ENGINEERING MATHEMATICS GATE Question Paper 2010

ELECTRONICS AND COMMUNICATION (EC) P1

Q. 1. The eigen values of a skew–symmetric matrix are

(a) always zero
(b) always pure imaginary
(c) either zero or pure imaginary
(d) always real

Q. 2. A function $n(x)$ satisfies the differential equation $\frac{d^2n(x)}{dx^2} - \frac{n(x)}{L^2} = 0$ where L is a constant. Boundary condition are : $n(0) = K$ and $n(\infty) = 0$. The solution to this equation is

(a) $n(x) = K\exp\left(\frac{x}{L}\right)$

(b) $n(x) = K\exp\left(\frac{-x}{\sqrt{L}}\right)$

(c) $n(x) = K^2\exp\left(\frac{-x}{L}\right)$

(d) $n(x) = K\exp\left(\frac{-x}{L}\right)$

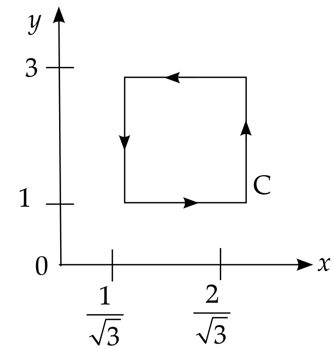
Q. 3. If $e^y = x^{\frac{1}{x}}$, then y has a

(a) maximum at $x = e$
(b) minimum at $x = e$
(c) maximum at $x = e^{-1}$
(d) minimum at $x = e^{-1}$

Q. 4. A fair coin is tossed independently four times. The probability of the event "the number of times heads show up is more than the number of times tails show up" is

(a) $\frac{1}{16}$ **(b)** $\frac{1}{8}$
(c) $\frac{1}{4}$ **(d)** $\frac{5}{16}$

Q. 5. If $\vec{A} = xy\hat{a}_x + x^2\hat{a}_y$, than $\oint_c \vec{A}\cdot d\vec{l}$ over the path shown in the figure is



(a) 0 **(b)** $\frac{2}{\sqrt{3}}$
(c) 1 **(d)** $2\sqrt{3}$

Q. 6. The residues of a complex function $X(z) = \frac{1-2z}{z(z-1)(z-2)}$ at its poles are

(a) $\frac{1}{2}, -\frac{1}{2}$ and 1 **(b)** $\frac{1}{2}, \frac{1}{2}$ and -1
(c) $\frac{1}{2}$, 1 and $-\frac{3}{2}$ **(d)** $\frac{1}{2}$, -1 and $\frac{3}{2}$

Q. 7. Consider a differential equation $\frac{dy(x)}{dx} - y(x) = x$ with the initial condition $y(0) = 0$. Using Euler's first order method with a step size of 0.1. The value of $y(0.3)$ is

(a) 0.01 **(b)** 0.031
(c) 0.0631 **(d)** 0.1

CIVIL ENGINEERING (CE) P1

Q. 8. The $\lim_{x\to 0} \frac{\sin\left[\frac{2}{3}x\right]}{x}$ is

(a) $\frac{2}{3}$ **(b)** 1
(c) $\frac{3}{2}$ **(d)** ∞

Q. 9. Two coins are simultaneously tossed. The probability of two heads simultaneously appearing is

(a) 1/8 **(b)** 1/6
(c) 1/4 **(d)** 1/2

Q. 10. The order and degree of the differential equation

$$\frac{d^3y}{dx^3}+4\sqrt{\left(\frac{dy}{dx}\right)^3+y^2}=0$$ are respectively

