ENGINEERING MATHEMATICS GATE Question Paper 2012

ELECTRONICS AND COMMUNICATION (EC) P1

Q. 1. With initial condition $x(1) = 0.5$, the solution of the differential equation, $t\frac{dx}{dt} + x = t$ is

(a) $x = t - \frac{1}{2}$ **(b)** $x = t^2 - \frac{1}{2}$

(c) $x = \frac{t^2}{2}$ **(d)** $x = \frac{t}{2}$

Q. 2. Given $f(z) = \frac{1}{z+1} - \frac{2}{z+3}$.

If C is a counter clockwise path in the z-plane such that $|z+1| = 1$, the value of $\frac{1}{2\pi i}\oint_C f(z)dz$ is

(a) –2 **(b)** –1

(c) 1 **(d)** 2

Q. 3. Two independent random variables X and Y are uniformly distributed in the interval [–1, 1]. The probability that max [X, Y] is less than $\frac{1}{2}$ is

(a) $\frac{3}{4}$ **(b)** $\frac{9}{16}$

(c) $\frac{1}{4}$ **(d)** $\frac{2}{3}$

Q. 4. If $x = \sqrt{-1}$, then the value of x^x is

(a) $e^{\frac{-\pi}{2}}$ **(b)** $e^{\frac{\pi}{2}}$

(c) x **(d)** 1

Q. 5. Consider the differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t) \text{ with}$$

$$y(t)\Big|_{t=0^-} = -2 \text{ and } \frac{dy}{dx}\Big|_{t=0^-} = 0.$$

The numerical value $\frac{dy}{dt}\Big|_{t=0^+}$ is

(a) –2 **(b)** –1

(c) 0 **(d)** 1

Q. 6. The direction of vector A is radially outward from the origin, with $|A| = kr^n$ where $r^2 = x^2 + y^2 + z^2$ and k is a constant. The value of n for which $\nabla \cdot A = 0$ is

(a) –2 **(b)** 2

(c) 1 **(d)** 0

Q. 7. A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is

(a) $\frac{1}{3}$ **(b)** $\frac{1}{2}$

(c) $\frac{2}{3}$ **(d)** $\frac{3}{4}$

Q. 8. The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval [1, 6] is

(a) 21 **(b)** 25

(c) 41 **(d)** 46

Q. 9. Given that

$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the value of A^3 is

(a) 15 A + 12 I **(b)** 19 A + 30 I

(c) 17 A + 15 I **(d)** 17 A + 21 I

MINING ENGINEERING (MN) P1

Q. 10. The coefficient of variation of a dataset is measured by

(a) $\frac{\text{mean}}{\text{standard deviation}}$

(b) $\frac{\text{mean}}{\text{variance}}$

(c) $\frac{\text{standard deviation}}{\text{mean}}$

(d) $\frac{\text{variance}}{\text{mean}}$

Q. 11. The value of $\int_0^1 \sin^{-1}(\cos x)\, dx$ is

(a) $\frac{(\pi-1)}{2}$ **(b)** $\frac{(\pi+1)}{2}$

(c) $\frac{(2\pi+1)}{2}$ **(d)** $\frac{(2\pi-1)}{2}$

Q. 12. Assuming sin (1) = 0.841 and sin (3) = 0.141, the Lagrangian linear interpolating polynomial, for the function $f(x)$ = sin (x) defined on the interval [1, 3] and passing through the end point of the interval, is

(a) $-0.35x + 1.19$
(b) $-3.05x + 11.92$
(c) $-35.00x + 119.10$
(d) $-40.50x + 219.19$

Q. 13. The 2nd order differential equation having a solution $y = \left(\frac{A}{x}\right) + B$, where A and B are constants, is

(a) $\frac{d^2y}{dydx} + \frac{2}{x}\frac{dy}{dx} = 0$

(b) $\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} = 0$

(c) $\left(\frac{d^2y}{dx^2}\right)^2 + \frac{2}{x}\frac{dy}{dx} = 0$

(d) $\frac{d^2y}{dydx} + \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Q. 14. An investment of ₹10,000, compounded annually, is estimated to return ₹20,000 after 6 years from the date of investment. The expected rate of return on this investment in percentage is

