ISC EXAMINATION PAPER – 2025
MATHEMATICS
Class–12th
(Solved)

Maximum Marks: 80 *Time Allotted: Three Hours* *Reading Time: Additional Fifteen Minutes*

Instructions to Candidates:
1. You are allowed an **additional fifteen minutes** for **only** reading the paper.
2. You must **NOT** start writing during reading time.
3. The Question Paper has **11 printed pages** and **one blank page.**
4. The Question Paper is divided into **three** sections and has **22 questions** in all.
5. **Section A** is compulsory and has **fourteen** questions.
6. You are required to attempt **all** questions either from **Section B or Section C.**
7. **Section B** and **Section C** have **four** questions each.
8. Internal choices have been provided in **two** questions of **2 marks, two** questions of **4 marks** and **two** questions of **6 marks** in **Section A.**
9. Internal choices have been provided in **one** question of **2 marks** and **one** question of **4 marks** each in **Section B and Section C.**
10. While attempting **Multiple Choice Questions in Section A, B and C,** you are required to **write only ONE option as the answer.**
11. All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
12. Mathematical tables and graph papers are provided.
13. The intended marks for questions or parts of questions are given in the brackets [].

SECTION–A (65 MARKS)

Question 1

In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed.

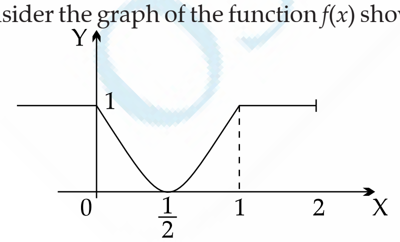
(i) If $A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$, then A^{16} is: **[1]**

(a) Unit matrix **(b)** Null matrix
(c) Diagonal matrix **(d)** Skew matrix

(ii) Which of the following is a homogenous differential equation? **[1]**

(a) $(4x^2 + 6y + 5)\, dy - (3y^2 + 2x + 4)\, dx = 0$
(b) $(xy)\, dx - (x^3 + y^3)\, dy = 0$
(c) $(x^3 + 2y^2)\, dx + 2xy\, dy = 0$
(d) $y^2\, dx + (x^2 - xy - y^2)\, dy = 0$

(iii) Consider the graph of the function $f(x)$ shown below:



Statement 1: The function $f(x)$ is increasing in $\left(\frac{1}{2}, 2\right)$.

Statement 2: The function $f(x)$ is strictly increasing in $\left(\frac{1}{2}, 1\right)$.

Which of the following is correct with respect to the above statements? **[1]**

(a) Statement 1 is true and Statement 2 is false.
(b) Statement 2 is true and Statement 1 is false.
(c) Both the statements are true.
(d) Both the statements are false.

(iv) $\int_0^1 \frac{x^4 - 1}{x^2 + 1}\, dx$ is equal to: **[1]**

(a) $\frac{2}{3}$ **(b)** $\frac{1}{3}$
(c) $\frac{-2}{3}$ **(d)** 0

(v) Assertion: Consider the two events A and B such that $n(A) = n(B)$ and $P\left(\frac{A}{B}\right) = P\left(\frac{B}{A}\right)$. **[1]**

Reason: The events A and B are mutually exclusive.

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true.

(vi) The existence of unique solution of the system of equations $x + y = \lambda$ and $5x + ky = 2$ depends on: **[1]**

(a) λ only **(b)** $\frac{\lambda}{k} = 1$
(c) both k and λ **(d)** k only

(vii) A cylindrical popcorn tub of radius 10 cm is being filled with popcorns at the rate of 314 cm^3 per minute. The level of the popcorns in the tub is increasing at the rate of: [1]

(a) 1 cm/minute **(b)** 0.1 cm/minute
(c) 1.1 cm/minute **(d)** 0.5 cm/minute

(viii) If $f(x)=\begin{cases} x+2 & x<0 \\ -x^2-2 & 0\le x<1 \\ x & x\ge 1 \end{cases}$

then the number of point(s) of discontinuity of $f(x)$, is / are: [1]

(a) 1 **(b)** 3
(c) 2 **(d)** 0

(ix) Assertion: If Set A has m elements, Set B has n elements and $n < m$, then the number of one−one function(s) from A → B is zero.

Reason: A function f: A → B is defined only if all elements in Set A have an image in Set B. [1]

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
(c) Assertion is true and Reason is false.
(d) Assertion is false and Reason is true.

(x) Let X be a discrete random variable. The probability distribution of X is given below:

X	30	10	−10
$P(X)$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{2}$

Then $E(X)$ will be: [1]

(a) 1 **(b)** 4
(c) 2 **(d)** 30

(xi) Statement 1: If 'A' is an invertible matrix, then $(A^2)^{-1} = (A^{-1})^2$

Statement 2: If 'A' is an invertible matrix, then $|A^{-1}| = |A|^{-1}$ [1]

(a) Statement 1 is true and Statement 2 is false.
(b) Statement 2 is true and Statement 1 is false.
(c) Both the statements are true.
(d) Both the statements are false.

(xii) Write the smallest equivalence relation from the set A to A, where A = {1, 2, 3}. [1]

(xiii) For what value of x, is $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew symmetric matrix? [1]

(xiv) Three critics review a book. Odds in favour of the book are 5:2, 4:3 and 3:4, respectively for the three critics. Find the probability that all critics are in favour of the book. [1]

(xv) Evaluate: $\int \frac{5}{\sqrt{2x+7}}\,dx$ [1]

Question 2 [2]

Find the point on the curve $y = 2x^2 - 6x - 4$ at which the tangent is parallel to the x-axis.

Question 3 [2]

Find the value of $\tan^{-1} x - \cot^{-1} x$, if $(\tan^{-1}x)^2 - (\cot^{-1}x) = \frac{5\pi}{8}$

Question 4 [2]

(i) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

OR

(ii) If $f(x) = \log(1 + x) + \frac{1}{1+x}$, show that $f(x)$ attains its minimum value at $x = 0$.

Question 5 [2]

Three shopkeepers Gaurav, Rizwan and Jacob use carry bags made of polythene, handmade paper and newspaper. The number of polythene bags, handmade bags and newspaper bags used by Gaurav, Rizwan and Jacob are (20, 30, 40), (30, 40, 20) and (40, 20, 30), respectively. One polythene bag costs ₹ 1, one handmade bag is for ₹ 5 and one newspaper bag costs ₹ 2. Gaurav, Rizwan and Jacob spend ₹ A, ₹ B and ₹ C, respectively on these carry bags.

Using the concepts of matrices and determinants, answer the following questions:

(i) Represent the above information in matrix form.
(ii) Find the values of ₹ A, ₹ B and ₹ C.

Question 6 [2]

(i) Differentiate $\sin^{-1}\left(\frac{2^{x+1}\,3^x}{1+(36)^x}\right)$ with respect to x.

OR

(ii) Show that $\tan^{-1}x + \tan^{-1}y = C$ is the general solution of the differential equation $(1 + x^2)\,dy + (1 + y^2)\,dx = 0$

Question 7 [4]

If $x + y + z = 0$ then show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = 0$, using properties of determinant.

Question 8 [4]

(i) Evaluate: $\int \frac{\cos x}{3\cos x - 5}\,dx$

OR

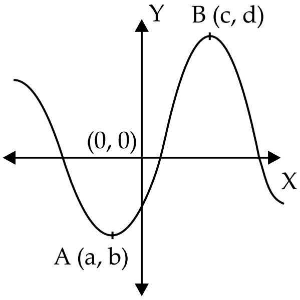
(ii) Evaluate: $\int (\log x)^2\,dx$

Question 9 [4]

(i) If $x = \tan\left(\frac{1}{a}\log y\right)$ then show that $(1 + x^2)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$

OR

(ii) The graph of $f(x) = -x^3 + 27x - 2$ is given below:



(a) Find the slope of the above graph. **[1]**

(b) Find the co-ordinates of turning points, A and B. **[2]**

(c) Evaluate $f''(-2)$, $f(0)$ and $f'(3)$ and arrange them in ascending order. **[1]**

Question 10

Pia, Sia and Dia displayed their paintings in an art exhibition. The three artists displayed 15, 5 and 10 of their paintings, respectively. A person bought three paintings from the exhibition.

(i) Find the probability that he bought one painting from each of them. **[2]**

(ii) Find the probability that he bought all the three paintings from the same person. **[2]**

Question 11 **[6]**

(i) Prove: $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x\,dx}{1+\sin x} = \left(\sqrt{2}-1\right)\pi$

OR

(ii) Evaluate: $\int \frac{x^2}{(x-1)^2\left(x^2+1\right)}dx$

Question 12 **[6]**

(i) Solve the differential equation:

$(x + 5y^2)\frac{dy}{dx} = y$ when $x = 2$ and $y = 1$

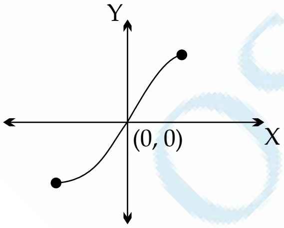
OR

(ii) Find the particular solution of the differential equation:

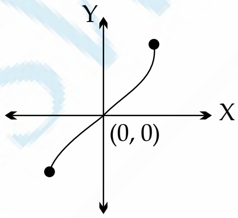
$(x^2 - 2y^2)\,dx + 2xydy = 0$, when $x = 1$ and $y = 1$

Question 13

Observe the two graphs, **Graph 1** and **Graph 2** given below and answer the questions that follow.



