ENGINEERING MATHEMATICS — GATE — Question Paper 2011

ELECTRONICS AND COMMUNICATION (EC) P1

Q. 1. The value of the integral $\oint_C \frac{-3z+4}{(z^2+4z+5)}dz$ where c is the circle $|z| - 1$ is given by

(a) 0 **(b)** $\frac{1}{10}$

(c) $\frac{4}{5}$ **(d)** 1

Q. 2. Consider a closed surface S surrounding a volume V. If $\vec{r}$ is the position vector of a point inside S, with $\hat{n}$ the unit normal on S, the value of the integral $\oiint_s 5\vec{r}\cdot\hat{n}\,dS$ is

(a) 3 V **(b)** 5 V

(c) 10 V **(d)** 15 V

Q. 3. The solution of the differential equation $\frac{dy}{dx} = ky,\ y(0) = c$ is

(a) $x = ce^{-ky}$ **(b)** $x = ke^{cy}$

(c) $y = ce^{kx}$ **(d)** $y = ce^{-kx}$

Q. 4. The system of equations

$x + y + z = 6$

$x + 4y + 6z = 20$

$x + 4y + \lambda z = \mu$

has No solution for values of λ and μ given by

(a) $\lambda = 6, \mu = 20$ **(b)** $\lambda = 6, \mu \neq 20$

(c) $\lambda \neq 6, \mu = 20$ **(d)** $\lambda \neq 6, \mu \neq 20$

Q. 5. A fair dice is tossed two times. The probability that the second toss results in a value that is higher than the first toss is

(a) $\frac{2}{36}$ **(b)** $\frac{2}{6}$

(c) $\frac{5}{12}$ **(d)** $\frac{1}{2}$

Q. 6. A numerical solution of the equation $f(x) = x + \sqrt{x} - 3 = 0$ can be obtained using Newton-Raphson method. If the starting value is $x = 2$ for the iteration, the value of x that is to be used in the next step is

(a) 0.306 **(b)** 0.739

(c) 1.694 **(d)** 2.306

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Q. 7. [A] is a square matrix which is neither symmetric nor skew–symmetric and $[A]^T$ is its transpose. The sum and difference of these matrices are defined as $[S] = [A] + [A]^T$ and $[D] = [A] - [A]^T$, respectively. Which of the following statement is TRUE ?

(a) Both [S] and [D] are symmetric

(b) Both [S] and [D] are skew – symmetric

(c) [S] is skew – symmetric and [D] is symmetric

(d) [S] is symmetric and [D] is skew – symmetric

Q. 8. The square root of a number N is to be obtained by applying the Newton Raphson iterations to the equation $x^2 - N = 0$. If i denotes the iteration index, the correct iterative scheme will be

(a) $x_{i+l} = \frac{1}{2}\left(x_i + \frac{N}{x_i}\right)$

(b) $x_{i+l} = \frac{1}{2}\left(x_i^2 + \frac{N}{x_i^2}\right)$

(c) $x_{i+l} = \frac{1}{2}\left(x_i + \frac{N^2}{x_i}\right)$

(d) $x_{i+l} = \frac{1}{2}\left(x_i - \frac{N}{x_i}\right)$

Q. 9. There are two containers, with one containing 4 Red and 3 Green balls and the other containing 3 Blue and 4 Green balls. One ball is drawn at random from each container. The probability that one of the balls is Red and the other is Blue will be

(a) $\frac{1}{7}$ **(b)** $\frac{9}{49}$

(c) $\frac{12}{49}$ **(d)** $\frac{3}{7}$

Q. 10. For an analytic function, $f(x + iy) = u(x, y) + i\,v(x, y)$, u is given by $u = 3x^2 - 3y^2$. The expression for v, considering K to be a constant is

(a) $3y^2 - 3x^2 + K$ **(b)** $6x - 6y + K$

(c) $6y - 6x + K$ **(d)** $6xy + K$

Q. 11. What should be the value of λ such that the function defined below is continuous at $x = \frac{\pi}{2}$?

$$f(x) = \begin{cases} \dfrac{\lambda \cos x}{\frac{\pi}{2} - x} & \text{if } x \neq \frac{\pi}{2} \\ 1 & \text{if } x = \frac{\pi}{2} \end{cases}$$

(a) 0 **(b)** $\frac{2}{\pi}$

(c) 1 **(d)** $\frac{\pi}{2}$

Q. 12. What is the value of the definite integral, $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$?

(a) 0 **(b)** $\frac{a}{2}$

(c) a **(d)** $2a$

Q. 13. If $\vec{a}$ and $\vec{b}$ are two arbitrary vectors with magnitudes a and b, respectively, $|\vec{a}\times\vec{b}|^2$ will be equal to