(a) 8.75 **(b)** 10.50
(c) 12.25 **(d)** 16.6

Q. 15. The angle between the tangents to the curve $\vec{R} = t^2\hat{i} + 2t\hat{j}$ at the point $t = \pm 1$ is

(a) $\frac{\pi}{2}$ **(b)** $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ **(d)** $\frac{\pi}{6}$

Q. 16. The cofactor matrix of $P = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ is

(a) $\begin{bmatrix} 21 & -5 & 2 \\ 2 & 21 & -5 \\ -5 & 2 & 21 \end{bmatrix}$ **(b)** $\begin{bmatrix} 7 & -5 & 1 \\ 1 & 7 & -5 \\ -5 & 1 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & 2 & 21 \\ 15 & 7 & 2 \\ -2 & 21 & -5 \end{bmatrix}$ **(d)** $\begin{bmatrix} 15 & 7 & 2 \\ -5 & 2 & 21 \\ -2 & 21 & -5 \end{bmatrix}$

CIVIL ENGINEERING (CE) P1

Q. 17. The estimate of $\int_{0.5}^{1.5} \frac{dx}{x}$ obtained using Simpson's rule with three-point function evaluation exceeds the exact value by

(a) 0.235 **(b)** 0.068
(c) 0.024 **(d)** 0.012

Q. 18. The annual precipitation data of a city is normally distributed with mean and standard deviation as 1000 mm 200 mm, respectively. The probability that the annual precipitation will be more than 1200 mm is

(a) $<50\%$ **(b)** 50%
(c) 75% **(c)** 100%

Q. 19. The infinite series $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ...$ corresponds to

(a) sec x **(b)** e^x
(c) cos x **(c)** $1 + \sin^2 x$

Q. 20. In an experiment, positive and negative values are equally likely to occur. The probability of obtaining at most one negative value in five trials is

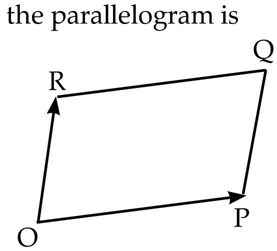
(a) $\frac{1}{32}$ **(b)** $\frac{2}{32}$

(c) $\frac{3}{32}$ **(d)** $\frac{6}{32}$

Q. 21. The eigenvalues of matrix $\begin{bmatrix} 9 & 5 \\ 5 & 8 \end{bmatrix}$ are

(a) –2.42 and 6.86 **(b)** 3.48 and 13.53
(c) 4.70 and 6.86 **(c)** 6.86 and 9.50

Q. 22. For the parallelogram OPQR shown in the sketch, $\overrightarrow{OP} = a\hat{i} + b\hat{j}$ and $\overrightarrow{OR} = c\hat{i} + d\hat{j}$. The area of the parallelogram is



(a) $a\,d - b\,c$ **(b)** $a\,c + b\,d$
(c) $a\,d + b\,c$ **(d)** $a\,b - c\,d$

Q. 23. The solution of the ordinary differential equation $\frac{dy}{dx} + 2y = 0$ for the boundary condition, $y = 5$ at $x = 1$ is

(a) $y = e^{-2x}$ **(b)** $y = 2e^{-2x}$

(c) $y = 10.95\, e^{-2x}$ **(d)** $y = 36.95\, e^{-2x}$

COMPUTER SCIENCE (CS) P1

Q. 24. Consider the following logical inferences.

I_1 : If it rains then the cricket match will not be played.
The cricket match was played.
Inference: There was no rain.

I_2 : If it rains then the cricket match will not be played.
It did not rain.
Inference: The cricket match was played.

Which of the following is TRUE?