Graph 1



Graph 2

(i) Which one of the graphs represents $y = \sin^{-1}x$? **[1]**

(ii) Write the domain and range of $y = \sin^{-1}x$. **[1]**

(iii) Prove that $\sin^{-1}\frac{1}{\sqrt{5}} + \sin^{-1}\frac{2}{\sqrt{5}} = \frac{\pi}{2}$ **[2]**

(iv) Find the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ **[2]**

Question 14 **[6]**

An international conference takes place in a metropolitan city. International leaders, scientists and industrialists participate in it.

The organisers of the conference appoint three agencies namely X, Y and Z for the security of the participants. The track record of the success of X, Y and Z in providing security services is 99%, 98.5% and 98%, respectively. The organisers assign the responsibility of ensuring the security of 1000 people to agency X, 2000 people to agency Y and 3000 people to agency Z.

At the end of the conference, one participant goes missing from the conference room. What is the probability that the missing participant was placed under the responsibility of the security agency X?

SECTION – B (15 MARKS)

Question 15

In subparts (i) and (ii) choose the correct options and in subparts (iii) and (iv), answer the questions as instructed.

(i) Assertion: $\left(\vec{a}+\vec{b}\right)^2 + \left(\vec{b}-\vec{a}\right)^2 = 2\,(a^2 + b^2)$ **[1]**

Reason: Dot product of any two vectors is commutative.

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

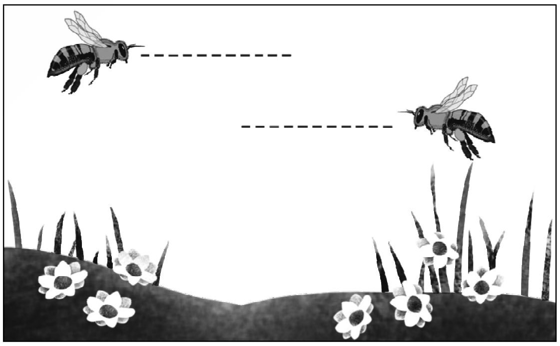
(ii) The angle between the two planes $x + y + 2z = 9$ and $2x - y + z = 15$ is: **[1]**

(a) $\frac{\pi}{2}$ **(b)** $\frac{\pi}{3}$

(c) π **(d)** $\frac{3\pi}{4}$

(iii) Show that points P(−2, 3, 5) Q(1, 2, 3) and R(7, 0, −1) are collinear. **[1]**

(iv) Two honeybees are flying parallel to each other in the garden to collect the nectar. The path traced by the bees is given $\vec{r} = \left(\hat{i}+2\hat{j}+3\hat{k}\right) + \lambda\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$.



(a) Write the above-mentioned equation in Cartesian form. **[1]**

(b) Find the equation of the path traced by the other honeybee passing through the point (2, 4, 5). **[1]**

Question 16 **[2]**

(i) Find the equation of the plane passing through the points (2, 2, −1), (3, 4, 2) and (7, 0, 6).

OR

(ii) Find the equation of the plane passing through the points (2, 3, 1), (4, −5, 3) and parallel to x-axis.

Question 17

Consider the position vectors of A, B and C as $\overrightarrow{OA} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\overrightarrow{OB} = \hat{i} + 2\hat{j} - 2\hat{k}$ and

$\overrightarrow{OC} = 2\hat{i} - \hat{j} + 4\hat{k}$

(i) Calculate $\overrightarrow{AB}$ and $\overrightarrow{BC}$. **[1]**

(ii) Find the projection of $\overrightarrow{AB}$ on $\overrightarrow{BC}$. **[1]**

(iii) Find the area of the triangle ABC whose sides are $\overrightarrow{AB}$ and $\overrightarrow{BC}$. **[2]**

Question 18

(i) The equation $y = 4 - x^2$ represents a parabola.

(a) Make a rough sketch of the graph of the given function. **[1]**

(b) Determine the area enclosed between the curve, the x-axis, the lines $x = 0$ and $x = 2$ **[2]**

(c) Hence, find the area bounded by the parabola and the x-axis. **[1]**

OR

(ii) A farmer has a field bounded by three lines $x + 2y = 2$, $y - x = 1$, $2x + y = 7$. Using integration, find the area of the region bounded by these lines. **[4]**

SECTION – C (15 MARKS)

Question 19

In subparts (i) and (ii) choose the correct options and in subpart (iii), answer the questions as instructed.

(i) The total revenue received from the sale of 'x' units of a product is $R(x) = 36x + 3x^2 + 5$. Then, the actual revenue for selling the 10th item will be: **[1]**

(a) 27 **(b)** 90

(c) 93 **(d)** 33

(ii) Read the following statements and choose the correct option. **[1]**

(I) The correlation coefficient and the regression coefficients are of the same sign

(II) The correlation coefficient is the arithmetic mean between the regression coefficients.

(III) The product of two regression coefficients is always equal to 1.

(IV) Both the regression coefficients cannot be numerically greater than unity.

(a) Only (IV) is correct.

(b) Only (I) and (II) are correct.

(c) Only (I) and (IV) are correct.

(d) Only (III) and (IV) are correct.

(iii) Consider the following data:

x	1	2	3	6
y	6	5	4	1
$x - \bar{x}$				
$y - \bar{y}$				

(a) Calculate $\bar{x}$ and $\bar{y}$ **[1]**

(b) Complete the table. **[1]**

(c) Calculate b_{xy} **[1]**

Question 20 **[2]**

(i) Find the regression line of best fit from the following data.

$\Sigma x = 24$, $\Sigma y = 44$, $\Sigma xy = 306$, $\Sigma x^2 = 164$, $\Sigma y^2 = 576$, $n = 4$

OR

(ii) Two lines of regression are given as $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Identify the line of regression of x on y.

Question 21

(i) A utensil manufacturer produces 'x' dinner sets per week and sells each set at ₹ p, where $x = \frac{600 - p}{8}$.

The cost of production of 'x' sets is ₹ $x^2 + 78x + 2000$

(a) Write the revenue function. **[1]**

(b) Write the profit function. **[1]**

(c) Calculate the number of dinner sets to be produced and sold per week to ensure maximum profit. **[2]**

OR

(ii) The Average Cost of producing 'x' units of commodity is given by:

$$AC = \frac{x^2}{200} - \frac{x}{50} - 30 + \frac{5000}{x}$$

(a) Find the Cost function. **[1]**

(b) Find the Marginal Cost function. **[1]**

(c) Find the Marginal Average Cost function. **[1]**

(d) Verify that $\frac{d}{dx}(AC) = \frac{MC - AC}{x}$ **[1]**

Question 22 **[4]**

Two different types of books have to be stacked in the shelf of a library. The first type of book weighs 1 kg and has a thickness of 6 cm. The second type of book weighs 1.5 kg and has a thickness of 4 cm. The shelf is 96 cm long and can support a maximum weight of 21 kg.

How should both the types of books be placed in the shelf to include the maximum number of books? Formulate a Linear Programming Problem and solve it graphically.

ANSWERS

SECTION-A (65 MARKS)

Question 1

1. **(i)Option (b) is correct.**

Explanation: Here,

$$A^2 = A \times A = \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

So, A^2 is the null matrix.

Since A^2 is the null matrix, any higher power of A will also be the null matrix:

$$A^3 = A^2 \times A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

This pattern continues for all higher exponents. Therefore, A^{16} is also the null matrix.

(ii) Option (d) is correct.

Explanation: A homogeneous differential equation is one in which every term has the same degree when all variables are considered.

Take $y^2dx + (x^2 - xy - y^2)\,dy = 0$

Degrees of terms:

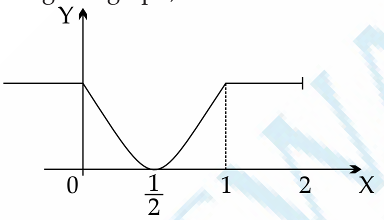
- y^2 has degree 2.
- x^2 has degree 2.
- $-xy$ has degree 2.
- $-y^2$ has degree 2.

All terms are of degree 2. This is homogeneous differential equation.

In the remaining options, the degree of terms is not the same.

(iii) Option (c) is correct.

Explanation: Observing the function's behaviour from the given graph, we see



- The function decreases from $x = 0$ to $x = \frac{1}{2}$.
- The function then increases from $x = \frac{1}{2}$ to $x = 1$.
- From $x = 1$ to $x = 2$, the function remains constant.

So, function increases from $\left(\frac{1}{2}, 2\right)$ but not strictly increases from $\left(\frac{1}{2}, 2\right)$.

Thus, Both the statement are true.

(iv) Option (c) is correct.

Explanation: $\int_0^1 \frac{x^4 - 1}{x^2 + 1}dx$

$$= \int_0^1 \frac{(x^2+1)(x^2-1)}{x^2+1}dx$$

$$= \int_0^1 (x^2 - 1)dx$$

$$= \int_0^1 x^2 dx - \int_0^1 1\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^1 - [x]_0^1$$

$$= \frac{1}{3} - 1 = -\frac{2}{3}$$

(v) Option (c) is correct.

Explanation: **For Assertion:**

Using the definition of conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Given that $P(A|B) = P(B|A)$, we equate these expressions:

$$\frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)}$$

Since $P(B) = P(A)$ (because $n(A) = n(B)$), the equation holds true.

