ENGINEERING MATHEMATICS — GATE — Question Paper 2013

ELECTRONICS AND COMMUNICATION (EC) P1

Q. 1. Consider a vector field $\vec{A}(\vec{r})$. The closed loop line integral $\oint \vec{A}\cdot d\vec{l}$ can be expressed as

(a) $\oiint(\nabla\times\vec{A})\cdot d\vec{s}$ over the closed surface bounded by the loop

(b) $\oiiint(\nabla\cdot\vec{A})d\vec{v}$ over the closed volume bounded by the loop

(c) $\iiint(\nabla\cdot\vec{A})d\vec{v}$ over the open volume bounded by the loop

(d) $\iint(\nabla\times\vec{A})\cdot d\vec{s}$ over the open surface bounded by the loop

Q. 2. The maximum value of θ until which the approximation sinθ ≈ θ holds to within 10% error is

(a) 10° **(b)** 18°
(c) 50° **(d)** 90°

Q. 3 The divergence of the vector field $\vec{A} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ is

(a) 0 **(b)** $\frac{1}{3}$
(c) 1 **(d)** 3

Q. 4 The minimum eigenvalue of the following matrix is

$$\begin{bmatrix} 3 & 5 & 2 \\ 5 & 12 & 7 \\ 2 & 7 & 5 \end{bmatrix}$$

(a) 0 **(b)** 1
(c) 2 **(d)** 3

Q. 5 Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively.

The probability $P(3V \geq 2U)$ is

(a) $\frac{4}{9}$ **(b)** $\frac{1}{2}$
(c) $\frac{2}{3}$ **(d)** $\frac{5}{9}$

Q. 6. Let A be an $m \times n$ matrix and B an $n \times m$ matrix. It is given that determinant (I_m + AB) = determinant (I_n + AB), where I_k is the $k \times k$ identity matrix. Using the above property, the determinant of the matrix given below is

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

(a) 2 **(b)** 5
(c) 8 **(d)** 16

ELECTRICAL ENGINEERING (EE) P1

Q. 7. A continuous random variable X has a probability density function

$f(x) = e^{-x}, 0 < x < \infty$. Then P {X > 1} is

(a) 0.368 **(b)** 0.5
(c) 0.632 **(d)** 1.0

Q. 8. The curl of the gradient of the scalar field defined by $V = 2x^2 y + 3y^2 z + 4z^2 x$ is

(a) $4xya_x + 6yza_y + 8zxa_z$

(b) $4a_x + 6a_y + 8a_z$

(c) $(4xy + 4z^2)a_x + (2x^2 + 6yz)a_y + (3y^2 + 8zx)az$

(d) 0

Q. 9. Square roots of $-i$, where $i = \sqrt{-1}$, are

(a) $i, -i$

(b) $\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right), \cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)$

(c) $\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right), \cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)$

(d) $\cos\left(\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right), \cos\left(-\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)$

Q. 10. Given a vector field $F = y^2xa_x - yza_y - x^2 a_z$, the line integral $\int F\cdot dl$ evaluated along a segment on the x-axis from $x = 1$ to $x = 2$ is

(a) –2.33 **(b)** 0
(c) 2.33 **(d)** 7

Q. 11. The equation $\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ has

(a) no solution

(b) only one solution $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(c) non-zero unique solution

(d) multiple solutions

Q. 12. A matrix has eigenvalues –1 and –2. The corresponding eigenvectors are $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ respectively. The matrix is

(a) $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$ **(d)** $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

Q. 13. $\int \frac{z^2-4}{z^2+4}\, dz$ evaluated anti-clockwise around the circle $|z-i| = 2$, where $i = \sqrt{-1}$, is

(a) -4π **(b)** 0

(c) $2+\pi$ **(d)** $2+2i$

Q. 14. A function $y = 5x^2 + 10x$ is defined over an open interval $x = (1, 2)$. At least at one point in this interval, $\frac{dy}{dx}$ is exactly

(a) 20 **(b)** 25

(c) 30 **(d)** 35

Q. 15. When the Newton-Raphson method is applied to solve the equation $f(x) = x^3 + 2x - 1 = 0$, the solution at the end of the first iteration with the initial guess value as $x_0 = 1.2$ is

(a) – 0.82 **(b)** 0.49

(c) 0.705 **(d)** 1.69

MECHANICAL ENGINEERING (ME) P1

Q. 16. The partial differential equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}$$ is a

(a) linear equation of order 2

(b) non-linear equation of order 1

(c) linear equation of order 1

(d) non-linear equation of order 2

Q. 17. The eigenvalues of a symmetric matrix are all

(a) complex with non-zero positive imaginary part.

(b) complex with non-zero negative imaginary part.

(c) real.

(d) pure imaginary.

Q. 18. Match the **CORRECT** pairs.

Numerical Integration Scheme	Order of Fitting Polynomial
P. Simpson's 3/8 Rule	1. First
Q. Trapezoidal Rule	2. Second
R. Simpson's 1/3 Rule	3. Third

(a) P – 2, Q – 1, R – 3

(b) P – 3, Q – 2, R – 1

(c) P – 1, Q – 2, R – 3

(d) P – 3, Q – 1, R – 2

Q. 19. Let X be a normal random variable with mean 1 and variance 4. The probability $P\{X < 0\}$ is

(a) 0.5

(b) greater than zero and less than 0.5

(c) greater than 0.5 and less than 1.0

(d) 1.0

Q. 20. Choose the **CORRECT** set of functions, which are linearly dependent.

(a) $\sin x$, $\sin^2 x$ and $\cos^2 x$

(b) $\cos x$, $\sin x$ and $\tan x$

(c) $\cos 2x$, $\sin^2 x$ and $\cos^2 x$

(d) $\cos 2x$, $\sin x$ and $\cos x$

Q. 21. The following surface integral is to be evaluated over a sphere for the given steady velocity vector field $F = xi + yj + zk$ defined with respect to a Cartesian coordinate system having i, j and k as unit base vectors.

