C.D.S.
Combined Defence Services

EXAM

II

2023

Elementary Mathematics

Time Allowed : 2 Hours **M.M. : 100**

Important Instructions :

1. *This Test Booklet contains **100** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ONLY ONE response for each item.*
2. *You have to mark all your responses ONLY on the separate Answer Sheet provided. See directions in the Answer Sheet.*
3. ***All** items carry equal marks.*
4. ***Penalty for wrong answers :***
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i) *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.*
   (ii) *If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii) *If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.*

1. If $a^2 - bc = \alpha$, $b^2 - ac = \beta$, $c^2 - ab = \gamma$, then what is $\frac{a\alpha + b\beta + c\gamma}{(a+b+c)(\alpha+\beta+\gamma)}$ equal to?
   (a) $a + b - c$ (b) $a - b + c$
   (c) $-a + b + c$ (d) 1
2. If $(x-1)^3$ is a factor of $x^4 + \alpha x^3 + \beta x^2 + \gamma x - 1$, then the other factor will be:
   (a) $x + 1$ (b) $x - 3$
   (c) $x + 2$ (d) x
3. A 2-digit number is such that the sum of the number and the number obtained by reversing the order of the digits of the number is 55. Further, the difference of the given number and the number obtained by reversing the order of the digits of the number is 45. What is the product of the digits?
   (a) 5 (b) 2
   (c) 1 (d) 0
4. If A and B can finish a work in 10 days, B and C can finish the same work in 12 days, C and A can finish the same work in 15 days; then in how many days can A, B and C together finish half of the work?
   (a) 8 days (b) 5 days
   (c) 4 days (d) 3 days
5. A person borrowed ₹10,000 at 12% rate of interest per annum compounded quarterly for a period of 9 months. What is the interest paid by him to settle his account after 9 months?
   (a) ₹927.27 (b) ₹947.47
   (c) ₹967.67 (d) ₹987.87
6. For what relation between a and b is the equation $\sin\theta = \frac{a+b}{2\sqrt{ab}}$ possible?
   (a) $a = b$ (b) $a \le b$
   (c) $a \ge b$ (d) $a > b$
7. Three persons A, B and C together can do a piece of work in 36 days. A and B together can do five times as much work as C alone; B and C together can do as much work as A alone. If A and C together can do n times as much work as B alone, then what is the value of n?
   (a) 1.5 (b) 2
   (c) 2.5 (d) 3
8. If $\frac{2a}{3} = \frac{4b}{5} = \frac{3c}{4}$, then what is the value of $\frac{18}{a}\sqrt{a^2 + c^2 - b^2}$?
   (a) $3\sqrt{5}$ (b) $\sqrt{355}$
   (c) $\sqrt{375}$ (d) $3\sqrt{15}$
9. The sum of deviations of n numbers from 10 and 20 are a, b respectively. If $\frac{b}{a} = -4$, then what is the mean of these n numbers?
   (a) 12 (b) 14
   (c) 16 (d) 18

10. If the median of observations 12, 1, 8, 54, 61, 28, 45, 35, 21, 17 is M, then what is the value of 2M + 5?
(a) 12 (b) 28
(c) 52 (d) 54

11. How many real roots does the equation $\sqrt{x+9} = x-3$ have?
(a) Only one (b) Only two
(c) Only three (d) None

12. If $x = 97 + 56\sqrt{3}$, then what is the value of $\sqrt[4]{x} + \frac{1}{\sqrt[4]{x}}$?
(a) 7 (b) 6
(c) 5 (d) 4

13. Let L be the LCM and H be the HCF of two given numbers. L and H are in the ratio 3 : 2. If the sum of the two numbers is 45, then what is the product of the numbers?
(a) 243 (b) 486
(c) 504 (d) Cannot be determined due to insufficient data

14. A man walks at an average speed of 3 km/h from his home and reaches office 40 minutes early. If he walks at an average speed of 2 km/h, he would reach office 40 minutes late. What is the distance between his home and office?
(a) 6 km (b) 8 km
(c) 10 km (d) 12 km

15. If $3^{x-1} + 3^{3-x} = 6$, then what is $2^{x-1} + 2^{3-x}$ equal to?
(a) 4 (b) 3
(c) 2 (d) 1

16. If $x\left(a-b+\frac{ab}{a-b}\right) = y\left(a+b-\frac{ab}{a+b}\right)$ and $x + y = 2a^3$, then what is $x - y$ equal to?
(a) $-2b^3$ (b) $-2ab^3$
(c) $2b^3$ (d) $2ab^3$

17. Which one of the following is a factor of $3\sqrt{3}x^3 + 2\sqrt{2}y^3 - 18xy + 6\sqrt{6}$?
(a) $\sqrt{3}x + \sqrt{2}y - \sqrt{3}$
(b) $\sqrt{3}x + \sqrt{2}y - \sqrt{6}$
(c) $3x^2 + 2y^2 - \sqrt{18}x - \sqrt{12}y - \sqrt{6}xy + 6$
(d) $3x^2 + 2y^2 + \sqrt{18}x + \sqrt{12}y - \sqrt{6}xy + 6$

18. What is the number of digits in the expansion of 125^{100}? (Given $\log_{10} 2 = 0.301$)
(a) 69 (b) 70
(c) 209 (d) 210

19. What is the HCF of $acx^3 + bcx^2 + adx^2 + acdx + bdx + bcd$ and $adx^3 + acx^2 + bdx^2 + bcx + acdx + bcd$ if HCF $(c, d) = 1$, $c \neq d$?
(a) $bx + c$ (b) $cx + x$
(c) $ax + d$ (d) $ax + b$

20. If $x^n - py^n + qz^n$ is divisible by $x^2 + abyz - bzx - axy$, then what is $\frac{p}{a^n} - \frac{q}{b^n}$ equal to?
(a) –1 (b) 0
(c) 1 (d) 2

21. Consider the following statements:
1. If $(a + b)$ is directly proportional to $(a - b)$, then $(a^2 + b^2)$ is directly proportional to ab.
2. If a is directly proportional to b, then $(a^2 - b^2)$ is directly proportional to ab.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

22. If $(3a + 6b + c + 2d) \times (3a - 6b - c + 2d) = (3a - 6b + c - 2d) \times (3a + 6b - c - 2d)$, then which one of the following is correct?
(a) $ab = cd$ (b) $ac = bd$
(c) $ad = bc$ (d) $ad + bc = 0$

23. If $3\sin\theta + 5\cos\theta = 5$, then what is the value of $5\sin\theta - 3\cos\theta$ equal to?
(a) 5 (b) –3
(c) –2 (d) 0

24. The combined age of a man and his wife is 6 times the combined age of their children. Two years ago their combined age was 10 times the combined age of their children; and six years later their combined age will be 3 times the combined age of their children. How many children do they have if each child is at least 2 years old?
(a) 2 (b) 3
(c) 4 (d) 5

25. What is $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin x)^6 + 4(\cos x)^6$ equal to?
(a) 9 (b) 11
(c) 13 (d) 15

26. What is the value of $\sin\theta + \cos\theta$, if satisfies the equation $\cot^2\theta - (\sqrt{3}+1)\cot\theta + \sqrt{3} = 0$; $0 < \theta < \frac{\pi}{4}$?
(a) $\sqrt{2}$ (b) 2
(c) $\frac{\sqrt{3}+1}{2}$ (d) $\frac{\sqrt{3}-1}{2}$

27. Which one of the following is a value of θ, if θ satisfies the equation $\tan 2\theta \tan 4\theta - 1 = 0$; $0 < \theta < \frac{\pi}{2}$?
(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{15}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{5}$

28. If $\tan x = \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$, $\frac{\pi}{4} < \theta < \frac{\pi}{2}$, then what is $\sqrt{2}\sin x$ equal to?
(a) $\sin\theta + \cos\theta$ (b) $\sin\theta - \cos\theta$
(c) $\frac{\sin\theta + \cos\theta}{2}$ (d) $\frac{\sin\theta - \cos\theta}{2}$

29. How many values of θ will satisfy the equation $(\sin^2\theta - 4\sin\theta + 3)(4 - \cos^2\theta + 4\sin\theta) = 0$, where $0 < \theta < \frac{\pi}{2}$?
(a) None (b) Only one
(c) Only two (d) Only three

30. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta - y\cos\theta = 0$, for every $\theta \in \left(0, \frac{\pi}{2}\right)$, then what is $x^2 + y^2$ equal to?
(a) 0 (b) 1
(c) 2 (d) 3

Consider the following for the next two (02) items that follow:

A flagstaff stands on the top of a vertical tower. The angle of elevation of the top of the flagstaff from a certain place on the same horizontal level with the base of the tower is found to be α. Advancing a distance *d* towards the tower in the same horizontal plane, the angle of elevation of the top of the flagstaff is observed to be β and that of the top of the tower is observed to be γ. Let H be the height of the top of the flagstaff from the base of the tower and h be the height of the tower.

31. Which one of the following is correct?
(a) $H\tan\gamma - h\tan\beta = 0$
(b) $h\tan\gamma - H\tan\beta = 0$
(c) $H\tan\gamma - h\tan\alpha = 0$
(d) $h\tan\gamma - H\tan\alpha = 0$

32. Which one of the following is correct?
(a) $d = \frac{H(\cot\alpha - \cot\beta)}{2}$ (b) $d = \frac{H(\tan\alpha - \tan\beta)}{2}$
(c) $d = H(\cot\alpha - \cot\beta)$ (d) $d = H(\tan\alpha - \tan\beta)$

Consider the following for the next two (02) items that follow:

AB is a straight road leading to the foot P of a tower of height *h*. Q is at distance *x* from P and R is at a distance *y* from Q (R is farther from P than Q; R, Q are on the same side). The angle of elevation of the top of the tower at Q is twice of that at R. (Use the formula $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$).

33. Which one of the following is correct?
(a) $x = y$
(b) $x < y$
(c) $x > y$
(d) Cannot be concluded due to insufficient data.

34. Which one of the following is correct?
(a) $h^2 = x^2 - y^2$ (b) $h^2 = x^2 + y^2$
(c) $h^2 = 2(y^2 - x^2)$ (d) $h^2 = y^2 - x^2$

Consider the following for the next two (02) items that follow:

Two parallel chords AB and CD of a circle are of lengths 60 cm and 80 cm respectively. They are on the same side of the centre O and 10 cm apart.

35. What is the diameter of the circle?
(a) 120 cm (b) 110 cm
(c) 100 cm (d) 90 cm

36. If the chord AB subtends an angle α and chord CD subtends an angle β at the centre O, then what is the value of $\tan\left(\frac{\beta}{2}\right) - \tan\left(\frac{\alpha}{2}\right)$?
(a) $\frac{3}{4}$ (b) $\frac{5}{12}$
(c) $\frac{1}{2}$ (d) $\frac{7}{12}$

Consider the following for the next two (02) items that follow:

Let $p = x^4 - y^2z^2$, $q = y^4 - z^2x^2$, $r = z^4 - x^2y^2$.

37. What is $px^2 + qy^2 + rz^2$ equal to?
(a) $(x^2 + y^2 + z^2)(p + q + r)$
(b) $-(x^2 + y^2 + z^2)(p + q + r)$
(c) $(y^2 + z^2 - x^2)(r - q - p)$
(d) $(x^2 + y^2 - z^2)(p - q - r)$

38. What is $x^2(px^2 + qy^2 + rz^2) + qr - p^2$ equal to?
(a) 0 (b) 1
(c) $p + q + r$ (d) $x^2 + y^2 + z^2$

Consider the following for the next two (02) items that follow:

A right conical cap just covers two spheres placed one above the other on a table such that it touches both the spheres. Let *r* be the radius of the smaller sphere and R be the radius of the bigger sphere. Let 2θ be the vertical angle of the cone.

39. What is the height of the cone?
(a) $\frac{2r^2}{R - r}$ (b) $\frac{2R^2}{R - r}$
(c) $\frac{2(r^2 + R^2)}{R - r}$ (d) $\frac{r^2 + R^2}{R - r}$

40. What is the radius of the base of the cone?
(a) $\frac{2r^2\tan\theta}{R - r}$ (b) $\frac{2R^2\tan\theta}{R - r}$
(c) $\frac{2(r^2 + R^2)\tan\theta}{R - r}$ (d) $\frac{(r^2 + R^2)\tan\theta}{R - r}$

Consider the following for the next two (02) items that follow:

A line segment AB is bisected at C and semi-circles S_1, S_2 and S_3 are drawn respectively on AB, AC and CB as diameters such that they all lie on same side of AB. A circle S is drawn touching internally S_1 and externally S_2 and S_3.

41. If *r* is the radius of S and R is the radius of S_2, then which one of the following is correct?

(a) R = 3*r* (b) R = 2*r*
(c) 3R = 4*r* (d) 2R = 3*r*

42. If *m* is the area of the circle S and *n* is the area of semi-circle S_1, then which one of the following is correct?
(a) $9m = 2n$ (b) $9m = 4n$
(c) $3m = 2n$ (d) $7m = 3n$

Consider the following for the next two (02) items that follow:

Let $\frac{(x-a)(x-b)}{(x-ma)(x-mb)} = \frac{(x+a)(x+b)}{(x+ma)(x+mb)}$; $m, a, b > 0$.

43. What is $\frac{x^2+ab}{x^2+m^2ab}$ equal to?
(a) $-\frac{1}{m^2}$ (b) $\frac{1}{m^2}$
(c) $\frac{2}{m}$ (d) $\frac{1}{m}$

44. What is x equal to?
(a) $\pm\sqrt{mab}$ (b) $\pm\sqrt{ab}$
(c) $\pm\sqrt{2mab}$ (d) $\pm\sqrt{2ab}$

Consider the following for the next two (02) items that follow:

The total monthly electricity bill for a house consists of the sum of two parts, one part is proportional to number of rooms and the other part is proportional to number of units consumed. ₹400 is the monthly electricity bill for a house with 8 rooms and consuming 240 units and ₹320 is the monthly electricity bill for a house with 6 rooms and consuming 200 units.