This confirms that the assertion is true.

For Reason:

Two events A and B are mutually exclusive if: $P(A \cap B) = 0$

If A and B were mutually exclusive, then:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = 0$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = 0$$

This would imply $P(A \backslash B) = P(B \backslash A) = 0$, which is a special case but not necessarily true for all cases where $P(A \backslash B) = P(B \backslash A)$.

Therefore, mutual exclusivity is not a necessary condition for the assertion to hold, meaning the reason is false.

(vi) Option (d) is correct.

Explanation:

The system can be written as: $\begin{pmatrix} 1 & 1 \\ 5 & k \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \lambda \\ 2 \end{pmatrix}$

The determinant of the coefficient matrix is:

$$\Delta = \begin{vmatrix} 1 & 1 \\ 5 & k \end{vmatrix} = (1 \times k) - (1 \times 5) = k - 5$$

For the system to have a unique solution, the determinant must be non-zero:

$\Delta \neq 0 \Rightarrow k-5 \neq 0 \Rightarrow k \neq 5$

The value of λ does not affect the determinant or the condition for a unique solution. It only affects the specific solution values of x and y.

The existence of a unique solution depends only on k (specifically, $k \neq 5$). The value of λ is irrelevant to the existence of a unique solution.

(vii) Option (a) is correct.

Explanation: The volume V of a cylinder is given by:

$V = \pi r^2 h$

Differentiate both sides of the equation with respect to time t:

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

Given, $\frac{dV}{dt}$ = 314 cm^3/minute, r = 10 cm, and $\pi \approx 3.14$.

Substitute these values into the above equation, we get

$$314 = \pi(10)^2 \frac{dh}{dt}$$

$$314 = 3.14 \times 100 \times \frac{dh}{dt}$$

$$314 = 314 \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = 1 \text{ cm/minute}$$

The level of popcorn in the tub is increasing at a rate of 1 cm/minute.

(viii) Option (c) is correct.

Explanation: Check continuity at $x = 0$

Left-hand limit (LHL) as $x \to 0^-$:

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (x+2) = 0+2 = 2$$

Right-hand limit (RHL) as $x \to 0^+$:

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (-x^2-2) = -0-2 = -2$$

Since, LHL ≠ RHL, $f(x)$ is discontinuous at $x = 0$.

Check continuity at $x = 1$

Left-hand limit (LHL) as $x \to 1^-$:

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (-x^2-2) = -1-2 = -3$$

Right-hand limit (RHL) as $x \to 1^+$:

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x = 1$$

Since, LHL ≠ RHL, $f(x)$ is discontinuous at $x = 1$.

(ix) Option (b) is correct.

Explanation: For Assertion

A one–one function (injective function) requires that each element of A maps to a unique element B. If $n < m$, there are not enough elements in B to uniquely map all elements of A.

Hence, no one–one function exists from $A \to B$ when $n < m$.

Thus, the assertion is true.

For Reason

Reason is true, as a function must map every element of the domain A to some element in the codomain B. However, this reason does not explain why the number of one–one functions is zero when $n < m$, only states a basic requirement for a function to exist.

(x) Option (b) is correct.

Explanation: The expected value $E(X)$ of a discrete random variable X is given by:

$$E(X) = \sum X_i P(X_i)$$

$$E(X) = \left(30 \times \frac{1}{5}\right) + \left(10 \times \frac{3}{10}\right) + \left(-10 \times \frac{1}{2}\right)$$

$= 6+3-5$

$= 4$

(xi) Option (c) is correct.

Explanation: **Statement 1:**

Since A is invertible, A^{-1} exists.

The inverse of A^2 is:

$(A^2)^{-1} = (A \cdot A)^{-1} = A^{-1} \cdot A^{-1} = (A^{-1})^2$

Thus, $(A^2)^{-1} = (A^{-1})^2$.

Thus, Statement 1 is true.

Statement 2: For any invertible square matrix A, we use the property of determinants: $|A^{-1}| = \frac{1}{|A|}$

Also, $AA^{-1} = I$

Taking determinants on both sides:

$|A| \cdot |A^{-1}| = |I| = 1$

Thus, $|A^{-1}| = \frac{1}{|A|} = |A|^{-1}$

Thus, Statement 2 is also true.

(xii) Given the set, A = {1,2,3}

The largest equivalence relation

R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2)}

Thus, the smallest equivalence relation is:

R = {(1, 1), (2, 2), (3, 3)}

This relation is:

Reflexive: Since (1, 1), (2, 2), (3, 3) are present.

Symmetric: Since there are no other elements besides the reflexive ones, symmetry holds automatically.

Transitive: Since there are no extra pairs, transitivity also holds.

(xiii) A matrix A is skew-symmetric if $A^T = -A$.

So, $$\begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -x & 3 & 0 \end{bmatrix}$$

Compare the matrices element by element,

from $x = 2$ and $-2 = -x$, we confirm that $x = 2$.

(xiv) The odds in favour of an event are given as $a : b$, where a is the number of favourable outcomes and b is the number of unfavourable outcomes.

The probability P of the event is $P = \frac{a}{a+b}$

Critic 1:
Odds in favour: 5 : 2.

Probability: $P_1 = \frac{5}{5+2} = \frac{5}{7}$

Critic 2:
Odds in favour: 4 : 3.

Probability: $P_2 = \frac{4}{4+3} = \frac{4}{7}$

Critic 3:
Odds in favour: 3 : 4.

Probability: $P_3 = \frac{3}{3+4} = \frac{3}{7}$

Since the critics' opinions are independent.

P (all in favour) = $P_1 \times P_2 \times P_3$

$P(\text{ all in favour}) = \frac{5}{7} \times \frac{4}{7} \times \frac{3}{7} = \frac{60}{343}$

The probability that all three critics are in favour of the book is $\frac{60}{343}$.

(xv) Let, $I = \int \frac{5}{\sqrt{2x+7}} dx$

Put $u = 2x + 7$

Differentiating both sides:

$\frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2}$

Rewriting the integral in terms of u:

$I = \int \frac{5}{\sqrt{u}} \cdot \frac{du}{2}$

$= \frac{5}{2} \int u^{-\frac{1}{2}} du$

$= \frac{5u^{\frac{1}{2}}}{2 \times \frac{1}{2}} + C = 5\sqrt{u} + C = 5\sqrt{2x+7} + C$

Question 2

Given curve is

$y = 2x^2 - 6x - 4$ ...(i)

The slope of the tangent at any point is given by the derivative:

$\frac{dy}{dx} = \frac{d}{dx}\left(2x^2 - 6x - 4\right) = 4x - 6$

Since the tangent is parallel to the x-axis, its slope is zero, so:

$4x - 6 = 0$

$4x = 6$

$x = \frac{3}{2}$

Substituting $x = \frac{3}{2}$ into the Eq. (i), we get

$y = 2\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) - 4$

$= 2 \times \frac{9}{4} - 9 - 4$

$= \frac{9}{2} - 13 = -\frac{17}{2}$

Thus, the required point is $\left(\frac{3}{2}, -\frac{17}{2}\right)$

Question 3:

We know that:

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ ...(i)

The given equation is:

$$\left(\tan^{-1} x\right)^2 - \left(\cot^{-1} x\right)^2 = \frac{5\pi}{8}$$

$$\Rightarrow \left(\tan^{-1} x - \cot^{-1} x\right)\left(\tan^{-1} x + \cot^{-1} x\right) = \frac{5\pi}{8}$$

$$\Rightarrow \left(\tan^{-1} x - \cot^{-1} x\right) \cdot \frac{\pi}{2} = \frac{5\pi}{8}$$

[Using Eq. (i)]

$$\Rightarrow \tan^{-1} x - \cot^{-1} x = \frac{\frac{5\pi}{8}}{\frac{\pi}{2}}$$

$$\Rightarrow \tan^{-1} x - \cot^{-1} x = \frac{5}{4}$$

Question 4:

(i) Given equation:

$x^y = e^{x-y}$

Taking logarithm on both sides:

$\log (x^y) = \log(e^{x-y})$

$y \log x = x - y$

$y(\log x + 1) = x$

$y = \frac{x}{\log x + 1}$

$$\frac{dy}{dx} = \frac{(\log x + 1) \cdot \frac{d}{dx}(x) - x \cdot \frac{d}{dx}(\log x + 1)}{(\log x + 1)^2}$$

$$\frac{dy}{dx} = \frac{(\log x + 1) \cdot 1 - x \cdot \frac{1}{x}}{(\log x + 1)^2}$$

$$\frac{dy}{dx} = \frac{\log x + 1 - 1}{(\log x + 1)^2} = \frac{\log x}{(\log x + 1)^2}$$

Hence Proved

OR

(ii) Given,

$f(x) = \log(1+x) + \frac{1}{1+x}$

$\therefore f'(x) = \frac{d}{dx}(\log(1+x)) + \frac{d}{dx}\left(\frac{1}{1+x}\right)$.

$f'(x) = \frac{1}{1+x} - \frac{1}{(1+x)^2}$

Put $f'(x) = 0$ to find critical points:

$$\frac{1}{1+x} - \frac{1}{(1+x)^2} = 0$$

$$\frac{(1+x)-1}{(1+x)^2} = 0$$

$$\frac{x}{(1+x)^2} = 0$$

This equation is satisfied only when $x=0$. Thus, $x=0$ is the critical point.