(a) Both I_1 and I_2 are correct inferences

(b) I_1 is correct but I_2 is not a correct inference

(c) I_1 is not correct but I_2 is a correct inference

(d) Both I_1 and I_2 are not correct inferences

Q. 25. Consider the function $f(x) = \sin(x)$ in the interval $x \in \left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$. The number and location(*s*) of the local minima of this function are

(a) One, at $\frac{\pi}{2}$

(b) One, at $\frac{3\pi}{2}$

(c) Two, at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

(d) Two, at $\frac{\pi}{4}$ and $\frac{3\pi}{2}$

Q. 26. Let A be the 2 × 2 matrix with elements $a_{11} = a_{12} = a_{21} = +1$ and $a_{22} = -1$. Then the eigenvalues of the matrix A^{19} are

(a) 1024 and −1024

(b) $1024\sqrt{2}$ and $-1024\sqrt{2}$

(c) $4\sqrt{2}$ and $-4\sqrt{2}$

(d) $512\sqrt{2}$ and $-512\sqrt{2}$

Q. 27. What is the correct translation of the following statement into mathematical logic ?
"Some real numbers are rational"

(a) $\exists x(\text{real}(x) \vee \text{rational}(x))$

(b) $\forall x(\text{real}(x) \rightarrow \text{rational}(x))$

(c) $\exists x(\text{real}(x) \wedge \text{rational}(x))$

(d) $\exists x(\text{rational}(x) \rightarrow \text{real}(x))$

Q. 28. Consider a random variable X that takes values + 1 and −1 with probability 0.5 each. The values of the cumulative distribution function F(x) at $x = -1$ and $+1$ are

(a) 0 and 0.5 **(b)** 0 and 1

(c) 0.5 and 1 **(d)** 0.25 and 0.75

Q. 29. The bisection method is applied to compute a zero of the function $f(x) = x^4 - x^3 - x^2 - 4$ in the interval [1, 9]. The method converges to a solution after ——— iterations.

(a) 1 **(b)** 3

(c) 5 **(d)** 7

Q. 30. Suppose a fair six-sided die is rolled once. If the value on the die is 1, 2, or 3, the die is rolled a second time. What is the probability that the sum total of values that turn up is at least 6 ?

(a) $\frac{10}{21}$ **(b)** $\frac{5}{12}$

(c) $\frac{2}{3}$ **(d)** $\frac{1}{6}$

Answer Key			
Q. No.	**Answer**	**Topic Name**	**Chapter Name**
1	(d)	1st Order Equation	Differential Equation
2	(c)	Cauchy's Integral	Complex Variable
3	(b)	Uniform Distribution	Probability and Statistics
4	(a)	Complex Power	Complex Analysis
5	(d)	Second Order Equation	Differential Equation
6	(a)	Divergence	Vector Calculus
7	(c)	Probability	Probability and Statistics
8	(c)	Maximum/Minimum	Calculus
9	(b)	Cayley Hamilton Theorem	Linear Algebra
10	(c)	Statistics	Probability and Statistics
11	(a)	Definite Integral	Calculus
12	(a)	Linear Interpolation	Numerical Analysis
13	(b)	2nd Order De	Differential Equation
14	(c)	Compound Interest	Calculus
15	(a)	Application of Derivatives	Linear Algebra
16	(b)	Cofactor	Linear Algebra
17	(d)	Simpson's Rule	Numerical Analysis
18	(a)	Normal Distribution	Probability and Statistics
19	(b)	Taylor Series	Calculus
20	(d)	Error	Numerical Analysis
21	(d)	Binomial Distribution	Probability and Statistics
22	(b)	Eigen Values	Linear Algebra
23	(a)	Area of Parallelogram	Vector Analysis
24	(d)	1st Order Equation	Differential Equation
25	(b)	Logic	Discrete Mathematics
26	(d)	Maxima/Minima	Calculus
27	(d)	Eigen Values	Linear Algebra
28	(c)	Logic	Discrete Mathematics
29	(c)	CDF	Probability and Statistics
30	(b)	Bisection Method	Numerical Analysis
31	(b)	Probability	Probability and Statistics