$$\iint_S \frac{1}{4}(F.n)\, dA$$

where S is the sphere, $x^2 + y^2 + z^2 = 1$ and n is the outward unit normal vector to the sphere. The value of the surface integral is

(a) π **(b)** 2π

(c) $3\frac{\pi}{4}$ **(d)** 4π

Q. 22. The function $f(t)$ satisfies the differential equation $\frac{d^2 f}{dt^2} + f = 0$ and the auxiliary conditions, $f(0) = 0, \frac{df}{dt}(0) = 4$. The Laplace transform of $f(t)$ is given by

(a) $\frac{2}{s+1}$ **(b)** $\frac{4}{s+1}$

(c) $\frac{4}{s^2+1}$ **(d)** $\frac{2}{s^4+1}$

Q. 23. The probability that a student knows the correct answer to a multiple choice question is $\frac{2}{3}$. If the student does not know the answer, then the student guesses the answer. The probability of the guessed answer being correct is $\frac{1}{4}$. Given that the student has answered the question correctly, the conditional probability that the student knows the correct answer is

(a) $\frac{2}{3}$ **(b)** $\frac{3}{4}$

(c) $\frac{5}{6}$ **(d)** $\frac{8}{9}$

Q. 24. The solution to the differential equation $\frac{d^2u}{dx^2} - k\frac{du}{dx} = 0$ where k is a constant, subjected to the boundary conditions $u(0) = 0$ and $u(\text{L}) = \text{U}$, is

(a) $u = \text{U}\frac{x}{\text{L}}$ **(b)** $u = \text{U}\left(\frac{1-e^{kx}}{1-e^{k\text{L}}}\right)$

(c) $u = \text{U}\left(\frac{1-e^{-kx}}{1-e^{-k\text{L}}}\right)$ **(d)** $u = \text{U}\left(\frac{1+e^{kx}}{1+e^{k\text{L}}}\right)$

Q. 25. The value of the definite integral $\int_1^e \sqrt{x}\ \ln(x)\ dx$ is

(a) $\frac{4}{9}\sqrt{e^3} + \frac{2}{9}$ **(b)** $\frac{2}{9}\sqrt{e^3} - \frac{4}{9}$

(c) $\frac{2}{9}\sqrt{e^3} + \frac{4}{9}$ **(d)** $\frac{4}{9}\sqrt{e^3} - \frac{2}{9}$

CIVIL ENGINEERING (CE) P1

Q. 26. There is no value of x that can simultaneously satisfy both the given equations.Therefore, find the 'least squares error' solution to the two equations, *i.e.*, find the value of x that minimizes the sum of squares of the errors in the two equations.

$2x = 3$

$4x = 1$

Q. 27. What is the minimum number of multiplications involved in computing the matrix product PQR?

Matrix P has 4 rows and 2 columns, matrix Q has 2 rows and 4 columns, and matrix R has 4 rows and 1 column.

Q. 28. $a(1 - h)$ rainfall of 10 cm magnitude at a station has a return period of 50 years. The probability that $a(1 - h)$ rainfall of magnitude 10 cm or more will occur in each of two successive years is:

(a) 0.04 **(b)** 0.2

(c) 0.02 **(d)** 0.0004

Q. 29. Find the magnitude of the error (correct to two decimal places) in the estimation of following integral using Simpson's $\frac{1}{3}$ Rule. Take the step length as 1.

$$\int_0^4 \left(x^4 + 10\right)dx$$

Q. 30. The solution for $\int_0^{\frac{\pi}{6}} \cos^4 3\theta \sin^3 6\theta\, d\theta$ is:

(a) 0 **(b)** $\frac{1}{15}$

(c) 1 **(d)** $\frac{8}{3}$

Q. 31. Find the value of λ such that the function $f(x)$ is a valid probability density function.

$f(x) = \lambda(x-1)(2-x)$ for $1 \le x \le 2$

$= 0$ otherwise

Q. 32. Laplace equation for water flow in soils is given below.

$$\frac{\partial^2 \text{H}}{\partial x^2} + \frac{\partial^2 \text{H}}{\partial y^2} + \frac{\partial^2 \text{H}}{\partial z^2} = 0$$

Head H does not vary in y and z directions.

Boundary conditions are: at $x = 0$, H = 5;and $\frac{d\text{H}}{dx} = -1$.

What is the value of H at $x = 1.2$? _________

COMPUTER SCIENCE (CS) P1

Q. 33. A binary operation $\oplus$ on a set of integers is defined as $x \oplus y = x^2 + y^2$. Which one of the following statements is **TRUE** about $\oplus$?

(a) Commutative but not associative

(b) Both commutative and associative

(c) Associative but not commutative

(d) Neither commutative nor associative

Q. 34. Suppose p is the number of cars per minute passing through a certain road junction between 5 PM and 6 PM, and p has a Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval?

(a) $\frac{8}{(2e^3)}$ **(b)** $\frac{9}{(2e^3)}$

(c) $\frac{17}{(2e^3)}$ **(d)** $\frac{26}{(2e^3)}$

Q. 35. Which one of the following does **NOT** equal $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$?

(a) $\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix}$

(b) $\begin{vmatrix} 1 & (x+1) & x^2+1 \\ 1 & (y+1) & y^2+1 \\ 1 & (z+1) & z^2+1 \end{vmatrix}$

(c) $\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$

(d) $\begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix}$

Q. 36. Which one of the following functions is continuous at $x = 3$?

(a) $f(x) = \begin{cases} 2 & if \quad x = 3 \\ x-1 & if \quad x > 3 \\ \frac{x+3}{3} & if \quad x < 3 \end{cases}$

(b) $f(x) = \begin{cases} 4, & \text{if} \quad x = 3 \\ 8-x & \text{if} \quad x \neq 3 \end{cases}$

(c) $f(x) = \begin{cases} x+3, & \text{if} \quad x \leq 3 \\ x-4 & \text{if} \quad x > 3 \end{cases}$

(d) $f(x) = \frac{1}{x^3-27}$, if $x \neq 3$

Q. 37. Function f is known at the following points:

x	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0
$f(x)$	0	0.09	0.36	0.81	1.44	2.25	3.24	4.41	5.76	7.29	9.00

The value of $\int_0^3 f(x)\, dx$ computed using the trapezoidal rule is

(a) 8.983 **(b)** 9.003

(c) 9.017 **(d)** 9.045

Q. 38. Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is $\frac{1}{2}$. What is the expected number of unordered cycles of length three?

