ENGINEERING MATHEMATICS — GATE — Question Paper 2014

ELECTRONICS AND COMMUNICATION (EC) P1

Q. 1. For matrices of same dimension M, N and scalar c, which one of these properties DOES NOT ALWAYS hold?

(a) $(M^T)^T = M$
(b) $(cM)^T = c(M)^T$
(c) $(M + N)^T = M^T + N^T$
(d) MN=NM

Q. 2. In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random, has a sibling is ______.

Q. 3. C is a closed path in the z–plane given by $|z| = 3$. The value of the integral

$$\oint_c \left(\frac{z^2 - z + 4j}{z + 2j}\right) dz \text{ is}$$

(a) − 4□(1 + j2) **(b)** 4□(3 − j2)
(c) − 4□(3 + j2) **(d)** 4□(1 − j2)

Q. 4. A real (4 × 4) matrix A satisfies the equation $A^2 = I$, where I is the (4 × 4) identity matrix. The positive eigen value of A is ______.

Q. 5. Let X_1, X_2, and X_3 be independent and identically distributed random variables with the uniform distribution on [0, 1]. The probability P{X_1 is the largest} is ______.

Q. 6. The Taylor series expansion of $3 \sin x + 2 \cos x$ is

(a) $2 + 3x - x^2 - \frac{x^3}{2} + \ldots$
(b) $2 - 3x + x^2 - \frac{x^3}{2} + \ldots$
(c) $2 + 3x + x^2 + \frac{x^3}{2} + \ldots$
(d) $2 - 3x - x^2 + \frac{x^3}{2} + \ldots$

Q. 7. For a function $g(t)$, it is given that

$$\int_{-\infty}^{+\infty} g(t)e^{-j\omega t}dt = \omega e^{-2\omega^2}$$

for any real value ω.

If $y(t) = \int_{-\infty}^{t} g(\tau)\,d\tau$, then $\int_{-\infty}^{+\infty} y(t)\,dt$ is

(a) 0 **(b)** $-j$
(c) $-\frac{j}{2}$ **(d)** $\frac{j}{2}$

Q. 8. The volume under the surface $z(x,y) = x + y$ and above the triangle in the x-y plane defined by $\{0 \le y \le x \text{ and } 0 \le x \le 12\}$ is ______.

Q. 9. Consider the matrix

$$J_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

which is obtained by reversing the order of the columns of the identity matrix I_6.
Let $P = I_6 + \alpha J_6$, where α is a non-negative real number. The value of α for which det(P) = 0 is ______.

ELECTRONICS AND COMMUNICATION (EC) P2

Q. 10. The determinant of matrix A is 5 and the determinant of matrix B is 40. The determinant of matrix AB is ______.

Q. 11. Let X be a random variable which is uniformly chosen from the set of positive odd numbers less than 100. The expectation, E [X], is ______.

Q. 12. For $0 \le t < \infty$, the maximum value of the function $f(t) = e^{-t} - 2e^{-2t}$ occurs at

(a) $t = \log_e 4$
(b) $t = \log_e 2$
(c) $t = 0$
(d) $t = \log_e 8$

Q. 13. The value of $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$ is

(a) ln 2 **(b)** 1.0
(c) e **(d)** ∞

Q. 14. If the characteristic equation of the differential equation

$$\frac{d^2y}{dx^2}+2\alpha\frac{dy}{dx}+y=0$$

has two equal roots, then the values of α are

(a) ±1 **(b)** 0,0

(c) $\pm j$ **(d)** ±1/2

Q. 15. The system of linear equations

$$\begin{pmatrix}2 & 1 & 3\\3 & 0 & 1\\1 & 2 & 5\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}5\\-4\\14\end{pmatrix}$$ has

(a) a unique solution

(b) infinitely many solutions

(c) no solution

(d) exactly two solutions

Q. 16. The real part of an analytic function $f(z)$ where $z = x + jy$ is given by $e^{-y}\cos(x)$. The imaginary part of $f(z)$ is

(a) $e^{y}\cos(x)$ **(b)** $e^{-y}\sin(x)$

(c) $-e^{y}\sin(x)$ **(d)** $-e^{-y}\sin(x)$

Q. 17. The maximum value of the determinant among all 2 × 2 real symmetric matrices with trace 14 is ________

Q. 18. If $\vec{r}=x\hat{a}_x+y\hat{a}_y+z\hat{a}_z$ and $|\vec{r}|=r$, then div $(r^2\nabla(\ln r))$ = ______.

ELECTRONICS AND COMMUNICATION (EC) P3

Q. 19. The maximum value of the function $f(x) = \ln(1 + x) - x$ (where $x > -1$) occurs at x = ______.

Q. 20. Which one of the following is a linear non-homogeneous differential equation, where x and y are the independent and dependent variables respectively?

(a) $\frac{dy}{dx}+xy=e^{-x}$ **(b)** $\frac{dy}{dx}+xy=0$

(c) $\frac{dy}{dx}+xy=e^{-y}$ **(d)** $\frac{dy}{dx}+e^{-y}=0$

Q. 21. Match the application to appropriate numerical method.

Application	Numerical Method
P1: Numerical integration	M1: Newton-Raphson Method
P2: Solution to a transcendental equation	M2: Runge-Kutta Method
P3: Solution to a system of linear equations	M3: Simpson's 1/3-rule
P4: Solution to a differential equation	M4: Gauss Elimination Method

(a) P1–M3, P2–M2, P3–M4, P4–M1

(b) P1–M3, P2–M1, P3–M4, P4–M2

(c) P1–M4, P2–M1, P3–M3, P4–M2

(d) P1–M2, P2–M1, P3–M3, P4–M4

Q. 22. An unbiased coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is

(a) 0.067 **(b)** 0.073

(c) 0.082 **(d)** 0.091

Q. 23. If $z = xy$ In (xy), then

(a) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=0$ **(b)** $y\frac{\partial z}{\partial x}=x\frac{\partial z}{\partial y}$

(c) $x\frac{\partial z}{\partial x}=y\frac{\partial z}{\partial y}$ **(d)** $y\frac{\partial z}{\partial x}+x\frac{\partial z}{\partial y}=0$

Q. 24. The maximum value of $f(x) = 2x^3 - 9x^2 + 12x - 3$ in the interval $0 \le x \le 3$ is ______.

Q. 25. Which one of the following statements is NOT true for a square matrix A?

(a) If A is upper triangular, the eigen values of A are the diagonal elements of it

(b) If A is real symmetric, the eigen values of A are always real and positive

(c) If A is real, the eigen values of A and A^T are always the same

(d) If all the principal minors of A are positive, all the eigen values of A are also positive

Q. 26. A fair coin is tossed repeatedly till both head and tail appear at least once. The average number of tosses required is ______.

Q. 27. Given the vector A = (cos$x$) (sin y) $\hat{a}_x$ +(sin x) (cos y) $\hat{a}_y$ where $\hat{a}_x,\hat{a}_y$ denote unit vectors along x, y directions, respectively. The magnitude of curl of A is ______.

ELECTRONICS AND COMMUNICATION (EC) P4

Q. 28. The series $\sum_{n=0}^{\infty}\frac{1}{n!}$ converges to

(a) 2 ln 2 **(b)** $\sqrt{2}$

(c) 2 **(d)** e

Q. 29. The magnitude of the gradient for the function $f(x,y,z) = x^2 + 3y^2 + z^3$ at the point (1,1,1) is ______.

Q. 30. Let X be a zero mean unit variance Gaussian random variable. E[|X|] is equal to ______.

Q. 31. If a and b are constants, the most general solution of the differential equation

$$\frac{d^2x}{dt^2}+2\frac{dx}{dt}+x=0 \text{ is}$$

(a) ae^{-t} **(b)** $ae^{-t}+bte^{-t}$

(c) $ae^{t}+bte^{-t}$ **(d)** ae^{-2t}

Q. 32. The directional derivative of

$f(x,y)=\frac{xy}{\sqrt{2}}(x+y)$ at (1, 1) in the direction of the unit vector at an angle of $\frac{\pi}{4}$ with y-axis, is given by ______ .

Q. 33. With initial values $y(0) = y'(0) = 1$, the solution of the differential equation

$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=0$ at $x = 1$ is ______.

Q. 34. Parcels from sender S to receiver R pass sequentially through two post-offices. Each post-office has a probability $\frac{1}{5}$ of losing an incoming parcel, independently of all other parcels. Given that a parcel is lost, the probability that it was lost by the second post-office is ______.

Q. 35. The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^2+s+1}$. Which one of the following is the unilateral Laplace transform of $g(t) = t \cdot f(t)$?

(a) $\frac{-s}{(s^2+s+1)^2}$ **(b)** $\frac{-(2s+1)}{(s^2+s+1)^2}$

(c) $\frac{s}{(s^2+s+1)^2}$ **(d)** $\frac{2s+1}{(s^2+s+1)^2}$

Q. 36. For a right angled triangle, if the sum of the lengths of the hypotenuse and a side is kept constant, in order to have maximum area of the triangle, the angle between the hypotenuse and the side is

(a) 12° (b) 36°

(c) 30° (d) 45°

ELECTRICAL ENGINEERING (EE) P1

Q. 37. Given a system of equations:

$x + 2y + 2z = b_1$

$5x + y + 3z = b_2$

Which of the following is true regarding its solutions

(a) The system has a unique solution for any given b_1 and b_2

(b) The system will have infinitely many solutions for any given b_1 and b_2

(c) Whether or not a solution exists depends on the given b_1 and b_2

(d) The system would have no solution for any values of b_1 and b_2

Q. 38. Let $f(x) = xe^{-x}$. The maximum value of the function in the interval $(0, \infty)$ is

(a) e^{-1} **(b)** e

(c) $1 - e^{-1}$ **(d)** $1 + e^{-1}$

Q. 39. The solution for the differential equation

$\frac{d^2x}{dt^2}=-9x$, with initial conditions $x(0) = 1$

and $\frac{dx}{dt}\Big|_{t=0}=1, is$

(a) $t^2 + t + 1$

(b) $\sin 3t+\frac{1}{3}\cos 3t+\frac{2}{3}$

(c) $\frac{1}{3}\sin 3t+\cos 3t$

(d) $\cos 3t + t$

Q. 40. Let $X(s) = \frac{3s+5}{s^2+10s+21}$ be the Laplace Transform of a signal $x(t)$. Then $x(0^+)$ is

(a) 0 **(b)** 3

(c) 5 **(d)** 21

Q. 41. Let S be the set of points in the complex plane corresponding to the unit circle. (That is,S $\{z:|z| = 1\}$). Consider the function $f(z) = z\,z^*$ where z^* denotes the complex conjugate of z. The $f(z)$ maps S to which one of the following in the complex plane

(a) unit circle

(b) horizontal axis line segment from origin to (1, 0)

(c) the point (1, 0)

(d) the entire horizontal axis

Q. 42. Let $g: [0,\infty) \to [0,\infty)$ be a function defined by $g(x) = x-[x]$,where $[x]$ represents the integer part of x. (That is, it is the largest integer which is less than or equal to x).