Again,

$$f''(x) = \frac{d}{dx}\left(\frac{1}{1+x} - \frac{1}{(1+x)^2}\right)$$

$$f''(x) = -\frac{1}{(1+x)^2} + \frac{2}{(1+x)^3}$$

At $x = 0$,

$$f''(0) = -\frac{1}{(1+0)^2} + \frac{2}{(1+0)^3} = -1+2 = 1 > 0$$

Since $f''(0) > 0$, the function $f(x)$ has a local minimum at $x = 0$. **Hence Proved**

Question 5

(i) Let,

U be the usage matrix, where rows represent shopkeepers and columns represent types of bags.

C be the cost matrix, where rows represent types of bags and columns represent costs.

The usage matrix U is:

$$U = \begin{bmatrix} 20 & 30 & 40 \\ 30 & 40 & 20 \\ 40 & 20 & 30 \end{bmatrix}$$

where,

Rows correspond to Gaurav, Rizwan and Jacob.

Columns correspond to polythene bags, handmade bags and newspaper bags.

The cost matrix C is:

$$C = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$$

where,

Rows correspond to polythene bags, handmade bags and newspaper bags.

The single column represents the cost of each type of bag.

(ii) The total expenditure of each shopkeeper is given by the matrix product $U \times C$. Let:

$S = U \times C$

where S is the expenditure matrix.

$$S = \begin{bmatrix} 20 & 30 & 40 \\ 30 & 40 & 20 \\ 40 & 20 & 30 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$$

$$S = \begin{bmatrix} (20\times1)+(30\times5)+(40\times2) \\ (30\times1)+(40\times5)+(20\times2) \\ (40\times1)+(20\times5)+(30\times2) \end{bmatrix} = \begin{bmatrix} 250 \\ 270 \\ 200 \end{bmatrix}$$

Or

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 250 \\ 270 \\ 200 \end{bmatrix}$$

Thus, values of A, B and C :

A = ₹250,

B = ₹270,

C = ₹200.

Question 6

(i) We have,

$$y = \sin^{-1}\left[\frac{2^{x+1}\cdot 3^x}{1+36^x}\right]$$

$$y = \sin^{-1}\left[\frac{2\cdot 6^x}{1+6^{2x}}\right]$$

Put $6^x = \tan\theta$

$\Rightarrow \theta = \tan^{-1}(6^x)$

Now,

$$y = \sin^{-1}\left[\frac{2\tan\theta}{1+\tan^2\theta}\right]$$

$y = \sin^{-1}[\sin 2\theta]$

$y = 2\theta$

$y = 2\tan^{-1}[6^x]$

$$\frac{dy}{dx} = \frac{2}{1+6^{2x}} \times \frac{d}{dx}(6^x)$$

$$\frac{dy}{dx} = \frac{2}{1+6^{2x}} \times 6^x \log 6$$

$$\frac{dy}{dx} = \frac{2(6^x \log 6)}{1+(36)^x}$$

OR

(ii) Given equation is: $\tan^{-1} x + \tan^{-1} y = C$

Differentiating w.r.t. x, we have

$$\frac{1}{1+x^2} + \frac{1}{1+y^2}\cdot\frac{dy}{dx} = 0$$

$$\Rightarrow \left(\frac{1}{1+y^2}\right)\frac{dy}{dx} = -\left(\frac{1}{1+x^2}\right)$$

$\Rightarrow (1 + x^2)\, dy = -(1 + y^2)\, dx$

$\Rightarrow (1 + x^2)\, dy + (1 + y^2)\, dx = 0$

Hence Proved

Question 7

$$\text{Let } D = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix}$$

Apply $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$D = \begin{vmatrix} 1 & 0 & 0 \\ x & y-x & z-x \\ x^3 & y^3-x^3 & z^3-x^3 \end{vmatrix}$$

$$D = \begin{vmatrix} 1 & 0 & 0 \\ x & y-x & z-x \\ x^3 & (y-x)(y^2+xy+x^2) & (z-x)(z^2+xz+x^2) \end{vmatrix}$$

[Using identity $a^3 - b^3 = (a-b)(a^2+ab+b^2)$]

$$D = (y-x)(z-x)\begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ x^3 & (y^2+xy+x^2) & (z^2+xz+x^2) \end{vmatrix}$$

[Taking $(y-x)$ and $(z-x)$ common from C_2 and C_3]

Now, expending along R_1, we get

$D = (y-x)(z-x)\,[1\{(z^2+xz+x^2)-(y^2+xy+x^2)\}] + 0 + 0$

$D = (y-x)(z-x)\,[(z^2+xz+x^2)-(y^2+xy+x^2)]$

$D = (y-x)(z-x)(z^2-y^2+xz-xy)$

$D = (y-x)(z-x)\,[(z+y)(z-y)+x(z-y)]$

$D = (y-x)(z-x)(z-y)(x+y+z)$

$D = 0$ [Given, $x+y+z=0$]

Question 8

(i) Let $I = \int \frac{\cos x}{3\cos x - 5}dx$

$$= \frac{1}{3}\int \frac{3\cos x - 5 + 5}{3\cos x - 5}dx$$

$$= \frac{1}{3}\left[\int \frac{3\cos x - 5}{3\cos x - 5}dx + \int \frac{5}{3\cos x - 5}dx\right]$$

$$= \frac{1}{3}\int 1dx + \frac{5}{3}\int \frac{1}{3\cos x - 5}dx$$

$$= \frac{x}{3} + \frac{5}{3}\int \frac{1}{3\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) - 5}dx$$

$$= \frac{x}{3} + \frac{5}{3}\int \frac{\sec^2\frac{x}{2}}{-2-8\tan^2\frac{x}{2}}dx$$

$$= \frac{x}{3} - \frac{5}{6}\int \frac{\sec^2\frac{x}{2}}{1+4\tan^2\frac{x}{2}}dx$$

Put $\tan\frac{x}{2} = u \Rightarrow \sec^2\frac{x}{2}\cdot\frac{1}{2}dx = du$

$$\therefore I = \frac{x}{3} - \frac{5}{6}\int \frac{2du}{1+4u^2}$$

$$= \frac{x}{3} - \frac{5}{3}\int \frac{du}{1+(2u)^2}$$

$$= \frac{x}{3} - \frac{5}{3}\times\frac{1}{2}\tan^{-1}2u + C$$

$$= \frac{x}{3} - \frac{5}{6}\tan^{-1}2\left(\tan\frac{x}{2}\right) + C$$

OR

(ii) Let

$I = \int (\log x)^2\,dx$

$I = \int (\log x)^2 \cdot 1dx$

$I = (\log x)^2 \int 1\cdot dx - \int\left[\frac{d}{dx}(\log x)^2 \int 1\cdot dx\right]dx$

$I = x(\log x)^2 - \int\left[2\log x\cdot\frac{1}{x}\cdot x\right]dx$

$I = x(\log x)^2 - 2\int[\log x\cdot 1]\,dx$

$I = x(\log x)^2 - 2\left[\log x\int 1\cdot dx - \int\left\{\frac{d}{dx}(\log x)\int 1\cdot dx\right\}dx\right]$

$I = x(\log x)^2 - 2\left[(\log x)x - \int\frac{1}{x}\cdot x\cdot dx\right]$

$I = x(\log x)^2 - 2[x\log x - \int 1\cdot dx]$

$I = x(\log x)^2 - 2(x\log x - x) + c$

$I = x[(\log x)^2 - 2\log x + 2] + c$

Question 9

(i) Given, $x = \tan\left(\frac{1}{a}\log y\right)$

$\Rightarrow \tan^{-1}x = \frac{1}{a}\log y$

$\Rightarrow a\tan^{-1}x = \log y$

Differentiating w.r.t. x, we get

$\Rightarrow \frac{a}{1+x^2} = \frac{1}{y}\cdot\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{ay}{1+x^2}$

$\Rightarrow (1+x^2)\frac{dy}{dx} = ay$

Differentiating w.r.t. x, we get

$$(1+x^2)\frac{d^2y}{dx^2} + 2x\cdot\frac{dy}{dx} = a\cdot\frac{dy}{dx}$$

$$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$$

Hence Proved

OR

(ii) We have, $(x) = -x^3 + 27x - 2$

(a) The slope of the graph at any point is given by the first derivative:

$f'(x) = \frac{d}{dx}(-x^3 + 27x - 2)$

$f'(x) = -3x^2 + 27$

(b) Turning points occur where $f'(x)=0$.

$-3x^2 + 27 = 0$

$$3x^2 = 27$$
$$x^2 = 9$$
$$x = \pm 3$$

For $x = 3$:

$f(3) = -(3)^3+27(3)-2$

$= -27 + 81 - 2 = 52$

For $x = -3$:

$f(-3) = -(-3)^3 + 27(-3)-2$

$= 27 - 81 - 2 = -56$

Thus, the turning points are:

$A(-3, -56)$, $B(3, 52)$

(c) The second derivative is:

$$f''(x) = \frac{d}{dx}\left(-3x^2+27\right)$$

$f''(x) = -6x$

Now, $f''(-2) = -6(-2) = 12$

$f(0) = -(0)^3 + 27(0)-2 = -2$

$f'(3) = -3(3)^2 + 27$

$= -27 + 27 = 0$

Thus, $f'(3)=0$, $f(0)=-2$, $f''(-2)=12$

Arranging in ascending order: $-2, 0, 12$

or $f(0) < f'(3) < f''(-2)$

Question 10

Given,

Pia displayed 15 paintings.