45. What is the monthly electricity bill for a house with *m* rooms and consuming *n* units?
(a) ₹$(40m + n)$ (b) ₹$(20m + n)$
(c) ₹$\frac{(40m+n)}{2}$ (d) ₹$\frac{(30m+n)}{2}$

46. What is the monthly electricity bill for a house with 7 rooms consuming 300 units?
(a) ₹500 (b) ₹440
(c) ₹340 (d) ₹300

Consider the following for the next two (02) items that follow:

A grouped frequency distribution is given below :

Weekly wages in Rupees (₹)	Numbers of workers
2050–2550	5
2550–3050	10
3050–3550	*k*
3550–4050	8
4050–4550	2
4550–5050	10

47. If average weekly wages earned by a worker is ₹3,520, then what is the value of k?
(a) 10 (b) 12
(c) 15 (d) 20

48. What is the median (approximate value) of the distribution?
(a) ₹ 3,263 (b) ₹3,383
(c) ₹3,413 (d) ₹3,483

Consider the following for the next two (02) items that follow:

A quadratic equation is given by $(a + b + c)x^2 - (2a + 2b)x + (a + b - c) = 0$. Where *a*, *b* and *c* are real and distinct.

49. What are the roots of the equation?
(a) $1, \frac{(a+b-c)}{(a+b+c)}$ (b) $1, \frac{(a-b+c)}{(a+b+c)}$
(c) $-1, \frac{(-a-b+c)}{(a+b+c)}$ (d) $-1, \frac{(a+b-c)}{(a+b+c)}$

50. Consider the following statements:
1. One of the roots of the equation is always less than 1 if *a*, *b* and *c* are all positive.
2. One of the roots of the equation is always negative if *a*, *b* and *c* are all negative.

Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

51. What is the radius of the sphere passing through the corners of the cuboid with edges 8 cm, 12 cm and 24 cm?
(a) 10.5 cm (b) 14 cm
(c) 21 cm (d) 28 cm

52. A lamp shade is in the shape of a part of a cone and its top and bottom ends are circles whose circumferences are respectively 30 cm and 40 cm. The perpendicular distance between the ends is 6 cm. If the cone were to be completed, then how far would its vertex be from the top end?
(a) 20 cm (b) 18 cm
(c) 12 cm (d) 9 cm

53. A sum of money at 20% rate of compound interest per annum becomes more than 100 times in *n* years. What is the least value of n? (Use $\log_{10} 2 = 0.301$, $\log_{10} 3 = 0.477$).
(a) 23 (b) 24
(c) 25 (d) 26

54. The corners of an equilateral triangular plate were cut in such a manner that it forms a regular hexagonal plate. What is the ratio of the area of the triangular plate to the area of the hexagonal plate?
(a) 2: 1 (b) 3: 2
(c) 4: 3 (d) 5: 3

55. Two equal arcs of different circles C_1 and C_2 subtend angles of 60° and 75° respectively, at the centres. What is the ratio of the radius of C_1 to the radius of C_2?
(a) 4 : 5 (b) 5 : 4
(c) 1 : 1 (d) 3 : 2

56. ABC is a triangle with sides AB = 41 cm, BC = 28 cm and CA = 15 cm. IF D, E and F are the mid-point of AB, BC and CA respectively, then what is the area of the triangle DEF?
(a) 63 square cm (b) 45 square cm
(c) 31.5 square cm (d) 22.5 square cm

57. A triangle has sides 13 cm, 14 cm and 15 cm long. What is the length of the smallest altitude of the triangle?
(a) 11 cm (b) 11.2 cm
(c) 12 cm (d) 12.2 cm

58. The circumference of a circle exceeds the diameter by 16.8 cm. What is the diameter of the circle? (Take $\pi = \frac{22}{7}$)
(a) 6.24 cm (b) 6.42 cm
(c) 7.64 cm (d) 7.84 cm

59. The hypotenuse AC of a right-angled ABC is $3\sqrt{5}$ cm. If AB is doubled and BC is tripled such that ABC remains a right-angled triangle, the hypotenuse becomes 15 cm. What is AB + BC equal to?
(a) 10 cm (b) 9 cm
(c) $2\sqrt{5}$ cm (d) 8 cm

60. What is the area of the region between two concentric circles if the chord of the outer circle of length 14 cm is a tangent of the inner circle? (Take $\pi = \frac{22}{7}$)
(a) 125 square cm (b) 132 square cm
(c) 144 square cm (d) 154 square cm

61. A pendulum swings through an angle of 9° and its end describes an arc of length 14.3 cm. What is the length of the pendulum? (Take $\pi = \frac{22}{7}$)
(a) 88 cm (b) 91 cm
(c) 95 cm (d) 98 cm

62. The arch of a bridge is in the form of an arc of a circle. If the span of the bridge is 40 m and height in the middle is 8 m, then what is the radius of curvature of the bridge?
(a) 25 m (b) 27 m
(c) 29 m (d) 31 m

63. If a, b and c are the sides of a right-angled triangle, where $a > b > c$, then what is the value of the expression
$(a + b + c)(a + b - c)(a - b + c)(a - b - c)$?
(a) $4b^2c^2$ (b) $-4b^2c^2$
(c) $-2a^2b^2$ (d) $-4a^2b^2$

64. The cube root of x varies inversely as the square root of y. $x = 8$ when $y = 3$. What is the value of x when $y = \sqrt[3]{3}$?
(a) 18 (b) 21
(c) 24 (d) 27

65. Three solid lead spheres of diameters 6 cm, 8 cm and 10 cm are melted together and recast as a solid sphere. What is the percentage diminution of the surface area as compared to the sum of the surface areas of the three spheres?
(a) 25% (b) 26%
(c) 27% (d) 28%

66. A solid sphere of radius 3 cm is melted to form a hollow cylinder of height 4 cm and external diameter 10 cm. What is the thickness of the cylinder?
(a) 0.42 cm (b) 0.46 cm
(c) 0.50 cm (d) 1.00 cm

67. Three glasses P, Q and R have capacities in the ratio 1 : 2 : 3. All these glasses are completely filled with mixtures of milk and water. The ratio of milk to water in P is 1 : 2, in Q it is 2 : 3 and in R it is 3 : 1. If the content of all three glasses are put into a bigger container, what will be the ratio of milk to water in the container?
(a) 203 : 117 (b) 203 : 157
(c) 172 : 91 (d) 165 : 88

68. What is the LCM of $x^4 + x^2y^2 + y^4$, $x^3 + y^3$, $x^3 - y^3$?
(a) $(x^2 - y^2)(x^4 + x^2y^2 + y^4)^2$
(b) $(x^2 - y^2)(x^4 + 2x^2y^2 + y^4)$
(c) $(x^6 - y^6)$
(d) $(x^6 + y^6)$

69. What is $\frac{x^2 - y^2 - z^2 - 2yz}{x^2 + y^2 - z^2 + 2xy} + \frac{x^2 - y^2 - z^2 - 2yz}{x^2 - y^2 + z^2 - 2xz}$ equal to?
(a) $\frac{x}{x + y - z}$ (b) $\frac{y + z}{x + y - z}$
(c) $\frac{2x}{x + y - z}$ (d) $\frac{2y + 2z}{x + y - z}$

70. If tan A + cot A = 2, where $0 < A < 90°$, then what is the value of $\tan^2A + \tan^3A + \tan^4A + \ldots + \tan^nA$?
(a) 1 (b) $n - 2$
(c) $n - 1$ (d) n

71. Which of the following is/are identity/identities?
1. $\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} + \sin\theta\cos\theta = 1;\ 0 < \theta < \frac{\pi}{2}$
2. $1 - \sin^6\theta = \cos^2\theta(\cos^4\theta + 3\sin^2\theta)$
Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

72. If $7\sin^4\theta + 9\cos^4\theta + 42\sin^2\theta = 16,\ 0 < \theta < \frac{\pi}{2}$, then what is the $\tan\theta$ equal to?
(a) 1 (b) $\sqrt{2}$
(c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

73. An isosceles triangle has its base length $2a$ and its height is h. On each side of the triangle a square is drawn external to the triangle. What is the area of the figure thus formed?

(a) $6a^2 + 2h^2 + 2ah$ (b) $6a^2 + 2h^2 + ah$
(c) $4a^2 + 2h^2 + ah$ (d) $6a^2 + h^2 + ah$

74. If $p = \frac{a^2}{(b-a)(c-a)}$, $q = \frac{b^2}{(c-b)(a-b)}$, $r = \frac{c^2}{(a-c)(b-c)}$, then what is $(p + q + r)^2$ equal to?
(a) 9 (b) 4
(c) 1 (d) 0

75. Which one of the following is a factor of $a^2 - b^2 - c^2 + 2bc + a + b - c$?
(a) $a + b + c + 1$ (b) $a - b - c + 1$
(c) $a + b + c - 1$ (d) $a - b + c + 1$

76. Let α and β be the roots of the equation $\frac{1}{x+a+b} = \frac{1}{x} + \frac{1}{a} + \frac{1}{b}$; $a \neq 0, b \neq 0, x \neq 0$. Which one of the following is a quadratic equation whose roots are α^2 and β^2?
(a) $x^2 + (a^2 + b^2)x + a^2b^2 = 0$
(b) $x^2 - (a^2 + b^2)x + a^2b^2 = 0$
(c) $x^2 - (a^2 + b^2)x - a^2b^2 = 0$
(d) $x^2 + (a^2 + b^2)x - a^2b^2 = 0$

77. If $x = \frac{6}{7 - \frac{6}{7 - \frac{6}{7 - \frac{6}{7-x}}}}$; $x > 1$, then what is the value of $x^2 - 3x + 2$ equal to?
(a) 0 (b) 1
(c) 18 (d) 20

78. A train completely overtakes two persons, walking in the same direction with speeds 3 km/h and 4 km/h in 9 seconds and $\frac{75}{8}$ seconds respectively. What is the length of the train?
(a) 60 m (b) 62.5 m
(c) 55 m (d) 67.5 m

79. A person bought an article and sold it at a profit of 20%. Had he bought it at 20% less, what would have been the profit percentage if the selling price had been the same?
(a) 25% (b) 40%
(c) 50% (d) 60%

80. If $2s = a + b + c$, then what is $s^2 + (s-a)(s-b) + (s-b)(s-c) + (s-c)(s-a)$ equal to?
(a) $(a + b + c)^2$ (b) $ab + bc + ca$
(c) $2(ab + bc + ca)$ (d) $3(ab + bc + ca)$

81. A sphere of radius 5 cm is dropped in a right circular cylindrical vessel partly filled with water. The radius of the cylindrical vessel is 10 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel?
(a) $\frac{5}{3}$ cm (b) $\frac{5}{2}$ cm
(c) 1 cm (d) $\frac{5}{6}$ cm

82. Consider the following statements :
1. The angle in a sector greater than a semi-circle is less than a right angle.
2. If two sides of a pair of opposite sides of a cyclic quadrilateral are equal, then its diagonals are also equal.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

83. If a, b, c, x, y, z are real numbers such that $(a + b + c)^2 - 3(ab + bc + ca) + 3(x^2 + y^2 + z^2) = 0$, then which one of the following is correct?
(a) $a = b = c, x = y = z \neq 0$
(b) $a = b = c = 0, x = y = z = 1$
(c) $a = b = c, x = y = z = 0$
(d) $a \neq b \neq c, x = y = z = 0$

84. In a triangle ABC, angle B = 90° and p is the length of the perpendicular from B to AC. If BC = 10 cm and AC = 12 cm, then what is the value of p?
(a) $\frac{5\sqrt{11}}{3}$ (b) $\frac{10\sqrt{11}}{3}$
(c) $\frac{40}{\sqrt{61}}$ (d) $\frac{12}{25}$

85. The mean of p, q, r, s and t is 280. If the mean of p, r and t is 240, what is the mean of q and s?
(a) 310 (b) 320
(c) 330 (d) 340

Consider the following for the next (05) items that follow:

A, B, C, D, E, F and G are cousins. D is thrice as old as A. Further, C is as many years younger to B, as G to E and E to D. The average age of D and G is 16 years; the average age of A and E is 11 years; the average age of B and C is also 11 years. B and C have equal weight. A's weight is 10 kg less than that of B; D is 4 kg heavier than E; E is 4 kg heavier than F; F is 4 kg heavier than G. Further, D has age-weight ratio of 9 : 20, where age is in years and weight in kg; A has age-weight ratio of 2 : 5. Moreover, none of them is more than 40 kg.

86. What is D's age (in years)?
(a) 15 (b) 16
(c) 17 (d) 18

87. What is the average age (in years) of B, C, D, E and G?
(a) 12 (b) 13
(c) 14 (d) 15

88. What is the difference between the weights (in kg) of G and C?
(a) 4 (b) 3
(c) 2 (d) 1

89. What is the average weight (in kg) of A, B, C, D, E, F and G?
(a) $\frac{201}{7}$ (b) $\frac{197}{7}$
(c) 30 (d) 32

90. Consider the following statements:
1. The age of F cannot be determined due to insufficient data.
2. The average weight of D and F is equal to weight of E.
3. The weight difference is maximum for D and A.
Which of the statements given above are correct?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

Consider the following for the next ten (10) items that follow:

Mark option (a) if the question can be answered by using one of the statements alone, but cannot be answered using the other statement alone.