(a) $\frac{1}{8}$ **(b)** 1

(c) 7 **(d)** 8

Q. 39. What is the logical translation of the following statement?

"None of my friends are perfect."

(a) $\exists x (F(x) \wedge \neg P(x))$

(b) $\exists x (\neg F(x) \wedge P(x))$

(c) $\exists x (\neg F(x) \wedge \neg P(x))$

(d) $\neg \exists x (F(x) \wedge P(x))$

Q. 40. Which one of the following is **NOT** logically equivalent to $\neg\exists x (\forall y (\alpha) \wedge \forall z (\beta))$?

(a) $\forall x (\exists z (\neg\beta) \rightarrow \forall y (\alpha))$

(b) $\forall x (\forall z (\beta) \rightarrow \exists y (\neg\alpha))$

(c) $\forall x (\forall y (\alpha) \rightarrow \exists z (\neg\beta))$

(d) $\forall x (\exists y (\neg\alpha) \rightarrow \exists z (\neg\beta))$

CHEMICAL ENGINEERING (CH) P1

Q. 41. The number of e-mails received on six consecutive days is 11, 9, 18, 18, 4 and 15, respectively. What are the median and the mode for these data?

(a) 18 and 11, respectively

(b) 13 and 18, respectively

(c) 13 and 12.5, respectively

(d) 12.5 and 18, respectively

Q. 42. For two rolls of a fair die, the probability of getting a 4 in the first roll and a number less than 4 in the second roll, up to 3 digits after the decimal point, is __________

Q. 43. Which of the following statements are **TRUE**?

P. The eigenvalues of a symmetric matrix are real

Q. The value of the determinant of an orthogonal matrix can only be + 1

R. The transpose of a square matrix A has the same eigenvalues as those of A

S. The inverse of an '$n \times n$' matrix exists if and only if the rank is less than 'n'

(a) P and Q only **(b)** P and R only
(c) Q and R only **(d)** P and S only

Q. 44. Evaluate

$$\int \frac{dx}{e^x - 1}$$

(Note: C is a constant of integration).

(a) $\frac{e^x}{e^x - 1} + C$ **(b)** $\frac{\ln(e^x - 1)}{e^x} + C$
(c) $\ln\left(\frac{e^x}{e^x - 1}\right) + C$ **(d)** $\ln(1 - e^{-x}) + C$

Q. 45. For the function

$$f(z) = \frac{1}{(2-z)(z+2)}$$

the residue at $z = 2$ is ______.

Q. 46. The solution of the differential equation $\frac{dy}{dx} - y^2 = 0$, given $y = 1$ at $x = 0$ is

(a) $\frac{1}{1+x}$ **(b)** $\frac{1}{1-x}$
(c) $\frac{1}{(1-x)^2}$ **(d)** $\frac{x^3}{3} + 1$

Q. 47. The solution of the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 0.25y = 0$, given $y = 0$ at $x = 0$ and $\frac{dy}{dx} = 1$ at $x = 0$ is

(a) $x e^{0.5x} - x e^{-0.5x}$ **(b)** $0.5x e^x - 0.5x e^{-x}$
(c) $x e^{0.5x}$ **(d)** $-x e^{0.5x}$

Q. 48. The value of the integral

$$\int_{0.1}^{0.5} e^{-x^3} dx$$

evaluated by Simpson's rule using 4 subintervals (up to 3 digits after the decimal point) is ______.

Answer Key			
Q. No.	**Answer**	**Topic Name**	**Chapter Name**
1	(d)	Theorems	Vector Calculus
2	(b)	Error	Numerical Analysis
3	(d)	Divergence	Vector Calculus
4	(a)	Eigen Value	Linear Algebra
5	(b)	Normal Distribution	Probability and Statistics
6	(b)	Determinant	Linear Algebra
7	(a)	PDF	Probability and Statistics
8	(d)	Curl	Vector Calculus
9	(b)	Square roots	Complex Analysis
10	(b)	Line Integral	Vector Calculus
11	(d)	Solution of Equations	Linear Algebra
12	(d)	Eigen Vectors	Linear Algebra
13	(a)	Residue Theorem	Complex Variable
14	(b)	Mean Value Theorem	Calculus
15	(c)	Newton Raphson Method	Numerical Analysis
16	(d)	Type and Order	Differential Equation
17	(c)	Eigen Value	Linear Algebra