The value of the constant term in the Fourier series expansion of $g(x)$ is ______

Q. 43. A fair coin is tossed n times. The probability that the difference between the number of heads and tails is $(n - 3)$ is

(a) 2^{-n} **(b)** 0

(c) $^{n}c_{n-3}2^{-n}$ **(d)** 2^{-n+3}

Q. 44. The line integral of function $\mathbf{F} = yzi$, in the counterclockwise direction, along the circle $x^2 + y^2 = 1$ at $z = 1$ is

(a) -2π **(b)** $-\pi$

(c) π **(d)** 2π

ELECTRICAL ENGINEERING (EE) P2

Q. 45. Which one of the following statements is true for all real symmetric matrices?

(a) All the eigen values are real.

(b) All the eigen values are positive.

(c) All the eigen values are distinct.

(d) Sum of all the eigen values is zero.

Q. 46. Consider a dice with the property that the probability of a face with n dots showing up is proportional to n. The probability of the face with three dots showing up is ______.

Q. 47. Minimum of the real valued function $f(x) = (x-1)^{\frac{2}{3}}$ occurs at x equal to

(a) $-\infty$ **(b)** 0

(c) 1 **(d)** ∞

Q. 48. All the values of the multi-valued complex function 1^i, where $i = \sqrt{-1}$, are

(a) purely imaginary.

(b) real and non-negative.

(c) on the unit circle.

(d) equal in real and imaginary parts.

Q. 49. Consider the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$. Which of the following is a solution to this differential equation for $x > 0$?

(a) e^x **(b)** x^2

(c) $\frac{1}{x}$ **(d)** $\ln x$

Q. 50. To evaluate the double integral $\int_0^8\left(\int_{(y/2)}^{(y/2)+1}\left(\frac{2x-y}{2}\right)dx\right)dy$, we make the substitution $u = \left(\frac{2x-y}{2}\right)$ and $v = \frac{y}{2}$. The integral will reduce to

(a) $\int_0^4(\int_0^2 2u\,du)\,dv$ **(b)** $\int_0^4(\int_0^1 2u\,du)\,dv$

(c) $\int_0^4(\int_0^1 u\,du)\,dv$ **(d)** $\int_0^4(\int_0^2 u\,du)\,dv$

Q. 51. Let X be a random variable with probability density function

$$f(x) = \begin{cases} 0.2, & \text{for } |x| \le 1 \\ 0.1, & \text{for } 1 < |x| \le 4 \\ 0, & \text{otherwise} \end{cases}$$

The probability $P(0.5 < x < 5)$ is ______.

Q. 52. The minimum value of the function $f(x) = x^3 - 3x^2 - 24x + 100$ in the interval $[-3, 3]$ is

(a) 20 **(b)** 28

(c) 16 **(d)** 32

ELECTRICAL ENGINEERING (EE) P3

Q. 53. Two matrices A and B are given below:

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}, \quad B = \begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix}$$

If the rank of matrix A is N, then the rank of matrix B is

(a) $\frac{N}{2}$ **(b)** N–1

(c) N **(d)** 2 N

Q. 54. A particle, starting from origin at $t = 0\ s$, is traveling along x-axis with velocity

$$v = \frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right) \text{ m/s}$$

At $t = 3\ s$, the difference between the distance covered by the particle and the magnitude of displacement from the origin is ______.

Q. 55. Let $\nabla.(f\,v) = x^2y + y^2z + z^2x$, where f and v are scalar and vector fields respectively. If $v = yi + zj + xk$, then $v.\nabla f$ is

(a) $x^2y + y^2z + z^2x$ **(b)** $2xy + 2yz + 2zx$

(c) $x + y + z$ **(d)** 0

Q. 56. Lifetime of an electric bulb is a random variable with density $f(x) = kx^2$, where x is measured in years. If the minimum and maximum lifetimes of bulb are 1 and 2 years respectively, then the value of k is ______.

Q. 57. Integration of the complex function

$$f(z) = \frac{z^2}{z^2-1},$$

in the counterclockwise direction, around $|z-1| = 1$

(a) $-\pi i$ **(b)** 0

(c) πi **(d)** $2\pi i$

Q. 58. The mean thickness and variance of silicon steel laminations are 0.2 mm and 0.02 respectively. The varnish insulation is applied on both the sides of the laminations. The mean thickness of one side insulation and its variance are 0.1 mm and 0.01 respectively. If the transformer core is made using 100 such varnish coated laminations, the mean thickness and variance of the core respectively are

(a) 30 mm and 0.22 **(b)** 30 mm and 2.44
(c) 40 mm and 2.44 **(d)** 40 mm and 0.24

Q. 59. The function $f(x) = e^x - 1$ is to be solved using Newton-Raphson method. If the initial value of x_0 is taken as 1.0, then the absolute error observed at 2nd iteration is ______.

MECHANICAL ENGINEERING (ME) P1

Q. 60. Given that the determinant of the matrix $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$ is –12, the determinant of the matrix $\begin{bmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{bmatrix}$ is

(a) –96 **(b)** –24
(c) 24 **(d)** 96

Q. 61. $\underset{x \to 0}{\text{Lt}} \dfrac{x - \sin x}{1 - \cos x}$ is

(a) 0 **(b)** 1
(c) 3 **(d)** not defined

Q. 62. The argument of the complex number $\dfrac{1+i}{1-i}$, where $i = \sqrt{-1}$, is

(a) $-\pi$ **(b)** $-\dfrac{\pi}{2}$
(c) $\dfrac{\pi}{2}$ **(d)** π

Q. 63. The matrix form of the linear system $\dfrac{dx}{dt} = 3x - 5y$ and $\dfrac{dy}{dt} = 4x + 8y$ is

(a) $\dfrac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 3 & -5 \\ 4 & 8 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(b) $\dfrac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 3 & 8 \\ 4 & -5 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(c) $\dfrac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 4 & -5 \\ 3 & 8 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(d) $\dfrac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 4 & 8 \\ 3 & -5 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

Q. 64. Which one of the following describes the relationship among the three vectors, $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 3\hat{j} + \hat{k}$, and $5\hat{i} + 6\hat{j} + 4\hat{k}$,?

(a) The vectors are mutually perpendicular
(b) The vectors are linearly dependent
(c) The vectors are linearly independent
(d) The vectors are unit vectors

Q. 65. The integral $\oint_c (ydx - xdy)$ is evaluated along the circle $x^2 + y^2 = \dfrac{1}{4}$ traversed in counter clockwise direction. The integral is equal to

(a) 0 **(b)** $-\dfrac{\pi}{4}$
(c) $-\dfrac{\pi}{2}$ **(d)** $\dfrac{\pi}{4}$

Q. 66. If $y = f(x)$ is the solution of $\dfrac{d^2y}{dx^2} = 0$ with the boundary conditions $y = 5$ at $x = 0$, and $\dfrac{dy}{dx} = 2$ at $x = 10$, $f(15)$ = ______.

Q. 67. In the following table , x is a discrete random variable and $p(x)$ is the probability density. The standard deviation of x is

x	1	2	3
$p(x)$	0.3	0.6	0.1

(a) 0.18 **(b)** 0.36
(c) 0.54 **(d)** 0.6

Q. 68. Using the trapezoidal rule, and dividing the interval of integration into equal subintervals, the definite integral $\int_{-1}^{+1} |x|\, dx$ is ______.

MECHANICAL ENGINEERING (ME) P2

Q. 69. One of the eigenvectors of the matrix $\begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$ is

(a) $\begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$ **(b)** $\begin{Bmatrix} -2 \\ 9 \end{Bmatrix}$
(c) $\begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$ **(d)** $\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

Q. 70. $\underset{x\to 0}{\text{Lt}}\left(\dfrac{e^{2x}-1}{\sin(4x)}\right)$ is equal to

(a) 0 **(b)** 0.5
(c) 1 **(d)** 2

Q. 71. Curl of vector $\vec{F} = x^2z^2\hat{i} - 2xy^2z\hat{j} + 2y^2z^3\hat{k}$ is

(a) $(4yz^3 + 2xy^2)\hat{i} + 2x^2z\hat{j} - 2y^2z\hat{k}$
(b) $(4yz^3 + 2xy^2)\hat{i} - 2x^2z\hat{j} - 2y^2z\hat{k}$
(c) $2xz^2\hat{i} - 4xyz\hat{j} + 6y^2z^2\hat{k}$
(d) $2xz^2\hat{i} + 4xyz\hat{j} + 6y^2z^2\hat{k}$

Q. 72. A box contains 25 parts of which 10 are defective. Two parts are being drawn simultaneously in a random manner from the box. The probability of both the parts being good is

(a) $\dfrac{7}{20}$ **(b)** $\dfrac{42}{125}$
(c) $\dfrac{25}{29}$ **(d)** $\dfrac{5}{9}$

Q. 73. The best approximation of the minimum value attained by $e^{-x}\sin(100x)$ for $x \geq 0$ is ______.

Q. 74. An analytic function of a complex variable $z = x + iy$ is expressed as $f(z) = u(x, y) + iv(x,y)$,
Where $i = \sqrt{-1}$. If $u(x,y) = 2xy$, then $v(x,y)$ must be.

(a) $x^2 + y^2$ + constant
(b) $x^2 - y^2$ + constant
(c) $-x^2 + y^2$ + constant
(d) $-x^2 - y^2$ + constant

Q.75. The general solution of the differential equation $\dfrac{dy}{dx} = \cos(x+y)$, with c as a constant, is

(a) $y + \sin(x+y) = x + c$
(b) $\tan\left(\dfrac{x+y}{2}\right) = y + c$
(c) $\cos\left(\dfrac{x+y}{2}\right) = x + c$
(d) $\tan\left(\dfrac{x+y}{2}\right) = x + c$

Q. 76. Consider an unbiased cubic dice with opposite faces coloured identically and each face coloured red, blue or green such that each colour appears only two times on the dice. If the dice is thrown thrice, the probability of obtaining red colour on top face of the dice at least twice is ______.

Q. 77. The value of $\int_{2.5}^{4} \ln(x)\, dx$ calculated using the Trapezoidal rule with five subintervals is.

MECHANICAL ENGINEERING (ME) P3

Q. 78. Consider a 3 × 3 real symmetric matrix S such that two of its eigen values are $a \neq 0$, $b \neq 0$ with respective eigenvectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$.
If $a \neq b$ then $x_1y_1 + x_2y_2 + x_3y_3$ equals

(a) a **(b)** b
(c) ab **(d)** 0

Q. 79. If a function is continuous at a point,

(a) the limit of the function may not exist at the point
(b) the function must be derivable at the point
(c) the limit of the function at the point tends to infinity
(d) the limit must exist at the point and the value of limit should be same as the value of the function at that point

Q. 80. Divergence of the vector field
$x^2z\hat{i} + xy\hat{j} - yz^2\hat{k}$ at (1, –1, 1)is

(a) 0 **(b)** 3
(c) 5 **(d)** 6

Q. 81. A group consists of equal number of men and women. Of this group 20% of the men and 50% of the women are unemployed. If a person is selected at random from this group, the probability of the selected person being employed is ______.

Q. 82. The definite integral $\int_1^2 \dfrac{1}{x}\, dx$ is evaluated using Trapezoidal rule with a step size of 1. The correct answer is ______.