Mark option (b) if the question can be answered by using either statement alone.

Mark option (c) if the question can be answered by using both the statements together, but cannot be answered using either statement alone.

Mark option (d) if the question cannot be answered even by using both the statements together.

91. Question: Is $m > n$ if m, n are real numbers?
Statement I: $m = (1 - p)(p^2 + p + 1)$ and $n = (p + 1)(p^2 - p + 1)$
Statement II:
$m = pn$

92. Question: What is the other root of the quadratic equation with real coefficients if one of the roots is $\frac{-4-\sqrt{10}}{2}$?
Statement I: The product of the roots is $-\frac{3}{2}\left(3+\sqrt{10}\right)$.
Statement II: The sum of roots of quadratic equation is –1.

93. Question : What is the 3-digit number which is divisible by 10?
Statement I: If the digits in hundred's place and ten's place of the number are interchanged, the resulting number is diminished by 180.
Statement II : If the digit in hundred's place is halved and digit in ten's place and unit place of the number are interchanged, the resulting number is diminished by 336.

94. Question: Are x, y, z equal, where x, y, z are real numbers?
Statement I: $x^2 + y^2 + z^2 - xy - yz - zx = 0$
Statement II: $x^3 + y^3 + z^3 - 3xyz = 0$

95. Question: What is the ratio $x : y : z$ equal to if x, y, $z \neq 0$?
Statement I: $\frac{x+z}{y} = \frac{z}{x}$
Statement II: $\frac{z-y}{x} = \frac{x}{z}$

96. Question: What is the sum of two natural numbers?
Statement I: LCM of the two numbers is 144.
Statement II: One of the numbers is 72.

97. Question: Is average of the largest and the smallest of 4 given numbers greater than the average of the 4 numbers?
Statement I: The difference between the largest and the second largest numbers is less than the difference between the second smallest and the smallest of the numbers.
Statement II: The difference between the largest and the smallest numbers is greater than the difference between the second largest and the second smallest of the numbers.

98. Question: Is $(a - b + c) > (a + b - c)$, where a, b and c are real numbers?
Statement I: b is negative.
Statement II: c is negative.

99. Question: What is the cost of 15 pens, 21 pencils and 18 note books?
Statement I: The cost of 7 pens, 6 pencils and 5 note books is ₹200.
Statement II: The cost of 3 pens, 8 pencils and 7 note books is ₹210.

100. Question: What is the area of the triangle inscribed in a semi-circle with the diameter as the base?
Statement I: The diameter of semi-circle is 20 cm.
Statement II: Two shorter sides of the triangle are 12 cm and 16 cm.

Answers			
Q No	**Answer Key**	**Topic Name**	**Chapter Name**
1	(d)	Factorisaton	Polynomials
2	(a)	Factorisaton	Polynomials
3	(d)	Practical Problems	Linear Equations in Two Variables
4	(c)	Work Related Problem	Linear Equations in Two Variables
5	(a)	Compound Interest	Interest
6	(a)	Trigonometric Ratio	Trigonometry
7	(b)	Wrok Related Problem	Linear Equations in Two Variables
8	(b)	Ratio	Ratio and Proportion
9	(a)	Mean	Statistics
10	(d)	Median	Statistics
11	(b)	Roots of Equation	Quadratic Equation
12	(d)	Simplification	Polynomials
13	(b)	LCM & HCF	Number System
14	(b)	Practical Problems	Linear Equations in Two Variables
15	(a)	Solution	Polynomials
16	(a)	Solution	Linear Equations in Two Variables
17	(c)	Factorisaton	Polynomials
18	(d)	Application	Logarithm
19	(d)	HCF	Polynomials
20	(c)	Factor Theorem	Polynomials
21	(a)	Proportion	Ratio and Proportion
22	(c)	Simplification	Algebraic Expression
23	(b)	Trigonometric Ratio	Trigonometry
24	(b)	Age Related Problem	Linear Equations in Two Variables
25	(c)	Trigonometric Identities	Trigonometry
26	(c)	Trigonometric Equation	Trigonometry
27	(a)	Trigonometric Equation	Trigonometry
28	(a)	Trigonometric Ratio	Trigonometry
29	(a)	Trigonometric Equation	Trigonometry
30	(b)	Trigonometric Equation	Trigonometry
31	(a)	Height and Distance	Trigonometry
32	(c)	Height and Distance	Trigonometry
33	(b)	Height and Distance	Trigonometry
34	(d)	Height and Distance	Trigonometry
35	(c)	Chord	Circle
36	(d)	Chord	Circle
37	(a)	Factorisaton	Polynomials
38	(a)	Factorisaton	Polynomials
39	(b)	Application	Trigonometry
40	(b)	Application	Trigonometry

41	(d)	Circle	Circle
42	(a)	Circle	Circle
43	(d)	Ratio	Ratio and Proportion
44	(a)	Ratio	Ratio and Proportion
45	(b)	Practical Problems	Linear Equations in Two Variables
46	(b)	Practical Problems	Linear Equations in Two Variables
47	(c)	Mean	Statistics
48	(b)	Median	Statistics
49	(a)	Roots of Euqation	Quadratic Equation
50	(a)	Nature of Roots	Quadratic Equation
51	(b)	Sphere	Mensuration
52	(b)	Cone	Mensuration
53	(d)	Compound Interest	Interest
54	(b)	Area	Mensuration
55	(b)	Area Related To Circle	Circle
56	(c)	Herons Formula	Mensuration
57	(b)	Area of Triangle	Mensuration
58	(d)	Circle	Circle
59	(b)	Pythagoras Theorem	Triangle
60	(d)	Area Related To Circle	Circle
61	(b)	Arc Length	Circle
62	(c)	Radius of Curvature	Circle
63	(b)	Triangle Inequality	Properties of Triangle
64	(c)	Proportion	Ratio and Proportion
65	(d)	Sphere	Mensuration
66	(d)	Sphere and Cylinder	Mensuration
67	(b)	Capacity	Mensuration
68	(c)	LCM & HCF	Polynomials
69	(c)	Simplification	Polynomials
70	(c)	Trigonometric Ratio	Trigonometry
71	(c)	Trigonometric Identities	Trigonometry
72	(d)	Trigonometric Equation	Trigonometry
73	(b)	Area of Triangle	Coordinate Geometry
74	(c)	Ratio	Ratio and Proportion
75	(d)	Factor Theorem	Polynomials
76	(b)	Quadratic Equation	Quadratic Equation
77	(d)	Solution	Quadratic Equation
78	(b)	Speed and Time	Linear Equations in Two Variables
79	(c)	Profit and Loss	Linear Equations in Two Variables
80	(b)	Herons Formula	Mensuraton
81	(a)	Sphere	Mensuration

82	(b)	Sector and Segment	Circle
83	(c)	Factorisaton	Polynomials
84	(a)	Area of Triangle	Triangle
85	(d)	Mean	Statistics
86	(d)	Practical Problems	Linear Equations in Two Variables
87	(c)	Practical Problems	Linear Equations in Two Variables
88	(b)	Practical Problems	Linear Equations in Two Variables
89	(a)	Practical Problems	Linear Equations in Two Variables
90	(d)	Practical Problems	Linear Equations in Two Variables
91	(d)	Divisibility	Number System
92	(b)	Roots of Equation	Quadratic Equation
93	(c)	Practical Problems	Linear Equations in Two Variables
94	(b)	Factorisaton	Polynomials
95	(d)	Ratio	Ratio and Proportion
96	(d)	HCF and LCM	Number System
97	(a)	Mean	Statistics
98	(d)	Linear Inequality	Linear Inequality
99	(c)	Practical Problems	Linear Equations in Variables
100	(a)	Area	Circle

C.D.S.
Combined Defence Services

EXAM II 2023

Elementary Mathematics

Answers with Explanation

1. Option (d) is correct.

$a^2 - bc = \alpha \Rightarrow \alpha a = a^3 - abc$

$b^2 - ac = \beta \Rightarrow \beta b = b^3 - abc$

$c^2 - ab = \gamma \Rightarrow c\gamma = c^3 - abc$

Now

$$\frac{a\alpha + b\beta + c\gamma}{(a+b+c)(\alpha+\beta+\gamma)} = \frac{a^3 + b^3 + c^3 - 3abc}{(a+b+c)(\alpha+\beta+\gamma)}$$

$$= \frac{(a+b+c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right)}{(a+b+c)(\alpha+\beta+\gamma)}$$

$$= \frac{(a+b+c)(\alpha+\beta+\gamma)}{(a+b+c)(\alpha+\beta+\gamma)}$$

$= 1$

2. Option (a) is correct.

$\because$ $(x-1)^3$ is a factor of $x^4 + \alpha x^3 + \beta x^2 + \gamma x - 1$.

$\therefore$ $x^4 + \alpha x^3 + \beta x^2 + \gamma x - 1 = (x + a)(x - 1)^3$.

Put $x = 0$

$-1 = a(0 - 1)^3 \Rightarrow a = 1$

$\therefore$ $(x + 1)$ is other factor.

3. Option (d) is correct.

Let the unit digit and tens place digit of number be x and y.

$\Rightarrow$ Number $= 10y + x$.

$\therefore$ According to the question

$10y + x + 10x + y = 55$

$\Rightarrow$ $x + y = 5$ ...(1)

Difference $= 45$

$10y + x - (10x + y) = 45$

$\Rightarrow$ $9y - 9x = 45$

$\Rightarrow$ $y - x = 5$ ...(2)

Solving (1) & (2)

$2y = 10 \Rightarrow y = 5$ & $x = 0$

$\therefore$ Required number is 50.

$\therefore$ Product of digits $= 0$

4. Option (c) is correct.

$\therefore$ Let A finishes the work in x days

Let B finishes the work in y days

Let C finishes the work in z days

Now according to question

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{10} \quad ...(1)$$

$$\frac{1}{y} + \frac{1}{z} = \frac{1}{12} \quad ...(2)$$

and $$\frac{1}{z} + \frac{1}{x} = \frac{1}{15} \quad ...(3)$$

Add (1) (2) (&) (3) we get

$$2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = \frac{1}{10} + \frac{1}{12} + \frac{1}{15}$$

$$= \frac{6 + 5 + 4}{60} = \frac{15}{60} = \frac{1}{4}$$

$\therefore$ $$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{8} \quad ...(4)$$

$\therefore$ They can complete the whole work completely together in 8 days.

$\therefore$ Half of work can be finished in 4 days.

5. Option (a) is correct.

P = ₹10,000; R = 12% per annum

Rat per quarter $= \frac{12}{4} = 3\%$

Time = 9 months = 3 quarters $\Rightarrow n = 3$

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^n$$

$$= 10,000\left(1 + \frac{3}{100}\right)^3$$

$$= 10,000\left(\frac{103}{100}\right)^3$$

$$= 10,000 \times \frac{103}{100} \times \frac{103}{100} \times \frac{103}{100}$$

$= 10927.27$

So, CI = 10927.97 – 10000 = ₹927.27

6. Option (a) is correct.

$$\sin\theta = \frac{a+b}{2\sqrt{ab}}$$

$$\frac{a+b}{2\sqrt{ab}} \le 1$$

$$a + b \le 2\sqrt{ab}$$

$$a + b - 2\sqrt{ab} \le 0$$

$\left(\sqrt{a}-\sqrt{b}\right)^2 \le 0$

It is possible only when

$\sqrt{a}-\sqrt{b}=0$

$\Rightarrow \sqrt{a}=\sqrt{b}$

$\Rightarrow a=b$

7. Option (b) is correct.

Let A finishes the piece of work in x days.
Let B finishes the piece of work in y days.
Let C finishes the piece of work in z days.

$\therefore$ ATQ, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{36}$ ...(1)

$\frac{1}{x}+\frac{1}{y}=\frac{5}{z}$ ...(2)

$\frac{1}{y}+\frac{1}{z}=\frac{1}{x}$ ...(3)

$\frac{1}{x}+\frac{1}{z}=\frac{n}{y}$ ...(4)

By (1) & (2)

$\frac{5}{z}+\frac{1}{z}=\frac{1}{36}$

$\Rightarrow \frac{6}{z}=\frac{1}{36}$

$\Rightarrow z=216$

By (1) & (3)

$\frac{2}{x}=\frac{1}{36} \Rightarrow x=72$

$\therefore$ By (2)

$\Rightarrow \frac{1}{72}+\frac{1}{y}=\frac{5}{216}$

$\Rightarrow \frac{1}{y}=\frac{5}{216}-\frac{1}{72}=\frac{5-3}{216}$

$\Rightarrow \frac{1}{y}=\frac{2}{216}$

$\Rightarrow y=108$

$\therefore$ By (4)

$\frac{1}{x}+\frac{1}{z}=\frac{n}{y} \Rightarrow \frac{1}{72}+\frac{1}{216}=\frac{n}{108}$

$\Rightarrow \frac{3+1}{216}=\frac{n}{108}$

$\Rightarrow \frac{4\times 108}{216}=n$

$\Rightarrow n=2$

8. Option (b) is correct.

Let $\frac{2a}{3}=\frac{4b}{5}=\frac{3c}{4}=k$

$\Rightarrow a=\frac{3k}{2}, b=\frac{5k}{4}, c=\frac{4k}{3}$

$\therefore \frac{18}{a}\sqrt{a^2+c^2-b^2}$

$=\frac{18}{3k/2}\sqrt{\frac{9k^2}{4}+\frac{16k^2}{9}-\frac{25k^2}{16}}$

$=\frac{36}{3k}\times\sqrt{\frac{324k^2+256k^2-225k^2}{16\times 9}}$

$=\frac{12}{k}\times\frac{k\sqrt{355}}{4\times 3}$

$=\sqrt{355}$

9. Option (a) is correct.

$\sum x_i-10=a \quad \& \quad \sum x_i-20=b.$

$10=a-b$ ...(1)

$\& \cdot \frac{b}{a}=-4 \Rightarrow b=-4a$

By (1), $10=5a \Rightarrow a=2$

$b=-8.$

$\therefore \sum x_i=a+10$

$=2+10$

$=12$

10. Option (d) is correct.

Given data: 12, 1, 8, 54, 61, 28, 45, 35, 21, 17

Arranging in descending order

61, 54, 45, 35, 28, 21, 17, 12, 8, 1

$n=10$

$\therefore$ Median $=\frac{5^{th}\text{ observation}+6^{th}\text{ observation}}{2}$

$=\frac{28+21}{2}=\frac{49}{2}$

$\therefore$ 2M + 5 = 49 + 5 = 54

11. Option (b) is correct.

$\sqrt{x+9}=x-3$ ...(1)

Define when $x+9>0$

$\Rightarrow x>-9$ and $x-3>0 \Rightarrow x>3$

Squaring both the sides

$x+9=(x-3)^2$

$\Rightarrow x+9=x^2-6x+9$

$\Rightarrow x^2=6x \Rightarrow x=0, 6$

$\therefore$ Only $x=6$ is possible.