Q. No.	Answer	Topic Name	Chapter Name
18	(d)	Numerical Integration	Numerical Analysis
19	(b)	Mean and Variance	Probability and Statistics
20	(c)	Linear Dependence	Calculus
21	(a)	Surface Integral	Vector Calculus
22	(c)	Laplace Transform	Transform Theory
23	(d)	Conditional Probability	Probability and Statistics
24	(b)	2nd Order	Differential Equation
25	(c)	Definite Integral	Calculus
26	0.5	Least Square Approximation	Numerical Analysis
27	16	Matrix	Linear Algebra
28	(d)	Probability	Probability and Statistics
29	0.53	Simpson's One Third Rule	Numerical Analysis
30	(b)	Definite Integral	Calculus
31	6	Probability Density Function	Probability and Statistics
32	3.8	Laplace Transform	Transform Theory
33	(a)	Set Theory	Discrete Mathematics
34	(c)	Poisson Distribution	Probability and Statistics
35	(a)	Determinant	Linear Algebra
36	(a)	Continuity	Calculus
37	(d)	Trapezoidal Rule	Numerical Analysis
38	(a)	Graph Theory	Discrete Mathematics
39	(d)	Logics	Discrete Mathematics
40	(a,b,c & d)	Logics	Discrete Mathematics
41	(b)	Mean and Mode	Probability and Statistics
42	0.083	Probability	Probability and Statistics
43	(b)	Eigen Value Properties	Linear Algebra
44	(d)	Integration	Calculus
45	–0.25	Residue Theorem	Complex Variable
46	(b)	1st Order	Differential Equation
47	(c)	2nd Order	Differential Equation
48	0.38	Simpson's Rule	Numerical Analysis

ANSWERS WITH EXPLANATIONS

1. Option (d) is correct.

According to Stokes' theorem

$\therefore \quad \oint \vec{A}.\vec{dl} = \iint \left(\nabla \times \vec{A}\right)\vec{ds}$

2. Option (b) is correct.

We know, $\lim_{\theta \to 0} \sin\theta \approx \theta$

For $\theta = 18^\circ = 0.314$ radians

$\sin 18^\circ = 0.309$

$$\% \text{ error} = \frac{0.314 - 0.309}{0.309} \times 100\% = -1\cdot612\%$$

For $\theta = 50 = 50^\circ \times \frac{\pi}{180^\circ} = 0.873$

$= \sin 50^\circ = 0.77$

$$\% \text{ error} = \frac{0.873 - 0.77}{0.77} \times 100\% = -12.25\%$$

But this is more than 10% and we require $<10\%$. So, option (b) is correct $\theta = 18^\circ$

3. Option (d) is correct.

$\vec{A} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$

$$\nabla.\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \frac{dx}{dx} + \frac{dy}{dy} + \frac{dz}{dz}$$

$= 1 + 1 + 1 = 3$

4. Option (a) is correct.

Minimum eigen value of given matrix.

$$|A| = \begin{vmatrix} 3 & 5 & 2 \\ 5 & 12 & 7 \\ 2 & 7 & 5 \end{vmatrix} = 0,$$

Since $R_1 + R_3 = R_2$ linear dependence

For singular matrix one of the eigen value will be zero and among all non negative options $\lambda = 0$ is minimum.

5. Option (b) is correct.

For $3V \geq 2U,\ 3V - 2U > 0$, Let $W = 3V - 2U$

U and V are independent variables and

$U \sim N\left(0, \frac{1}{4}\right)$

$V \sim N\left(0, \frac{1}{9}\right)$, then W will also follow Normal Distribution

$\therefore \quad \sigma_W = 3^2\sigma_V^2 + 2^2\sigma_U^2$

$= 1 + 1 = 2$

$W \sim N(0, 2)$ that is W has $\mu = 0$ and $\sigma^2 = 2$

$$\therefore \quad P(W \geq 0) = P\left(\frac{w - \mu}{\sigma} \geq \frac{0 - 0}{\sqrt{2}}\right)$$

$= P(Z \geq 0)$,

Z is standard normal variable.

$= 0.5 = \frac{1}{2}$, so $P(3V > 2U) = \frac{1}{2}$

6. Option (b) is correct.

Let $A = [1 \quad 1 \quad 1 \quad 1]_{1\times 4}$, $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}_{[4\times(-1)]}$, $I_1 = 1$,

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} = I_4 + BA = C$$

$m = 1, n = 4$

$\text{Det}(I_1 + AB) = \text{Det}(I_4 + BA)$

$$\Rightarrow \quad \text{Det}[5] = \text{Det of} \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

So, $\quad \text{Det } c = 5$

7. Option (a) is correct.

The PDF of given random variable x is.

$f(x) = e^{-x}; \qquad 0 < x < \infty$

$$\therefore \quad P\ (x > 1) = \int_1^\infty e^{-x} dx$$

$$= \left[\frac{e^{-x}}{-1}\right]_1^\infty = e^{-1} = 0.368$$

8. Option (d) is correct.

Curl of a gradient is zero.

9. Option (b) is correct.

As $-i = e^{-i\pi/2}$ or $e^{i3\pi/2}$

$\because$ $$-i = \cos\left(\frac{-\pi}{2}\right) + i\sin\left(\frac{-\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)$$

So, $$(-i)^{\frac{1}{2}} = \left\{\cos\left(\frac{-\pi}{2}\right) + i\sin\left(\frac{-\pi}{2}\right)\right\}^{\frac{1}{2}} \text{ or } \left\{\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right\}^{\frac{1}{2}}$$

$$= \left\{\cos\left(\frac{-\pi}{4}\right) + i\sin\left(\frac{-\pi}{4}\right)\right\} \text{ or } \left\{\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right\}$$

[by De Moivre's Theorem]

10. Option (b) is correct.

$\vec{F} = y^2 x\hat{a}_x - yz\hat{a}_y - x^2\hat{a}_z$

The line segment along x – axis

$x = 0$ to $x = 1$

$\vec{dl} = dx\hat{a}_x$, $dy = 0$, $dz = 0$

$\vec{F}.\vec{dl} = y^2 x dx$

$\because$ on x – axis $y = 0, z = 0$

$\int \vec{F}.\vec{dl} = 0$

11. Option (d) is correct.

Given, $\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\because$ $A = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$

$|A| = 0$, rank $A < n$, $n = 2$

For homogeneous set of linear equations.

So, there will be multiple solution.