ENGINEERING MATHEMATICS GATE Solved Paper 2012

ANSWERS WITH EXPLANATIONS

1. Option (d) is correct.

Given, DE is $t\frac{dx}{dt}+x=t$

$$\frac{dx}{dt}+\frac{x}{t}=1$$

with $P=\frac{1}{t}, Q=1$

$$IF=e^{\int\frac{1}{t}dt}=e^{\log t}=t$$

Solution is $x(IF)=\int(IF)t\,dt$

$$xt=\int t\,dt$$

$\Rightarrow$ $xt=\frac{t^2}{2}+c$

Given, $x(1)=0.5$

$\Rightarrow$ $0.5=\frac{1}{2}+c$

$\Rightarrow$ $c=0$

Solution : $xt=\frac{t^2}{2}$

$$x=\frac{t}{2}$$

2. Option (c) is correct.

$$I=\frac{1}{2\pi i}\left[\underbrace{\oint_C\frac{1}{z+1}dz}_{I_1}-\underbrace{\frac{z}{z+3}dz}_{I_2}\right]$$

$z=-1$ is a single pole in circle. And $z=-3$ is a single pole outside the circle, where $c:|z+1|=1$

By Cauchy's integral formula

$$I_2=\oint_C\frac{z}{z+3}dz=0$$

$$I_1=\oint_C\frac{1}{z+1}dz=1\ 2\pi i f(z_0)$$

where $f(z)=1, z_0=-1$

$\therefore$ $I_1=2\pi i$

$\Rightarrow$ $I=\frac{1}{2\pi i}\{2\pi i-0\}=1$

3. Option (b) is correct.

Uniform distribution X, Y on [1, –1]

$$f(x)=f(y)=\frac{1}{1-(-1)}=\frac{1}{2}$$

$$P(\max(x,y))\le\frac{1}{2}$$

$$=P\left(X=\frac{1}{2},-1\le Y\le\frac{1}{2}*\right)$$

$$P\left(-1\le X\le\frac{1}{2},Y=\frac{1}{2}\right)$$

$$\int_{-1}^{\frac{1}{2}}\frac{1}{2}dx\int_{-1}^{\frac{1}{2}}\frac{1}{2}dy=\frac{1}{4}\cdot[x]_{-1}^{\frac{1}{2}}\cdot[y]_{-1}^{\frac{1}{2}}=\frac{1}{4}\cdot\frac{3}{2}\cdot\frac{3}{2}=\frac{9}{16}$$

4. Option (a) is correct.

It is given that $x=\sqrt{-1}=i$

$$x^x=i^i$$

$\because$ $e^{i\theta}=\cos\theta+i\sin\theta \Rightarrow e^{i\frac{\pi}{2}}=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}=i$

$\therefore$ $i^i=\left(e^{i\frac{\pi}{2}}\right)^i=e^{-\frac{\pi}{2}}$

5. Option (d) is correct.

$$\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}+y(t)=\delta(t)$$

Taking laplace transform

$$s^2y(s)-sy(0)-y'(0)$$
$$+2[sy(s)-y(0)]+y(s)=1$$

$$[s^2+2s+1]y(s)+2s+4=1$$

$$y(s)=\frac{-3-2s}{(s^2+2s+1)}=\frac{-3-2s}{(s+1)^2}=\frac{-2}{s+1}\frac{-1}{(s+1)^2}$$

Taking inverse laplace transform

$$y(t)=\left[-2e^{-t}-te^{-t}\right]u(t)$$

$$\frac{dy(t)}{dt}=2e^{-t}+te^{-t}-e^{-t}$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0^+}=2-1=1$$

6. Option (a) is correct.

$$\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right)$$

$$\nabla.\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k r^n\right)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(k r^{n+2}\right) = \frac{k}{r^2}(n+2)r^{n+1}$$

$$= k(n+2)r^{n-1}$$

For $\nabla.\vec{A} = 0 \Rightarrow n+2=0$

$\therefore \quad n = -2$

7. Option (c) is correct.

Probability of odd tosses to get a head

P (odd tosses) $= P(H) + P(TTH) + P(TTTTH) + ...$

$$= \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + ...$$

$$= \frac{1}{2}\left(1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + ...\right), \text{GP}$$

$$= \frac{1}{2}\left(\frac{1}{1-\frac{1}{4}}\right) = \frac{1}{2}\times\frac{4}{3} = \frac{2}{3}$$

8. Option (c) is correct.

$$f(x) = x^3 - 9x^2 + 24x + 5$$

For stationary points $f'(x) = 0$

$\Rightarrow \quad 3x^2 - 18x + 24 = 0$

$\Rightarrow \quad x = 2, 4$

Function value at stationary and end points:

$$f(2)_{max} = (2)^3 - 9(2)^2 + 24(2) + 5 = 25$$

$$f(4) = 4^3 - 9(4)^2 + 24(4) + 5 = 18$$

$$f(1) = 1 - 9 + 24 + 5 = 21$$

$\therefore \quad f(6) = (6)^3 - 9(6)^2 + 24(6) + 5 = 41$

So, maximum value of function is 41.