Sia displayed 5 paintings.

Dia displayed 10 paintings.

Thus, the total number of paintings in the exhibition is: $15 + 5 + 10 = 30$

A person buys 3 paintings randomly.

(i) Probability of buying one painting from each artist

Choose 1 painting from Pia's 15 paintings: $\left(15_{C_1}\right) = 15$

Choose 1 painting from Sia's 5 paintings: $\left(5_{C_1}\right) = 5$

Choose 1 painting from Dia's 10 paintings:

$$\left(10_{C_1}\right) = 10$$

Thus, the number of ways to select 3 paintings, one from each artist, is: $15 \times 5 \times 10 = 750$

Total ways to select any 3 paintings from 30:

$$\left(30_{C_3}\right) = \frac{30!}{3!(30-3)!} = \frac{30\times29\times28}{3\times2\times1} = 4060$$

Thus, the probability $= \frac{750}{4060} = \frac{75}{406}$

(ii) Probability of buying all three paintings from the same person:

There are three possible cases:

- All 3 paintings from Pia
- All 3 paintings from Sia
- All 3 paintings from Dia

Case 1: All 3 paintings from Pia

Ways to choose 3 from Pia's 15:

$$\left(15_{C_3}\right) = \frac{15!}{3!(12!)} = \frac{15\times14\times13}{3\times2\times1} = 455$$

Case 2: All 3 paintings from Sia

Ways to choose 3 from Sia's 5 :

$$\left(5_{C_3}\right) = \frac{5!}{3!(2!)} = \frac{5\times4\times3}{3\times2\times1} = 10$$

Case 3: All 3 paintings from Dia

Ways to choose 3 from Dia's 10:

$$\left(10_{C_3}\right) = \frac{10!}{3!(7!)} = \frac{10\times9\times8}{3\times2\times1} = 120$$

Total ways to pick all 3 paintings from the same artist:

$455 + 10 + 120 = 585$

Thus, the probability $= \frac{585}{4060} = \frac{117}{812}$

Question 11

(i) $$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}dx$$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)}{1+\sin x\left(\frac{3\pi}{4}+\frac{\pi}{4}-x\right)}dx$$

$$\left[\text{Using } \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{(\pi-x)}{1+\sin x}dx$$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi}{1+\sin x}dx - \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}dx$$

$$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi}{1+\sin x}dx$$

$$I = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\sin x}$$

$$I = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\frac{1-\sin x}{\cos^2 x}\right)dx$$

[Multiplying Nr. & Dr. by $(1 + \sin x)$]

$$I = \frac{\pi}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\sec^2 x - \tan x\sec x\right)dx$$

$$I = \frac{\pi}{2}\left[\tan x - \sec x\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \frac{\pi}{2}\left[\left(-1+\sqrt{2}\right)-\left(1-\sqrt{2}\right)\right]$$

$$I = \pi\left[\left(\sqrt{2}-1\right)\right]$$

OR

(ii) Given integral:

$$\int \frac{x^2}{(x-1)^2\left(x^2+1\right)}dx$$

Here, we use partial fraction decomposition.

$$\frac{x^2}{(x-1)^2(x^2+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}$$

$x^2 = A(x-1)(x^2+1) + B(x^2+1) + (Cx+D)(x-1)^2$

$x^2 = A(x^3 - x^2 + x - 1) + B(x^2+1) + (Cx+D)(x^2 - 2x + 1)$

$x^2 = Ax^3 - Ax^2 + Ax - A + Bx^2 + B + Cx^3 - 2Cx^2 + Cx + Dx^2 - 2Dx + D$

$x^2 = (A+C)x^3 + (-A+B-2C+D)x^2 + (A+C-2D)x + (-A+B+D)$

Equate coefficients of corresponding powers of x:

$x^3: A + C = 0.$

$x^2: -A + B - 2C + D = 1.$

$x: A + C - 2D = 0.$

Constant term: $-A + B + D = 0.$

From $A + C = 0$, we get $C = -A$.

From $A + C - 2D = 0$, substitute $C = -A$:

$A - A - 2D = 0 \Rightarrow -2D = 0 \Rightarrow D = 0$

From $-A + B - 2C + D = 1$, substitute $C = -A$ and $D = 0$:

$-A + B - 2(-A) + 0 = 1 \Rightarrow -A + B + 2A = 1$

$\Rightarrow A + B = 1$

From $-A + B + D = 0$, substitute $D = 0$:

$-A + B = 0 \Rightarrow B = A$

Substitute $B = A$ into $A + B = 1$:

$A + A = 1 \Rightarrow 2A = 1 \Rightarrow A = \frac{1}{2}$

Thus,

$B = \frac{1}{2}, C = -\frac{1}{2}, D = 0$

Substitute A, B, C and D into the partial fractions:

$$\frac{x^2}{(x-1)^2(x^2+1)} = \frac{\frac{1}{2}}{x-1} + \frac{\frac{1}{2}}{(x-1)^2} + \frac{-\frac{1}{2}x}{x^2+1}$$

$$\frac{x^2}{(x-1)^2(x^2+1)} = \frac{1}{2(x-1)} + \frac{1}{2(x-1)^2} - \frac{x}{2(x^2+1)}$$

Now, integrate each term separately:

$$\int \frac{x^2}{(x-1)^2(x^2+1)}dx = \frac{1}{2}\int \frac{1}{x-1}dx + \frac{1}{2}\int \frac{1}{(x-1)^2}dx - \frac{1}{2}\int \frac{x}{x^2+1}dx$$

First term:

$$\frac{1}{2}\int \frac{1}{x-1}dx = \frac{1}{2}\ln|x-1| + C_1$$

Second term:

$$\frac{1}{2}\int \frac{1}{(x-1)^2}dx = \frac{1}{2}\cdot\left(-\frac{1}{x-1}\right) + C_2 = -\frac{1}{2(x-1)} + C_2$$

Third term:

$$-\frac{1}{2}\int \frac{x}{x^2+1}dx$$

Let $u = x^2 + 1$, then $du = 2xdx$,

so $xdx = \frac{1}{2}du$:

$$-\frac{1}{2}\int \frac{x}{x^2+1}dx = -\frac{1}{2}\cdot\frac{1}{2}\ln|u| + C_3 = -\frac{1}{4}\ln(x^2+1) + C_3$$

Combining all terms, we get

$$\int \frac{x^2}{(x-1)^2(x^2+1)}dx = \frac{1}{2}\ln|x-1| - \frac{1}{2(x-1)} - \frac{1}{4}\ln(x^2+1) + C$$

Question 12

(i) Rewriting the given differential equation in a separable form:

$$(x+5y^2)\frac{dy}{dx} = y \Rightarrow \frac{dy}{dx} = \frac{y}{x+5y^2}$$

$$\frac{dx}{dy} = \frac{x+5y^2}{y}$$

$$\frac{dx}{dy} = \frac{x}{y} + 5y$$

$$\frac{dx}{dy} - \frac{1}{y}x = 5y$$

It is of the form: $\frac{dx}{dy} + P(y)x = Q(y)$

where $P(y) = -\frac{1}{y}$ and $Q(y) = 5y$.

The integrating factor $IF(y)$ is:

$$IF(y) = e^{\int P(y)dy} = e^{-\int \frac{1}{y}dy} = e^{-\ln y} = \frac{1}{y}$$

Multiply both sides of the differential equation by $IF(y)$:

$$\frac{1}{y}\frac{dx}{dy} - \frac{1}{y^2}x = 5$$

$$\frac{d}{dy}\left(\frac{x}{y}\right) = 5$$

Integate both sides with respect to:

$$\frac{x}{y} = 5y + C$$

where C is the constant of integration.

$x = 5y^2 + Cy$

Use the initial condition $x = 2$ and $y = 1$:

$2 = 5(1)^2 + C(1) \Rightarrow 2 = 5 + C \Rightarrow C = -3$

Thus, the solution is:

$x = 5y^2 - 3y$

OR

(ii) Rewriting the given differential equation as

$$\frac{dy}{dx}=\frac{2y^2-x^2}{2xy}$$

This is a homogeneous differential equation, since each term is of degree 2.

Let $y = vx$

Differentiating both sides:

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

Substituting $y = vx$ into the given differential equation, we get

$$v+x\frac{dv}{dx}=\frac{2v^2x^2-x^2}{2vx^2}$$

$$v+x\frac{dv}{dx}=\frac{2v^2-1}{2v}$$

$$v+x\frac{dv}{dx}=v-\frac{1}{2v}$$

$$x\frac{dv}{dx}=-\frac{1}{2v}$$

$$2v\,dv=-\frac{dx}{x}$$

Integrating both sides, we get

$$2\int v\,dv=-\int\frac{dx}{x}$$

$$2\frac{v^2}{2}=-ln|x|+C$$

$v^2 + ln|x| = C$

$$\left(\frac{y}{x}\right)^2+ln|x|=C \text{ [Since, } v=\frac{y}{x}]$$

At $x = 1$ and $y = 1$

$1 + ln1 = C$

$\Rightarrow C = 1$

Thus, solution is

$$\left(\frac{y}{x}\right)^2+ln|x|=1$$

Question 13

(i) Graph 2 represents $y = \sin^{-1}x$

(ii) Domain: The graph should only exist for $x\in[-1,1]$.