12. Option (d) is correct.

$x=97+56\sqrt{3}$

$=97+2(28)\sqrt{3}$

$=97+2(7)\left(4\sqrt{3}\right)=49+48+2(7)(4\sqrt{3})$

$=\left(7+4\sqrt{3}\right)^2$

$=\left[(2+\sqrt{3})^2\right]^2=\left(2+\sqrt{3}\right)^4$

$\therefore \sqrt[4]{x}+\frac{1}{\sqrt[4]{x}}=2+\sqrt{3}+\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$

$=2+\sqrt{3}+2-\sqrt{3}=4$

13. Option (b) is correct.

$a : b = 3x : 2x$

HCF $(a, b) = x$

LCM $(a, b) = 6x$

Now, $a + b = 3x + 2x$

$= 5x$

Given,

$a + b = 45$

$5x = 45$

$x = 9$

So, the number be 27, 18

Product of $(a, b) = 27 \times 18 = 486$

14. Option (b) is correct.

Let the distance b/w office and home is x km.

$$\therefore \quad \frac{x}{2}-\frac{x}{3}=\frac{(40+40)\min}{60}$$

$$\Rightarrow \quad \frac{x}{6}=\frac{80}{60}$$

$$\Rightarrow \quad x=\frac{80}{10}=8\,\text{km}$$

15. Option (a) is correct.

$$3^{x-1}+3^{3-x}=6$$

$$\Rightarrow \quad \frac{3^x}{3}+\frac{3^3}{3^x}=6$$

Let $3^x = t$

So, $\frac{t}{3}+\frac{27}{t}=6$

$\Rightarrow \quad t^2 + 81 = 18t$

$\Rightarrow \quad t^2 - 18t + 81 = 0$

$\Rightarrow \quad (t - 9)^2 = 0$

$\Rightarrow \quad t = 9$

$\Rightarrow \quad 3^x = 9$

$\Rightarrow \quad x = 2$

$\Rightarrow \quad 2^{x-1} + 2^{3-x} = 2^1 + 2^1 = 4$

16. Option (a) is correct.

$$x\left(a-b+\frac{ab}{a-b}\right)=y\left(a+b-\frac{ab}{a+b}\right) \qquad ...(1)$$

$x + y = 2a^3$ ...(2)

In eqn (1)

$$\Rightarrow \quad x\left(\frac{a^2+b^2-2ab+ab}{a-b}\right)=y\left(\frac{a^2+b^2+2ab-ab}{a+b}\right)$$

$$\Rightarrow \quad x\left(\frac{a^2+b^2-ab}{a-b}\right)-y\left(\frac{a^2+b^2+ab}{a+b}\right)=0$$

$$\Rightarrow \quad x\frac{(a+b)\left(a^2+b^2-ab\right)}{a^2-b^2}-\frac{y(a-b)\left(a^2+b^2+ab\right)}{a^2-b^2}=0$$

$$\Rightarrow \quad x\left(a^3+b^3\right)-y\left(a^3-b^3\right)=0$$

$$\Rightarrow \quad x=y\frac{\left(a^3-b^3\right)}{a^3+b^3}$$

Also, $x + y = 2a^3$

$$\Rightarrow \quad y\left(\frac{a^3-b^3+a^3+b^3}{a^3+b^3}\right)=2a^3$$

$$\Rightarrow \quad y=\frac{2a^3}{2a^3}\left(a^3+b^3\right)=a^3+b^3$$

$$\therefore \quad x=a^3-b^3$$

$$x-y=-2b^3$$

17. Option (c) is correct.

$$3\sqrt{3}x^3+2\sqrt{2}y^3-18xy+6\sqrt{6}$$

$$\left(\sqrt{3}x\right)^3+\left(\sqrt{2}y\right)^3+\left(\sqrt{6}\right)^3-3\left(\sqrt{3}x\right)\left(\sqrt{2}y\right)\left(\sqrt{6}\right)$$

$$=\left(\sqrt{3}x+\sqrt{2}y+\sqrt{6}\right)\left(3x^2+2y^2+6\ -\sqrt{6}xy-\sqrt{12}y-\sqrt{18}x\right)$$

18. Option (d) is correct.

Let $P = (125)^{100}$

$\log_{10} P = 100 \log_{10} 125$

$= 300 \log_{10} 5 \ = 300(\log_{10} 10 - \log_{10} 2$

$= 300\ (1 - 0.301)$

$= 300 \times 0.699$

$= 3 \times 69.9$

$= 209.7$

$\therefore$ No. of digits = 210

19. Option (d) is correct.

$acx^3 + bcx^2 + adx^2 + acdx + bdx + bcd$

$= cx^2(ax + b) + adx^2 + bdx + acdx + bcd$

$= cx^2(ax + b) + dx(ax + b) + cd(ax + b)$

$= (ax + b)\left(cx^2 + dx + cd\right)$

and $adx^3 + bdx^2 + acx^2 + bcx + acdx + bcd$

$= dx^2(ax + b) + cx(ax + b) + cd(ax + b)$

$= (ax + b)\left(dx^2 + cx + cd\right)$

$\therefore$ HCF is $(ax + b)$

20. Option (c) is correct.

$x^n - py^n + qz^n$ is divisible by

$x^2 + abyz - bzx - axy.$

Now

$x^2 - axy + abyz - bzx$

$= x(x - ay) - bz(x - ay)$

$= (x - ay)\ (x - bz)$

$x^n - py^n + qz^n = f(x)\ (x - ay)\ (x - bz)$

Put $y = \frac{x}{a} \& z = \frac{x}{b}$

$$x^n-p\frac{x^n}{a^n}+q\left(\frac{x^n}{b^n}\right)=0$$

$$\Rightarrow \quad 1-\frac{p}{a^n}+\frac{q}{b^n}=0$$

$$\therefore \frac{p}{a^n}-\frac{q}{b^n}=1$$

21. Option (a) is correct.
For statement 1
$(a + b) \propto (a - b) \Rightarrow a + b = k(a - b)$

$$\therefore \quad a^2 + b^2 = \frac{(a+b)^2 + (a-b)^2}{2}$$

$$= \frac{[k(a-b)]^2 + (a-b)^2}{2}$$

$$= (a-b)^2\left(\frac{1+k^2}{2}\right)$$

$$\Rightarrow \quad a^2 + b^2 = \left(a^2 + b^2 - 2ab\right)\left(\frac{1+k^2}{2}\right)$$

$$\Rightarrow \quad a^2 + b^2\left(\frac{1+k^2}{2} - 1\right) = ab\left(1+k^2\right)$$

$$\Rightarrow \quad a^2 + b^2 = 2\frac{\left(k^2+1\right)}{\left(k^2-1\right)}ab$$

$\Rightarrow$ $(a^2 + b^2) \propto ab$
$\Rightarrow$ statement 1 true.
For statement 2
$a \propto b \Rightarrow a = kb$
$\therefore$ $a^2 - b^2 = (a - b)(a + b)$
$= (kb - b)(kb + b)$
$= b^2(k^2 - 1)$
$\therefore$ statement 2 is false.

22. Option (c) is correct.
$(3a + 6b + c + 2d) \times (3a - 6b - c + 2d)$
$= (3a - 6b + c - 2d) \times (3a + 6b - c - 2d)$
$\Rightarrow \{(3a + 2d) + (6b + c)\}\{(3a + 2d) - (6b + c)\}$
$= \{(3a - 2d) + (c - 6b)\}\ \{3a - 2d\} - (c - 6b)\}$
$\Rightarrow (3a + 2d)^2 - (6b + c)^2 = (3a - 2d)^2 - (c - 6b)^2$
$12ad - 12bc = -12ad + 12bc$
$24bc = 24ad$
$bc = ad$

23. Option (b) is correct.
$3\sin\theta + 5\cos\theta = 5$ ...(1)
$5\sin\theta - 3\cos\theta = P$ ...(2)
Squaring (1) & (2) and then add
$9\sin^2\theta + 25\cos^2\theta + 30\sin\theta\cos\theta + 25\sin^2\theta + 9\cos^2\theta - 30\sin\theta\cos\theta = 25 + P^2$
$\therefore$ $9 + 25 = 25 + P^2$
$9 = P^2 \Rightarrow P = \pm 3$

24. Option (b) is correct.
Let the combined present ages of a man and his wife is x and their children is y and number of children is 'n'.
According to question
$x = 6y$ ...(1)
Two years ago
$x - 2 - 2 = 10(y - 2n)$
$\Rightarrow$ $x - 4 = 10(y - 2n)$ ...(2)
Six years later,
$x + 6 + 6 = 3(y + 6n)$
$\Rightarrow$ $x + 12 = 3(y + 6n)$ ...(3)
By (1) & (2)
$6y - 4 = 10y - 20n$
$\Rightarrow$ $4y = 20n - 4$
$\Rightarrow$ $y = 5n - 1$
By (1) & (3)
$6y + 12 = 3(y + 6n)$
$\Rightarrow$ $6y + 12 = 3y + 18n$
$\Rightarrow$ $3y = 18n - 12$
$\Rightarrow$ $y = 6n - 4$
So, $5n - 1 = 6n - 4$
$\Rightarrow$ $n = 3$

25. Option (c) is correct.

$$3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin x)^6 + 4(\cos x)^6$$

$$= 3(1 - \sin 2x)^2 + 6(1 + \sin 2x) + 4\left((\sin^2 x)^3 + (\cos^2 x)^3\right)$$

$$= 3 + 3\sin^2 2x - 6\sin 2x + 6 + 6\sin 2x + 4\left(\sin^2 x + \cos^2 x\right)\left(\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x\right)$$

$$= 9 + 3\sin^2 2x + 4\left(1 - 3\sin^2 x\cos^2 x\right)$$

$$= 9 + 3\sin^2 2x + 4 - 12\sin^2 x\cos^2 x$$

$$= 9 + 12\sin^2 x\cos^2 x + 4 - 12\sin^2 x\cos^2 x$$

$= 13$

26. Option (c) is correct.

$$\cot^2\theta - \left(\sqrt{3} + 1\right)\cot\theta + \sqrt{3} = 0$$

$$\Rightarrow \quad \cot^2\theta - \sqrt{3}\cot\theta - \cot\theta + \sqrt{3} = 0$$

$$\Rightarrow \quad \cot\theta\left(\cot\theta - \sqrt{3}\right) - 1\left(\cot\theta - \sqrt{3}\right) = 0$$

$$\Rightarrow \quad \left(\cot\theta - \sqrt{3}\right)(\cot\theta - 1) = 0$$

$$\Rightarrow \quad \cot\theta = \sqrt{3} \text{ or } \cot\theta = 1$$

$$0 < \theta < \frac{\pi}{4}$$

$$\Rightarrow \quad \theta = \frac{\pi}{6}$$

$$\therefore \quad \sin\theta + \cos\theta = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{3}+1}{2}$$

27. Option (a) is correct.
$\tan 2\theta \cdot \tan 4\theta - 1 = 0$

$$\because \tan 4\theta = \frac{2\tan 2\theta}{1 - \tan^2 2\theta} \text{ and let } \tan 2\theta = t$$

$$\Rightarrow t\cdot\left(\frac{2t}{1-t^2}\right) - 1 = 0$$

$$\Rightarrow \quad \frac{2t^2}{1-t^2} = 1 \Rightarrow 2t^2 = 1 - t^2$$

$$\Rightarrow 3t^2 = 1 \Rightarrow t^2 = \frac{1}{3}$$

$$\Rightarrow \quad t = \pm\frac{1}{\sqrt{3}}$$

$$\tan 2\theta = \pm\frac{1}{\sqrt{3}}$$

$\because 0 < \theta < \frac{\pi}{2} \Rightarrow 0 < 2\theta < \pi$

$\Rightarrow \quad 2\theta = \frac{\pi}{6}$

$\Rightarrow \quad \theta = \frac{\pi}{12}$

28. Option (a) is correct.

$\tan x = \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}, \quad \frac{\pi}{4} < \theta < \frac{\pi}{2}$

$= \frac{1+\tan\theta}{\tan\theta - 1} = -\frac{1+\tan\theta}{1-\tan\theta}$

$\tan x = -\frac{\tan\left(\frac{\pi}{4}+\theta\right)}{1} = \frac{P}{B}$

$\because \frac{\pi}{4} < \theta < \frac{\pi}{2}$

$\frac{\pi}{2} < \frac{\pi}{4} + \theta < \frac{3\pi}{4}$

$\therefore$ tan x will be −ve

$\sin x = \frac{\tan\left(\frac{\pi}{4}+\theta\right)}{\sqrt{1+\tan^2\left(\frac{\pi}{4}+\theta\right)}} = \frac{\tan\left(\frac{\pi}{4}+\theta\right)}{\sec\left(\frac{\pi}{4}+\theta\right)}$

$\Rightarrow \quad \sin x = \sin\left(\frac{\pi}{4}+\theta\right)$

$\Rightarrow \quad \sin x = \frac{\sin\theta + \cos\theta}{\sqrt{2}}$

$\Rightarrow \quad \sqrt{2}\sin x = \sin\theta + \cos\theta$

29. Option (a) is correct.

$\left(\sin^2\theta - 4\sin\theta + 3\right)\left(4 - \cos^2\theta + 4\sin\theta\right) = 0$

$\sin^2\theta - 4\sin\theta + 3 = 0$

$\Rightarrow \quad (\sin\theta - 1)(\sin\theta - 3) = 0$

$\Rightarrow \quad \sin\theta = 1, 3$

$4 - \cos^2\theta + 4\sin\theta = 0$

$\Rightarrow \quad 4 - 1 + \sin^2\theta + 4\sin\theta = 0$

$\Rightarrow \quad \sin^2\theta + 4\sin\theta + 3 = 0$

$\Rightarrow \quad (\sin\theta + 3)(\sin\theta + 1) = 0$

$\Rightarrow \quad \sin\theta = -1, -3$

$\because 0 < \theta < \frac{\pi}{2}$

$\therefore$ No value of θ.