12. Option (d) is correct.

Eigenvalue problem

$AX = \lambda X$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

for $\lambda = -1$ as eigen vector is $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = -1\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$a - b = -1$ ...(i)

$c - d = 1$ ...(ii)

for $\lambda = -2$, eigen vector is $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = -2\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$a - 2b = -2$ ...(iii)

$c - 2d = 4$ ...(iv)

From equation (i) and (iii)

$a = 0, b = 1$

From equation (ii) and (iv)

$c = -2, d = -3$

$\therefore$ Required matrix

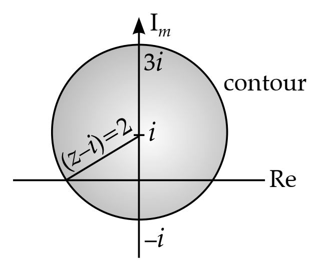
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

13. Option (a) is correct.

Given, $\oint \frac{z^2 - 4}{z^2 + 4} dz$

poles at $z = \pm 2i$

$C : |z - i| = 2$



Encloses $z = 2i$ whereas $z = -2i$ is outside of the contour.

Residue $f(2i) = \left.\frac{z^2 - 4}{z + 2i}\right|_{z=2i} = \frac{(2i)^2 - 4}{2i + 2i} = 2i$

[By Residue Theorem]

$$\oint \frac{z^2 - 4}{z^2 + 4} dz = 2\pi i(\text{sum of residues})$$

$$= 2\pi i(2i) = -4\pi$$

14. Option (b) is correct.

Given, $y = f(x) = 5x^2 + 10x$ in the interval

$x = (1, 2)$

$\because$ $f(x)$ is continuous and differentiable in the interval (1, 2) as it is a polynomial so, according to langrage's mean value theorem there exists at least a point where :

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

for $a = 1, b = 2$

$$y = f(a) = f(1) = 5(1)^2 + 10(1) = 15$$

$$y = f(b) = f(2) = 5(2)^2 + 10(2) = 40$$

$$f'(c) = \frac{40 - 15}{2 - 1} = 25$$

25 is the value of $\frac{dy}{dx}$ at some point $x = c$ in the interval $x = (1, 2)$.

15. Option (c) is correct.

$$f(x) = x^3 + 2x + 1$$

$$f'(x) = 3x^2 + 2$$

$$x_0 = 1.2$$

$$f(x_0) = (1.2)^3 + 2(1.2) - 1 = 3.128$$

$$f'(x_0) = 3(1.2)^2 + 2 = 6.32$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1.2 - \frac{3.128}{6.32} = 0.705$$

16. Option (d) is correct.

$$\frac{\partial^2 u}{\partial x^2} - u\frac{\partial u}{\partial x} - \frac{\partial u}{\partial t} = 0$$

The differential equation has second order derivative as highest order.

So, it is a 2nd order partial differential equation.

As there is product of u with $\frac{\partial u}{\partial x}$ it is non linear.

17. Option (c) is correct.

Eigen value are real for real symmetric matrices.

18. Option (d) is correct.

Numerical Integration Scheme	Order of Fitting Polynomial
P. Simpson's (3/8)th Rule	3. Third order
Q. Trapezoidal Rule	1. First order
R. Simpson's (1/3)rd Rule	2. Second order

19. Option (b) is correct.

The normal random variable $X = N(\mu, \sigma)$

$$\mu = 1, \sigma = 2$$

$$P(X < 0) = P\left(\frac{x - \mu}{\sigma} < \frac{0 - 1}{2}\right)$$

$$= P(z < -0.5)$$

z = standard normal variable.

$\therefore$ $P(z = 0) = 0.5$

$P(z < 0) < 0.5$

$\therefore$ Probability $P\{X < 0\}$ is > 0 and < 0.5.

20. Option (c) is correct.

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\Rightarrow C_1 \cos 2x + C_2 \cos^2 x + C_3 \sin^2 x = 0$$

with C_1, C_2, C_3 constants

$\cos 2x$, $\cos^2 x$ and $\sin^2 x$ are linearly dependent.

21. Option (a) is correct.

$$F = x\hat{i} + y\hat{j} + z\hat{k}$$

$$S : x^2 + y^2 + z^2 = 1$$

The Gauss divergence theorem–

$$\oiint_S (F.n)\,dA = \iiint_V (\nabla.F)\,dV$$

$$\nabla.F = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right).\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$

$$= 1 + 1 + 1 = 3$$

$$\iiint_V (\nabla.F)\,dV = \iiint_V 3\,dV$$

$$= 3 \times \text{Vol. of Sphere}$$

$$= 3 \times \frac{4}{3}\pi r^3, \text{ with } r = 1 \text{ for sphere}$$

$$\therefore \iint_S \frac{1}{4}(F.n)\,dA = \frac{1}{4} \times 4\pi = \pi$$

22. Option (c) is correct.

For $\frac{d^2 f}{dt^2} + f = 0$

Applying Laplace Transform

$$\left\{s^2 F(s) - s\,f(0) - f'(0)\right\} + F(s) = 0$$

$$\Rightarrow \quad F(s)(s^2 + 1) - 0.5 - 4 = 0$$

$$\Rightarrow \quad F(s) = \frac{4}{s^2 + 1}$$

23. Option (d) is correct.

Let event A be student knows the answer and event C be student guesses the answer.

$$P(A) = P(A \cap B) = \frac{2}{3}$$

$$P(C) = \frac{2}{3}$$

$$P(\text{student does not know answer}) = \frac{1}{3}$$

$$P(\text{correct guess}) = \frac{1}{4}$$

among 4 options

Total probability of correct answer

$$P(B) = \frac{2}{3} + \frac{1}{3} \times \frac{1}{4} = \frac{9}{12}$$

Conditional Probability that student know the correct answer.