(a) 3 and 2 **(b)** 2 and 3

(c) 3 and 3 **(d)** 3 and 1

Q. 11. The solution to the ordinary differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0$ is

(a) $y=c_1e^{3x}+c_2e^{-2x}$ **(b)** $y=c_1e^{3x}+c_2e^{2x}$

(c) $y=c_1e^{-3x}+c_2e^{2x}$ **(d)** $y=c_1e^{-3x}+c_2e^{-2x}$

Q. 12. The inverse of the matrix $\begin{bmatrix}3+2i & i\\ -i & 3-2i\end{bmatrix}$ is

(a) $\frac{1}{12}\begin{bmatrix}3+2i & -i\\ i & 3-2i\end{bmatrix}$

(b) $\frac{1}{12}\begin{bmatrix}3-2i & -i\\ i & 3+2i\end{bmatrix}$

(c) $\frac{1}{14}\begin{bmatrix}3+2i & -i\\ i & 3-2i\end{bmatrix}$

(d) $\frac{1}{14}\begin{bmatrix}3+2i & -i\\ i & 3+2i\end{bmatrix}$

Q. 13. The table below gives values of a function F(x) obtained for values of x at intervals of 0.25.

x	0	0.25	0.5	0.75	1.0
F(x)	1	0.9412	0.8	0.64	0.50

The value of the integral of the function between the limits 0 to 1 using Simpson's rule is

(a) 0.7854 **(b)** 2.3562

(c) 3.1416 **(d)** 7.5000

Q. 14. The partial differential equation that can be formed from

$z = ax + by + ab$ has the form

(with $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$)

(a) $z = px + qy$ **(b)** $z = px + pq$

(c) $z = px + qy + pq$ **(d)** $z = qy + pq$

Q. 15. Given a function

$f(x, y) = 4x^2 + 6y^2 - 8x - 4y + 8$

The optimal value of $f(x, y)$

(a) is a minimum equal to $\frac{10}{3}$

(b) is a maximum equal to $\frac{10}{3}$

(c) is a minimum equal to $\frac{8}{3}$

(d) is a maximum equal to $\frac{8}{3}$

COMPUTER SCIENCE (CS) P1

Q. 16. Newton–Raphson method is used to compute a root of the equation $x^2 - 13 = 0$ with 3.5 as the initial value. The approximation after one iteration is

(a) 3.575 **(b)** 3.677

(c) 3.667 **(d)** 3.607

Q. 17. What is the possible number of reflexive relations on a set of 5 elements?

(a) 2^{10} **(b)** 2^{15}

(c) 2^{20} **(d)** 2^{25}

Q. 18. Consider the set S = {l, ω, ω^2}, where ω and ω^2 are cube roots of unity. If * denotes the multiplication operation, the structure {S, *} forms

(a) a group

(b) a ring

(c) an integral domain

(d) a field

Q. 19. What is the value of $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n}$?

(a) 0 **(b)** e^{-2}

(c) $e^{\frac{-1}{2}}$ **(d)** 1

Q. 20 Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q. What is the probability of a computer being declared faulty ?

(a) $pq + (1–p)(1–q)$ **(b)** $(1–q)p$

(c) $(1–p)q$ **(d)** pq

Q. 21. What is the probability that a divisor of 10^{99} is a multiple of 10^{96}?

(a) $\frac{1}{625}$ **(b)** $\frac{4}{625}$

(c) $\frac{12}{625}$ **(d)** $\frac{16}{625}$

Q. 22. The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequence can not be the degree sequence of any graph?

I. 7, 6, 5, 4, 4, 3, 2, 1

II. 6, 6, 6, 6, 3, 3, 2, 2

III. 7, 6, 6, 4, 4, 3, 2, 2

IV. 8, 7, 7, 6, 4, 2, 1, 1

(a) I and II **(b)** III and IV

(c) IV only **(d)** II and IV

Q. 23. Consider the following matrix

$$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$$

If the eigen values of A are 4 and 8, then

(a) $x = 4, y = 10$ **(b)** $x = 5, y = 8$

(c) $x = -3, y = 9$ **(d)** $x = -4, y = 10$

Q. 24. Suppose the predicate F (x, y, t) is used to represent the statement that person x can fool person y at time t. which one of the statements below expresses best the meaning of the formula

$\forall x \exists y \exists t (\neg F(x, y, t))$?

(a) Everyone can fool some person at some time.

(b) No one can fool everyone all the time.

(c) Everyone cannot fool some person all the time.