9. Option (b) is correct.

$$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$\Rightarrow \quad \begin{vmatrix} -5-\lambda & -3 \\ 2 & 0-\lambda \end{vmatrix} = 0$

$\Rightarrow \quad (-5-\lambda)(-\lambda) + 6 = 0$

$\Rightarrow \quad 5\lambda + \lambda^2 + 6 = 0$

$\Rightarrow \quad \lambda^2 = -5\lambda - 6$

$$\lambda^3 = -5\lambda^2 - 6\lambda$$

$$= -5(-5\lambda - 6) - 6\lambda$$

$$= 25\lambda + 30 - 6\lambda = 19\lambda + 30$$

$\therefore \quad A^3 = 19A + 30I$

[By Cayley Hamilton Theorem]

10. Option (c) is correct.

The coefficient of variation of a dataset is measured as ratio of standard deviation to the mean.

11. Option (a) is correct.

Given, $\quad I = \int_0^1 \sin^{-1}(\cos x)\,dx$

$$= \int_0^1 \sin^{-1}\left(\sin\left(\frac{\pi}{2} - x\right)\right) dx$$

$$I = \int_0^1 \left(\frac{\pi}{2} - x\right)dx$$

$$I = \left[\frac{\pi}{2}x - \frac{x^2}{2}\right]_0^1 = \frac{\pi - 1}{2}$$

12. Option (a) is correct.

Lagrange interpolation formula

$$f(x) = \frac{(x-x_1)(x-x_2)...(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)...(x_0-x_n)}f(0)$$

$$+ \frac{(x-x_0)(x-x_2)...(x-x_n)}{(x_1-x_0)(x_1-x_2)...(x_1-x_n)}f(1) + ...$$

$$\frac{(x-x_0)(x-x_1).......(x-x_{n-1})}{(x_n-x_0)(x_n-x_2)....(x_n-x_{n-1})}f(n)$$

$$\sin x = \frac{x-3}{1-3}\times 0.841 + \frac{x-1}{3-1}\times 0.141$$

$$\sin(x) = -0.35x + 1.19$$

13. Option (b) is correct.

$$y = \frac{A}{x} + B \quad ...(i)$$

Differentiating (i) wrt x

$$\frac{dy}{dx} = -\frac{A}{x^2}$$

$$x^2\frac{dy}{dx} = -A$$

Again, Differentiating wrt to x.

$$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$$

Dividing by x^2

$$\frac{d^2y}{dx^2} + 2\frac{1}{x}\frac{dy}{dx} = 0$$

14. Option (c) is correct.

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$20000 = 10000\left(1+\frac{r}{100}\right)^6$$

$$2 = \left(1+\frac{r}{100}\right)^6$$

$$\frac{r}{100} = 2^{\frac{1}{6}} - 1$$

$$r = 12.25\%$$

15. Option (a) is correct.

$$\vec{R} = t^2\hat{i} + 2t\hat{j}$$

$$\frac{d\vec{R}}{dt} = R'(t) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\Rightarrow \quad R'(t) = 2t\hat{i} + 2\hat{j}$$

$$\left(R'(t)\right)_{t=1} = 2\hat{i} + 2\hat{j}$$

$$\left(R'(t)\right)_{t=-1} = -2\hat{i} + 2\hat{j}$$

$$\cos\theta = \frac{\left(R'(t)\right)_{t=1}\cdot\left(R'(t)\right)_{t=-1}}{\left|\left(R'(t)\right)_{t=1}\right|\cdot\left|\left(R'(t)\right)_{t=-1}\right|}$$