(a) $a^2b^2 - (\vec{a}\cdot\vec{b})^2$ **(b)** $ab - \vec{a}\cdot\vec{b}$

(c) $a^2b^2 + (\vec{a}\cdot\vec{b})^2$ **(d)** $ab + \vec{a}\cdot\vec{b}$

Q. 14. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x$, with the condition that $y = 1$ at $x = 1$, is

(a) $y = \frac{2}{3x^2} + \frac{x}{3}$ **(b)** $y = \frac{x}{2} + \frac{1}{2x}$

(c) $y = \frac{2}{3} + \frac{x}{3}$ **(d)** $y = \frac{2}{3x} + \frac{x^2}{3}$

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Q. 15. If two fair coins are flipped and at least one of the outcomes is known to be a head, what is the probability that both outcomes are heads?

(a) $\frac{1}{3}$ **(b)** $\frac{1}{4}$

(c) $\frac{1}{2}$ **(d)** $\frac{2}{3}$

Q. 16. If the difference between the expectation of the square of a random variable ($E[X^2]$) and the square of the expectation of the random variable ($E[X^2]$) is denoted by R, then

(a) $R = 0$ **(b)** $R < 0$

(c) $R \geq 0$ **(d)** $R > 0$

Q. 17. Which of the following option is **CORRECT** given three positive integers x, y and z, and a predicate

$$P(x) = \neg(x = 1) \wedge \forall y (\exists z (x = y * z) \Rightarrow (y = x) \vee (y = 1))$$

(a) P (x) being true means that x is a prime number

(b) P (x) is true means that x is a number other than 1

(c) P (x) is always true irrespective of the value of x

(d) P (x) being true means that x has exactly two factors other than 1 and x

Q. 18. Given $i = \sqrt{-1}$, what will be the evaluation of the definite integral $\int_0^{\pi/2} \frac{\cos x + i\sin x}{\cos x - i\sin x}dx$?

(a) 0 **(b)** 2

(c) $-i$ **(d)** i

Q. 19. A deck of 5 cards (each carrying a distinct number from 1 to 5) is shuffled thoroughly. Two cards are then removed one at a time from the deck. What is the probability that the two cards are selected with the number on the first card being one higher than the number on the second card ?

(a) $\frac{1}{5}$ **(b)** $\frac{4}{25}$

(c) $\frac{1}{4}$ **(d)** $\frac{2}{5}$

Q. 20. Four matrices M_1, M_2, M_3 and M_4 of dimensions $p \times q$, $q \times r$, $r \times s$, $s \times t$, respectively can be multiplied in several ways with different number of total scalar multiplications. For example when multiplied as $((M_1 \times M_2) \times (M_3 \times M_4))$, the total number of scalar multiplications is $pqr + rst + prt$. When multiplied as $(((M_1 \times M_2) \times M_3) \times M_4)$, the total number of scalar multiplication is $pqr + prs + pst$. If $p = 10$, $q = 100$, $r = 20$, $s = 5$, and $t = 80$, then the minimum number of scalar multiplications needed is

(a) 248000 **(b)** 19000

(c) 44000 **(d)** 25000

Q. 21. Consider the matrix as given below.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix}$$

Which one of the following options provides the **CORRECT** values of the eigenvalues of the matrix?

(a) 1, 4, 3 **(b)** 3, 7, 3

(c) 7, 3, 2 **(d)** 1, 2, 3

Answer Key			
Q. No.	**Answer**	**Topic Name**	**Chapter Name**
1	(a)	Cauchy's Integral	Complex Variable
2	(d)	Divergence Theorem	Vector Calculus
3	(c)	1st Order	Differential Equation
4	(b)	System of Equation	Linear Algebra
5	(c)	Probability	Probability and Statistics
6	(c)	Newton Raphson Method	Numerical Analysis
7	(d)	Matrix	Linear Algebra
8	(a)	Newton Raphson Method	Numerical Analysis
9	(c)	Probability	Probability and Statistics
10	(d)	Analytic Function	Complex Variable
11	(c)	Continuity	Calculus
12	(b)	Definite Integral	Calculus
13	(a)	Cross Product	Vector Calculus
14	(d)	1st Order	Differential Equation
15	(a)	Probability	Probability and Statistics
16	(c)	Mean and Variance	Probability and Statistics
17	(a)	Propositional-Logic	Discrete Mathematics
18	(d)	Definite Integral	Calculus
19	(a)	Probability	Probability and Statistics
20	(c)	Matrix	Linear Algebra
21	(a)	Eigen Values	Linear Algebra

ANSWERS WITH EXPLANATIONS

1. Option (a) is correct.

$$\oint_C \frac{-3z+4}{z^2+4z+5}dz = 0$$

Poles for $f(z)$ will be $z = -2 \pm j$ which lies outside the unit circle.