Q. 83. An analytic function of a complex variable $z = x + iy$ is expressed as $f(z) = u(x, y) + iv(x, y)$, where $i = \sqrt{-1}$. If $u(x, y) = x^2 - y^2$, then expression for $v(x, y)$ in terms of x, y and a general constant c would be

(a) $xy + c$ **(b)** $\dfrac{x^2+y^2}{2} + c$
(c) $2xy + c$ **(d)** $\dfrac{(x-y)^2}{2} + c$

Q. 84. Consider two solutions $x(t) = x_1(t)$ and $x(t) = x_2(t)$ of the differential equation

$$\frac{d^2x(t)}{dt^2} + x(t) = 0, t > 0,$$

such that $x_1(0) = 1, \left.\frac{dx_1(t)}{dt}\right|_{t=0} = 0, x_2(0) = 0, \left.\frac{dx_2(t)}{dt}\right|_{t=0} = 1.$

The Wronskian $W(t) = \begin{vmatrix} x_1(t) & x_2(t) \\ \frac{dx_1(t)}{dt} & \frac{dx_2(t)}{dt} \end{vmatrix}$ at $t = \frac{\pi}{2}$ is

(a) 1 **(b)** −1

(c) 0 **(d)** $\frac{\pi}{2}$

Q. 85. A machine produces 0, 1 or 2 defective pieces in a day with associated probability of $\frac{1}{6}$ $\frac{2}{3}$ and $\frac{1}{6}$, respectively. The mean value and the variance of the number of defective pieces produced by the machine in a day, respectively, are

(a) 1 and $\frac{1}{3}$ **(b)** $\frac{1}{3}$ and 1

(c) 1 and $\frac{4}{3}$ **(d)** $\frac{1}{3}$ and $\frac{4}{3}$

Q. 86. The real root of the equation $5x - 2\cos x - 1 = 0$ (up to two decimal accuracy) is ______.

MECHANICAL ENGINEERING (ME) P4

Q. 87. Which one of the following equations is a correct identity for arbitrary 3 × 3 real matrices P, Q and R?

(a) P(Q + R) = PQ + RP

(b) $(P - Q)^2 = P^2 - 2PQ + Q^2$

(c) det (P + Q) = det P + det Q

(d) $(P + Q)^2 = P^2 + PQ + QP + Q^2$

Q. 88. The value of the integral

$$\int_0^2 \frac{(x-1)^2 \sin(x-1)}{(x-1)^2 + \cos(x-1)} dx \text{ is}$$

(a) 3 **(b)** 0

(c) –1 **(d)** –2

Q. 89. The solution of the initial value problem

$$\frac{dy}{dx} = -2xy;\ y(0) = 2 \text{ is}$$

(a) $1 + e^{-x^2}$ **(b)** $2e^{-x^2}$

(c) $1 + e^{x^2}$ **(d)** $2e^{x^2}$

Q. 90. A nationalized bank has found that the daily balance available in its savings accounts follows a normal distribution with a mean of ₹500 and a standard deviation of ₹50. The percentage of savings account holders, who maintain an average daily balance more than ₹500 is ______ .

Q. 91. Laplace transform of cos (ω*t*) is $\frac{s}{s^2 + \omega^2}$. The Laplace transform of e^{-2t} cos (4*t*) is

(a) $\frac{s-2}{(s-2)^2 + 16}$

(b) $\frac{s+2}{(s-2)^2 + 16}$

(c) $\frac{s-2}{(s+2)^2 + 16}$

(d) $\frac{s+2}{(s+2)^2 + 16}$

Q. 92. If *z* is a complex variable, the value of $\int_5^{3i} \frac{dz}{z}$ is

(a) − 0.511 − 1.57i

(b) − 0.511 + 1.57*i*

(c) 0.511 − 1.57*i*

(d) 0.511 + 1.57*i*

Q. 93. The value of the integral $\int_0^2 \int_0^x e^{x+y}\, dy\, dx$ is

(a) $\frac{1}{2}(e-1)$ **(b)** $\frac{1}{2}(e^2-1)^2$

(c) $\frac{1}{2}(e^2-e)$ **(d)** $\frac{1}{2}\left(e - \frac{1}{e}\right)^2$

Q. 94. The number of accidents occurring in a plant in a month follows Poisson distribution with mean as 5.2. The probability of occurrence of less than 2 accidents in the plant during a randomly selected month is

(a) 0.029 **(b)** 0.034

(c) 0.039 **(d)** 0.044

Q. 95. Consider an ordinary differential equation. $\frac{dx}{dt} = 4t + 4$ If $x = x_0$ at $t = 0$, the increment in *x* calculated using Runge-Kutta fourth order multi-step method with a step size of $\Delta t = 0.2$ is ______.

(a) 0.22 **(b)** 0.44

(c) 0.66 **(d)** 0.88

CIVIL ENGINEERING (CE) P1

Q. 96. $\lim_{x\to\infty}\left(\frac{x+\sin x}{x}\right)$ equals to

(a) $-\infty$ **(b)** 0
(c) 1 **(d)** ∞

Q. 97. Given the matrices $J=\begin{bmatrix}3&2&1\\2&4&2\\1&2&6\end{bmatrix}$ and

$K=\begin{bmatrix}1\\2\\-1\end{bmatrix}$, the product $K^T JK$ is ______.

Q. 98. The probability density function of evaporation E on any day during a year in a watershed is given by ______.

$$f(E)=\begin{cases}\frac{1}{5} & 0\le E\le 5\,\text{mm / day}\\ 0 & \text{otherwise}\end{cases}$$

The probability that E lies in between 2 and 4 mm/day in a day in the watershed is (in decimal) ______.

Q. 99. The sum of Eigen values of the matrix, [M] is

where $[M]=\begin{bmatrix}215&650&795\\655&150&835\\485&355&550\end{bmatrix}$

(a) 915 **(b)** 1355
(c) 1640 **(d)** 2180

Q. 100. With reference to the conventional Cartesian (x, y) coordinate system, the vertices of a triangle have the following coordinates: $(x_1, y_1) = (1, 0)$; $(x_2, y_2) = (2, 2)$; and $(x_3, y_3) = (4, 3)$. The area of the triangle is equal to

(a) $\frac{3}{2}$ **(b)** $\frac{3}{4}$
(c) $\frac{4}{5}$ **(d)** $\frac{5}{2}$

Q. 101. A traffic office imposes on an average 5 number of penalties daily on traffic violators. Assume that the number of penalties on different days is independent and follows a Poisson distribution. The probability that there will be less than 4 penalties in a day is ______.

CIVIL ENGINEERING (CE) P2

Q. 102. A fair (unbiased) coin was tossed four times in succession and resulted in the following outcomes: (i) Head, (ii) Head, (iii) Head, (iv) Head. The probability of obtaining a 'Tail' when the coin is tossed again is

(a) 0 **(b)** $\frac{1}{2}$
(c) $\frac{4}{5}$ **(d)** $\frac{1}{5}$

Q. 103. The determinant of matrix $\begin{bmatrix}0&1&2&3\\1&0&3&0\\2&3&0&1\\3&0&1&2\end{bmatrix}$ is ______.

Q. 104. $z=\frac{2-3i}{-5+i}$ can be expressed as

(a) $-0.5-0.5i$ **(b)** $-0.5+0.5i$
(c) $0.5-0.5i$ **(d)** $0.5+0.5i$

Q. 105. The integrating factor for the differential equation $\frac{dP}{dt}+k_2 P = k_1L_0e^{-k1t}$ is.

(a) e^{-k_1t} **(b)** e^{-k_2t}
(c) e^{k_1t} **(d)** e^{k_2t}

Q. 106. If $\{x\}$ is a continuous, real valued random variable defined over the interval $(-\infty, +\infty)$ and its occurrence is defined by the density function given as:

$f(x) = \frac{1}{\sqrt{2\pi}\times b}e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2}$ where 'a' and 'b' are the statistical attributes of the random variable $\{x\}$. The value of the integral $\int_{-\infty}^{a}\frac{1}{\sqrt{2\pi}\times b}e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2}dx$ is ______

(a) 1 **(b)** 0.5
(c) π **(d)** $\frac{\pi}{2}$

Q. 107. The expression $\lim_{\alpha\to 0}\frac{x^\alpha-1}{\alpha}$ is equal to

(a) $\log x$ **(b)** 0
(c) $x\log x$ **(d)** ∞

Q. 108. An observer counts 240 veh/h at a specific highway location. Assume that the vehicle arrival at the location is Poisson distributed, the probability of having one vehicle arriving over a 30-second time interval is ______.

Q. 109. The rank of the matrix $\begin{bmatrix}6&0&4&4\\-2&14&8&18\\14&-14&0&-10\end{bmatrix}$ is______.

COMPUTER SCIENCE (CS) P1

Q. 110. Consider the statement

"Not all that glitters is gold"

Predicate glitters (x) is true if x glitters and predicate gold (x) is true if x is gold. Which one of the following logical formulae represents the above statement?

(a) $\forall x$: glitters (x) $\Rightarrow \neg$gold (x)

(b) $\forall x$: gold (x) $\Rightarrow$ glitters (x)

(c) $\exists x$: gold (x) $\wedge \neg$ glitters (x)

(d) $\exists x$: glitters (x) $\wedge \neg$ gold (x)

Q. 111. Suppose you break a stick of unit length at a point chosen uniformly at random. Then the expected length of the shorter stick is ______.

Q. 112. Consider the following system of equations:

$3x + 2y = 1$

$4x + 7z = 1$

$x + y + z = 3$

$x - 2y + 7z = 0$

The number of solutions for this system is ______.

Q. 113. The value of the dot product of the eigenvectors corresponding to any pair of different eigen values of a 4-*by*-4 symmetric positive definite matrix is ______.

Q. 114. The function $f(x) = x \sin x$ satisfies the following equation : $f''(x) + f(x) + t \cos x = 0$. The value of t is ______.

Q. 115. A function $f(x)$ is continuous in the interval [0,2]. It is known that $f(0) = f(2) = -1$ and $f(1) = 1$. Which one of the following statements must be true?

(a) There exists a y in the interval (0, 1) such that $f(y) = f(y + 1)$

(b) For every y in the interval (0, 1), $f(y) = f(2 - y)$

(c) The maximum value of the function in the interval (0, 2) is 1

(d) There exists a y in the interval (0,1) such that $f(y) = -f(2 - y)$

Q. 116. Four fair six – sided dice are rolled. The probability that the sum of the results being 22 is $\frac{x}{1296}$. The value of x is ______.

Q. 117. A pennant is a sequence of numbers, each number being 1 or 2. An n-pennant is a sequence of numbers with sum equal to n. For example, (1,1,2) is a 4-pennant. The set of all possible 1-pennants is {(1)}, the set of all possible 2-pennants is {(2), (1,1)} and the set of all 3-pennants is {(2,1), (1,1,1), (1,2)}. Note that the pennant (1,2) is not the same as the pennant (2,1). The number of 10-pennants is ______.

Q. 118. Let S denote the set of all functions $f:\{0,1\}^4 \to \{0,1\}$. Denote by N the number of functions from S to the set {0,1}. The value of $\log_2 \log_2$ N is ______.

COMPUTER SCIENCE (CS) P2

Q. 119. The security system at an IT office is composed of 10 computers of which exactly four are working. To check whether the system is functional, the officials inspect four of the computers picked at random (without replacement). The system is deemed functional if at least three of the four computers inspected are working. Let the probability that the system is deemed functional be denoted by p. Then $100\,p$ = ______.