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{2}{3} \div \frac{9}{12} = \frac{8}{9}$$

24. Option (b) is correct.

Given DE is $\left(D^2 - kD\right)u = 0$

$$AE \rightarrow m^2 - km = 0$$

$$\Rightarrow \quad m(m-k) = 0 \Rightarrow m = 0, k$$

solution $u = C_1 e^{0x} + C_2 e^{kx}$

$$\Rightarrow \quad u = C_1 + C_2 e^{kx} \quad \text{...(i)}$$

Given, $u(0) = 0 \Rightarrow 0 = C_1 + C_2$

$$\Rightarrow C_1 + C_2 = 0 \quad \text{...(ii)}$$

also $u(L) = U \Rightarrow U = C_1 + C_2 e^{kL}$ ...(iii)

Subtracting equation (ii) and (iii)

$$U = C_2\left(e^{kL} - 1\right)$$

$$C_2 = \frac{U}{\left(e^{kL} - 1\right)}$$

From equation (ii)

$$C_1 = -C_2 = \frac{-U}{\left(e^{kL} - 1\right)}$$

Putting the value of C_1 and C_2 in (i)

$$u = \frac{-U}{\left(e^{kL} - 1\right)} + \frac{U}{\left(e^{kL} - 1\right)} e^{kx}$$

$$u = U\left(\frac{1 - e^{kx}}{1 - e^{kL}}\right)$$

25. Option (c) is correct.

Applying ILATE Rule

$$I = \int_1^e \underbrace{\sqrt{x}}_{II} \underbrace{\ln(x)}_{I} dx$$

$$I = \ln(x)\int_1^e \sqrt{x}dx - \int_1^e \frac{1}{x} \times \frac{2}{3}x^{\frac{3}{2}} dx$$

$$I = \left[\ln(x)\frac{2}{3}x^{\frac{3}{2}}\right]_1^e - \left[\frac{2}{3} \times \frac{2}{3}x^{\frac{3}{2}}\right]_1^e$$

$$= \left[\frac{2}{3}e^{\frac{3}{2}} - 0\right] - \frac{4}{9}\left[e^{\frac{3}{2}} - 1\right]$$

$$= \frac{2}{3}e^{\frac{3}{2}} - \frac{4}{9}e^{\frac{3}{2}} + \frac{4}{9}$$

$$= \frac{2}{9}e^{\frac{3}{2}} + \frac{4}{9}$$

$$= \frac{2}{9}\sqrt{e^3} + \frac{4}{9}$$

26. Correct answer is [0.5].

$$2x = 3$$

$$4x = 1$$

$$\Rightarrow \quad \begin{bmatrix} 2 \\ 4 \end{bmatrix}[x] = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

For the least square approximation

$$A^T A\hat{X} = A^T b$$

$$\Rightarrow \begin{bmatrix} 2 \\ 4 \end{bmatrix}^T \begin{bmatrix} 2 \\ 4 \end{bmatrix}[x] = \begin{bmatrix} 2 \\ 4 \end{bmatrix}^T \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$[20]\,[x] = [10]$$

$$\Rightarrow \quad 20x = 10$$

$$\Rightarrow \quad x = \frac{10}{20} = \frac{1}{2} = 0.5$$

27. Correct answer is [16].

By multiplying Matrix Q and R first

$\rightarrow Q_{2\times4} \times R_{4\times1}$ having multiplication no.

$$2 \times 4 \times 1 = 8$$

$\rightarrow P_{4\times2}\,(QR)_{2\times1}$ having no. of multiplication

$$= 4 \times 2 \times 1 = 8$$

$P_{4\times2}\,(QR)_{2\times1}$ will have minimum no. of multiplication = (8 + 8) = 16

28. Option (d) is correct.

Given Return period of rainfall (T) = 50 years.

∴ Probability of occurrence of rainfall one time in a year $= \frac{1}{50} = 0.02 = p$

Now, Probability of occurrence of rainfall in each of 2 successive year $= p^2 = (0.02)^2$

$$= 0.0004$$

29. Correct answer is [0.53].

Given $f(x) = x^4 + 10$

Approximate value

x	0	1	2	3	4
$f(x)$	10 y_0	11 y_1	26 y_2	91 y_3	266 y_4

$$I = \frac{h}{3}\{y_0 + 4(y_1 + y_3) + 2y_2 + y_4\}$$

$$\int_0^4 \left(x^4 + 10\right)dx = \frac{1}{3}$$

$$\left[(10+266) + 2(26) + 4(11+91)\right] = 245.33$$

Exact value

$$I = \int_0^4 \left(x^4 + 10\right)dx = \left(\frac{x^5}{5} + 10x\right)_0^4$$

$$= \frac{4^5}{5} + 10 \times 4 = 244.8$$

$\therefore$ Error = 245.33 – 244.8 = 0.53

30. Option (b) is correct.

$$I = \int_0^{\frac{\pi}{6}} \cos^4 3\theta \sin^3 6\theta d\theta$$

$\because$ $\sin 2\theta = 2\sin\theta\cos\theta$

$$I = \int_0^{\frac{\pi}{6}} \cos^4 3\theta \left(2\sin 3\theta \cos 3\theta\right)^3 d\theta$$

$$= 8\int_0^{\frac{\pi}{6}} \cos^4 3\theta \sin^3 3\theta \cos^3 3\theta d\theta$$

$$= 8\int_0^{\frac{\pi}{6}} \cos^7 3\theta \sin^3 3\theta d\theta$$

Putting $3\theta = t$

$$d\theta = \frac{dt}{3},\ 3.\frac{\pi}{6} = \frac{\pi}{2}$$

$$= \frac{8}{3}\int_0^{\frac{\pi}{2}} \cos^7 t \sin^3 t dt$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^m \theta \sin^n \theta d\theta = \frac{(m-1)(m-3)(m-5)\ldots\ldots(n-1)(n-3)(n-5)\ldots\ldots}{(m+n)(m+n-2)(m+n-4)\ldots\ldots}$$

$$\left[\frac{\pi}{2};\ \text{if } m \text{ and } n \text{ are even}\right]$$

with $m = 7, n = 3$

$$\therefore \quad I = \frac{8}{3} \times \frac{6.4.2.2}{10.8.6.4.2} = \frac{1}{15}$$