30. Option (b) is correct.

$x \sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ ...(1)

$x\sin\theta - y\cos\theta = 0$

$\Rightarrow x\sin\theta = y\cos\theta$

So, (1) becomes

$(x\sin\theta)\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow (y\cos\theta)\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow (y\cos\theta)\left(\sin^2\theta + \cos^2\theta\right) = \sin\theta\cos\theta$

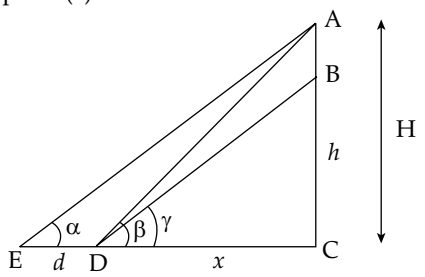
$\Rightarrow \quad y\cos\theta = \sin\theta\cos\theta$

$\Rightarrow \quad y = \sin\theta$

$\therefore x = \cos\theta$

$\Rightarrow \quad x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

31. Option (a) is correct.



In ΔACE

$\frac{AC}{CE} = \tan\alpha \Rightarrow \frac{H}{d+x} = \tan\alpha$ ...(1)

In ΔACD

$\frac{AC}{CD} = \tan\beta \Rightarrow \frac{H}{x} = \tan\beta$ ...(2)

In ΔBCD

$\frac{BC}{CD} = \tan\gamma \Rightarrow \frac{h}{x} = \tan\gamma$ ...(3)

By (2) & (3)

$\frac{H}{h} = \frac{\tan\beta}{\tan\gamma}$

$\Rightarrow H\tan\gamma - h\tan\beta = 0$

32. Option (c) is correct

From Q. 31

By (1) & (2)

$\frac{x}{d+x} = \frac{\tan\alpha}{\tan\beta} \Rightarrow \frac{d+x}{x} = \frac{\tan\beta}{\tan\alpha}$

$\Rightarrow 1 + \frac{d}{x} = \frac{\tan\beta}{\tan\alpha}$

$\Rightarrow \quad \frac{d}{x} = \frac{\tan\beta}{\tan\alpha} - 1$ ...(3)

(3) by (2)

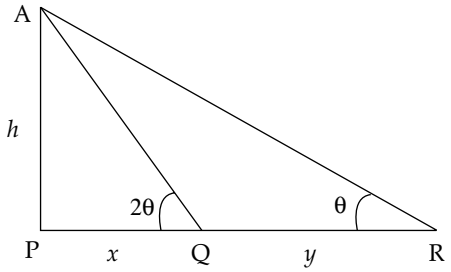
$\frac{H}{H} = \tan\beta$ ...(4)

$\therefore \frac{d}{H} = \frac{\tan\beta - \tan\alpha}{\tan\alpha\tan\beta}$

$\frac{d}{H} = \cot\alpha - \cot\beta$

$d = H(\cot\alpha - \cot\beta)$

33. Option (b) is correct.



ΔAPR

$$\tan\theta = \frac{h}{x+y} \qquad ...(1)$$

In ΔAPQ, $\tan 2\theta = \frac{h}{x}$

divide (1) by (2)

$$\frac{\tan\theta}{\tan 2\theta} = \frac{x}{x+y}$$

$$\Rightarrow \frac{\tan\theta\left(1-\tan^2\theta\right)}{2\tan\theta} = \frac{x}{x+y}.$$

$$\Rightarrow \frac{1-\tan^2\theta}{2} = \frac{x}{x+y}$$

$$\Rightarrow \frac{2}{1-\tan^2\theta} = \frac{x+y}{x}$$

$$\Rightarrow \frac{2}{1-\tan^2 x} = 1+\frac{y}{x}$$

$$\Rightarrow \frac{2}{1-\tan^2\theta} - 1 = \frac{y}{x}$$

$$\Rightarrow \frac{2-1+\tan^2\theta}{1-\tan^2\theta} = \frac{y}{x}$$

$$\Rightarrow \frac{1+\tan^2\theta}{1-\tan^2\theta} = \frac{y}{x}$$

$$\Rightarrow \frac{x}{y} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\Rightarrow \frac{x}{y} = \cos 2\theta$$

$$\therefore \cos 2\theta < 1 \Rightarrow \frac{x}{y} < 1$$

$$\Rightarrow x < y$$

34. Option (d) is correct.

From Q 33

$$\frac{x}{y} = \cos 2\theta \Rightarrow x = y\cos 2\theta$$

$$\therefore \frac{h}{x} = \tan 2\theta$$

$$\Rightarrow h = x\tan 2\theta$$

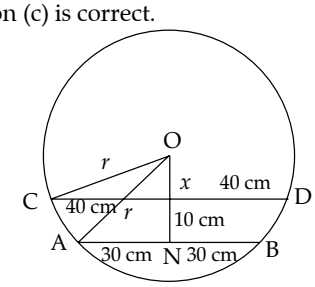
$$= y\cdot\cos 2\theta\cdot\frac{\sin 2\theta}{\cos 2\theta}$$

$$h = y\sin 2\theta$$

$$\therefore h^2 + x^2 = y^2$$

$$\Rightarrow h^2 = y^2 - x^2$$

35. Option (c) is correct.



In Δ OMC

$$x^2 + 40^2 = r^2 \qquad ...(1)$$

In ΔONA

$$(x+10)^2 + (30)^2 = r^2 \qquad ...(2)$$

By (1) & (2)

$$x^2 + 1600 = x^2 + 100 + 20x + 900$$

$$\Rightarrow 20x = 600$$

$$\Rightarrow x = 30$$

$$\therefore r^2 = (40)^2 + (30)^2$$

$$= 2500$$

$$\Rightarrow r = 50$$

$\therefore$ Diameter = 100 cm

36. Option (d) is correct.

From question 35

$$\because \angle AOB = \alpha \Rightarrow \angle AON = \frac{\alpha}{2}$$

$$\angle COD = \beta \Rightarrow \angle COM = \frac{\beta}{2}$$

In ΔAON

$$\tan\frac{\alpha}{2} = \frac{30}{40} = \frac{3}{4}$$

In ΔCOM

$$\tan\frac{\beta}{2} = \frac{40}{30} = \frac{4}{3}$$

$$\tan\frac{\beta}{2} - \tan\frac{\alpha}{2}$$

$$= \frac{4}{3} - \frac{3}{4}$$

$$= \frac{16-9}{12} = \frac{7}{12}$$

37. Option (a) is correct.

$$p = x^4 - y^2z^2$$

$$q = y^4 - z^2x^2$$

$$r = z^4 - x^2y^2$$

$$\therefore px^2 + qy^2 + rz^2$$

$$= x^6 - x^2y^2z^2 + y^6 - x^2y^2z^2 + z^6 - x^2y^2z^2$$

$$= x^6 + y^6 + z^6 - 3x^2y^2z^2$$

$$= \left(x^2+y^2+z^2\right)\left(x^4+y^4+z^2-x^2y^2-y^2z^2-x^2z^2\right)$$

$\left\{\because a^3+b^3+c^3-3abc=(a+b+c)\right.$
$\left.\left(a^2+b^2+c^2-ab-bc-ca\right)\right\}$

$=\left(x^2+y^2+z^2\right)(p+q+r)$

38. Option (a) is correct.
From Q.37

$x^2\left(px^2+qy^2+rz^2\right)+qr-p^2$

$= px^4+qx^2y^2+rx^2z^2+qr-p^2$

$= p\left(x^4-p\right)+qx^2y^2+r\left(q+x^2z^2\right)$

$= p\left(y^2z^2\right)+qx^2y^2+ry^4.$

$= y^2\left(pz^2+qx^2+ry^2\right)$

$\because pz^2=x^4z^2-y^2z^4$

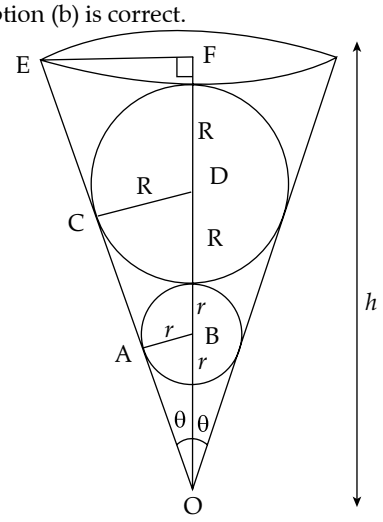
$qx^2=y^4x^2-z^2x^4$

$ry^2=z^4y^2-x^2y^4$

$\because pz^2+qx^2+ry^2=0$

$\therefore$ Required value is 0

39. Option (b) is correct.



$\therefore$ In Δ AOB

$$\sin\theta=\frac{r}{h-(2R+r)} \quad ...(1)$$

In Δ DOC

$$\sin\theta=\frac{R}{h-R} \quad ...(2)$$

By (1) and (2)

$$\frac{r}{h-2R-r}=\frac{R}{h-R}$$

$\Rightarrow$ $hr-Rr=Rh-2R^2-rR$
$\Rightarrow$ $Rh-rh=2R^2+rR-Rr$
$\Rightarrow$ $h(R-r)=2R^2$

$$h=\frac{2R^2}{R-r}$$

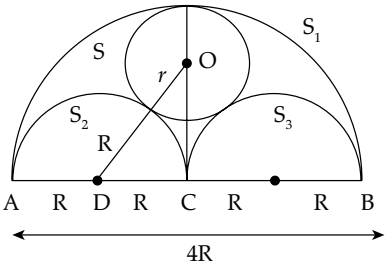
40. Option (b) is correct.
From question 39
Δ EFO

$\Rightarrow$ $\tan\theta=\frac{r}{h}$

$\Rightarrow$ $r=h\tan\theta$

$\Rightarrow$ $r=\frac{2R^2\tan\theta}{R-r}$

41. Option (d) is correct.



In Δ OCD

$$OC = 2R - r$$
$\Rightarrow$ $OC^2 + CD^2 = OD^2$
$\Rightarrow$ $(2R-r)^2+R^2=(R+r)^2$
$\Rightarrow$ $4R^2+r^2-4Rr+R^2=R^2+r^2+2Rr$
$\Rightarrow$ $4R^2=6Rr$
$\Rightarrow$ $2R=3r$

42. Option (a) is correct.
From question no. 41

$m=\pi r^2, n=\frac{1}{2}\pi(2R)^2=2\pi R^2$

$\therefore$ $\frac{m}{n}=\frac{\pi r^2}{2\pi R^2}$

$=\frac{r^2}{2R^2}$

$=\frac{r^2}{2\left(\frac{3r}{2}\right)^2}=\frac{2}{9}$

$\Rightarrow 9m=2n$

43. Option (d) is correct.

$$\frac{(x-a)(x-b)}{(x-ma)(x-mb)}=\frac{(x+a)(x+b)}{(x+ma)(x+mb)};m,a,b>0$$

$$\Rightarrow \frac{(x-a)(x-b)}{(x+a)(x+b)}=\frac{(x-ma)(x-mb)}{(x+ma)(x+mb)}$$

$$\Rightarrow \frac{x^2-x(a+b)+ab}{x^2+x(a+b)+ab}=\frac{x^2-x(ma+mb)+m^2ab}{x^2+x(ma+mb)+m^2ab}$$

By componendo and dividendo

$$\frac{2\left(x^2+ab\right)}{2(x(a+b))}=\frac{2\left(x^2+m^2ab\right)}{2(x(ma+mb))}$$

$$\Rightarrow \frac{x^2+ab}{x^2+m^2ab}=\frac{x(a+b)}{xm(a+b)}=\frac{1}{m}.$$

44. Option (a) is correct.
From solution 43

$$\frac{x^2+ab}{x^2+m^2ab}=\frac{1}{m}$$

$\Rightarrow \quad mx^2+mab=x^2+m^2ab$

$\Rightarrow \quad mx^2-x^2=m^2ab-mab$

$\Rightarrow \quad x^2(m-1)=mab(m-1)$

$\Rightarrow \quad x^2=mab$

$\Rightarrow \quad x=\pm\sqrt{mab}$

45. Option (b) is correct.
Total monthly electricity bill = Part A + Part B
(bill) of Part A = x (no. of rooms)
(bill) of Part B = y (no. of units).
According to question

$$400 = 8x + 240y$$
$$50 = x + 30y \quad ...(1)$$

And

$$320 = 6x + 200y$$
$$160 = 3x + 100y \quad ...(2)$$

By (1) and (2)

$$160 - 150 = 100y - 90y \Rightarrow 10y = 10$$
$$\Rightarrow \quad y = 1$$
$$\therefore \quad x=\frac{320-200}{6}=20$$

