18$

$10 = 2a$

$a = 5$

22. $I = \int 2x^3 e^{x^2} dx$

We can write it as $\int 2\cdot x\cdot x^2 \cdot e^{x^2} dx$

Take $x^2 = t$

So we get $2x = \frac{dt}{dx}$

$\Rightarrow 2x\,dx = dt$

It can be written as $\int 2x\cdot x^2\cdot e^{x^2} dx = \int t\cdot e^t\,dt$

By integrating w.r.t. t taking the first function as t and second function as e^t

$$\int te^t\,dt = t\int e^t\,dt - \int\left(\frac{dt}{dt}\cdot\int e^t\,dt\right)dt$$

$$= te^t - \int 1\cdot e^t\,dt$$

Now by replacing with

$$x^2e^{x^2} - e^{x^2} + c$$

By taking as common

$$e^{x^2}(x^2 - 1) + c$$

24. Here, $|\vec{a}| = 2$, $|\vec{b}| = 3$, and $\vec{a}\cdot\vec{b} = 4$

So, $|\vec{a} + 2\vec{b}|^2 = \vec{a}\cdot\vec{a} + 4\vec{a}\cdot\vec{b} + 4\vec{b}\cdot\vec{b}$

$= |\vec{a}|^2 + 4\vec{a}\cdot\vec{b} + 4|\vec{b}|^2$

$= 4 + 16 + 36$

$= 56$

$|\vec{a} + 2\vec{b}|^2 = 56$

$\Rightarrow$ $|\vec{a} + 2\vec{b}| = 2\sqrt{14}$

27. $R = \{(x, y) : x - y + \sqrt{3}$ is an irrational number$\}$

(i) For reflexive:

$(x, x) \in R$

$x - x + \sqrt{3} = \sqrt{3}$

And $\sqrt{3}$ is an irrational number

So, R is reflexive.

(ii) For symmetric

Let $x = \sqrt{3}$, $y = 1$

$x - y + \sqrt{3} = 2\sqrt{3} - 1$ is irrational

$\therefore$ $(x, y) \in R$

$y - x + \sqrt{3} = 1$ is rational.

$\therefore$ $(y, x) \notin R$

$(x, y) \in R \not\Rightarrow (y, x) \in R$

$\therefore$ R is not symmetric.

(iii) For transitive

Let $x = 1$, $y = \sqrt{2}$, $z = \sqrt{3}$

$x - y + \sqrt{3} = 1 - \sqrt{2} + \sqrt{3}$ is irrational

$\therefore$ $(x, y) \in R$

$y - z + \sqrt{3} = \sqrt{2} - \sqrt{2} + \sqrt{3}$

$= \sqrt{2}$ is irrational

$\therefore$ $(y, z) \in R$

$x - z + \sqrt{3} = 1 - \sqrt{3} + \sqrt{3}$

$= 1$ is rational

$\therefore$ $(x, z) \notin R$

xRy and $yRz \not\Rightarrow xRz$

$\therefore$ R is not transitive.

28.

$z = 2x + y$

$3x + y \geq 9$

$x + y \geq 7$

$x + 2y \geq 8$

$x, y \geq 0$

(This implies that the feasible region lies in the first quadrant.)

To plot the constraints, we first convert them into equations:

$3x + y = 9$

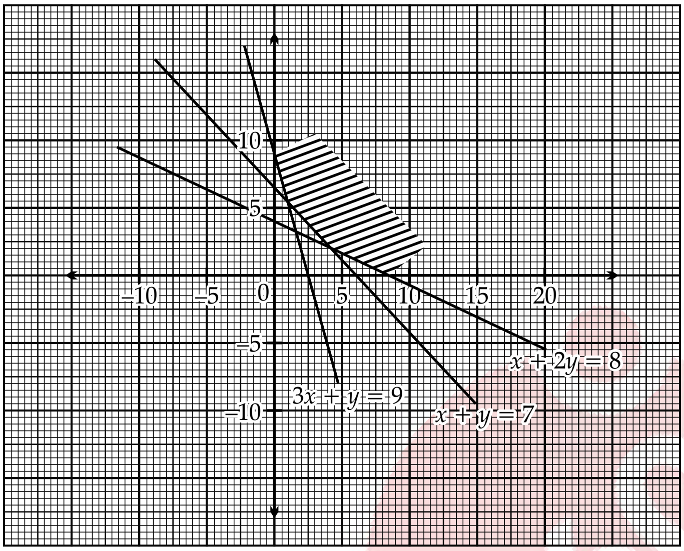
x	0	3
y	9	0

$x + y = 7$

x	0	7
y	7	0

$x + 2y = 8$

x	8	0
y	0	4

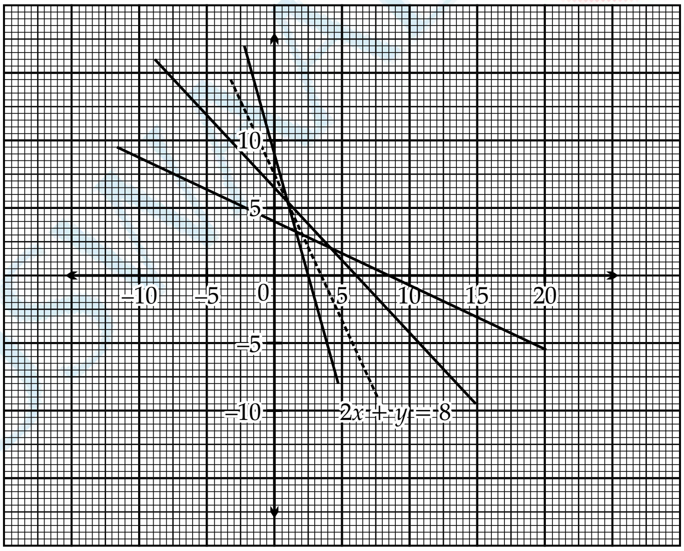


The corner points are: (8, 0), (6, 1), (1, 6), (0, 9)

Corner points	Value of $z = 2x + y$
(8, 0)	16
(6, 1)	13
(1, 6)	8 = Minimum
(0, 9)	9

Since, the region that is feasible that is unbounded.
Hence 8 may or may not be the minimum value of Z.
We need to graph inequality: $2x + y < 8$

x	0	4
y	8	0



There is no common point between feasible region and inequality.
Therefore, $z = 8$ is minimum on all points joining line (0, 9) and (8, 0)
i.e., $z = 8$ will be minimum on $z = 2x + y$

31. Here $\int_a^b x^3\,dx = 0$ and $\int_a^b x^2\,dx = \dfrac{2}{3}$

$$\int_a^b x^3\,dx = 0$$

$$\left[\frac{x^4}{4}\right]_a^b = 0$$

$$\frac{b^4}{4} - \frac{a^4}{4} = 0$$

$$b^4 - a^4 = 0 \times 4$$

$$b^4 = a^4$$

$\therefore$ $$b = \pm a$$

Now, $\left[\frac{x^3}{3}\right]_a^b = \frac{2}{3}$

$$\frac{b^3}{3} - \frac{a^3}{3} = \frac{2}{3}$$

$$b^3 - a^3 = 2$$

Now, if $b = a$

$$b^3 - a^3 = 0 \Rightarrow \text{not possible}$$

Now if $b = -a$

$$-a^3 - a^3 = 2$$

$$-2a^3 = 2$$

$$a = -1 \text{ and } b = 1$$

33. $\int (\sqrt{\tan x} + \sqrt{\cot x})\,dx$

$$= \int \left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}} \right) dx$$

$$= \int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx$$

$$= \sqrt{2} \int \frac{\sin x + \cos x}{\sqrt{2 \sin x \cos x}} dx$$

$$= \sqrt{2} \int \frac{\sin x + \cos x}{\sqrt{2 \sin x \cos x + 1 - 1}} dx$$

$$= \sqrt{2} \int \frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}} dx$$

Put $\sin x - \cos x = t$

$(\sin x + \cos x)dx = dt$

$$= \sqrt{2} \int \frac{dt}{\sqrt{1 - t^2}}$$

$$= \sqrt{2} \sin^{-1} t + c$$

$$= \sqrt{2} \sin^{-1}(\sin x - \cos x) + c$$

❑❑