31. Correct answer is [6].

Given probability density function

$$f(x) = \begin{cases} \lambda\left(-x^2 + 3x - 2\right) & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

And we know $\int_{-\infty}^{\infty} f(x)dx = 1$

$$\Rightarrow \int_1^2 \lambda\left(-x^2 + 3x - 2\right)dx = 1$$

$$\Rightarrow \lambda\left[-\frac{x^3}{3} + 3\frac{x^2}{2} - 2x\right]_1^2 = 1$$

$$\lambda\left[-\left(\frac{8}{3} - \frac{1}{3}\right) + \frac{3}{2}(4-1) - 2(2-1)\right] = 1$$

$$\lambda\left[-\frac{7}{3} + \frac{9}{2} - 2\right] = 1$$

$$\lambda = 6$$

32. Correct answer is [3.8].

$$\frac{\partial^2 H}{\partial x^2} = 0$$

No variation of y and z direction

$$\frac{d^2 H}{dy^2} = 0,\ \frac{d^2 H}{dz^2} = 0$$

Integrating both side

$$\frac{\partial H}{\partial x} = C_1$$

$$H = C_1 x + C_2 \qquad \ldots(i)$$

At $x = 0, H = 5 \Rightarrow C_2 = 5$

At $x = 0, \frac{\partial H}{\partial x} = -1 \Rightarrow C_1 = -1$

$$H = -x + 5$$

At $x = 1.2$ m, $H = -1.2 + 5 = 3.8$ m

33. Option (a) is correct.

$$y \oplus x = y^2 + x^2 = x^2 + y^2 = x \oplus y$$

For associativity

$$(x \oplus y) \oplus z = \left(x^2 + y^2\right) \oplus z = \left(x^2 + y^2\right)^2 + z^2$$

$$x \oplus (y \oplus z) = x \oplus \left(y^2 \oplus z^2\right) = x^2 \oplus \left(y^2 \oplus z^2\right)^2$$

$$\because \left((x \oplus y) \oplus z\right) \ne \left(x \oplus (y \oplus z)\right)$$

$\therefore$ not associative

Hence it is commutative but not associative.

34. Option (c) is correct.

Poisson probability density function $= \dfrac{\lambda^k}{e^{\lambda} k!}$,

with $\lambda = 3$. For x be no. of car observed

$p(x < 3) = p(x = 0) + p(x = 1) + p(x = 2)$

$$p(x<3) = \frac{3^0}{c^3 \cdot 0!} + \frac{3^1}{c^3 \cdot 1!} + \frac{3^2}{c^3 \cdot 2!}$$

$$p(x<3) = \left(\frac{1}{e^3} + \frac{3}{e^3} + \frac{9}{2e^3}\right) = \frac{17}{2e^3}$$

35. Option (a) is correct.

For option A

$$\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix} = \begin{vmatrix} 1 & x^2+x & x+1 \\ 1 & y^2+y & y+1 \\ 1 & z^2+z & z+1 \end{vmatrix}$$

Here C_2 is $C_2 + C_3$ from original matrix

C_3 is $C_2 + C_1$ from original matrix

obtained after elementary transformation and exchange of C_2 and C_3

$\therefore \det = -|A|$, (negative of original determinant)

36. Option (a) is correct.

For option A

LHL : $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3}\left(\dfrac{x+3}{3}\right) = \dfrac{3+3}{3} = 2$

RHL : $\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3}(x-1) = 3-1 = 2$

also $f(x = 3) = 2$

with LHL = RHL $f(x)$ at $x = 3$

the function is continuous.

Option B : $\lim\limits_{x\to 3} f(x) = 8-3 = 5 \neq 4 \neq f(x=3)$

Option C :

$\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3}(x+3) = 6$, limit does not exist

$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3}(x-4) = -1$

Option D : $f(x)$ is not defined at $x = 3$

37. Option (d) is correct.

According to trapezoidal rule

$$\int_a^b f(x)dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \ldots y_{n-1})\right]$$

$h = \dfrac{b-a}{n} = \dfrac{3-0}{10} = 10$, n = number of interval

$$\int_0^3 f(x)dx = \frac{0.3}{2}\left[(0+9) + 2(0.09 + 0.36 + \ldots + 7.29)\right]$$

$$= \frac{0.3}{2}\left[9 + 2(25.65)\right]$$

$$= \frac{0.3}{2}\left[9 + 51.3\right]$$

$$= 9.045$$

38. Option (c) is correct.

A cycle of length 3 requires 3 vertices.

No. of ways in which we can choose 3 vertices out of 8 = ${}^8C_3 = 56$, $P(\text{edge}) = \dfrac{1}{2}$

Probability that 3 vertices form a cycle

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

Expected no. of unordered cycles of length three = $56 \times \dfrac{1}{8} = 7$

39. Option (d) is correct.

F(x): x is my friend.

P(x): x is perfect.

"None of my friends are perfect" can be written as –

$\forall x[F(x) \Rightarrow \neg P(x)]$

$\equiv \forall x[\neg F(x) \vee \neg P(x)]$ $[\because (p - q) \equiv \neg p \vee q]$

[De Morgan's Law]

$\equiv \forall x \neg [F(x) \wedge P(x)]$

$\equiv \neg \exists x [F(x) \wedge P(x)]$

40. Option (a) and (d) is correct.

Important rule $\forall x(\alpha) = \neg\exists(x)(\neg\alpha)$

$$p \to \neg q \equiv \neg p \vee \neg q \equiv \neg (p \wedge q)$$

From option (a)