Now m be the rooms and n be units
$\therefore$ bill $= m \times x + n \times y$
$= ₹(20m + n)$

46. Option (b) is correct.
From question 45
Total bill = $7 \times 20 + 300 \times 1$
$= 140 + 300$
$= ₹440$

47. Option (c) is correct.

₹	No. of workers	x_i	$u_i=\frac{x_i-3800}{500}$	f_iu_i
2050 – 2550	5	2300	–3	–15
2550 – 3050	10	2800	–2	–20
3050 – 3550	k	3300	–1	$–k$
3550 – 4050	8	3800	0	0
4050 – 4550	2	4300	1	2
4550 – 5050	10	4800	2	20
	$35 + k$		Total = $–k – 13$	

$$\text{Mean}(\bar{\alpha})=\Delta+\frac{\sum f_iu_i}{\sum f_i}\times h$$

$$\Rightarrow 3520=3800+\frac{(-k-13)}{35+k}\times 500$$

$$\Rightarrow -\frac{280}{500}=\frac{-(k+13)}{35+k}$$

$$\Rightarrow \frac{14}{25}=\frac{k+13}{35+k}$$

$$\Rightarrow 14\times 35+14k=25k+13\times 25$$

$$\Rightarrow 490-325=11k \Rightarrow k=\frac{165}{11}=15$$

48. Option (b) is correct.

₹	f	Cf
2050 – 2550	5	5
2550 – 3050	10	15
3050 – 3550	15	30
3550 – 4050	8	38
4050 – 4550	2	40
4550 – 5050	10	50

N = 50

$$\Rightarrow \frac{N}{2}=25$$

$\therefore$ Median class = 3050 – 3550

So, Median $=l+\frac{\frac{N}{2}-cf}{f}\times h$

$$=3050+\frac{25-15}{15}\times 500$$

$$=3050+\frac{1000}{3}=3383.3$$

49. Option (a) is correct.
$(a + b + c)x^2 - (2a + 2b)x + (a + b - c) = 0$
$(a + b + c)x^2 - (a + b + c)x - (a + b - c)x + (a + b - c) = 0$
$(a + b + c)x(x - 1) - (a + b - c)(x - 1) = 0$
$(x - 1)\{(a + b + c)x - (a + b - c)\} = 0$

$$\therefore x=1, \quad \frac{a+b-c}{a+b+c}$$

50. Option (a) is correct.
From Q. 49

$$x=1, \quad \frac{a+b-c}{a+b+c}$$

$$1,\frac{a+b+c}{a+b+c}-\frac{2c}{a+b+c}$$

$$1,1-\frac{2c}{a+b+c}$$

$\therefore$ S_1 is true but S_2 is false.

51. Option (b) is correct.

$$\text{Diagonal}=\sqrt{l^2+b^2+h^2}$$

$$\Rightarrow 2r=\sqrt{8^2+12^2+24^2}$$

$$\Rightarrow 2r=\sqrt{64+144+576}$$

$$\Rightarrow 2r=\sqrt{784}$$

$$\Rightarrow r=\sqrt{196}$$

$$\Rightarrow r=14\,\text{cm}$$

52. Option (b) is correct.
$2\pi r_1 = 30$

$2\pi r_2 = 40$

$\Rightarrow \frac{r_1}{r_2} = \frac{3}{4}$

$\Delta ABC \sim \Delta ADE$

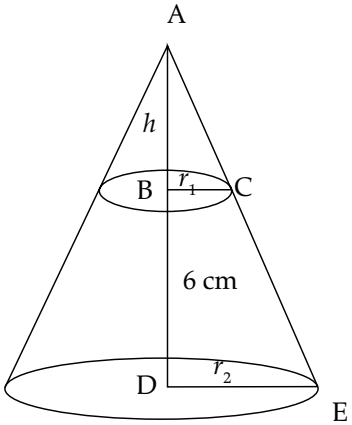
$\frac{AB}{BC} = \frac{AD}{DE}$

$\Rightarrow \frac{h}{r_1} = \frac{h+6}{r_2}$

$\Rightarrow \frac{h}{h+6} = \frac{r_1}{r_2} = \frac{3}{4}$

$\Rightarrow 4h = 3h + 18$

$\Rightarrow h = 18$ cm



53. Option (d) is correct.

$100P = P\left(1 + \frac{R}{100}\right)^n$

$\Rightarrow 100 = \left(1 + \frac{20}{100}\right)^n$

$\Rightarrow 100 = \left(\frac{12}{10}\right)^n$

$\Rightarrow \log_{10} 100 = n \log \frac{12}{10}$

$\Rightarrow 2 = n(2\log 2 + \log 3 - 1)$

$2 = n(0.602 + 0.477 - 1)$

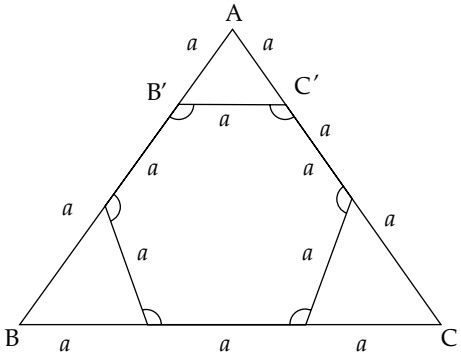
$2 = n(1.079 - 1)$

$n = \frac{2}{0.079} = \frac{2000}{79}$

$= 25.32$

$\therefore$ Least value of $n = 26$

54. Option (b) is correct.



Area of $\Delta = \frac{\sqrt{3}}{4}(3a)^2 = \frac{9\sqrt{3}}{4}a^2$

Area of regular hexagon

= Area of $\Delta ABC - 3 \times$ Area of $\Delta AB'C'$

$= \frac{\sqrt{3}}{4}(3a)^2 - 3 \times \frac{\sqrt{3}}{4}a^2$

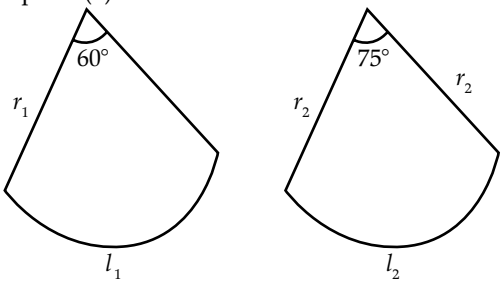
$= \frac{\sqrt{3}}{4}\left(9a^2 - 3a^2\right)$

$= \frac{6\sqrt{3}}{4}a^2$

$\therefore$ Required ratio $= \frac{9\sqrt{3}}{4}a^2 : \frac{6\sqrt{3}}{4}a^2$

$= 9 : 6 = 3 : 2$

55. Option (b) is correct.



$\because l_1 = l_2$

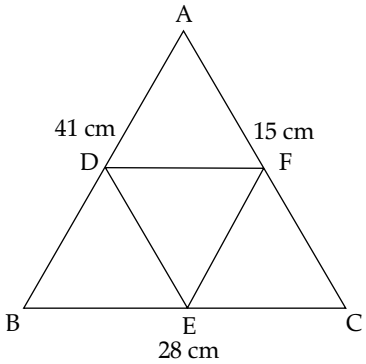
$\Rightarrow \frac{2\pi r_1 \theta}{360°} = \frac{2\pi r_2 \theta}{360°}$

$\Rightarrow r_1\theta = r_2\theta$

$\Rightarrow r_1(60°) = r_2(75°)$

$\Rightarrow \frac{r_1}{r_2} = \frac{75°}{60°} = \frac{5}{4}$

56. Option (c) is correct.



$s = \frac{28 + 41 + 15}{2} = \frac{84}{2} = 42$ cm

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-28)(42-41)(42-15)}$

$= \sqrt{7 \times 2 \times 3 \times 2 \times 7 \times 1 \times 3 \times 3 \times 3}$

$= 7 \times 2 \times 9 = 126$ cm^2

For ΔDEF

$s = \frac{\frac{41}{2} + \frac{28}{2} + \frac{15}{2}}{2} = \frac{84}{4} = 21$ cm

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\frac{42}{2} \times \left(\frac{42}{2} - \frac{28}{2}\right)\left(\frac{42}{2} - \frac{41}{2}\right)\left(\frac{42}{2} - \frac{15}{2}\right)}$

$= \frac{126}{4} = 31.5 \text{ cm}^2$

57. Option (b) is correct.

$s = \frac{13+14+15}{2}$

$= 21$ cm

Area of Δ

$= \sqrt{21(21-13)(21-14)(21-15)}$

$= \sqrt{21\times 8\times 7\times 6}$

$= 7 \times 3 \times 4$

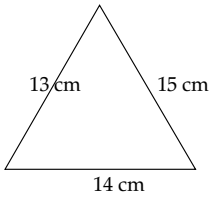
$= 84 \text{ cm}^2$

For smallest altitude base will be largest

$\frac{1}{2} \times b \times h = 84$

$\frac{1}{2} \times 15 \times h = 84$

$h = \frac{84\times 2}{15} = \frac{56}{5} = 11.2$ cm



58. Option (d) is correct.

$2\pi r - 2r = 16.8$

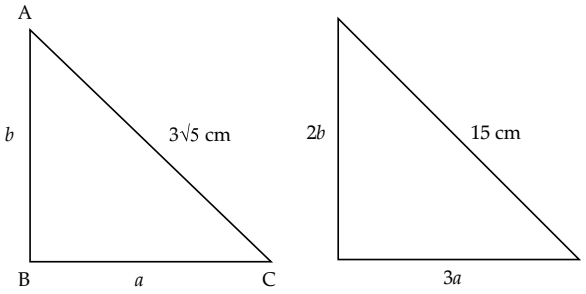
$\Rightarrow 2r(\pi - 1) = 16.8$

$\Rightarrow 2r\left(\frac{22}{7} - 1\right) = 16.8$

$\Rightarrow 2r = \frac{16.8\times 7}{15}$

$D = \frac{39.2}{5} = 7.84$ cm

59. Option (b) is correct.



$a^2 + b^2 = \left(3\sqrt{5}\right)^2$ (using Pythagoras theorem)

$\Rightarrow a^2 + b^2 = 45$ ...(1)

Also, $(3a)^2 + (2b)^2 = (15)^2$

$\Rightarrow 9a^2 + 4b^2 = 225$ ...(2)

By (1) & (2)

$9a^2 + 4b^2 = 225$

$\underline{4a^2 + 4b^2 = 180}$

$5a^2 = 45$

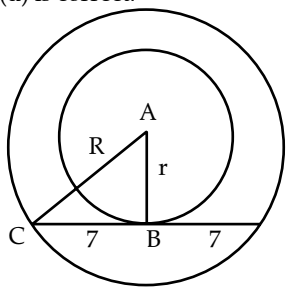
$\Rightarrow a^2 = 9$

$\Rightarrow a = 3$

By (1), $b^2 = 36 \Rightarrow b = 6$

$\therefore$ AB + BC = $b + a = 6 + 3 = 9$ cm

60. Option (d) is correct.



$\therefore R^2 = 7^2 + r^2$

$\therefore R^2 - r^2 = 49$

Area of ring = $\pi (R^2 - r^2)$

$= \frac{22}{7} \times 49 = 154 \text{ cm}^2$

61. Option (b) is correct.

$\theta = 9°$, Arc length = 14.3 cm

Length of pendulum = r (say)

Arc length = $\frac{2\pi r\theta}{360°}$

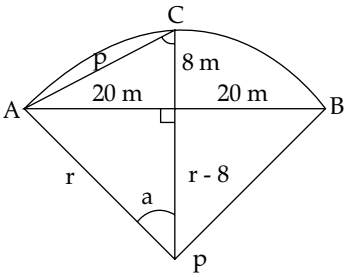
$\Rightarrow 14.3 = 2 \times \frac{22}{7} \times r \times \frac{9}{360}$

$\Rightarrow \frac{14.3\times 7\times 40}{2\times 22} = r$

$\Rightarrow 1.3 \times 7 \times 10 = r$

$\Rightarrow 91 = r$

62. Option (c) is correct.



$r^2 = (r-8)^2 + 400$ (using Pythagoras theorem)

$\Rightarrow r^2 = r^2 - 16r + 464$

$\Rightarrow 16r = 464$

$\Rightarrow r = \frac{464}{16} = 29$ cm

63. Option (b) is correct.

a, b, c – sides of right angled triangle

$\because a > b > c$

$\therefore a$ will be hypotenuse.