Q. 120. Each of the nine words in the sentence **"The quick brown fox jumps over the lazy dog"** is written on a separate piece of paper. These nine pieces of paper are kept in a box. One of the pieces is drawn at random from the box. The *expected* length of the word drawn is ______. (The answer should be rounded to one decimal place.)

Q. 121. The maximum number of edges in a bipartite graph on 12 vertices is ______.

Q. 122. If the matrix A is such that

$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} [1 \quad 9 \quad 5]$$

then the determinant of A is equal to _____.

Q. 123. A non-zero polynomial $f(x)$ of degree 3 has roots at $x = 1$, $x = 2$ and $x = 3$. Which one of the following must be TRUE?

(a) $f(0) f(4) < 0$

(b) $f(0) f(4) > 0$

(c) $f(0) + f(4) > 0$

(d) $f(0) + f(4) < 0$

Q. 124. In the Newton-Raphson method, an initial guess of $x_0 = 2$ is made and the sequence $x_0, x_1, x_2 \ldots$ is obtained for the function

$0.75x^3 - 2x^2 - 2x + 4 = 0$

Consider the statements

(I) $x_3 = 0$.

(II) The method converges to a solution in a finite number of iterations.

Which of the following is **TRUE**?

(a) Only I **(b)** Only II

(c) Both I and II **(d)** Neither I nor II

Q. 125. The product of the non-zero eigen values of the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$ is ______.

Q. 126. The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is ______.

Q. 127. The number of distinct positive integral factors of 2014 is ______.

COMPUTER SCIENCE (CS) P3

Q. 128. Consider the following statements:

P: Good mobile phones are not cheap

Q: Cheap mobile phones are not good

L: P implies Q

M: Q implies P

N: P is equivalent to Q

Which one of the following about L, M, and N is **CORRECT**?

(a) Only L is TRUE.

(b) Only M is TRUE.

(c) Only N is TRUE.

(d) L, M and N are TRUE.

Q. 129. Let X and Y be finite sets and $f : X \rightarrow Y$ be a function. Which one of the following statements is TRUE?

(a) For any subsets A and B of X, $|f(A \cup B)| = |f(A)| + |f(B)|$

(b) For any subsets A and B of X, $f(A \cap B) = f(A) \cap f(B)$

(c) For any subsets A and B of X, $|f(A \cap B)| = \min\{|f(A)|, |f(B)|\}$

(d) For any subsets S and T of Y, $|f^{-1}(S \cap T)| = f^{-1}(S) \cap f^{-1}(T)$

Q. 130. Let G be a group with 15 elements. Let L be a subgroup of G. It is known that L ≠ G and that the size of L is at least 4. The size of L is ______.

Q. 131. Which one of the following statements is TRUE about every $n \times n$ matrix with only real eigen values?

(a) If the trace of the matrix is positive and the determinant of the matrix is negative, at least one of its eigen values is negative.

(b) If the trace of the matrix is positive, all its eigen values are positive.

(c) If the determinant of the matrix is positive, all its eigen values are positive.

(d) If the product of the trace and determinant of the matrix is positive, all its eigen values are positive.

Q. 132. If, $\int_0^{2\pi} |x \sin x|\, dx = k\pi$ then the value of k is equal to ______.

Q. 133. With respect to the numerical evaluation of the definite integral, $K = \int_a^b x^2 dx$, where a and b are given, which of the following statements is/ are **TRUE**?

I. The value of K obtained using the trapezoidal rule is always greater than or equal to the exact value of the definite integral .

II. The value of K obtained using the Simpson's rule is always equal to the exact value of the definite integral.

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

Q. 134. The value of the integral given below is

$\int_0^{\pi} x^2 \cos x\, dx$,

(a) -2π **(b)** π

(c) $-\pi$ **(d)** 2π

Q. 135. Let S be a sample space and two mutually exclusive event A and B be such that A ∪ B = S. If P(·) denotes the probability of the event , the maximum value of P (A) P (B) is ______.

Q. 136. Consider the set of all functions $f : \{0, 1, \ldots, 2014\} \rightarrow \{0, 1, \ldots, 2014\}$ such that $f(f(i)) = i$, for all $0 \le i \le 2014$. Consider the following statements:

P. For each such function it must be the case that for every $i, f(i) = i$.

Q. For each such function it must be the case that for some $i, f(i) = i$.

R . Each such function must be onto.

Which one of the following is CORRECT?

(a) P, Q and R are true

(b) Only Q and R are true

(c) Only P and Q are true

(d) Only R is true

Answer Key			
Q. No.	Answer	Topic Name	Chapter Name
1	(d)	Matrix Properties	Linear Algebra
2	0.667	Probability	Probability and Statistics
3	(c)	Cauchy's Integral	Complex Variable
4	1	Eigen Values	Linear Algebra
5	0.33	Statistics	Probability and Statistics
6	(a)	Taylor Series	Calculus
7	(b)	Fourier Transform	Transform Theory
8	864	Determinant	Calculus
9	1	Determinant	Linear Algebra
10	200	Determinant	Linear Algebra
11	50	Mean	Probability and Statistics
12	(a)	Maximum/Minimum	Calculus
13	(c)	Limit	Calculus
14	(a)	2nd Order	Differential Equation
15	(b)	System of Linear Equation	Linear Algebra
16	(b)	Analytic Function	Complex Variable
17	49	Determinant	Linear Algebra
18	3	Divergence	Vector Calculus
19	0	Maximum/Minimum	Calculus
20	(a)	1st Order DE	Differential Equation
21	(b)	Properties	Numerical Analysis
22	(c)	Probability	Probability and Statistics
23	(c)	Partial Differentiation	Calculus
24	6	Maximum/Minimum	Calculus
25	(b)	Properties	Linear Algebra
26	3	Probability	Probability and Statistics
27	0	Curl	Vector Calculus
28	(d)	Series	Calculus
29	7	Gradient	Vector Calculus
30	0.8	Normal Distribution	Probability and Statistics
31	(b)	2nd Order DE	Differential Equation
32	3	Directional Derivative	Vector Calculus
33	0.54	2nd Order DE	Differential Equation
34	0.44	Probability	Probability and Statistics
35	(d)	Laplace Transform	Transform Theory
36	(c)	Application of Derivative	Calculus

Q. No.	Answer	Topic Name	Chapter Name
37	(b)	System of Equation	Linear Algebra
38	(a)	Maximum/Minimum	Calculus
39	(c)	2nd Order	Differential Equation
40	(b)	Laplace Transform	Transform Theory
41	(c)	Complex Mapping	Complex Variable
42	0.5	Fourier Series	Transform Theory
43	(b)	Probability	Probability and Statistics
44	(b)	Line Integral	Vector Calculus
45	(a)	Eigen Value Property	Linear Algebra
46	0.14	Probability	Probability and Statistics
47	(c)	Maximum/Minimum	Calculus
48	(b)	Properties	Complex Variable
49	(c)	2nd Order	Differential Equation
50	(b)	Double Integration	Calculus
51	0.4	Random Variable	Probability and Statistics
52	(b)	Maximum/Minimum	Calculus
53	(c)	Rank	Linear Algebra
54	2	Application in Integration	Calculus
55	(a)	Divergence	Vector Analysis
56	0.43	Random Variable	Probability and Statistics
57	(c)	Complex Integration	Complex Variable
58	(d)	Mean and Variance	Probability and Statistics
59	0.06	Newton Raphson	Numerical Analysis
60	(a)	Determinant	Linear Algebra
61	(a)	Limit	Calculus
62	(c)	Argument	Complex Variable
63	(a)	System of Linear Equations	Linear Algebra
64	(b)	Properties	Vector Analysis
65	(c)	Line Integral	Vector Analysis
66	35	2nd Order	Differential Equation
67	(d)	Standard Deviation	Probability and Statistics
68	1.111	Trapezoidal Rule	Numerical Analysis
69	(d)	Eigen Values	Linear Algebra
70	(b)	Limit	Calculus
71	(a)	Curl	Vector Analysis
72	(a)	Probability	Probability and Statistics
73	–0.954	Maximum/Minimum	Calculus

Q. No.	Answer	Topic Name	Chapter Name
74	(c)	Analytic Function	Complex Variable
75	(d)	1st Order DE	Differential Equation
76	0.2592	Probability	Probability and Statistics
77	1.7533	Trapezoidal Rule	Numerical Analysis
78	(d)	Eigen Vectors	Linear Algebra
79	(d)	Functions	Calculus
80	(c)	Divergence	Vector Analysis
81	0.65	Probability	Probability and Statistics
82	1.165	Trapezoidal Rule	Numerical Analysis
83	(c)	Analytic Function	Complex Variable
84	(a)	Second Order DE	Differential Equation
85	(a)	Mean and Variance	Probability and Statistics
86	0.54	Newton Rapshon	Numerical Analysis
87	(d)	Properties	Linear Algebra
88	(b)	Integral	Calculus
89	(b)	First Order OdE	Differential Equations
90	50	Normal Distribution	Probability and Statistics
91	(b)	Lapalce Transform	Transform Theory
92	(b)	Complex Integral	Complex Variable
93	(b)	Double Integral	Calculus
94	(b)	Poisson Distribution	Probability and Statistics
95	(d)	Runga Kutta Method	Numerical Analysis
96	(c)	Limit	Calculus
97	23	Multiplication	Linear Algebra
98	0.4	Probability	Probability and Statistics
99	(a)	Eigen Values	Linear Algebra
100	(a)	Area of Triangle	Vector Analysis
101	0.265	Poisson Distribution	Probability and Statistics
102	(b)	Probability	Probability and Statistics
103	88	Determinant	Linear Algebra
104	(b)	Complex Number	Complex Variable
105	(d)	1st Order DE	Differential Equation
106	(b)	Normal Distribution	Probability and Statistics
107	(a)	Limit	Calculus
108	0.27	Poisson Distributed	Probability and Statistics
109	2	Rank	Linear Algebra
110	(d)	Logic	Discrete Mathematics

Q. No.	Answer	Topic Name	Chapter Name
111	0.25	Mean	Probability and Statistics
112	1	System of Equations	Linear Algebra
113	0	Eigen Values	Linear Algebra
114	–2	Derivative	Calculus
115	(a)	Functions	Calculus
116	10	Probability	Probability and Statistics
117	89	Combinatory	Discrete Mathematics
118	16	Set	Discrete Mathematics
119	11.90	Probability	Probability and Statistics
120	3.9	Probability	Probability and Statistics
121	36	Graph Theory	Discrete Mathematics
122	0	Determinant	Linear Algebra
123	(a)	Functions	Calculus
124	(a)	Newton Raphson Method	Numerical Analysis
125	6	Eigen Values	Linear Algebra
126	0.26	Probability	Probability and Statistics
127	8	Combinatory	Discrete Mathematics
128	(d)	Logic	Discrete Mathematics
129	(d)	Set Theory	Discrete Mathematics
130	5	Set Theory	Discrete Mathematics
131	(a)	Eigen Values	Linear Algebra
132	4	Integration	Calculus
133	(c)	Numerical Integration	Numerical Analysis
134	(a)	Integral	Calculus
135	0.25	Probability	Probability and Statistics
136	(b)	Functions	Calculus

ANSWERS WITH EXPLANATIONS

1. Option (d) is correct.

Generally, matrix multiplication is not commutative. Matrix multiplication may not even be defined if we reverse the order.