(d) No one can fool some person at some time.

Answer Key			
Q. No.	**Answer**	**Topic Name**	**Chapter Name**
1	(c)	Eigen Values	Linear Algebra
2	(d)	2nd Order	Differential Equation
3	(a)	Maximum/Minimum	Calculus
4	(d)	Probability	Probability and Statistics
5	(c)	Line Integral	Vector Calculus
6	(c)	Residue Theorem	Complex Variable
7	(b)	Eular Method	Numerical Analysis
8	(a)	Eular Method	Calculus
9	(c)	Probability	Probability and Statistics
10	(a)	Order and Degree	Differential Equation
11	(c)	2nd Order	Differential equation
12	(b)	Inverse of Matrix	Linear Algebra
13	(a)	Simpson's Rule	Numerical Analysis
14	(c)	Partial Differential Equations	Differential Equation
15	(a)	Maximum/Minimum	Calculus
16	(d)	Newton Raphson Method	Numerical Analysis
17	(c)	Set Theory	Discrete Mathematics
18	(a)	Set Theory	Discrete Mathematics
19	(b)	Limit	Calculus
20	(a)	Probability	Probability and Statistics
21	(a)	Probability	Probability and Statistics
22	(d)	Graph Theory	Discrete Mathematics
23	(d)	Eigen Values	Linear Algebra
24	(b)	Propositional-Logic	Discrete Mathematics

ANSWERS WITH EXPLANATIONS

1. Option (c) is correct.

If a matrix is skew symmetric then the elements in diagonal should be zero.

$$A = \begin{bmatrix} 0 & q \\ -q & 0 \end{bmatrix}$$

$|A - \lambda I| = 0 \qquad \Rightarrow \lambda^2 + q^2 = 0$

Solving this we get the eigen values either zero or pure imaginary values.

2. Option (d) is correct.

Differential equation : $D^2 n - \dfrac{n}{L^2} = 0,\ D = \dfrac{d}{dx}$

$\therefore \quad$ AE $\quad m^2 - \dfrac{1}{L^2} = 0$

$\Rightarrow \qquad m = \pm \dfrac{1}{L}$

So, $\qquad n = A\ e^{\frac{x}{L}} + B\ e^{\frac{x}{L}}$

given, $\qquad n(0) = k;\ A + B = K \qquad$ ...(i)

also $\qquad n(\infty) = 0;\ Ae^{\infty} + Be^{-\infty} = 0$

possible if $\qquad A = 0$

in (i) $\qquad 0 + B = K \Rightarrow B = K$

$\therefore \qquad n(x) = k \cdot e^{\frac{x}{L}}$

3. Option (a) is correct.

Since, $\qquad e^y = x^{\frac{1}{x}}$

$$\ln e^y = \ln x^{\frac{1}{x}}$$

$$y = \frac{1}{x}\ln x$$

$$\frac{dy}{dx} = \frac{1}{x}.\frac{1}{x} + \ln x\left(\frac{-1}{x^2}\right) = \frac{1}{x^2} - \frac{\ln}{x^2}$$

For stationary points

$$\frac{dy}{dx} = \frac{1}{x^2}(1 - \ln x) = 0$$

$\Rightarrow \qquad \ln x = 1 \rightarrow x = e^1$

also $\qquad \dfrac{d^2y}{dx^2} = \dfrac{-2}{x^3} - \ln x\left(\dfrac{-2}{x^3}\right) - \dfrac{1}{x^2}\left(\dfrac{1}{x}\right)$

$$\left.\frac{d^2y}{dx^2}\right|_{x=e^1} = -\frac{2}{e^3} + \frac{2}{e^3} - \frac{1}{e^3} < 0$$

Function y has maximum at $x = e^1$.

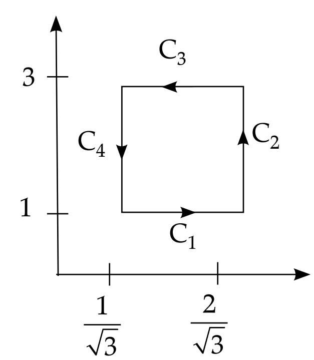
4. Option (d) is correct.