$\therefore a^2 = b^2 + c^2$

$a^2 - b^2 - c^2 = 0$

$\therefore (a+b+c)(a+b-c)(a-b+c)(a-b-c)$

$= (a+b+c)[a-(b+c)](a+(b-c))[a-(b-c)]$

$\Rightarrow [a^2 - (b+c)^2][a^2 - (b-c)^2]$

$= (a^2 - b^2 - c^2 - 2bc)(a^2 - b^2 - c^2 + 2bc)$

$= (-2bc)(2bc)$

$= -4b^2c^2$

64. Option (c) is correct.

$$\sqrt[3]{x} \propto \frac{1}{\sqrt{y}}$$

$$\Rightarrow \sqrt[3]{x} = \frac{k}{\sqrt{y}}$$

$\because$ $x = 8$ when $y = 3$

$$\therefore \quad \sqrt[3]{8} = \frac{k}{\sqrt{3}} \Rightarrow k = 2\sqrt{3}$$

$$\therefore \quad \sqrt[3]{x} = \frac{2\sqrt{3}}{\sqrt{y}}$$

$\therefore$ At $y = \sqrt[3]{3} = 3^{1/3}$

$$\sqrt[3]{x} = \frac{2\sqrt{3}}{\sqrt{3^{1/3}}}$$

Cubing both sides,

$$x = \frac{8 \times 3\sqrt{3}}{\sqrt{3}} = 24$$

65. Option (d) is correct.

$r_1 = 6$ cm

$r_2 = 8$ cm

$r_3 = 10$ cm

Let r be the radius of new sphere

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3$$

$\Rightarrow r^3 = 6^3 + 8^3 + 10^3$

$= 216 + 512 + 1000$

$= 1728$

$\Rightarrow r = 12$ cm

S = surface area $= 4\pi (12)^2 = 576\pi$

$S_1 + S_2 + S_2 = 4\pi (r_1^2 + r_2^2 + r_3^2)$

$= 4\pi (36 + 64 + 100)$

$= 800\pi$

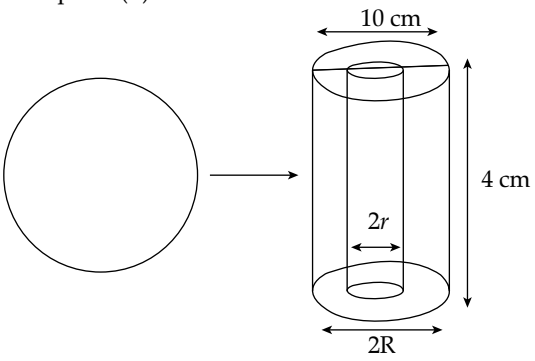
% diminution in surface area

$$= \frac{800\pi - 576\pi}{800\pi} \times 100$$

$$= \frac{224}{800} \times 100$$

$= 28\%$

66. Option (d) is correct.



$r = 3$ cm (radius of sphere)

So, according to question

$$\frac{4}{3}\pi r^3 = \pi h (R^2 - r^2)$$

$$\Rightarrow \frac{4}{3} \times 3 \times 3 \times 3 = 4 \times \left(5^2 - r^2\right)$$

$\Rightarrow 9 = 25 - r^2 \Rightarrow r^2 = 16$

$\Rightarrow r = 4$

$\therefore$ Thickness $= 5 - 4 = 1$ cm

67. Option (b) is correct.

Ratio $= 1 : 2 : 3 = x : 2x : 3x$

Ratio of milk : water

Glass P – $1/3 \ : \ 2/3$

Glass Q – $\frac{2}{5} \ : \ \frac{3}{5}$

Glass R – $\frac{3}{4} \ : \ \frac{1}{4}$

Quantity of water

$$= \frac{\frac{2}{3} \times x + \frac{3}{5} \times 2x + \frac{1}{4} \times 3x}{6x}$$

$$= \frac{\frac{2}{3} + \frac{6}{5} + \frac{3}{4}}{6}$$

$$= \frac{40 + 72 + 45}{60 \times 6} = \frac{157}{360}$$

Quantity of milk

$$= \frac{\frac{1}{3} \times x + \frac{2}{5} \times 2x + \frac{3}{4} \times 3x}{6x}$$

$$= \frac{\frac{1}{3} + \frac{4}{5} + \frac{9}{4}}{6}$$

$$= \frac{20 + 48 + 135}{360}$$

$$= \frac{203}{360}$$

$\therefore$ Required ratio

$= 203 : 157$

68. Option (c) is correct.

Let

$$P_1 = x^4 + x^2 y^2 + y^4 = (x^2 + y^2 - xy)\left(x^2 + y^2 + xy\right)$$

$$P_2 = x^3 + y^3 = (x + y)\left(x^2 + y^2 - xy\right)$$

$$P_3 = x^3 - y^3 = (x - y)\left(x^2 + y^2 + xy\right)$$

LCM of P_1, P_2, P_3

$$= (x + y)(x - y)\left(x^4 + x^2 y^2 + y^4\right)$$

$$= \left(x^2 - y^2\right)\left(x^4 + x^2 y^2 + y^4\right)$$

$$= \left(x^2\right)^3 - \left(y^2\right)^3$$

$$= x^6 - y^6$$

69. Option (c) is correct.

$$\frac{x^2-y^2-z^2-2yz}{x^2+y^2-z^2+2xy}+\frac{x^2-y^2-z^2-2yz}{x^2-y^2+z^2-2xz}$$

$$=\frac{x^2-(y+z)^2}{(x+y)^2-z^2}+\frac{x^2-(y+z)^2}{(x-z)^2-y^2}$$

$$=\frac{(x+y+z)(x-y-z)}{(x+y+z)(x+y-z)}+\frac{(x-y-z)(x+y+z)}{(x+y-z)(x-y-z)}$$

$$=\frac{x-y-z}{x+y-z}+\frac{x+y+z}{x+y-z}$$

$$=\frac{x-y-z+x+y+z}{x+y-z}=\frac{2x}{x+y-z}$$

70. Option (c) is correct.

$\tan A + \cot A = 2$

$\Rightarrow A = 45°$

$\therefore \quad \tan^2 A + \tan^3 A + \ldots\ldots\ldots + \tan^n A$

$= 1 + 1 + \ldots\ldots + 1$

$= (n - 1)$

71. Option (c) is correct.

S1 $\dfrac{\sin^3\theta+\cos^3\theta}{\sin\theta+\cos\theta}+\sin\theta\cos\theta$

$$=\frac{(\sin\theta+\cos\theta)\left(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta\right)}{\sin\theta+\cos\theta}+\sin\theta\cos\theta$$

$= 1 - \sin\theta\cos\theta + \sin\theta\cos\theta$

$= 1 = RHS$

$\therefore \quad S_1$ is correct

S2

$1 - \sin^6\theta$

$= 1^3 - (\sin^2\theta)^3$

$= (1 - \sin^2\theta)(1^2 + \sin^4\theta + \sin^2\theta)$

$= \cos^2\theta (1 + (1 - \cos^2\theta)^2 + \sin^2\theta)$

$=\cos^2\theta\left(1+1+\cos^4\theta-2\cos^2\theta+\sin^2\theta\right)$

$=\cos^2\theta\left(\cos^4\theta+2\left(1-\cos^2\theta\right)+\sin^2\theta\right)$

$= \cos^2\theta (\cos^4\theta + 3\sin^2\theta)$

$\therefore \quad S_2$ is correct

72. Option (d) is correct.

$7\sin^4\theta + 9\cos^4\theta + 42\sin^2\theta = 16$

$\Rightarrow 7\sin^4\theta + 9(1 - \sin^2\theta)^2 + 42\sin^2\theta = 16$

$\Rightarrow 7\sin^4\theta + 9\sin^4\theta + 9 - 18\sin^2\theta + 42\sin^2\theta - 16 = 0$

$\Rightarrow 16\sin^4\theta + 24\sin^2\theta - 7 = 0$

Let $\sin^2\theta = t$

$\Rightarrow 16t^2 + 24t - 7 = 0$

$\Rightarrow 16t^2 + 28t - 4t - 7 = 0$

$\Rightarrow 4t(4t + 7) - 1(4t + 7) = 0$

$\Rightarrow (4t + 7)(4t - 1) = 0$

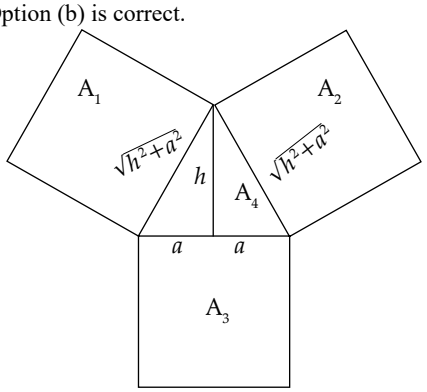
$\Rightarrow t = -\dfrac{7}{4}, \quad \dfrac{1}{4}$

$\sin^2\theta = -\dfrac{7}{4}$ (Not possible)

$\therefore \quad \sin^2\theta = \dfrac{1}{4} \Rightarrow \sin\theta = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{6}$

$\therefore \quad \tan\theta = \dfrac{1}{\sqrt{3}}$

73. Option (b) is correct.



Area of figure $= A_1 + A_2 + A_3 + A_4$

$$=\left(\sqrt{h^2+a^2}\right)^2+\left(\sqrt{h^2+a^2}\right)^2+(2a)^2+\frac{1}{2}\times 2a\times h$$

$= h^2 + a^2 + h^2 + a^2 + 4a^2 + ah$

$= 2h^2 + 6a^2 + ah$

74. Option (c) is correct.

$$p=\frac{a^2}{(b-a)(c-a)} \quad q=\frac{b^2}{(c-b)(a-b)} \quad r=\frac{c^2}{(a-c)(b-c)}$$

So, $(p+q+r)^2=\left(\dfrac{a^2}{(b-a)(c-a)}+\dfrac{b^2}{(c-b)(a-b)}+\dfrac{c^2}{(a-c)(b-c)}\right)^2$

$$=\left[\frac{(c-b)a^2+b^2(a-c)+c^2(b-a)}{(a-b)(b-c)(c-a)}\right]^2$$

$$=\left(\frac{a^2c-a^2b+ab^2-b^2c+bc^2-ac^2}{(a-b)(b-c)(c-a)}\right)^2$$

$$=\left[\frac{\left(a^2c-b^2c\right)+\left(ab^2-a^2b\right)+\left(bc^2-ac^2\right)}{(a-b)(b-c)(c-a)}\right]$$

$$=\left[\frac{(a-b)\left\{(a+b)c-ab-c^2\right\}}{(a-b)(b-c)(c-a)}\right]^2$$

$$=\left[\frac{ac+bc-ab-c^2}{(b-c)(c-a)}\right]^2=(1)^2=1$$

75. Option (d) is correct.

$a^2 - b^2 - c^2 + 2bc + a + b - c$

$= \left[a^2 - (b - c)^2\right] + (a + b - c)$

$= (a + b - c)(a - b + c) + (a + b - c)$

$(a + b - c)(a - b + c + 1)$

76. Option (b) is correct.

$$\frac{1}{x+a+b}=\frac{1}{x}+\frac{1}{a}+\frac{1}{b}$$

$$\Rightarrow \frac{1}{x+a+b}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$$

$$\Rightarrow \frac{x-x-a-b}{x(x+a+b)}=\frac{a+b}{ab}$$

$$\Rightarrow (-a-b)ab=\left[x^2+(a+b)x\right](a+b)$$

$$\Rightarrow x^2+(a+b)x+ab=0$$

Since, α, β are the roots
So, $\alpha+\beta=-(a+b)$
and $\alpha\beta=ab$
New eqn having roots α^2 & β^2
So, $S=\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$
$=a^2+b^2$
$P=\alpha^2\beta^2=a^2b^2$
$\therefore$ Eqn is

$$x^2-\left(a^2+b^2\right)x+a^2b^2=0$$

77. Option (d) is correct.

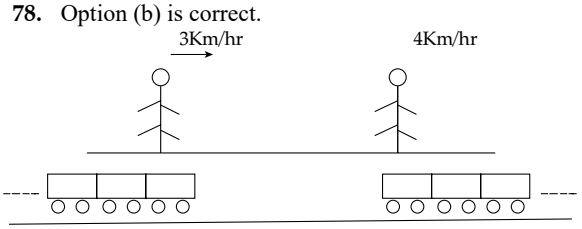
$$x=\cfrac{6}{7-\cfrac{6}{7-\cfrac{6}{7-\cfrac{6}{7-x}}}}$$

$\therefore$ We can write

$$x=\frac{6}{7-x}\Rightarrow 7x-x^2=6$$

$\Rightarrow x^2-7x+6=0$
$\Rightarrow (x-6)(x-1)=0$
$\Rightarrow x=6, 1$
$\because x>1$
$\therefore x=6$
So, $x^2-3x+2=36-18+2=20$

78. Option (b) is correct.



$$3\,\text{km/h}=3\times\frac{5}{18}\text{m/s}=\frac{5}{6}\text{m/s}.$$

$$4\,\text{km/h}=4\times\frac{5}{18}\text{m/s}=\frac{10}{9}\text{m/s}$$

Let length of train be x m and its speed be y m/s.

$$\frac{x}{y-\frac{5}{6}}=9\ \&\ \frac{x}{y-\frac{10}{9}}=\frac{75}{8}$$

$$x=9y-\frac{9\times5}{6}\ \&8x=75y-\frac{10}{9}\times75$$

$$2x=^{3}18y-15\ \&\ 24x=225y-250$$
$$\text{(1)}\qquad\qquad\text{(2)}$$

$$24x=225y-250$$
$$\underline{24x=216y-180}$$
$$0=9y-70$$

$$\Rightarrow y=\frac{70}{9}$$

$$2x=18\times\frac{70}{9}-15$$

$=140-15=125$
$x=62.5$

79. Option (c) is correct.
Let person bought an article in ₹x
$\therefore$ Selling amount = $1.2x$
Now it it bought it at 20% less
$\therefore$ Article buying price = $0.8x$
$\therefore$ Profit amount = $1.2x-0\ 8x$
$=0.4x$