So, MN ≠ NM

2. Correct answer is [0.667].

Let E_1 = One child family.

E_2 = Two children family.

A is the event of picking a child.

Let x is the number of families.

According to Bayes' theorem.

$$P\left(\frac{E_2}{A}\right) = \frac{2\cdot x}{2\cdot x + 1\cdot x}$$

$$= \frac{2}{3} = 0.667$$

3. Option (c) is correct.

$z = -2j$ is a singularity lying inside C,

where C : $|z| = 3$

By Cauchy's integral formula.

$$\oint_C \frac{z^2 - z + 4j}{z + 2j} dz = 2\pi j\left[z^2 - z + 4j\right]_{z=-2j}$$

$$= 2\pi j\left\{(-2j)^2 - (-2j) + 4j\right\}$$

$$= 2\pi j(-4 + 2j + 4j)$$

$$2\pi j(-4 + 6j) = -4\pi[3 + 2j]$$

4. Correct answer is [1].

For $A^2 = I$, I being Identity matrix of order 4 eigen values of A^2 will be $\lambda = 1, 1, 1, 1$

For eigen values of A be λ_1

then $(\lambda_1)^2$ = eigen value of A^2 $\left[\because A^K = A^2,\ k = 2\right]$

$\therefore \quad \lambda_1^2 = 1$

$\Rightarrow \quad \lambda_1 = \pm\sqrt{1} = \pm 1$

So, positive eigen values of A is 1.

5. Correct answer is [0.33].

There can be 6 possible cases.

$(X_1 > X_2 > X_3)$

$(X_1 > X_3 > X_2)$

$(X_2 > X_1 > X_3)$

$(X_2 > X_3 > X_1)$

$(X_3 > X_2 > X_1)$

$(X_3 > X_1 > X_2)$

All three random variables are independent and identical, therefore all cases are equally likely and hence (X_1) being largest is $(X_1 > X_2 > X_3)$ or $(X_1 > X_3 > X_2)$

$\therefore (X_1)$ is the largest $= \frac{2}{6} = \frac{1}{3} = 0.33$

6. Option (a) is correct.

$f(x) = 3\sin x + 2\cos x$ ...(i)

Taylor's series expansion for trigonometric function as

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$$

Substitute the values in equation (i)

$3\sin x + 2\cos x$

$$= 3\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\right) + 2\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\right)$$

$$= 2 + 3x - x^2 - \frac{x^3}{2} + \ldots.$$

7. Option (b) is correct.

$$\int_{-\infty}^{\infty} g(t).e^{-j\omega t} dt = \omega e^{-2\omega^2} \text{ (let } G(j\omega))$$

$$\int_{-\infty}^{\infty} g(t) dt = 0 \quad [\text{for } \omega = 0]$$

$$y(t) = \int_{-\infty}^{t} g(z) dz$$

$$\Rightarrow \quad y(t) = \int_{-\infty}^{\infty} g(z) u(t - z) dz$$

$y(t) = g(t) * u(t)$

[$u(t) \rightarrow$ unit step function]

$$Y(jw) = G(j\omega)\cdot U(j\omega)$$

$$Y(jw) = \int_{-\infty}^{\infty} y(t).e^{-j\omega t}dt$$

$$\Rightarrow \quad Y(j0) = \int_{-\infty}^{\infty} y(t), \; w = 0$$

$$= \left[\omega e^{-2\omega^2}\left[\frac{1}{j\omega} + \pi\delta(\omega)\right]\right]_{\omega=0}$$

$$= \frac{1}{j} = -j$$

8. Correct answer is [864].

$$z(x, y) = x + y$$

$$V = \iint_R Z(x, y) dy dx$$

$$= \int_{x=0}^{12}\int_{y=0}^{x} (x + y) dy\, dx$$

$$= \int_0^{12}\left[xy + \frac{y^2}{2}\right]_0^x dx$$

$$= \int_0^{12} \frac{3}{2}x^2 dx = \frac{1}{2}\left[x^3\right]_0^{12}$$

$$= 864$$

9. Correct answer is [1].

$$P = I_6 + \alpha J_6$$

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \alpha \\ 0 & 1 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 1 & \alpha & 0 & 0 \\ 0 & \alpha & 0 & 0 & 1 & 0 \\ \alpha & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

if $\alpha = 1$ $\quad C_1 = C_6, \quad C_2 = C_5, \quad C_3 = C_4$

with linearly dependent columns det (P) = 0

$[\alpha = 1]$

10. Correct answer is [200].

Given, $|A| = 5$

$$|B| = 40, |A\cdot B| = |A|\cdot|B|$$

$$\therefore \quad |A\cdot B| = (5)\cdot(40) = 200$$

11. Correct answer is [50].

$$X = 1, 3, 5, 7, 9, ...99$$

$$\Rightarrow \quad n = 50 \rightarrow \text{No. of observation}$$

$$\therefore \quad E(X) = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \left[\because\; p(x_i) = \frac{1}{50} \text{ for all } i\right]$$

$$= \frac{1}{50}[1 + 3 + 5 + 7 + ...99]$$

$$E(x) = \frac{1}{50}[50]^2 = 50$$

$$\left[\because\; S_n = \frac{n}{2}[a + l] = \frac{50}{2}[1 + 99]\right]$$

12. Option (a) is correct.

$$f(t) = e^{-t} - 2e^{-2t}, \; f'(t) = 0$$

$$\Rightarrow f'(t) = -e^{-t} + 4e^{-2t} = 0$$

$$\Rightarrow e^{-t}\left[4e^{-t} - 1\right] = 0 \Rightarrow e^{-t} = \frac{1}{4}$$

$$\Rightarrow \quad t = \log_e 4, \; f''(t) = e^{-t}\left[-4e^{-t} + 1 - 4e^{-t}\right]$$

$f''(t) < 0$ at $t = \log_e 4$

13. Option (c) is correct.

$$\lim_{x\to\infty}\left(x + \frac{1}{x}\right)^x = e$$

14. Option (a) is correct.

AE : $m^2 + 2\,\alpha\, m + 1 = 0$

Given, 2nd ODE has equal roots, if it's discriminant is zero.

$$\Rightarrow \quad 4\alpha^2 - 4 = 0 \qquad \Rightarrow \alpha^2 - 1 = 0$$

$$\Rightarrow \quad \alpha = \pm 1$$

15. Option (b) is correct.

Given, matrix is in the form of $AX = B$.

$$\left[\frac{A}{B}\right] = \left[\begin{array}{ccc|c} 2 & 1 & 3 & 5 \\ 3 & 0 & 1 & -4 \\ 1 & 2 & 5 & 14 \end{array}\right]$$

$R_2 \rightarrow 2R_3 - 3R_1$

$R_3 \rightarrow 2R_3 - R_1$

$$= \left[\begin{array}{ccc|c} 2 & 1 & 3 & 5 \\ 0 & -3 & -7 & -23 \\ 0 & 3 & 7 & 23 \end{array}\right];$$

$$R_3 \rightarrow R_3 + R_2 \rightarrow \left[\begin{array}{ccc|c} 2 & 1 & 3 & 5 \\ 0 & -3 & -7 & -23 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\because \rho(A) = \rho\left(\frac{A}{B}\right) <$ No. of unknown

$\therefore$ System of linear equations has infinitely many solutions.

16. Option (b) is correct.

It is given that real part of $f(z)$ $u = e^{-y}\cos x$

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$$

$$= -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$$

(using Cauchy's Reimann equation)

$$= e^{-y}\cos x dx - e^{-y}\sin x dy$$
$$= d\left[e^{-y}\sin x\right]$$

On integration we get

$$V = e^{-y}\sin x$$

17. Correct answer is [49].

In General, 2×2 real symmetric matrix $\begin{bmatrix} y & x \\ x & z \end{bmatrix}$

$\Rightarrow$ det $= yz - x^2$ and trace $= y + z = 14$

$$z = 14 - y \qquad ...(i)$$

Let $f = yz - x^2\,(det)$

for maximum value of f, x^2 will be zero as it cannot be negative

$= -y^2 + 14\,y$ from (i)

$$\frac{df}{dy} = -2y + 14 = 0$$

$\Rightarrow \quad y = 7$

$\therefore$ Maximum value of the determinant is

det $= yz = 7\,(14 - 7) = 49$

18. Correct answer is [3].

For $\quad \vec{r} = X\,\hat{a}x + Y\,\hat{a}y + Z\,\hat{a}z$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\nabla(\ln r) = \frac{1}{r}\nabla r = \nabla(\ln r) = \frac{\vec{r}}{r^2}$$

$$\Rightarrow div\left(r^2\nabla(\ln r)\right) = div\left(\vec{r}\right) = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 1$$

$$\because \left[\nabla(\ln r) = \sum \hat{a}_x \frac{\partial}{\partial x}(\ln r)\right.$$
$$\left. = \sum \hat{a}_x \left(\frac{1}{r}\right)\left(\frac{x}{r}\right) = \frac{1}{r^2}\sum \hat{a}_x x = \frac{\vec{r}}{r^2}\right]$$

19. Correct answer is [0].

$$f(x) = \ln(1+x) - x$$

$$f'(x) = \frac{1}{(1+x)} - 1$$

For f_{max}, $\quad f'(x) = 0$

$$\Rightarrow \quad \frac{1}{(1+x)} - 1 = 0$$

$$\Rightarrow \quad \frac{-x}{1+x} = 0 \Rightarrow x = 0$$

$$f''(x) = \frac{-1}{(1+x)^2}\; f'' < 0$$

$$f''(0) < 0$$

$\therefore \quad x = 0$ is maxima

20. Option (a) is correct.

Linear non-homogeneous differential equation is of the form

$$\frac{dy}{dx} + PY = Q$$

Where P, Q are function of independent variable x and $Q \neq 0$

So, Option (a) with $P = x$, $Q = e^{-x}$ is correct answer.

21. Option (b) is correct.

P1—M3, P2—M1, P3—M4, P4—M2

22. Option (c) is correct.

P[4^{th} head appears at the 10^{th} toss] = P [Getting 3 heads in the first 9 tosses and *a* head at 10^{th} toss]

$$= \left[{}^9C_3\left(\frac{1}{2}\right)^9\right] \times \left(\frac{1}{2}\right) = \frac{84}{1024}$$
$$= 0.082$$

23. Option (c) is correct.

$$\frac{dz}{dx} = xy \cdot \frac{1}{xy} \cdot y + 1 \cdot y \cdot \ln xy$$
$$= y(1 + \ln xy)$$

Similarly $\dfrac{\partial z}{\partial y} = x(1 + \ln xy)$

$$\Rightarrow \quad x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$$

24. Correct answer is [6].

$$f(x) = 2x^3 - 9x^2 + 12x - 3$$
$$f'(x) = 6x^2 - 18x + 12 = 0$$

$\Rightarrow \quad x^2 - 3x + 2 = 0$

$\Rightarrow \quad x = 1, 2 \in [0, 3]$

$f(x)$ at end points and stationary points

$$f(0) = -3$$
$$f(3) = 6$$
$$f(1) = 2$$
$$f(2) = 1$$

$\therefore\ f(x)$ has maximum value of 6.