Probability of getting head 3 times or 4 times

$$= {}^4C_4\left(\frac{1}{2}\right)^4 + {}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)$$

$$= 1\cdot\frac{1}{16} + 4\left(\frac{1}{8}\right)\left(\frac{1}{2}\right) = \frac{5}{16}$$

5. Option (c) is correct.



$$\vec{A} = xy\hat{a}_x + x^2\hat{a}_y$$

$$\vec{dl} = dx\hat{a}_x + dy\hat{a}_y$$

$$\oint_C A.dl = \oint_C \left(xy\hat{a}_x + x^2\hat{a}_y\right).\left(dx\hat{a}_x + dy\hat{a}_y\right)$$

$$= \oint_C \left(xydx + x^2dy\right)$$

$$C_1 : y = 1, dy = 0 \; x : \frac{1}{\sqrt{3}} \text{ to } \frac{2}{\sqrt{3}}$$

$$C_2 : x = \frac{2}{\sqrt{3}}, \; dx = 0 \; y : 1 \text{ to } 3$$

$$C_3 : y = 3, \; dy = 0 \; x : \frac{2}{\sqrt{3}} \text{ to } \frac{1}{\sqrt{3}}$$

$$C_4 : x = \frac{1}{\sqrt{3}}, \; dx = 0 \; y : 3 \text{ to } 1$$

$$\int_{C_1:\frac{1}{\sqrt{3}}}^{\frac{2}{\sqrt{3}}} xdx + \int_{C_3:\frac{2}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} 3xdx + \int_{C_2:1}^{3} \frac{4}{3}dy + \int_{C_4:3}^{1} \frac{1}{3}dy$$

$$= \frac{1}{2}\left[\frac{4}{3} - \frac{1}{3}\right] + \frac{3}{2}\left[\frac{1}{3} - \frac{4}{3}\right] + \frac{4}{3}[3-1] + \frac{1}{3}[1-3]$$

$= 1$

6. **Option (c) is correct.**

$$x(z) = \frac{1-2z}{z(z-1)(z-2)}$$

X(z) has simple poles at $z = 0, z = 1, z = 2$

At $z = 0$ residue is

$$\text{Res } x(0) = z \cdot x(z)\Big|_{z=0} = \frac{1-2(0)}{(0-1)(0-2)} = \frac{1}{2}$$

$$\text{Res } x(1) = (z-1) \cdot x(z)\Big|_{z=1} = \frac{1-2(1)}{1(1-2)} = 1$$

$$\text{Res } x(2) = (z-2) \cdot x(z)\Big|_{z=2} = \frac{1-2(2)}{2(2-1)} = \frac{-3}{2}$$

7. **Option (b) is correct.**

Step size $h = 0.1$; $y(0) = 0$

x	y	$\frac{dy}{dx} = x + y$	$y_{i+1} = y_i + h\frac{dy}{dx}$
0	0	0	$y_1 = 0 + 0.1(0) = 0$
0.1	0	0.1	$y_2 = 0 + 0.1(0.1) = 0.01$
0.2	0.01	0.21	$y_3 = 0.01 + 0.21(0.1) = 0.031$

So, $y(x = 0.3) = y_3 = 0.031$

8. **Option (a) is correct.**

We know $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$$\therefore \quad \lim_{x\to 0} \frac{\left[\sin \frac{2}{3}x\right]}{x} = \lim_{x\to 0} \frac{\left[\sin \frac{2}{3}x\right]}{\frac{2}{3}x} \times \frac{2}{3}$$

$$= 1 \times \frac{2}{3} = \frac{2}{3}$$

9. **Option (c) is correct.**

When two coins are tossed simultaneously then exhaustive number of cases = 4 = n.

{(T,H), (H,T), (T,T), (H,H)}

Favourable no. of cases is one which is (H,H).

So, probability = $\frac{1}{4}$

10. **Option (a) is correct.**

Degree : Degree of highest ordered derivative occurring in equation.