So, that $\oint f(z)\, dz = 0$ [By Cauchy's Integral]

2. Option (d) is correct.

Using divergence theorem

$$\oiint_S 5\vec{r}.\vec{n}dx = \iiint_V 5\nabla.\vec{r}\, d\mathrm{V}, \text{as}\left\{\nabla.x\hat{i}+y\hat{j}+z\hat{k}=3\right\}$$

$$= 5(3)\iiint_V d\mathrm{V} = 15\mathrm{V}$$

3. Option (c) is correct.

From the given differential equation $\frac{dy}{dx} = ky$

$$\frac{dy}{y} = kdx$$

$$\ln y = kx + c_1$$

$$y = e^{kx+c_1}$$

at $x = 0, y = c$

$$c = e^0 \cdot e^{c_1}$$

So, $y = c \cdot e^{kx}$

4. Option (b) is correct.

For $\mathrm{A}x = \mathrm{B}$

$$\begin{bmatrix}1&1&1\\1&4&6\\1&4&\lambda\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}6\\20\\\mu\end{bmatrix}$$

$$[\mathrm{A}|\mathrm{B}] = \begin{bmatrix}1&1&1&6\\1&4&6&20\\1&4&\lambda&\mu\end{bmatrix}$$

For $\mathrm{R}_3 \to \mathrm{R}_3 - \mathrm{R}_2$

$$\begin{bmatrix}1&1&1&6\\1&4&6&20\\0&0&\lambda-6&\mu-20\end{bmatrix}$$

If $\lambda - 6 = 0$ and $\mu - 20 \neq 0$

i.e. $\lambda = 6$ and $\mu \neq 20$

then rank (A) < 3

rank (A|B) = 3

So, system will have no solution.

5. Option (c) is correct.

For a fair dice tossed twice $n(\mathrm{S}) = 6 \times 6 = 36$

Favourable out comes : 2nd toss value higher than 1st toss

Event: (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (5, 6),

$n(\mathrm{E}) = 15$

$$\therefore \mathrm{P(E)} = \frac{n(\mathrm{E})}{n(\mathrm{S})} = \frac{15}{36} = \frac{5}{12}$$

6. Option (c) is correct.

$$f(x) = x + \sqrt{x} - 3$$

$$f(2) = 2 + \sqrt{2} - 3 = \sqrt{2} - 1 = 0.4142$$

$$f'(x) = 1 + \frac{1}{2} \times \frac{1}{\sqrt{x}}$$

$$f'(2) = 1 + \frac{1}{2\sqrt{2}} = 1.353$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{0.4142}{1.353} = 1.6939$$

7. Option (d) is correct.

According to property [S] is symmetric and [D] is skew symmetric.

8. Option (a) is correct.

$$f(x) = x^2 - \mathrm{N} = 0$$

$$f'(x) = 2x$$

Newton – Raphson formula is

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$= x_i - \frac{(x_i^2 - \mathrm{N})}{2x_i} = \frac{(2x_i^2 - x_i^2 + \mathrm{N})}{2x_i}$$

$$\Rightarrow \quad x_{i+1} = \frac{1}{2}\left[x_i + \frac{\mathrm{N}}{x_i}\right]$$

9. Option (c) is correct.

Given, container 1 contains 4 red and 3 green.

Container 2 contains 3 blue and 4 green.

$$P(\text{Red} \cap \text{Blue}) = P(\text{Red}) \cdot P(\text{Blue})$$

$$= \frac{4}{7} \times \frac{3}{7} = \frac{12}{49}$$

10. Option (d) is correct.

$$f(x+iy) = u(x,y) + iv(x,y)$$

$$f = u + iv$$

$$u = 3x^2 - 3y^2$$

For f to be analytic function it should satisfy Cauchy's Riemann equation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\frac{\partial u}{\partial x} = 6x \text{ and } \frac{\partial u}{\partial y} = -6y$$

$$\frac{\partial v}{\partial y} = 6x$$

Integrating

$$v = 6xy + \phi(x)$$

$$\frac{\partial v}{\partial x} = 6y + \phi'(x)$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$6y = 6y + \phi'(x)$$

Integrating

$$\phi(x) = K$$

$$\therefore \quad v = 6xy + K$$

11. Option (c) is correct.

According to given condition.