$\therefore$ Profit % $=\frac{0.4x}{0.8x}\times100=50\%$

80. Option (b) is correct.
$2s=a+b+c$

$$s^2+(s-a)(s-b)+(s-b)(s-c)+(s-c)(s-a)$$

$$=4s^2-s(a+b+b+c+c+a)+ab+bc+ca$$

$$=4s^2-2s(a+b+c)+ab+bc+ca$$

$$=4s^2-2s(2s)+ab+bc+ca$$

$$=4s^2-4s^2+ab+bc+ca$$
$=ab+bc+ca$

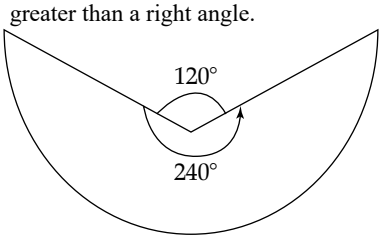
81. Option (a) is correct.
Let the height of water level raised in cylindrical vessel be 'h' cm.
Radius of sphere, $r=5$ cm
Radius of cylinder, R = 10 cm
$\therefore$ Volume of sphere = Volume of water displaced in cylinder

$$\Rightarrow\frac{4}{3}\pi r^3=\pi R^2h$$

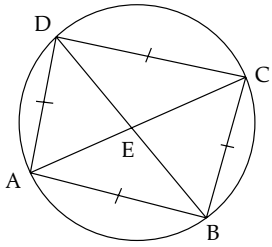
$$\Rightarrow\ \frac{4}{3}\times5^3=10^2h$$

$$\Rightarrow h=\frac{4}{3\times100}\times5\times5\times5=\frac{5}{3}\text{ cm}$$

82. Option (b) is correct.
S1 The angle in a sector greater than a semi circle is greater than a right angle.



S2



$\angle BAC = \angle BDC$ and $\angle ABD = \angle ACD$

(Angles in same segment)

$\because \Delta ABE \cong \Delta DCE$

$AE = DE$ ...(1)

$BE = CE$

$CE = BE$ ...(2)

Add (1) and (2)

$AE + CE = DE + BE$

$\Rightarrow AC = BD$

83. Option (c) is correct.

$$(a+b+c)^2 - 3(ab+bc+ca) + 3\left(x^2+y^2+z^2\right) = 0$$

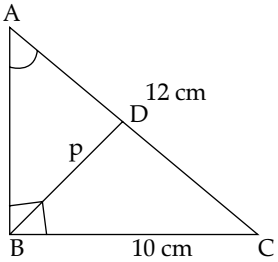
$$\Rightarrow a^2+b^2+c^2-ab-bc-ca+3\left(x^2+y^2+z^2\right)=0$$

$$\Rightarrow \frac{1}{2}\left\{(a-b)^2+(b-c)^2+(c-a)\right\}^2 + 3\left(x^2+y^2+z^2\right)=0$$

It is possible when

$a = b = c$ and $x = y = z = 0$

84. Option (a) is correct.



Using Pythagoras theroem,

$AB = \sqrt{144-100} = \sqrt{44} = 2\sqrt{11}$

$\Delta ABC \sim \Delta ADB$

$$\frac{AB}{AD} = \frac{BC}{BD} = \frac{AC}{AB}$$

$$\frac{2\sqrt{11}}{AD} = \frac{10}{p} = \frac{12}{2\sqrt{11}}$$

$$p = \frac{2\sqrt{11}}{12} \times 10$$

$$p = \frac{5\sqrt{11}}{3}$$

85. Option (d) is correct.

$$\bar{x} = \frac{p+q+r+s+t}{5} = 280$$

$$\Rightarrow \quad p+q+r+s+t = 1400$$

$$\text{and } \frac{p+r+t}{3} = 240$$

$p + r + t = 720$ ...(2)

By (1) – (2) $\Rightarrow q + s = 680$

$$\therefore \quad \text{mean of } q \text{ \& } s = \frac{q+s}{2}$$

$$= \frac{680}{2} = 340$$

(86–90):

A	B	C	D	E	F	G
↓	↓	↓	↓	↓	↓	↓
a	b	c	d	e	f	g

ATQ

$d = 3a$ ...(1)

$b - c = e - g = d - e$ ...(2)

We get $d + g = 2e$ ...(3)

$$\frac{d+g}{2} = 16 \Rightarrow d+g = 32 \quad \text{...(4)}$$

$$\frac{a+e}{2} = 11 \Rightarrow a+e = 22 \quad \text{...(5)}$$

$$\frac{b+c}{2} = 11 \Rightarrow b+c = 22 \quad \text{...(6)}$$

By (3) and (4) $\Rightarrow 2e = 32 \Rightarrow e = 16$

By (5) $a = 22 - 16 \Rightarrow a = 6$

By (1) $d = 3a \Rightarrow d = 18$

By (4) $g = 32 - 18 \Rightarrow g = 14$

By (2) $b - c = 16 - 14$

$b - c = 2$

By (6) $b + c = 22$

$2b = 24 \Rightarrow b = 12$ and $c = 10$

Similarly for weight

$b = c$ ...(1)

$b - a = 10$ ...(2)

$d - e = 4$ ...(3)

$e - f = 4$ ...(4)

$f - g = 4$ ...(5)

$$\frac{d_{age}}{d_{\omega t}} = \frac{9}{20} \quad \text{...(6)}$$

$$\frac{a_{age}}{a_{\omega t}} = \frac{2}{5} \quad \text{...(7)}$$

By (6) $\frac{18}{d_{\omega t}} = \frac{9}{20} \Rightarrow d_{wt} = 40 \rightarrow d$

By (7) $\frac{6}{a_{\omega t}} = \frac{2}{5} \Rightarrow a_{wt} = 15 \rightarrow a$

By (2) $b = 10 + a \Rightarrow b = 25 = c$

By (3) $e = d - 4 = 40 - 4 = 36$

$e = 36$

By (4) $f = 32$

By (5) $g = 28$

86. Option (d) is correct.

Age of D = 18 years

87. Option (c) is correct.

$$\text{Average age} = \frac{b+c+d+e+g}{5}$$

$$= \frac{12+10+18+16+14}{5}$$

$$= \frac{70}{5} = 14$$

88. Option (b) is correct.

$g_{\omega t} - C_{\omega t}$

$= 28 - 25$

$= 3$ kg

89. Option (a) is correct.

$$\text{Average weight} = \frac{a+b+c+d+e+f+g}{7}$$

$$= \frac{15+25+25+40+36+32+28}{7}$$

$$= \frac{201}{7}$$

90. Option (d) is correct.

S1 is correct

S2 Avg. weight of D & $F = \frac{40+32}{2} = 36$

$=$ weight of E

$\therefore$ S_2 is correct

S3 Weight difference of D & A $= 40 - 15 = 25$ which is max

$\therefore$ $\mathbf{S_3}$ is correct.

91. Option (d) is correct.

S1 $m = (1-p)(p^2+p+1)$

$= 1 - p^3$

$n = (p+1)(p^2-p+1)$

$= 1 + p^3$

S2 $m = pn$

$(1-p^3) = p(1+p^3)$

$1 - p^3 = p^4 + p$

$p^4 + p^3 + p - 1 = 0$

Both statements are not able to answered the question.

92. Option (b) is correct.

one root is $\frac{-4-\sqrt{10}}{2}$

S1

$$\text{Product of roots} = -\frac{3}{2}\left(3+\sqrt{10}\right)$$

$$\left(-\frac{4-\sqrt{10}}{2}\right)\beta = \frac{-3}{2}\left(3+\sqrt{10}\right)$$

$$\beta = \frac{3\left(3+\sqrt{10}\right)}{4+\sqrt{10}}$$

S2

Sum of roots $= -1$

$\alpha + \beta = -1$

$$\beta = -1 + \frac{4+\sqrt{10}}{2} = \frac{2+\sqrt{10}}{2}$$

$\therefore$ (b) is correct

93. Option (c) is correct.

3 digit no. divisible by 10

S1

Let 3 digit no is abc

$\therefore$ Number is $= 100a + 10b + c$

New number $\Rightarrow bac$

$\therefore$ Number is $100b + 10a + c$

$\therefore$ $100a + 10b + c - 100b - 10a - c = 180$

$90a - 90b = 180$

$a - b = 2$ ...(1)

S2

$abc \rightarrow \frac{a}{2}cb$

$$100a + 10b + c - 100\left(\frac{a}{2}\right) - 10c - b = 336$$

$50a + 9b - 9c = 336$

$\therefore$ Number is divisible by 10

$\therefore$ $c = 0$

$50a + 9b = 336$ ...(2)

$9a - 9b = 18$

$59a = 354$

$a = 6$

$b = 4$

$\therefore$ Both are required.

94. Option (b) is correct.

S1 $x^2 + y^2 + z^2 - xy - yz - zx = 0$

$$\Rightarrow \frac{1}{2}\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right] = 0$$

$\therefore$ $x = y = z$.

S2 $x^3 + y^3 + z^3 - 3xyz = 0$

$$\Rightarrow \frac{1}{2}(x+y+z)\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right] = 0$$

$x + y + z = 0$

$x = y = z$

$\therefore$ (b) is correct.

95. Option (d) is correct.

$x : y : z$

S1 $\frac{x+z}{y} = \frac{z}{x} = k$ (say)

$x + z = ky$

and $z = kx$

So, $x + kx = ky$

$\Rightarrow$ $x(k+1) = ky$

$$\Rightarrow \quad y = \frac{k+1}{k}x$$

$\therefore$ $x : y : z$

$$= \quad x : \left(\frac{k+1}{k}\right)x : kx$$

$= k : k+1 : k$

S2: $\frac{z-y}{x} = \frac{x}{z} = k_1$ (say)

$\Rightarrow$ $z - y = k_1 x$

and $x = k_1 z$

So, $y = z - k_1 x$

$= z - k_1^2 z$

$= z\left(1 - k_1^2\right)$

$\therefore \quad x : y : z$

$= k_1 z : z\left(1 - k_1^2\right) : z$

$= k_1 : \ 1 - k_1^2 : 1$

By using both

$\therefore \quad k = \dfrac{1}{k_1}$

$kk_1 = 1 \Rightarrow k_1 = \dfrac{1}{k}$

$\therefore \dfrac{1}{k} : 1 - \dfrac{1}{k^2} : 1$

$k : k^2 - 1 : k^2$

$\dfrac{k}{k} = \dfrac{k+1}{k^2-1} = \dfrac{k}{k^2}$

$k^2 - 1 = k + 1, \quad k^2 = k$

$k = -1, 2 \qquad k = 0, 1$

Which is not possible

$\therefore$ (d) is correct

96. Option (d) is correct.

Sum of two numbers = ?

S_1 LCM = 144

S_2 One number = 72 = a

$a \times b$ = LCM

$b = \dfrac{144}{72} = 2$

$\therefore \quad a = 72, b = 2$

97. Option (a) is correct.

S_1 $a > b > c > d$

$a - b > c - d \Rightarrow a + d > b + c$ ...(1)

S_2 $a - d > b - c$

$a + c > b + d$ ...(2)

Now avg. of largest & smallest numbers

$\dfrac{a+d}{2} = p$ & avg. of 4 numbers.= $\dfrac{a+b+c+d}{4} = q$

$\Rightarrow \dfrac{a+d}{2} > \dfrac{a+b+c+d}{4}$

$4a + 4d > 2a + 2b + 2c + 2d$

$a + d > b + c$

$\therefore$ S_1 is sufficient.

98. Option (d) is correct.

$a - b + c > a + b - c \quad ; \quad a, b, c \in R$

S_1 $b < 0$

S_2 $c < 0$

Not giving any such possibility

$\therefore$ Option (d) is correct

99. Option (c) is correct.

Cost of 15 pens + 21 pencils + 18 note books.

S_1 7 pens + 6 pencils + 5 note books = 200

S_2 3pens + 8 pencils + 7 note books = 210

Let Cost of 1 pen = x

1 pencil = y

1 note book = z

So, $15x + 21y + 18z = ?$

$7x + 6y + 5z = 200$

$3x + 8y + 7z = 210$

Let

$15x + 21y + 18z = \lambda(7x + 6y + 5z) + \mu\,(3x + 8y + 7z)$

$15 = 7\lambda + 3\mu$ (×8) | $21 = 6\lambda + 8\mu$ (×3) | $18 = 5\lambda + 7\mu$

$120 = 56\lambda + 24\mu$

$63 = 18\lambda + 24\mu$

$57 = 38\lambda$

$\lambda = \dfrac{57}{38} = \dfrac{3}{2}$

$\mu = \dfrac{21 - 6 \times \frac{3}{2}}{8}$

$= \dfrac{21 - 9}{8}$

$= \dfrac{12}{8} = \dfrac{3}{2}$

$\therefore \quad \lambda = \dfrac{3}{2} = \mu$

So, $5\lambda + 7\mu = \dfrac{15}{2} + \dfrac{21}{2}$

$= \dfrac{36}{2} = 18$

$\therefore \quad \lambda = \mu = \dfrac{3}{2}$ is accepteble

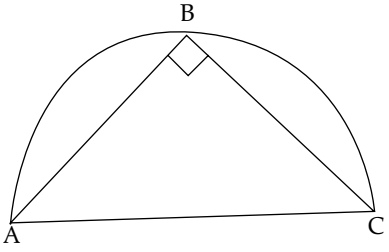
$\therefore$ We get cost of required pens, pencils and note books is

$= \dfrac{3}{2} \times 200 + \dfrac{3}{2} \times 210$

$= 300 + 315 = ₹615$

100. Option (a) is correct.

Area of Δ inside in a semicircle = ?



S_1 AC = 20 cm

S_2 AB = 12

BC = 16

$\therefore \quad A = \dfrac{1}{2} \times 12 \times 16 = 96$