Order : Order of the highest ordered derivative occurring in equation.

To find the degree and order equation should be rationalised and cleared of fraction.

$$\frac{d^3y}{dx^3} = -4\sqrt{\left(\frac{dy}{dx}\right)^3 + y^2}$$

$$\left(\frac{d^3y}{dx^3}\right)^2 = 16\left(\left(\frac{dy}{dx}\right)^3 + y^2\right)$$

$\therefore$ We have third order derivative raised to the power 2.

Hence, Order = 3, Degree = 2

11. Option (c) is correct.

$$\left(D^2 + D - 6\right)y = 0$$

$$AE \rightarrow m^2 + m - 6 = 0$$

$$\Rightarrow \quad (m+3)(m-2) = 0$$

$$\Rightarrow \quad m = -3, 2$$

$$\therefore \quad y = CF = C_1 e^{-3x} + C_2 e^{2x}$$

12. Option (b) is correct.

$$A = \begin{bmatrix} 3+2i & i \\ -i & 3-2i \end{bmatrix}$$

$$\text{Minor} = \begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$$

$$\text{Co-factor} = \begin{bmatrix} 3-2i & i \\ -i & 3+2i \end{bmatrix}$$

$$\text{Adj}(A) = C^T$$

$$\text{Adj (A)} = \begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{Adj}(A)$$

$$|A| = \begin{vmatrix} 3-2i & -i \\ i & 3+2i \end{vmatrix}$$

$$= 9 - 4i^2 + i^2 = 9 + 4 - 1 = 12$$

$$\therefore \quad A^{-1} = \frac{1}{12}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$$

13. Option (a) is correct.

x	0	0.25	0.5	0.75	1
F(x)	1	0.9412	0.8	0.64	0.50
	y_0	y_1	y_2	y_3	y_4

Simpson's (1/3)rd Rule

$$\int_{x_0}^{x+nh} f(x)dx = \frac{h}{3}\left[\left(y_0 + y_n\right) + 4\left(y_1 + y_3\right) + 2\left(y_2\right)\right]$$

$$h = \frac{1-0}{4} = 0.25$$

$$\int_0^1 f(x)dx = \frac{0.25}{3}\begin{bmatrix} (1+0.50) + 4(0.9412 + 0.64) \\ +2(0.8) \end{bmatrix}$$

$$= 0.7854$$

14. Option (c) is correct.

Given $z = ax + by + ab$

$$\frac{\partial z}{\partial x} = a \quad but \quad \frac{\partial z}{\partial x} = p \Rightarrow a = p \qquad \text{...(i)}$$

$$\frac{\partial z}{\partial y} = b \quad but \quad \frac{\partial z}{\partial y} = q \Rightarrow b = q \qquad \text{...(ii)}$$

We want to exclude constant $a.b$

$z = px + qy + pq$

15. Option (a) is correct.

$$f(x, y) = 4x^2 + 6y^2 - 8x - 4y + 8$$

$$\frac{\partial f}{\partial x} = 8x - 8$$

$$\frac{\partial f}{\partial y} = 12y - 4$$

$$\frac{\partial f}{\partial x} = 0 \Rightarrow x = 1$$

$$\frac{\partial f}{\partial y} = 0 \Rightarrow y = \frac{1}{3}$$

$\therefore \quad \left(1, \frac{1}{3}\right)$ is only stationary point.

$$r = \left[\frac{\partial^2 f}{\partial x^2}\right]_{\left(1, \frac{1}{3}\right)} = 8 > 0$$

$$t = \left[\frac{\partial^2 f}{\partial y^2}\right]_{\left(1, \frac{1}{3}\right)} = 12 > 0$$

$$s = \left[\frac{\partial^2 f}{\partial x \partial y}\right]_{\left(1,\frac{1}{3}\right)} = 0$$

$\therefore \quad rt - s^2 = 96 > 0$

$\because \left(1, \frac{1}{3}\right)$ is a point of minima.