Range: The y-values should range from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

(iii) $\text{LHS}=\sin^{-1}\frac{1}{\sqrt5}+\sin^{-1}\frac{2}{\sqrt5}$

$$=\sin^{-1}\left(\frac{1}{\sqrt5}\times\sqrt{1-\left(\frac{2}{\sqrt5}\right)^2}+\frac{2}{\sqrt5}\times\sqrt{1-\left(\frac{1}{\sqrt5}\right)^2}\right)$$

[Using $\sin^{-1}x+\sin^{-1}y=\sin^{-1}\left(x\times\sqrt{1-y^2}+y\times\sqrt{1-x^2}\right)$]

$$=\sin^{-1}\left(\frac{1}{\sqrt5}\times\sqrt{1-\frac{4}{5}}+\frac{2}{\sqrt5}\times\sqrt{1-\frac{1}{5}}\right)$$

$$=\sin^{-1}\left(\frac{1}{\sqrt5}\times\frac{1}{\sqrt5}+\frac{2}{\sqrt5}\times\frac{2}{\sqrt5}\right)$$

$$=\sin^{-1}\left(\frac{1}{5}+\frac{4}{5}\right)$$

$$=\sin^{-1}\frac{5}{5}$$

$$=\sin^{-1}1=\frac{\pi}{2}=\text{ RHS}$$

(iv) Given $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt3}{2}\right)\right]$

$$\Rightarrow\tan^{-1}\left[2\sin\left(2\times\frac{\pi}{6}\right)\right]$$

$$\Rightarrow\tan^{-1}\left[2\times\frac{\sqrt3}{2}\right]$$

$$\Rightarrow\tan^{-1}\sqrt3$$

$$\Rightarrow\frac{\pi}{3}$$

Question 14

Let X, Y and Z represent the events that a participant is under the responsibility of agencies X, Y and Z respectively.

Let M represent the event that a participant goes missing.

Total number of participants: 1000+2000+3000=6000

Probability of a participant being under each agency:

$$P(X)=\frac{1000}{6000}=\frac{1}{6}$$

$$P(Y)=\frac{2000}{6000}=\frac{1}{3}$$

$$P(Z)=\frac{3000}{6000}=\frac{1}{2}$$

Probability of a participant going missing under each agency:

Agency x: $P(M|X)=1-0.99=0.01$

Agency y: $P(M|Y)=1-0.985=0.015$

Agency z: $P(M|Z)=1-0.98=0.02$.

Using Baye's theorem, required probability is given as $P(X|M)=\frac{P(M|X)\cdot P(X)}{P(M)}$

where $P(M)$ is the total probability of a participant going missing.

Here,

$P(M) = P(M|X)\cdot P(X) + P(M|Y)\cdot P(Y) + P(M|Z)\cdot P(Z)$

Substitute the values.

$$P(M)=\left(0.01\cdot\frac{1}{6}\right)+\left(0.015\cdot\frac{1}{3}\right)+\left(0.02\cdot\frac{1}{2}\right)$$

$$P(M)=\frac{0.01}{6}+\frac{0.015}{3}+\frac{0.02}{2}$$

$P(M) = 0.001667 + 0.005 + 0.01 = 0.016667$

Now,

$$P\left(\frac{X}{M}\right)=\frac{P(M|X)\cdot P(X)}{P(M)}=\frac{0.01\cdot\frac{1}{6}}{0.016667}=\frac{0.001667}{0.016667}=0.1$$

The probability that the missing participant was under the responsibility of agency X is 0.1.

SECTION B

Question 15

(i) Option (b) is correct.

Explanation:

Assertion:

$$(\vec{a}+\vec{b})^2=(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})$$

$$(\vec{b}-\vec{a})^2=(\vec{b}-\vec{a})\cdot(\vec{b}-\vec{a})$$

Using the distributive property of the dot product:

$$(\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})=\vec{a}\cdot\vec{a}+2\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}=a^2+2\vec{a}\cdot\vec{b}+b^2$$

$$(\vec{b}-\vec{a})\cdot(\vec{b}-\vec{a})=\vec{b}\cdot\vec{b}-2\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{a}=b^2-2\vec{a}\cdot\vec{b}+a^2$$

Add the expanded forms:

$$(\vec{a}+\vec{b})^2+(\vec{b}-\vec{a})^2=(a^2+2\vec{a}\cdot\vec{b}+b^2)+(b^2-2\vec{a}\cdot\vec{b}+a^2)$$

$$=2a^2+2b^2=2(a^2+b^2)$$

The assertion is true.

Reason: The dot product of any two vectors is commutative, i.e., $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$

The reason is true, but it is not the correct explanation for the assertion.

(ii) Option (b) is correct.

Explanation:

Let

Plane 1: $x+y+2z=9$,

Plane 2: $2x-y+z=15$,

The angle between two planes is equal to the angle between their normal vectors.

The normal vector $\vec{n}_1$ of Plane 1 is the coefficients of x, y and z :

$\vec{n}_1=<1,1,2>$.

The normal vector $\vec{n}_2$ of Plane 2 is:

$\vec{n}_2=<2,-1,1>$

The angle θ between the two planes is given by:

$$\cos\theta=\frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}|\cdot|\vec{n_2}|}$$

Now,

$$\vec{n_1}\cdot\vec{n_2}=(1)(2)+(1)(-1)+(2)(1)=2-1+2=3$$

$$|\vec{n}_1|=\sqrt{1^2+1^2+2^2}=\sqrt{1+1+4}=\sqrt{6}$$

$$|\vec{n}_2|=\sqrt{2^2+(-1)^2+1^2}=\sqrt{4+1+1}=\sqrt{6}$$

Substitute into the formula:

$$\cos\theta=\frac{3}{\sqrt{6}\cdot\sqrt{6}}=\frac{3}{6}=\frac{1}{2}$$

$$\theta=\cos^{-1}\left(\frac{1}{2}\right)=60^\circ=\frac{\pi}{3}$$

(iii) If the points (x_1, y_1, z_1) (x_2, y_2, z_2) and (x_3, y_3, z_3) are collinear

Then $\begin{vmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix}=0$

Where $x_1=-2 \quad y_1=3 \quad z_1=5$

$x_2=1 \quad y_2=2 \quad z_2=3$

$x_3=7 \quad y_3=0 \quad z_3=-1$

Now,

$$\begin{vmatrix}-2 & 3 & 5\\ 1 & 2 & 3\\ 7 & 0 & -1\end{vmatrix}=-2(-2-1)-3(-1-21)+5(0-14)$$

$\Rightarrow +4+66-70=0$

Hance, the point P, Q, R are collinear.

(iv) **(a)** The general vector equation of a straight line is:

$\vec{r}=\vec{a}+\lambda\vec{b}$

where:

$\vec{a}=(1,2,3)$ is a point on the line.

$\vec{b}=(2,3,4)$ is the direction vector.

The parametric equations of the line are:

$x=1+2\lambda$, $y=2+3\lambda$, $z=3+4\lambda$

To eliminate λ, solve for λ in terms of x, y and z:

$$\lambda=\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

Thus, the Cartesian equation of the first bee's path is:

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

(b) The second honeybee's path is parallel to the first one, so it has the same direction vector (2,3,4), but it passes through (2,4,5).

Using the vector equation:

$$\vec{r}=(2\hat{i}+4\hat{j}+5\hat{k})+\mu(2\hat{i}+3\hat{j}+4\hat{k})$$

Expanding into parametric form:

$x=2+2\mu$, $y=4+3\mu$, $z=5+4\mu$

Eliminating μ, we get the Cartesian equation:

$$\frac{x-2}{2}=\frac{y-4}{3}=\frac{z-5}{4}$$

Question 16

(i) The general equation of a plane through $(2, 2\hat{i}-1)$ is

$a(x-2)+b(y-2)+c(z+1)=0$ ...(i)

It will pass through $B(3,4,2)$ and $C(7,0,6)$ if

$a(3-2)+b(4-2)+c(2+1)=0$ and

$a(7-2)+b(0-2)+c(6+1)=0$

$\Rightarrow a+2b+3c=0$ ...(ii)

And $5a-2b+7c=0$ ...(iii)

Solving (i) and (ii) by cross-multiplication, we get

$$\frac{a}{14+6}=\frac{b}{15-7}=\frac{c}{-2-10}$$

$$\Rightarrow \frac{a}{20}=\frac{b}{8}=\frac{c}{-12}$$

$$\Rightarrow \frac{a}{5}=\frac{b}{2}=\frac{c}{-3}=\lambda \text{ (say)}$$

$\Rightarrow a = 5\lambda,\ b = 2\lambda$ and $c = -3\lambda$

Substituting the values of a, b and c in Eq. (i), we get

$$5\lambda(x-2)+2\lambda(y-2)-3\lambda(z+1) = 0$$

$$\Rightarrow 5(x-2)+2(y-2)-3(z+1) = 0$$

$$\Rightarrow 5x+2y-3z = 17$$

which is the required equation of the plane.