25. Option (b) is correct.

Let $A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ is real symmetric matrix.

We know characteristic equation is

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{bmatrix} -1-\lambda & 1 \\ 1 & -1-\lambda \end{bmatrix} = 0$$

$\Rightarrow \quad (-1-\lambda)^2 - 1 = 0$

$\Rightarrow \quad (1+\lambda)^2 - 1 = 0$

$\Rightarrow \quad \lambda + 1 = \pm 1$

$\Rightarrow \quad \lambda = 0, -2$ (non positive)

26. Correct answer is [3].

Let the 1st toss is head and x denote the no. of tosses (after getting 1st head) to get first tail.

Event	X	Probability (P(x))
T	1	$\frac{1}{2}$
HT	2	$\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$
HHT	3	$\frac{1}{8}$
So on….		

$$E(X) = \sum_{x=1}^{\infty} xP(x)$$

$$= 1\times\frac{1}{2} + 2\times\frac{1}{4} + 3\times\frac{1}{8}...$$

$$= \frac{1}{2}\left(1 + 2\cdot\frac{1}{2} + 3\cdot\frac{1}{2^2} + ...\right)$$

$$\left[\because \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + ...\right]$$

$$E(x) = \frac{1}{2}\cdot\frac{1}{\left(1-\frac{1}{2}\right)^2}$$

$$= \left(\frac{1}{\frac{1}{2}}\right)$$

$$\Rightarrow E(X) = 2$$

The expected number of tosses (after 1st head) to get 1st tail is 2 and same can be applicable if 1st toss results in tail.

The average no. of tosses is 1+2 = 3.

27. Correct answer is [0].

$$\vec{A} = \cos x \sin y\, \hat{a}_x + \sin x \cos y\, \hat{a}_y$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos x \sin y & \sin x \cos y & 0 \end{vmatrix}$$

$$= \hat{a}_x(0) + \hat{a}_y(0) + \hat{a}_z(\cos x \cos y - \cos x \cos y) = 0$$

$$\therefore \quad \nabla \times A = 0$$

28. Option (d) is correct.

$$\sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1!} + \frac{1}{2!} + = e, \quad \text{for } x = 0$$

as $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + ..., \forall\, x$ in R

29. Correct answer is [7].

$$\nabla f = \frac{df}{dx}\hat{i} + \frac{df}{dy}\hat{j} + \frac{df}{dz}\hat{k}$$

$$(\nabla f)_{(1,1,1)} = \left(\hat{i}(2x) + \hat{j}(6y) + \hat{k}(3z^2)\right)_{(1,1,1)}$$

$$= \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right)$$

$$\left|(\nabla f)_{(1,1,1)}\right| = \sqrt{2^2 + 6^2 + 3^2} = 7$$

30. Correct answer is [0.8].

X is Gaussian RV with

$\mu = 0, \sigma^2 = 1$

$$\Rightarrow \quad f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}, \; -\infty < x < \infty$$

$$\therefore \quad E\{|x|\} = \int_{-\infty}^{\infty} |x|.f(x)\,dx$$

$$= \frac{2}{\sqrt{2\pi}} \int_0^{\infty} x e^{\frac{-x^2}{2}} dx \quad \{|x| \cdot f(x) \text{ is even}\}$$

For $\quad u = \frac{x^2}{2}, \; du = x\,dx$

$$= \frac{2}{\sqrt{2\pi}} \int_0^{\infty} e^{-u} du = \sqrt{\frac{2}{\pi}}\left[-e^{-u}\right]_0^{\infty}$$

$$= \sqrt{\frac{2}{\pi}}$$

$= 0.797 \simeq 0.8$

31. Option (b) is correct.

$$\left(D^2 + 2D + 1\right)x = 0$$

$AE \to m^2 + 2m + 1 = 0 \Rightarrow m = -1, -1$

real and repeated roots

$\therefore x = (a + bt)e^{-t} = ae^{-t} + bte^{-t}$

32. Correct answer is [3].

$$f(x, y) = \frac{1}{\sqrt{2}}\left(x^2 y + xy^2\right)$$

$$\nabla f = \hat{i}\left[\frac{2xy + y^2}{\sqrt{2}}\right] + \hat{j}\left[\frac{x^2 + 2xy}{\sqrt{2}}\right]$$

$$(\nabla f)_{(1,1)} = \frac{3}{\sqrt{2}}\hat{i} + \frac{3}{\sqrt{2}}\hat{j}$$

$\hat{e}$ = unit vector at an angle of $\frac{\pi}{4}$ with y–axis.

$$\left(\sin\frac{\pi}{4}\right)\vec{i}+\vec{j}\left(\cos\frac{\pi}{4}\right)=\frac{i}{\sqrt{2}}+\frac{j}{\sqrt{2}}$$

$$\text{Directional derivative} = \frac{3}{\sqrt{2}}(i+j)\cdot\frac{1}{\sqrt{2}}(i+j)$$

$$=2\left(\frac{3}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)=3$$

33. Correct answer is [0.54].

$$\left(D^2+4D+4\right)y=0$$

$$AE \to m^2+4m+4=0$$

$$(m+2)^2=0 \Rightarrow m=-2,-2$$

$\therefore$ general solution

$$y=(a+bx)e^{-2x} \quad \text{...(i)}$$

$$y(0)=1 \Rightarrow a=1$$

$$y'=(a+bx)(-2)e^{-2x}+be^{-2x} \quad \text{...(ii)}$$

$$y'(0)=1 \Rightarrow b=3$$

$$\therefore \quad y=(1+3x)e^{-2x} \quad \text{...(iii)}$$

at $x = 1 \Rightarrow y = 4e^{-2} = 0.541 \cong 0.54$

34. Correct answer is [0.44].

Parcel will be lost if

I. when lost at 1st post office.

II. passed by 1st post office and lost by 2nd post office.

Probability that parcel is lost by first post office $=\frac{1}{5}$

P (Parcel lost by 2nd post office if it passes 1st post office)= P (Parcel passed by 1st Post office) × P(Parcel lost by 2nd post office)

$$\frac{4}{5}\times\frac{1}{5}=\frac{4}{25}$$

$$\text{Probability (parcel is lost)} = \frac{1}{5}+\frac{4}{5}\times\frac{1}{5}=\frac{9}{25}$$

Probability (parcel lost by 2nd post office parcel lost) $=\dfrac{\frac{4}{25}}{\frac{9}{25}}=\frac{4}{9}=0.44$

35. Option (d) is correct.

We know if $f(t) \leftrightarrow F(S)$

And according to differentiation property

$$tf(t) \leftrightarrow -\frac{d}{ds}F(S)$$

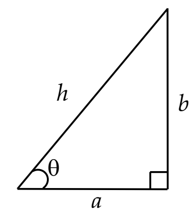
$$-\frac{d}{ds}\left(\frac{1}{s^2+s+1}\right)=-\frac{-(2s+1)}{\left(s^2+s+1\right)^2}=\frac{(2s+1)}{\left(s^2+s+1\right)^2}$$

36. Option (c) is correct.

Given, $a + h = c$, constant

then by Pythagoras theorem

$$b=\sqrt{h^2-a^2}$$



$$\text{Area of } \Delta = \frac{1}{2}a\cdot b$$

$$=\frac{1}{2}a\sqrt{h^2-a^2}=\frac{1}{2}a\sqrt{(c-a)^2-a^2}$$

$$A = \frac{1}{2}a\sqrt{c^2-2ac}, \text{ then}$$

$$A^2=\frac{a^2}{2}\left(c^2-2ac\right)$$

For A_{max}, A^2 will also have maximum value.

$$\frac{dA^2}{da}=0 \Rightarrow \frac{1}{2}\left\{2a\left(c^2-2ac\right)+a^2(-2c)\right\}=0$$

$$\Rightarrow ac^2-2a^2c-a^2c=0$$

$$\Rightarrow ac^2-3a^2c=0$$

$$\Rightarrow ac(c-3a)=0 \qquad \left[\therefore a=0 \text{ or } a=\frac{c}{3}\right]$$

ignoring $a = 0$,

for $a=\frac{c}{3}$, $\quad h=c-\frac{c}{3}=\frac{2c}{3}$

In right angled triangle $\cos\theta=\dfrac{\text{base}}{\text{hypotenuse}}$

$$\theta=\cos^{-1}\left(\frac{c/3}{2c/3}\right)=60°$$

37. Option (b) is correct.

For given system of equation

$$[A|B]=\begin{bmatrix}1 & 2 & 2 & b_1\\ 5 & 1 & 3 & b_2\end{bmatrix}_{(2\times4)}$$

with minor $\begin{vmatrix}1 & 2\\ 5 & 1\end{vmatrix}\neq 0$,

$\text{Rank}\,A=\text{Rank}\,(A|B)=2<3$ {no. of unknowns}

So, it has infinitely many solution for any b_1, b_2.

38. Option (a) is correct.

$$f(x)=xe^{-x}$$

for f_{max}, $f'(x)=-xe^{-x}+e^{-x}=0$

$$\Rightarrow \quad e^{-x}(-x+1)=0$$

$\Rightarrow \quad x = 1, \infty$

$$f''(x) = xe^{-x} - e^{-x} - e^{-x}$$

$$= xe^{-x} - 2e^{-x}$$

$$f''(1) = (1)e^{-(1)} - 2e^{-(1)} = -e^{-1} < 0$$

Therefore, $f(x)$ is maximum at $x = 1$

$$f(1) = e^{-1}$$

39. Option (c) is correct.

$$\frac{\partial^2 x}{\partial t^2} = 9x$$

Taking Laplace Transform on both sides.

$$s^2X(s) + sx(0) - x'(0) = -9X(s)$$

$$\Rightarrow \quad s^2X(s) - s(1) - (1) = -9X(s)$$

$$\Rightarrow \quad X(s)(s^2 + 9) = s + 1$$

$$\Rightarrow \quad X(s) = \frac{s+1}{(s^2+9)} = \frac{s}{(s^2+9)} + \frac{1}{(s^2+9)}$$

Taking Inverse Laplace transform

$$x(t) = \cos 3t + \frac{1}{3}\sin 3t$$

40. Option (b) is correct.

$$X(s) = \frac{3s+5}{s^2+10+21}$$

Using Initial Value Theorem

$$x(t) = \lim_{s\to\infty} sX(s)$$

$$x(0^+) = \lim_{s\to 0} sX(s)$$

$$= \lim_{s\to\infty} s\left(\frac{3s+5}{s^2+10+21}\right)$$

$$= \lim_{s\to\infty} s\frac{s\left(3+\frac{5}{s}\right)}{s^2\left(1+\frac{10}{s}+\frac{21}{s^2}\right)}$$

$$= \frac{3+\frac{5}{\infty}}{1+\frac{10}{\infty}+\frac{21}{\infty^2}} = 3$$

41. Option (c) is correct.

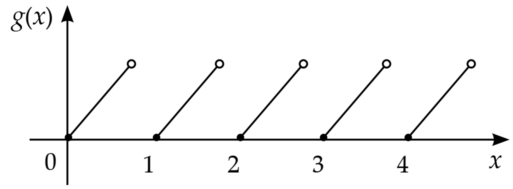
Given, set of points $S = \{z : |z| = 1\}$ and the function $f(z) = zz^*$

$\because \quad zz^* = |z|^2$

For given S, $f(z) = 1$ $\quad [\because |z| = 1]$

$f(z)$ maps S to the point (0, 1) in the complex plane.