$$f\left(1,\frac{1}{3}\right) = 4(1)^2 + 6\left(\frac{1}{3}\right)^2 - 8(1) - 4\left(\frac{1}{3}\right) + 8 = \frac{10}{3}$$

16. Option (d) is correct.

$f(x) = x^2 - 13$

$f'(x) = 2x, \quad x_0 = 3.5$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Applying the above formula, we get

$x_1 = 3.5 - (3.5 * 3.5 - 13) / 2 * 3.5$

$x_1 = 3.607$

17. Option (c) is correct.

Total number of reflexive relations $= 2^{(n^2 - n)}$

for $n = 5$, $2^{(5^2-5)} = 2^{20}$

18. Option (a) is correct.

Cayley Table

	1	ω	ω²
1	1	ω	ω²
ω	ω	ω²	1
ω²	ω²	1	ω

The Structure (S,*) satisfies the closure property, associativity and commutative property. The structure also has an identity element (= 1) and an inverse for each element. So the structure is an Abelian group.

19. Option (b) is correct.

$$y = \lim_{n\to\infty}\left(1 - \frac{1}{n}\right)^{2n}$$

$$y = \left\{\lim_{n\to\infty}\left(1 - \frac{1}{n}\right)^{n}\right\}^2$$

$$= \left(e^{-1}\right)^2 \qquad \left[\because \lim_{x\to\infty}\left(1 - \frac{1}{x}\right)^x = e^{-1}\right]$$

So, $\quad y = e^{-2}$

20. Option (a) is correct.

A computer can be declared faulty in two cases.

1. It is actually faulty and correctly declared so $(p * q)$.
2. It is not faulty and incorrectly declared faulty $(1 - p) * (1 - q)$.

So, required probability = sum of probabilities $= pq + (1 - p)(1 - q)$

21. Option (a) is correct.

Prime factorization of $10 = 2 \times 5$

So, $10^{99} = 2^{99} \times 5^{99}$

No. of possible factors of 10^{99} = No. of ways in which prime factors can be combined

$= 100 \times 100$

$10^{99} = 10^{96} \times 1000$, $1000 = 2^2 * 5^3$

So, no. of multiples of 10^{96} which divides 10^{99} = No. of possible factors of 1000.

$= 4 \times 4$

$= 16$

So, required probability $= \frac{16}{10000} = \frac{1}{625}$.

22. Option (d) is correct.

As a degree sequence $d_1, d_2, \ldots d_n$ of non-negative integer is graphical if it is a degree sequence of graph.

I. 7, 6, 5, 4, 4, 3, 2, 1
5, 4, 3, 3, 2, 1, 0
3, 2, 2, 1, 0
1, 1, 0
0, 0

II. 6, 6, 6, 6, 3, 3, 2, 2
5, 5, 5, 2, 2, 1, 2
5, 5, 2, 2, 2, 1
4, 4, 1, 1, 1, 1
3, 0, 0, 0, 1

III. 7, 6, 6, 4, 4, 3, 2, 2
5, 5, 3, 3, 2, 1, 1
4, 2, 2, 1, 0, 1
4, 2, 2, 1, 1, 0
1, 1, 0, 0, 0
0, 0, 0, 0

IV. 8, 7, 7, 6, 4, 2, 1, 1

not possible as number of vertices are 8, and max degree is 8.

So, option II, IV are not graphic sequence.

23. Option (d) is correct.

Sum of eigen values is equal to trace and product of eigen values is equal to the determinant of matrix.

So, $2 + y = 8 + 4 \Rightarrow y = 10$

$\text{det} = 2y - 3x = 32$

Solving above equations $y = 10$, $x = -4$

24. Option (b) is correct.

$F(x, y, t) \Rightarrow$ person x can fool person y at time t.

$\forall x \exists y \exists t (\neg F(x, y, t)) \equiv \neg \exists x \forall y \forall t (F(x, y, t))$

[By applying De Morgan's law.]

Now $\neg \exists x \forall y \forall t (F(x, y, t)) \Rightarrow$ there does not exist a person who can fool everyone all the time.

$\Rightarrow$ No one can fool everyone all the time.