OR

(ii) The equation of a plane parallel to x-axis is given as

$by + cz + d = 0$ ...(i)

As it passes through the points (2,3,1) and (4,−5,3)

$\therefore 3b + c + d = 0$ ...(ii)

and $-5b + 3c + d = 0$ ...(iii)

Solving (i) and (ii) by cross–multiplication, we get

$$\frac{b}{1-3}=\frac{c}{-5-3}=\frac{d}{9+5}$$

$$\Rightarrow \frac{b}{-2}=\frac{c}{-8}=\frac{d}{14}=\lambda(\text{say})$$

$b=-2\lambda,\ c=-8\lambda$ and $d=14\lambda$

Substituting the values of a, b and c in Eq. (i), we get

$$-2\lambda y-8\lambda z+14\lambda = 0$$

$$\Rightarrow -2y-8z+14 = 0$$

$$\Rightarrow y+4z-7 = 0$$

$\therefore$ Equation of plane is $y + 4z = 7$

Question 17

(i) Given,

The position vectors of A, B and C are

$\overrightarrow{OA}=2\hat{i}-2\hat{j}+\hat{k},\ \overrightarrow{OB}=\hat{i}+2\hat{j}-2\hat{k}$ and

$$\overrightarrow{OC}=2\hat{i}-\hat{j}+4\hat{k}$$

$$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\left(\hat{i}+2\hat{j}-2\hat{k}\right)-\left(2\hat{i}-2\hat{j}+\hat{k}\right)$$

$$=-\hat{i}+4\hat{j}-3\hat{k}$$

$$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=\left(2\hat{i}-\hat{j}+4\hat{k}\right)-\left(\hat{i}+2\hat{j}-2\hat{k}\right)$$

$$=\hat{i}-3\hat{j}+6\hat{k}$$

(ii) Projection of $\overrightarrow{AB}$ on BC $=\dfrac{\overrightarrow{AB}\cdot\overrightarrow{BC}}{\left|\overrightarrow{BC}\right|}$

Now,

$$\overrightarrow{AB}\cdot\overrightarrow{BC} = (-1)(1)+(4)(-3)+(-3)(6)$$

$$= -1-12-18 = -31$$

and $\left|\overrightarrow{BC}\right|=\sqrt{(1)^2+(-3)^2+(6)^2}$

$$=\sqrt{1+9+36}=\sqrt{46}$$

Projection of $\overrightarrow{AB}$ on $\overrightarrow{BC}=\dfrac{-31}{\sqrt{46}}$

(iii) The area of the triangle ABC :

$$\text{Area}=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{BC}\right|$$

Now,

$$\overrightarrow{AB}\times\overrightarrow{BC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 4 & -3\\ 1 & -3 & 6\end{vmatrix}$$

$$\overrightarrow{AB}\times\overrightarrow{BC}=\hat{i}\left(4\cdot 6-(-3)\cdot(-3)\right)-\hat{j}\left(-1\cdot 6-(-3)\cdot 1\right)+\hat{k}\left(-1\cdot(-3)-4\cdot 1\right)$$

$$\overrightarrow{AB}\times\overrightarrow{BC}=\hat{i}(24-9)-\hat{j}(-6+3)+\hat{k}(3-4)$$

$$\overrightarrow{AB}\times\overrightarrow{BC}=15\hat{i}+3\hat{j}-\hat{k}$$

$$\left|\overrightarrow{AB}\times\overrightarrow{BC}\right|=\sqrt{15^2+3^2+(-1)^2}=\sqrt{225+9+1}=\sqrt{235}$$

Thus, the area of the triangle is:

$$\text{Area}=\frac{1}{2}\sqrt{235}\text{ sq. units}$$

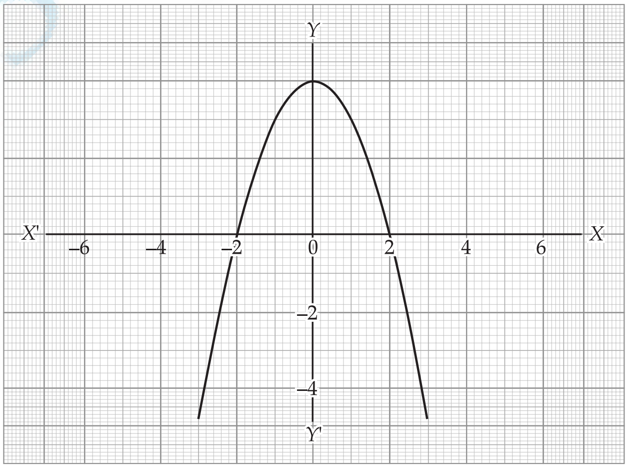
Question 18

(i) (a) The equation $y = 4 - x^2$ represents a downward-opening parabola with:

Vertex at (0,4),

Roots at $x = \pm 2$ (since $4 - x^2 = 0$ when $x = \pm 2$),

Symmetry about the y-axis.



(b) The area under the curve $y=4-x^2$, bounded by the x-axis and the lines $x=0$ and $x=2$, is given by the definite integral:

$$A = \int_0^2 \left(4 - x^2\right) dx$$

$$= \int_0^2 4dx - \int_0^2 x^2 dx$$

$$= 4x\Big|_0^2 - \frac{x^3}{3}\Big|_0^2$$

$$= \{4(2) - 4(0)\} - \left(\frac{2^3}{3} - \frac{0^3}{3}\right)$$

$$= 8 - \frac{8}{3} = \frac{24}{3} - \frac{8}{3} = \frac{16}{3} \text{ sq. units}$$

(c) The area under the curve $y=4-x^2$, bounded by the x-axis and the lines $x=-2$ and $x=2$, is given by the definite integral:

$$A = \int_{-2}^{2} \left(4 - x^2\right) dx$$

$$= \int_{-2}^{2} 4\, dx - \int_{-2}^{2} x^2 dx$$

$$= 4x\Big|_{-2}^{2} - \frac{x^3}{3}\Big|_{-2}^{2}$$

$$= \{4(2) - 4(-2)\} - \left(\frac{2^3}{3} - \frac{(-2)^3}{3}\right)$$

$$= 16 - \left(\frac{8}{3} + \frac{8}{3}\right) = 16 - \frac{16}{3} = \frac{32}{3} \text{ sq. units}$$

OR

(ii) Given, $x + 2y = 2$ … (i)

$y - x = 1$ … (ii)

$2x + y = 7$ ….. (iii)

On solving Eqs. (i) and (ii), we get

Coordinate of $A \equiv (0, 1)$.

Similarly,

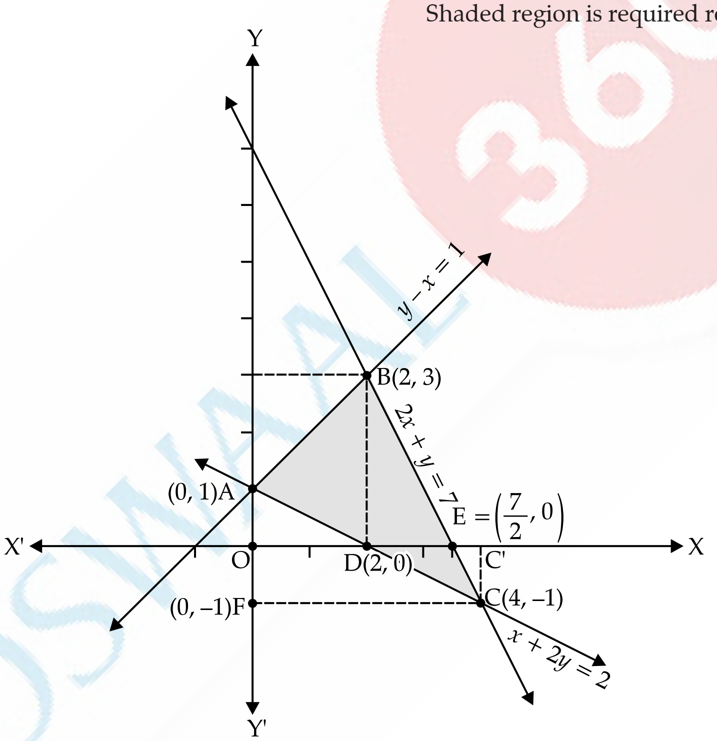
By solving Eq. (ii) and (iii), we get

Co-ordinate of $B \equiv (2,3)$

By solving Eq. (i) and (iii), we get

Co-ordinate of $C \equiv (4, -1)$

Shaded region is required region.



∴ Area of required region

$$= \int_{-1}^{3} \frac{7-y}{2} dy - \int_{-1}^{1} (2 - 2y) dy - \int_{1}^{3} (y-1) dy$$

$$= \frac{1}{2}\left[7y - \frac{y^2}{2}\right]_{-1}^{3} - \left[2y - y^2\right]_{-1}^{1} - \left[\frac{y^2}{2} - y\right]_{1}^{3}$$

$$= \frac{1}{2}\left(21 - \frac{9}{2} + 7 + \frac{1}{2}\right) - (2 - 1 + 2 + 1) - \left(\frac{9}{2} - 3 - \frac{1}{2} + 1\right)$$

$=12-4-2=6$ sq units

SECTION C

Question 19

(i) Option (b) is correct.

Explanation: The total revenue function is:

$R(x) = 36x + 3x^2 + 5$

Differentiate $R(x)$ with respect to x:

$$R'(x) = \frac{d}{dx}(36x + 3x^2 + 5) = 36 + 6x$$

Substituting $x = 9$:

$R'(9) = 36 + 6(9) = 36 + 54 = 90$

(ii) Option (c) is correct.

Explanation: Statement (I) is true. The correlation coefficient (r) and the regression coefficients (b_{yx} and b_{xy}) always share the same sign. If r is positive, both regression coefficients are positive, and if r is negative, both regression coefficients are negative.