$$= \frac{\left(2\hat{i}+2\hat{j}\right)\cdot\left(-2\hat{i}+2\hat{j}\right)}{\left|2\hat{i}+2\hat{j}\right|\cdot\left|-2\hat{i}+2\hat{j}\right|} = 0$$

$$\therefore \quad \theta = \frac{\pi}{2}$$

16. Option (b) is correct.

No answer matching from options

$$\text{For } P = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\text{Minors of } P = \begin{bmatrix} 7 & 5 & 1 \\ -1 & 7 & 5 \\ -5 & -1 & 7 \end{bmatrix}$$

$$\text{Cofactors of } P = \begin{bmatrix} 7 & -5 & 1 \\ 1 & 7 & -5 \\ -5 & 1 & 7 \end{bmatrix}$$

17. Option (d) is correct.

$$f(x) = \frac{1}{x}$$

x	0.5	1	1.5
$y = f(x)$	2 y_0	1 y_1	$\frac{2}{3}$ y_2

By Simpson's (1/3)rd rule

$$\int_{0.5}^{1.5} \frac{1}{x}dx = \frac{h}{3}\left[y_0 + 4y_1 + y_2\right], \text{ with } h = 0.5$$

$$= \frac{1}{6}\left[2 + 4 + 0.666\right] = 1.111$$

By direct integration

$$\int_{0.5}^{1.5} \frac{1}{x}dx = \log_e x\Big|_{0.5}^{1.5} = \ln 1.5 - \ln 0.5 = 1.0986$$

$$\text{Error} = \left|\text{Estimated value - Exact value}\right|$$

$$= 1.111 - 1.0986 = 0.012$$

18. Option (a) is correct.

Given, $\mu = 1000$ mm

$\sigma = 200$ mm

Converting into standard normal form

$$Z = \frac{X-\mu}{\sigma}$$

$$= \frac{X - 1000}{200}$$

Now, at $x = 1200$,

$$= \frac{1200-1000}{200} = 1$$

P (X > 1200) = P(z > 1) is < 50% as we know for standard normal variate Z, P(Z>0) = 50%.

19. Option (b) is correct.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

20. Option (d) is correct.

Using Bernoulli's theorem of Probability.

n = number of trials

p = probability of success

q = probability of failure

x = number of successes

for case in most x.

$$P(x \le \text{most } x) = \sum_{x=0}^{x}\binom{n}{x}p^x q^{n-x}$$

For most one cases x will 0 and 1, for two equally events $p = q = 0.5$.

$$P(x \le 1) = {}^5C_0\,(0.5)^0\,(0.5)^5 + {}^5C_1\,(0.5)^1\,(0.5)^4$$

$$= \frac{6}{32}$$

21. Option (b) is correct.

$$A = \begin{bmatrix} 9 & 5 \\ 5 & 8 \end{bmatrix}$$

Characteristics Equations $|A - \lambda I| = 0$

$$\begin{vmatrix} 9-\lambda & 5 \\ 5 & 8-\lambda \end{vmatrix} = 0$$

$$(9-\lambda)(8-\lambda) - 25 = 0 \;\Rightarrow\; \lambda^2 - 17\lambda + 47 = 0$$

$$\lambda = 3.48, 13.53$$

22. Option (a) is correct.

Area of parallelogram is given by cross product of its sides.

$$\text{area} = \overrightarrow{OP} \times \overrightarrow{OR}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & 0 \\ c & d & 0 \end{vmatrix}$$

$$= \hat{k}\,(ad - bc)$$

23. Option (d) is correct.

From the given differential equation $\frac{dy}{dx} + 2y = 0$

$\Rightarrow \quad (D+2)y = 0$

$AE \to m + 2 = 0$

$\Rightarrow \quad m = -2$

$y = CF = C_1 e^{-2x}$

At $\quad x = 1, y = 5$

$5 = C_1 e^{-2(1)}$

$C_1 = 5e^2$

$y = 5e^2 e^{-2x}$

$y = 36.95\, e^{-2x}$

24. Option (b) is correct.

As if it rains cricket match cannot be played, so I_1 inference is correct.