$\forall x(\exists z(\neg\beta) \to \forall y(\alpha))$

$\Rightarrow \forall x(\neg\exists z(\neg\beta) \vee \forall y(\alpha))$

$\Rightarrow \forall x(\forall z(\beta) \vee \forall y(\alpha))$

$\Rightarrow \neg\exists x \neg(\forall z(\beta) \vee \forall y(\alpha))$

$\Rightarrow \neg\exists x(\neg\forall z(\beta) \wedge \neg\forall y(\alpha))$

$\not\equiv \neg\exists x(\forall y(\alpha) \wedge \forall z(\beta))$

Option (b)

$\forall x(\forall z(\beta) \to \exists y(\neg\alpha))$

$\Rightarrow \forall x(\neg\forall z(\beta) \vee \exists y(\neg\alpha))$

$\Rightarrow \neg\exists x \neg(\neg\forall z(\beta) \vee \exists y(\neg\alpha))$

$\Rightarrow \neg\exists x(\forall z(\beta) \wedge \neg\exists y(\neg\alpha))$ [De Morgan's law]

$\Rightarrow \neg\exists x(\forall z(\beta) \wedge \forall y(\alpha))$

Option (c)

$\forall x(\forall y(\alpha) \to \exists z(\neg\beta))$

$\Rightarrow \forall x(\neg\forall y(\alpha) \vee \exists z(\neg\beta))$

$\Rightarrow \neg\exists x\neg(\neg\forall y(\alpha)\vee\exists z(\neg\beta))$

$\Rightarrow \neg\exists x(\forall y(\alpha)\wedge\neg\exists z(\neg\beta))$ [De Morgan's law]

$\Rightarrow \neg\exists x(\forall y(\alpha)\wedge\forall z(\beta))$

Option (d)

$\forall x(\exists y(\neg\alpha)\rightarrow\exists z(\beta))$

$\Rightarrow \forall x(\neg\exists y(\neg\alpha)\vee\exists z(\beta))$

$\Rightarrow \forall x(\forall y(\alpha)\vee\exists z(\beta))$

$\Rightarrow \neg\exists x\neg(\forall y(\alpha)\vee\exists z(\beta))$

$\Rightarrow \neg\exists x(\neg\forall y(\alpha)\wedge\neg\exists z(\beta))$ [De Morgan's law]

$\Rightarrow \neg\exists x(\neg\forall y(\alpha)\wedge\forall z(\neg\beta))$

$\not\equiv \neg\exists x(\neg\forall y(\alpha)\wedge\forall z(\beta))$

Therefore, option (a) and (d) are not logically equivalent to the given statement.

41. Option (b) is correct.

The numbers in ascending order: 4, 9, 11, 15, 18, 18.

For median we choose middle number

$$\text{Median} = \frac{11+15}{2} = 13\left\{\frac{3^{\text{rd}}\text{term}+4^{\text{th}}\text{term}}{2}\right\}$$

Most repeated number is the mode *i.e.* 18

42. Correct answer is [0.083].

Total outcomes 36, $n(s) = 36$

A: Roll 1 : = 4 (6 Possibilities)

B: Roll 2 : < 4 (18 Possibilities)

Possible cases: (4, 1),(4, 2),(4, 3).

$n(A\cap B) : 3$

$$P(A\cap B) = \frac{3}{36} = 0.083$$

43. Option (b) is correct.

P and R are correct.

The determinant of an orthogonal matrix can be ±1 both.

[De Morgan's Law]

Inverse of matrix doesn't exist if the matrix is singular or the rank is less than n.

44. Option (d) is correct.

$$\int\frac{1}{e^x-1}dx = \int\frac{e^{-x}}{1-e^{-x}}dx$$

Let $t = 1-e^{-x}$

$dt = e^{-x}dx$

$$\int\frac{e^{-x}}{1-e^{-x}}dx = \int\frac{dt}{t} = \ln t + c$$

$$= \ln\left(1-e^{-x}\right)+c$$

45. Correct answer is [–0.25].

$$\operatorname*{Res}_{z=z_0} f(z) = \lim_{z\to z_0}(z-z_0)f(z) \quad \text{[Residue Theorem]}$$

$$\operatorname*{Res}_{z=2} f(z) = (z-2)\frac{1}{(2-z)(z+2)} = \frac{-1}{(2+2)}$$

$$= -0.25$$

46. Option (b) is correct.

$$\frac{dy}{dx} = y^2$$

$$\frac{dy}{y^2} = dx$$

$$\int\frac{dy}{y^2} = \int dx$$

$$\Rightarrow \quad \frac{-1}{y} = x+c$$

$\because y = 1$ at $x = 0 \Rightarrow c = -1$

$$\frac{-1}{y} = x-1$$

$$\Rightarrow \quad y = \frac{1}{1-x}$$

47. Option (c) is correct.

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}+0.25y = 0$$

Applying Laplace Transform

$\{Y(s)\,s^2 - sy(0) - y'(0)\} - \{Y(s)\,s - y(0)\} + 0.25\,Y(s) = 0$

$\Rightarrow Y(s)\,\{s^2 - s + 0.25\} = 1$ [$\because y(0) = 0\; y'(0) = 1$]

$$\Rightarrow Y(s) = \frac{1}{\left(s-\frac{1}{2}\right)^2}$$

Laplace Inverse $f(x) = x\cdot e^{x/2}$

48. Correct answer is [0.38].

Given, $f(x) = e^{-x3}$

x	0.1	0.2	0.3	0.4	0.5
y	$e^{-(0.1)^3}$ y_0	$e^{-(0.2)^3}$ y_1	$e^{-(0.3)^3}$ y_2	$e^{-(0.4)^3}$ y_3	$e^{-(0.5)^3}$ y_4

$$h = \frac{b-a}{n} = \frac{0.5-0.1}{4} = \frac{0.4}{4} = 0.1$$

$$I = \int_a^b f(x)dx = \frac{0.1}{3}\{y_0+4y_1+2y_2+4y_3+y_4\}$$

$$= \frac{0.1(11.548)}{3}$$

$$\int_{0.1}^{0.5} e^{-x^3}dx = 0.385$$

■■■■■