Statement (II) is false. The correlation coefficient is not the arithmetic mean of the regression coefficients. Instead, it is the geometric mean of the two regression coefficients:

$$r = \sqrt{b_{yx} . b_{xy}}$$

Statement (III) is false. The product of the two regression coefficients is equal to the square of the correlation coefficient: $b_{yx} \cdot b_{xy} = r^2$

This product is not always equal to 1 unless $r = \pm 1$.

Statement (IV) is true. The regression coefficients are bounded by the correlation coefficient (r), which lies between -1 and 1. Therefore, the regression coefficients cannot exceed 1 in the absolute value.

(iii) From the given data:

(a) $\bar{x} = \dfrac{1+2+3+6}{4} = \dfrac{12}{4} = 3$

$\bar{y} = \dfrac{6+5+4+1}{4} = \dfrac{16}{4} = 4$

(b)

x	1	2	3	6
y	6	5	4	1
$x - \bar{x}$	-2	-1	0	3
$y - \bar{y}$	2	1	0	-3

(c) The formula for the regression coefficient b_{xy} is:

$$b_{xy} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma\ (x-\bar{x})^2}$$

Here,

$$\Sigma(x-\bar{x})(y-\bar{y}) = (-4)+(-1)+0+(-9) = -14$$

$$\Sigma(x-\bar{x})^2 = 4+1+0+9 = 14$$

Substituting the values:

$$b_{xy} = \frac{-14}{14} = -1$$

Question 20

(i) Given, $\Sigma x = 24$, $\Sigma y = 44$, $\Sigma xy = 306$, $\Sigma x^2 = 164$, $\Sigma y^2 = 576$, $n = 4$

So,

$$\bar{x} = \frac{24}{4} = 6$$

$$\bar{y} = \frac{44}{4} = 11$$

The regression equation of y on x is given by:

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

We know that,

$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$= \frac{4(306)-(24)(44)}{4(164)-(24)^2}$$

$$= \frac{1224-1056}{656-576}$$

$$= \frac{168}{80} = 2.1$$

Therefore, the regression line of y on x is:

$$y - 11 = 2.1(x-6)$$
$$y - 11 = 2.1x - 12.6$$
$$y = 2.1x - 1.6$$

Thus, the regression line of y on x is:

$y = 2.1x - 1.6$

This is the line of best fit based on the given data.

OR

(ii) A regression equation of x on y is of the form:

$x = b_{xy}y + c$

where b_{xy} is the regression coefficient of x on y.

Equation 1: $4x + 3y + 7 = 0$

$3y = -4x - 7$

$$y = -\frac{4}{3}x - \frac{7}{3}$$

So, the slope of this equation is $b_{yx} = -\dfrac{4}{3}$

Equation 2: $3x + 4y + 8 = 0$

$3x = -4y - 8$

$$x = -\frac{4}{3}y - \frac{8}{3}$$

So, the slope of this equation is $b_{xy} = -\dfrac{4}{3}$

Since the equation $x = -\dfrac{4}{3}y - \dfrac{8}{3}$ is already in the form $x = b_{xy}y + c$, this represents the regression line of x on y.

Question 21

(i) Given,

The number of dinner sets sold per week: $x = \dfrac{600-p}{8}$

The cost function:$(x) = x^2 + 78x + 2000$

(a) Revenue is given by:

$R(x) = (\text{ Price per unit }) \times (\text{ Number of units sold })$

Since $p = 600 - 8x$, we get:

$R(x) = p \cdot x = (600 - 8x)x$

$R(x) = 600x - 8x^2$

Thus, the revenue function is $R(x) = 600x - 8x^2$.

(b) Profit is given by: $P(x) = R(x) - C(x)$

$P(x) = (600x - 8x^2) - (x^2 + 78x + 2000)$

$P(x) = 600x - 8x^2 - x^2 - 78x - 2000$

$P(x) = -9x^2 + 522x - 2000$

Thus, the profit function is:

$P(x) = -9x^2 + 522x - 2000$

(c) To maximise profit, we take the derivative of $P(x)$ and put it to zero:

$$\frac{dP}{dx} = -18x + 522 = 0$$

$$18x = 522$$

$$x = 29$$

To confirm it's a maximum, we check the second derivative:

$$\frac{d^2P}{dx^2} = -18$$

Since $-18 < 0$, the function has a maximum at $x=29$. Thus, the manufacturer should produce and sell 29 dinner sets per week to ensure maximum profit.

OR

(ii) (a) The Average Cost (AC) is defined as:

$$AC = \frac{C(x)}{x}$$

Thus, the total Cost Function is:

$C(x)=AC\times x$

$$C(x) = x\left(\frac{x^2}{200} - \frac{x}{50} - 30 + \frac{5000}{x}\right)$$

$$C(x) = \frac{x^3}{200} - \frac{x^2}{50} - 30x + 5000$$

(b) The Marginal Cost (MC) is the derivative of the cost function $C(x)$:

$$MC = \frac{d}{dx}\left(\frac{x^3}{200} - \frac{x^2}{50} - 30x + 5000\right)$$

$$MC = \frac{3x^2}{200} - \frac{2x}{50} - 30$$

$$MC = \frac{3x^2}{200} - \frac{x}{25} - 30$$

(c) The Marginal Average Cost (MAC) is the derivative of the Average Cost (AC) function:

$$MAC = \frac{d}{dx}\left(\frac{x^2}{200} - \frac{x}{50} - 30 + \frac{5000}{x}\right)$$

$$MAC = \frac{2x}{200} - \frac{1}{50} + \frac{d}{dx}\left(\frac{5000}{x}\right)$$

$$MAC = \frac{x}{100} - \frac{1}{50} - \frac{5000}{x^2}$$

(d) To verify $\frac{d}{dx}(AC) = \frac{MC - AC}{x}$, we need to verify:

$$MAC = \frac{MC - AC}{x}$$

From (c), we have:

$$MAC = \frac{x}{100} - \frac{1}{50} - \frac{5000}{x^2}$$

Now,

$$\frac{MC - AC}{x} = \frac{\left(\frac{3x^2}{200} - \frac{x}{25} - 30\right) - \left(\frac{x^2}{200} - \frac{x}{50} - 30 + \frac{5000}{x}\right)}{x}$$

$$= \frac{\frac{3x^2}{200} - \frac{x}{25} - 30 - \frac{x^2}{200} + \frac{x}{50} + 30 - \frac{5000}{x}}{x}$$

$$= \frac{\left(\frac{3x^2}{200} - \frac{x^2}{200}\right) + \left(-\frac{x}{25} + \frac{x}{50}\right) - \frac{5000}{x}}{x}$$

$$= \frac{\frac{2x^2}{200} - \frac{x}{50} - \frac{5000}{x}}{x}$$

$$= \frac{x}{100} - \frac{1}{50} - \frac{5000}{x^2}$$

$$= MAC$$

Hence, verified.

Question 22

Let x books of first type and y books of second type. Number of books cannot be negative.

Therefore, $x, \geq 0$. According to question, the given information can be tabulated as:

	Thickness (cm)	Weight (kg)
First type (x)	6	1
Second type (y)	4	1.5
Capacity of shelf	96	21

Therefore, the constraints are $6x + 4y \leq 96$

$x + 1.5y \leq 21$

Or, $3x + 2y \leq 48$

$2x + 3y \leq 42$

Number of books $= Z = x + y$ which is to be maximised. Thus, the mathematical formulation of the given linear programming problem is

Maximise $Z = x + y$

subject to $3x + 2y \leq 48$

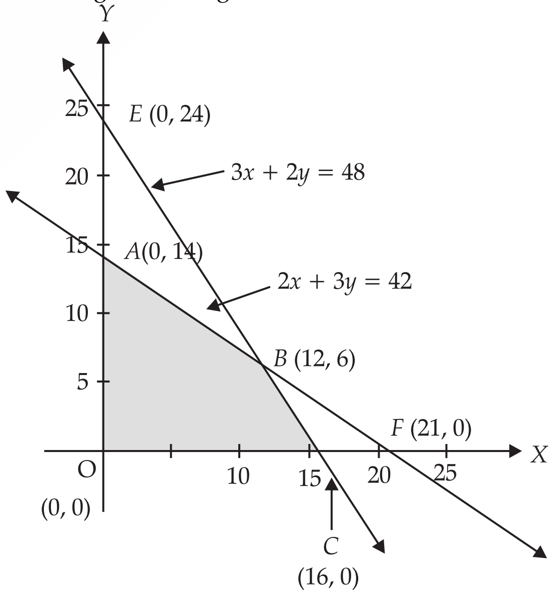
$2x + 3y \leq 42$

On considering the inequalities as equations, we get

$3x + 2y = 48$ ... (i)

$2x + 3y = 42$ ... (ii)

On plotting the above equations, we get the following shaded region.



From the graph, *OABCD* is the feasible region which is bounded.

The corner points are $O(0,0)$, $A(0,14)$, $B(12,16)$, $C(16,0)$.

And the value of Z at corner points is

Corner points	**Value of $Z = x + y$**
$O(0,0)$	$Z = 0 + 0 = 0$
$A(0,14)$	$Z = 0 + 14 = 14$
$B(12,6)$	$Z = 12 + 6 = 18$ (maximum)
$C(16,0)$	$Z = 16 + 0 = 16$

From the table, the maximum value of Z is 18 at (12,6).

Hence, the maximum number of books of I type is 12 and books of II type is 6.