42. Correct answer is [0.5].



$$g(x) = x - [x]$$

$$g(x) = \{x\}$$

It is a periodic function, so the constant term will be

$$\text{Average value} = (a_0) = \frac{1}{T}\int_0^T f(x)dx$$

$$= \frac{1}{T}\int_0^1 xdx \qquad \{T = 1\}$$

$$= \left[1 \times \frac{x^2}{2}\right]_0^1 = \frac{1}{2} = 0.5$$

43. Option (b) is correct.

A fair coin is tossed n times.

Let head appears x times and tail appears y times.

then $|x - y| = n - 3 \quad$ ...(i)

$x + y = n \quad$ ...(ii)

Here we can have two cases.

Case I: $x > y$

$$x - y = n - 3$$
$$x + y = n$$
$$2x = 2n - 3$$
$$x = \frac{2n-3}{2} = n - \frac{3}{2}$$

With both x and n being integer this is an invalid result

Case II: $x < y$

$$-x + y = n - 3$$
$$x + y = n$$
$$y = n - \frac{3}{2}$$

y is an integer and n is also an integer value. This is also contradictory relation. Therefore, the probability is zero.

44. Option (b) is correct.

$$\vec{F} = yz\hat{i}$$

C : $\quad x^2 + y^2 = 1$

Now the line integral of $\vec{F}$ along the circle at $z = 1$.

From Green's theorem

$$\oint_0 \vec{F}.\vec{dl} = \iint_R \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) dxdy \quad ...(i)$$

F_x and F_y are component of $\vec{F}$ along x and y directions.

$$F_x = yz$$
$$F_y = 0$$

Substituting value in (1) we get

$$\oint_C \vec{F}.\vec{dl} = \iint_R \left[0 - \frac{\partial(yz)}{\partial y}\right] dxdy = \iint_R$$

At $z = 1$ along the circle.

$$\oint_C \vec{F}.\vec{dl} = -\iint_R 1dxdy = -\iint_R dxdy = -\pi(1)^2$$

(Area of circle)

$$= -\pi$$

45. Option (a) is correct.

For real symmetric matrix all its eigen values will be real.

46. Correct answer is [0.14].

For probability proportional to no. of dots
probability of one dot showing up be x
probability of two dot showing up be $2x$
⋮
probability of six dot showing up be bx
then sum of all probabilities be 1
$\Rightarrow x + 2x + 3x + 4x + 5x + 6x = 1$

$\Rightarrow 21\,x = 1$ or $x = \frac{1}{21}$

So, probability of three dots showing up

$$= 3x = \frac{3}{21} = 0\cdot 1428$$

47. Option (c) is correct.

$$f(x) = (x-1)^{\frac{2}{3}} = \left\{(x-1)^{\frac{1}{3}}\right\}^2$$

being a square $f(x)$, will be non negative and it will be minimum when it is zero, which happens at $x = 1$

48. Option (b) is correct.

It can be represented as

$1 = \cos(2n\pi) + i\sin(2n\pi)$; n is integer.

$$1 = e^{i(2n\pi)} \quad \text{[Using Euler's Formula]}$$

$$\therefore \quad 1^i = \left[e^{i(2n\pi)}\right]^i$$

$$\Rightarrow \quad 1^i = e^{i^2(2n\pi)}$$

$$\Rightarrow \quad 1^i = e^{-(2n\pi)}$$

$\therefore$ 1^i is non–negative and real number.

49. Option (c) is correct.

Putting $x = e^t$ and $x > 0$

$$t = \ln x$$

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{dt}\left(\frac{1}{x}\right)^t$$

$$x\frac{dy}{dx} = \frac{dy}{dt} \quad ...(i)$$

$$x^2\frac{\partial^2 y}{\partial x^2} = \left(\frac{\partial^2 y}{\partial t^2} - \frac{dy}{dt}\right) \quad ...(ii)$$

From (i) and (ii) $\frac{\partial^2 y}{\partial t^2} - \frac{dy}{dt} + \frac{dy}{dt} - y = 0$

$$\frac{\partial^2 y}{\partial t^2} - y = 0$$

Auxiliary equation $m^2 - 1 = 0 \Rightarrow m = \pm 1$

Roots are real and distinct. Solution is

$$y = Ae^t + Be^{-t}$$

$$y = e^{-t}$$

$$y = Ax + B\cdot\frac{1}{x}$$

$y = \frac{1}{x}$ is a possible solution for A = 0.

50. Option (b) is correct.

$$\int_0^8 \left(\int_{y/2}^{\frac{y}{2}+1} \left(\frac{2x-y}{2}\right) dx\right) dy = ?$$

For $\frac{2x-y}{2} = u$

$\Rightarrow \quad x - \frac{y}{2} = u \quad [\therefore dx = du]$

Limit $x = \frac{y}{2}$ to $x = \frac{y}{2} + 1$

$$u = 0 \text{ to } 1$$

$\therefore$ Integral $\int_0^8 \left(\int_0^1 udu\right) dy$

For $v = \frac{y}{2}$

$$dv = \frac{dy}{2}$$

$$dy = 2dv$$

At $y = 0 \Rightarrow v = 0$

At $y = 8 \Rightarrow v = 4$

$$\therefore \quad \int_0^4 \left[\int_0^1 udu\right] 2dv = \int_0^4 \left[\int_0^1 2udu\right] dv$$

51. Correct answer is [0.4]

$$f(x) = \begin{cases} 0.2 & \text{for } |x| \le 1 \\ 0.1 & \text{for } 1 < |x| \le 4 \\ 0 & \text{otherwise} \end{cases}$$

$$f(x) = \begin{cases} 0.2 & x = (-1,1) \\ 0.1 & x = (-4,-1) \cup (1,4) \\ 0 & \text{otherwise} \end{cases}$$

Simplifying the above inequalities

$$P = \int_{0.5}^{5} f(x) = \int_{0.5}^{1} 0.2dx + \int_{1}^{4} 0.1dx$$

$$= 0.2[x]_{0.5}^{1} + 0.1[x]_1^4$$

$$= 0.2 \times 0.5 + 0.1 \times 3$$

$$P = 0.1 + 0.3 = 0.4$$

52. Option (b) is correct.

$f(x) = x^3 - 3x^2 - 24x + 100$

Polynomial function is continuous everywhere to get the critical point we have

$f'(x) = 3x^2 - 6x - 24$

For $f'(x) = 0 \Rightarrow 3(x^2 - 2x - 8) = 0$

$$x^2 - 2x - 8 = 0$$

$$\Rightarrow \quad x = 4, -2$$

But the given interval is [–3,3] so we consider the value of $x = -2$

Function value for critical point and end points

$$f(-2) = (-2)^3 - 3(-2)^2 - 24(-2) + 100$$

$$\Rightarrow \quad f(-2) = 128$$

$$f(-3) = (-3)^3 - 3(-3)^2 - 24(-3) + 100$$

$$\Rightarrow \quad f(-3) = 118$$

$$f(3) = (3)^3 - 3(3)^2 - 24(3) + 100 = 28$$

Minimum value is 28 at $x = 3$.

53. Option (c) is correct.

Here, $B = A \cdot A^T$

$\because \ \rho(AA^T) = \rho(A)$

$\therefore \ \rho(B) = \rho(A) = N$

54. Correct answer is [2].

For $v = \frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)$ m/s

$$\text{Displacement} = \int_{t=0}^{3} v\, dt = \int_0^3 \frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)dt$$

$$= \left[\sin\frac{\pi}{2}t\right]_0^3 = -1$$

Displacement = $|-1|$ = 1

$$\text{Distance} = \int_{t=0}^{3} |v|\, dt = \int_0^1 \frac{\pi}{2}\cos\frac{\pi}{2}t\,dt - \int_1^3 \frac{\pi}{2}\cos\frac{\pi}{2}t\,dt$$

$$= \left[\sin\frac{\pi}{2}t\right]_0^1 - \left[\sin\frac{\pi}{2}t\right]_1^3$$

Distance = 1 – (–1 –1) = 3

Distance – Displacement = 3 – 1 = 2

55. Option (a) is correct.

$$\nabla \cdot \left(f\vec{v}\right) = f \cdot \left(\nabla \cdot \vec{v}\right) + \nabla f \cdot \vec{v} \qquad \text{...(i)}$$

$$\nabla . \vec{v} = 0 + 0 + 0 = 0$$

$$\therefore \quad x^2y + y^2z + z^2x = f \cdot (0) + \vec{v} \cdot \nabla f$$

$$\Rightarrow \quad \vec{v} \cdot \nabla f = x^2y + y^2z + z^2x$$

56. Correct answer is [0.43]

Given, Life time of an electric bulb is random variable with probability density function

$$f(x) = kx^2$$

Using the property of random variable

$$\int_{-\infty}^{\infty} f(x)dx = 1 \qquad \text{...(i)}$$

$\because$ minimum and maximum lifetime of bulb are 1 and 2 years.

$\therefore$ equation (1) becomes $\int_1^2 f(x)dx = 1$

$$\Rightarrow \quad \int_1^2 kx^2dx = 1 \Rightarrow k\left[\frac{x^3}{3}\right]_1^2 = 1$$

$$\Rightarrow \quad k\left[\frac{8}{3} - \frac{1}{3}\right] = 1$$

$$\Rightarrow \quad k = \frac{3}{7} = 0.428 \approx 0.43$$

57. Option (c) is correct.

Given, $f(z) = \frac{z^2}{z^2 - 1}$

$z = -1, 1$ are simple pole of $f(z)$ and $z = 1$ lies inside the circle.

$$|z - 1| = 1$$

$$\therefore \quad \oint_C f(z)dz = 2\pi i\left[\underset{z=1}{\text{Res}}\, f(z)\right]$$

$$= 2\pi i\left[\lim_{z\to 1}(z-1)f(z)\right]$$

$$= 2\pi i\left[\lim_{z\to 1}\left\{(z-1) \cdot \frac{z^2}{(z+1)(z-1)}\right\}\right]$$

$$= 2\pi i\left[\lim_{z\to 1}\frac{z^2}{z+1}\right] = \pi i$$

58. Option (d) is correct.

Given that mean thickness of Si steel laminations = 0.2 mm and variance = 0.02

Vanish insulation applied both sides of laminations.

Mean thickness of one side insulation = 0.1 mm and variance = 0.01.

$\therefore$ mean thickness of core = mean thickness of Si steel + mean thickness of two side insulation. = 0.2 + 2 (0.1)

$$= 0.2 + 0.2 = 0.4 \text{ mm}$$

As 100 laminations are used so Mean thickness of core is

$$= 0.4 \times 100 = 40 \text{ mm}$$

$$\text{Var (core)} = 100 \text{ Var (steel)} + 200 \text{ Var (Insulation)}$$
$$= 100\,(0.02)^2 + 200\,(0.01)^2$$

Var (core) = 0.06

$$\therefore \quad \sigma_c = \sqrt{0.06} \approx 0.2449$$

$\therefore$ Mean thickness and variance of the core = 40 mm and 0.24.