But if it doesn't rain it is not necessary that cricket will be played, so inference I_2 is not correct.

25. Option (d) is correct.

$f(x) = \sin x$

$f'(x) = \cos x = 0$ gives roots $\frac{\pi}{2}$ and $3\frac{\pi}{2}$ in $\left[\frac{\pi}{4}, 7\frac{\pi}{4}\right]$

$f''(x) = -\sin x$

$f''\left(\frac{\pi}{2}\right) = -1 < 0$

$x = \frac{\pi}{2}$ is local maxima

$f''\left(3\frac{\pi}{2}\right) = 1 > 0$

$x = 3\frac{\pi}{2}$ is local maxima

at $\frac{\pi}{2}$ it is local maxima, so before its graph is strictly increasing, so $\frac{\pi}{4}$ is also local minima.

So, there are two local at $\frac{\pi}{4}$ and $3\frac{\pi}{2}$.

26. Option (d) is correct.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Characteristic equation

$$|A - \lambda I| = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad (1-\lambda)(-1-\lambda) - 1 = 0$$

$$\Rightarrow \quad -1 - \lambda + \lambda + \lambda^2 - 1 = 0$$

$$\Rightarrow \quad \lambda^2 - 2 = 0$$

$$\Rightarrow \quad \lambda = \sqrt{2} \text{ and } -\sqrt{2}$$

According to properties of eigen values.

Eigen values of A^{19} = (eigen values of A)^{19}

$$= \left(\sqrt{2}\right)^{19} \text{ and } \left(-\sqrt{2}\right)^{19}$$

$$= 512\sqrt{2},\; -512\sqrt{2}$$

27. Option (c) is correct.

(a) "There exist some numbers which are either real or rational"

(b) "All real numbers are rational"

(c) "There exist some numbers which are both real and rational"

(d) "There exist some numbers for which rational implies real"

Clearly, answer (c) is correct among all.

28. Option (c) is correct.

$P(-1) = 0.5$ and $P(1) = 0.5$.

So, $F(-1) = P(X \le -1) = P(X = -1) = 0.5$

$F(1) = P(X \le 1)$

$\Rightarrow \quad F(1) = P(-1) + P(1)$

$= 0.5 + 0.5 = 1$

29. Option (b) is correct.

$f(x) = x^4 - x^3 - x^2 - 4,\ x \in [1, a]$

$f(1) = -5 < 0,\ f(a) = 5747 > 0$

Iiteration $x_2 = \frac{x_0 + x_1}{2} = \frac{1+9}{2} = 5$

$f(5) = 471 > 0,$

$\therefore$ root lies in [1, 5]

II iteration $x_3 = \frac{x_0 + x_2}{2} = \frac{1+5}{2} = 3$

$f(3) = 41 > 0,$

$\therefore$ root lies in [1, 3]

III iteration $x_4 = \frac{x_0 + x_3}{2} = \frac{1+3}{2} = 2$

$f(2) = 0$

So, $x_4 = 2$ is a root, at third iteration.

30. Option (b) is correct.

Event : getting sum total at least 6.

E_1 : A six in first roll, $P(E_1) = \frac{1}{6}$

E_2 : getting 1, 2, 3 and rolling again

E_{21} : 1 in first roll, (5 or 6) in second roll

$$P(E_{21}) = \frac{1}{6}\left\{\frac{2}{6}\right\} = \frac{2}{36}$$

E_{22} : 2 in first roll, (4, 5 or 6) in second roll

$$P(E_{22}) = \frac{1}{6}\left\{\frac{3}{6}\right\} = \frac{3}{36}$$

E_{23} : 3 in first roll, (3, 4, 5 or 6) in second roll

$$P(E_{23}) = \frac{1}{6}\left\{\frac{4}{6}\right\} = \frac{4}{36}$$

$P(E_2) = P(E_{21}) + P(E_{22}) + P(E_{23})$

$$= \frac{2+3+4}{36} = \frac{1}{4}$$

So, $P(E) = P(E_1) + P(E_2) = \frac{1}{6} + \frac{1}{4} = \frac{5}{12}$