$$\lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \quad \lim_{x \to \frac{\pi}{2}} \left[\frac{\lambda \cos x}{\left(\frac{\pi}{2} - x\right)}\right] = 1$$

$$= \lim_{x \to \frac{\pi}{2}} \left[\frac{\lambda \cos x}{\frac{\pi}{2} - x}\right] \left(\frac{0}{0} \to \text{form}\right)$$

$$= \lim_{x \to \frac{\pi}{2}} \left[\frac{-\lambda \sin x}{-1}\right]$$

$$= \lambda$$

$$\therefore \quad \lambda = 1$$

12. Option (b) is correct.

Let $\quad I = \dfrac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx \quad$ ...(i)

Using the property

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{a-(a-x)}} dx$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{a-a+x}} dx$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx \quad \text{...(ii)}$$

Adding (i) and (ii)

$$I + I = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx + \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx$$

$$\Rightarrow \quad 2I = \int_0^a \frac{\sqrt{x} + \sqrt{a-x}}{\sqrt{x} + \sqrt{a-x}} dx$$

$$\Rightarrow \quad 2I = \int_0^a 1.dx$$

$$\Rightarrow \quad 2I = [x]_0^a$$

$$\Rightarrow \quad 2I = (a-0)$$

$$\Rightarrow \quad I = \frac{a}{2}$$

13. Option (a) is correct.

By Lagrange's equation $|\vec{a}.\vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 . |\vec{b}|^2$

Given magnitude of $\vec{a}$ and $\vec{b}$ is a and b.

$$|\vec{a}| = a$$

$$|\vec{b}| = b$$

$$|\vec{a} \cdot \vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = a^2 b^2$$

$$\Rightarrow \quad |\vec{a} \times \vec{b}|^2 = a^2 b^2 - (\vec{a} \cdot \vec{b})^2$$

14. Option (d) is correct.

Given DE equation is of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Here, $\quad P(x) = \dfrac{1}{x}; \; Q(x) = x$

$$IF = e^{\int P dx}$$

$$IF = e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$$

Solution, $y.\text{IF} = \int \text{IF}.\text{Q}(x)dx + c$

$\Rightarrow \quad y.x = \int x.xdx + c$

$\Rightarrow \quad y.x = \frac{x^3}{3} + c$

at $x = 1, y = 1$

$1 = \frac{1}{3} + c \Rightarrow c = 1 - \frac{1}{3} = \frac{2}{3}$

$y.x = \frac{x^3}{3} + \frac{2}{3}$

$\Rightarrow \quad y = \frac{x^2}{3} + \frac{2}{3x}$

15. Option (a) is correct.

P(B) = Probability (at least one head) = $\frac{3}{4}$

P(A∩B) = Probability of getting both head = $\frac{1}{4}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

16. Option (c) is correct.

The difference between $(E[X])^2$ and $(E[X])^2$ is variance of a random variable and as variance is always non-negative therefore $R \geq 0$.

17. Option (a) is correct.

$$P(x) = (\neg(x=1) \wedge \forall y(\exists z(x = y * z) \Rightarrow ((y = x) \vee (y = 1))$$

Statement: x is not equal to 1 and for all y if there exists some z such that product of y and z is x, then y is either x or 1. This means x has no other factir other than 1 and itself.

So, x is prime number.

18. Option (d) is correct.

Using Euler's Formula integration becomes

$$\int_0^{\frac{\pi}{2}} \frac{e^{ix}}{e^{-ix}} dx = \int_0^{\frac{\pi}{2}} e^{2ix} dx = \left(\frac{e^{2ix}}{2i}\right)_0^{\frac{\pi}{2}} = \frac{1}{2i}\left[e^{i\pi} - 1\right]$$

$$= \frac{1}{2i}\left[-1 + 0 - 1\right] = \frac{-2}{2i} = i$$

19. Option (a) is correct.

Total ways of picking 2 cards one at a time without replacement is $5 * 4$

Events $(2,1), (3,2), (4,3), (5,4), n(E) = 4$

Required probability $= \frac{4}{5 \times 4} = \frac{4}{20} = \frac{1}{5}$

20. Option (c) is correct.

Multiplying as $(M_1 \times (M_2 \times M_3)) \times M_4$

The total number of scalar multiplications

$= qrs + pqs + pst$

$= 10000 + 5000 + 4000 = 19000$

This gives the minimum number of scalar multiplication in comparison to all other options.

21. Option (a) is correct.

Given matrix is upper triangular matrix and we know its diagonal elements are its eigen values = 1, 4, 3.