59. Corrcet answe is [0.06].

$$f(x) = e^x - 1$$
$$f'(x) = e^x$$
$$x_0 = 1$$
$$f(x_0) = e^1 - 1$$
$$f'(x_0) = e^1$$
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$
$$= 1 - \frac{e^1 - 1}{e^1} = e^{-1} = 0.3678$$
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= \frac{1}{e} - \frac{\left(e^{\frac{1}{e}} - 1\right)}{e^{\frac{1}{e}}}$$
$$= \frac{1}{e} + \frac{1}{e^{\frac{1}{e}}} - 1$$
$$= 0.3678 + 0.6922 - 1$$
$$= 0.06$$

Therefore, absolute value of error at 2nd iteration is $|0 - 0.06| = 0.06$.

60. Option (a) is correct.

Let $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$

$$|A| = -12$$

and let $B = \begin{bmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{bmatrix}$

$$|B| = \begin{vmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{vmatrix}$$
$$= 2\begin{vmatrix} 1 & 6 & 0 \\ 2 & 12 & 8 \\ -1 & 0 & 4 \end{vmatrix}$$
$$= 2^2\begin{vmatrix} 1 & 3 & 0 \\ 2 & 6 & 8 \\ -1 & 0 & 4 \end{vmatrix}$$
$$= 2^3\begin{vmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{vmatrix}$$
$$= 8 \times (-12) = -96$$

61. Option (a) is correct.

$$\lim_{x\to 0}\left[\frac{x - \sin x}{1 - \cos x}\right] \to \frac{0}{0} \text{ form}$$

Applying L' Hospital rule

$$\lim_{x\to 0}\left[\frac{1 - \cos x}{\sin x}\right] \to \frac{0}{0} \text{ form}$$

Again applying L' Hospital rule

$$\lim_{x\to 0}\left[\frac{\sin x}{\cos x}\right] = \lim_{x\to 0} \tan x = 0$$

62. Option (c) is correct.

Given $z = \dfrac{1+i}{1-i}$

Let $z_1 = 1 + i$ and $z_2 = 1 - i$

According to argument property

$$\therefore \quad z = \frac{z_1}{z_2}$$

$$\text{Arg } z = \text{Arg } z_1 - \text{Arg } z_2$$
$$\text{Arg } z_1 = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$$
$$\text{Arg } z_2 = \tan^{-1}\left(\frac{-1}{1}\right) = \frac{-\pi}{4}$$
$$\text{Arg } z = \frac{\pi}{4} - \left(\frac{-\pi}{4}\right) = \frac{\pi}{2}$$

63. Option (a) is correct.

$$\frac{dx}{dt} = 3x - 5y$$

$$\frac{dy}{dt} = 4x + 8y$$

$$\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 3 & -5 \\ 4 & 8 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$$

64. Option (b) is correct.

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{c} = 5\hat{i} + 6\hat{j} + 4\hat{k}$

For vector to be perpendicular $\vec{a}.\vec{b} = 0$, $\vec{b}.\vec{c} = 0$, $\vec{a}.\vec{c} = 0$.

$$\vec{a}.\vec{b} = (\hat{i} + \hat{j} + \hat{k})(2\hat{i} + 3\hat{j} + \hat{k}) = 2 + 3 + 1 \neq 0$$

Clearly, vectors are not perpendicular.

For linear dependency.

$$\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 5 & 6 & 4 \end{vmatrix}$$

$$= 1(12-6) - 1(8-5) + 1(12-15)$$

$$= 6 - 3 - 3$$

$$= 0$$

$\therefore$ vectors are linearly dependent.

65. Option (c) is correct.

$$\oint_C (ydx - xdy)$$

Circle is $x^2 + y^2 = \frac{1}{4}$

Using Green's theorem

$$\oint_C (\text{M}dx - \text{N}dy) = \iint_A \left(\frac{\partial \text{N}}{\partial x} - \frac{\partial \text{M}}{\partial y}\right) dxdy$$

$\text{M} = y, \quad \text{N} = -x$

$\frac{\partial \text{M}}{\partial y} = 1 \qquad \frac{\partial \text{N}}{\partial x} = -1$

$$\iint_A (-1-1)\,dxdy = -2\iint_A dxdy$$

$\iint_A dxdy$ = Area of circle with radius is equal to 1/2.

$$= -2\left[\pi\left(\frac{1}{2}\right)^2\right] = -\frac{\pi}{2}$$

66. Correct answer is [35].

Given, DE $D^2 = 0$

AE $\rightarrow m^2 = 0$, $m = 0$, 0 real and equal roots

$y = \text{CF} + \text{PI}$

$\text{PI} = 0$

$y = (\text{C}_1 + \text{C}_2 x)e^{mx} \qquad \because \quad m = 0$

$\therefore \quad y = (\text{C}_1 + \text{C}_2 x) \qquad$ ...(i)

$at\ x = 0, y = 5$

$\therefore \quad 5 = (\text{C}_1 + \text{C}_2(0)) \Rightarrow \text{C}_1 = 5$

$\frac{dy}{dx} = \text{C}_2 \Rightarrow \text{C}_2 = 2 \qquad \left[\because \frac{dy}{dx} = 2 \text{ at } x = 10\right]$

So, $\quad y = 5 + 2x$

$y(15) = 5 + 2(15) = 35$

67. Option (d) is correct.

Given,

x	1	2	3
$p(x)$	0.3	0.6	0.1

$$\text{E}(\text{X}) = \sum_{i=1}^{n} p_i x_i$$

$$= \sum_{i=1}^{3} p_i x_i = (0.3 \times 1) + (0.6 \times 2) + (0.1 \times 3)$$

$$= 0.3 + 1.2 + 0.3 = 1.8$$

$$\text{variance}(\sigma^2) = \sum_{i=1}^{n} p_i x_i^2 - (\bar{x})^2$$

$$= \left[(0.3 \times 1^2) + (0.6 \times 2^2) + (0.1 \times 3^2)\right] - (1.8)^2$$

$$= 0.3 + 2.4 + 0.9 - 3.24 = 0.36$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{0.36} = 0.6$$

68. Correct answer is [1.111].

$$\int_{-1}^{1} |x|\,dx$$

$n = 3 =$ number of intervals.

$$h = \frac{b-a}{n} = \frac{1-(-1)}{3} = \frac{2}{3}$$

x	x_0 -1 $=-1$	x_1 $-1+\frac{2}{3}$ $=-\frac{1}{3}$	x_2 $-1+2\left(\frac{2}{3}\right)$ $=\frac{1}{3}$	x_3 $-1+3\left(\frac{2}{3}\right)$ $=1$
$y = \lvert x \rvert$	1 y_0	$\frac{1}{3}$ y_1	$\frac{1}{3}$ y_2	1 y_3

Using Trapezoidal Rule

$$\int_a^b f(x)dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+...y_{n-1})\right]$$

$$= \frac{h}{2}\left[(y_0+y_3)+2(y_1+y_2)\right]$$

$$= \frac{2/3}{2}\left[1+1+2\left(\frac{1}{3}+\frac{1}{3}\right)\right]$$

$$= \frac{1}{3}\left[2+\frac{4}{3}\right]$$

$$= \frac{1}{3}\left[\frac{6+4}{3}\right]$$

$$= \frac{10}{9} = 1.111$$

69. Option (d) is correct.

$$A = \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -5-\lambda & 2 \\ -9 & 6-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-5-\lambda)(6-\lambda)+18 = 0$$

$$\Rightarrow \lambda^2 - \lambda - 12 = 0$$

$$\lambda = 4, -3$$

$$\lambda = -3$$

$$\begin{bmatrix} -2 & 2 \\ -9 & 9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + 2x_2 = 0 \Rightarrow 2x_1 = 2x_2$$

$$-9x_1 + 9x_2 = 0 \Rightarrow 9x_1 = 9x_2$$

$$x_1 = x_2$$

Therefore, eigen vector is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

70. Option (b) is correct.

$$\lim_{x\to 0}\frac{e^{2x}-1}{\sin(4x)} = \frac{e^{2x}-1}{2x}\frac{2x}{\sin(4x)}$$

$$= \lim_{x\to 0}\underbrace{\frac{e^{2x}-1}{2x}}_{1}\lim_{x\to 0}\frac{2x}{\sin(4x)}$$

$$\left[\because \lim_{x\to 0}\left(\frac{e^x-1}{x}\right) = 1\right]$$

$$= \lim_{x\to 0}\frac{2x}{\sin(4x)}\times\frac{2}{2}$$

$$= \frac{4x}{\sin(4x)}\left(\frac{1}{2}\right) = 0.5 \quad \left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

71. Option (a) is correct.

$$\vec{F} = x^2z^2\hat{i} - 2xy^2z\hat{j} + 2y^2z^3\hat{k}$$

$$curl = \nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2z^2 & -2xy^2z & 2y^2z^3 \end{vmatrix}$$

$$= \hat{i}\left[4yz^3 + 2xy^2\right] - \hat{j}\left[0 - 2x^2z\right] + \hat{k}\left[-2y^2z - 0\right]$$

$$= \left[4yz^3 + 2xy^2\right]\hat{i} + \left[2x^2z\right]\hat{j} - \left[2y^2z\right]\hat{k}$$

72. Option (a) is correct.

$$\text{Probability}(P) = \frac{\text{Favourable cases}}{\text{Total no. of cases}}$$

$$\frac{\text{Selecting two parts from 15 non = defective parts}}{\text{Selecting two parts from overall 25 parts}} = \frac{^{15}C_2}{^{25}C_2} = \frac{105}{300} = \frac{7}{20}$$

73. Correct answer is [–0.954].

$$f(x) = e^{-x}\sin(100x) \quad \text{for } x \ge 0$$

$$f'(x) = e^{-x}\cos(100x)(100) - e^{-x}\sin(100x)$$

For minima

$$f'(x) = 0 \Rightarrow e^{-x}\cos(100x)(100) - e^{-x}$$

$$\sin(100x) = 0$$

$$\Rightarrow 100\cos(100x) - \sin(100x) = 0$$

$$\Rightarrow \tan(100x) = 100$$

$\Rightarrow 100\ x = \tan^{-1} 100$, as $\tan\theta$ has cycle of π

$$100x = \tan^{-1}(100) + \pi = 4.702$$

$$x = 0.047$$

$$\therefore f(x) = e^{-0.047}\cdot\sin(100(0.047)) = -0.954$$

74. Option (c) is correct.

$$u = 2xy$$

C R equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \Rightarrow 2y = \frac{dv}{dy}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad \Rightarrow 2x = -\frac{dv}{dx}$$

$$\Rightarrow \quad v(x,y) = y^2 + \phi(x) + c \quad ...(i)$$

$$\Rightarrow \quad v(x,y) = -x^2 + \phi(y) + c \quad ...(ii)$$

From (i) and (ii)

$$v(x,y) = -x^2 + y^2 + c$$

75. Option (d) is correct.

$$\frac{dy}{dx}=\cos(x+y)$$

Let $(x+y)=t$

Differentiating wrt x

$$\Rightarrow 1+\frac{dy}{dx}=\frac{dt}{dx}$$

$$\frac{dy}{dx}=\frac{dt}{dx}-1$$

$$\frac{dt}{dx}-1=\cos t$$

$$\frac{dt}{dx}=\cos t+1$$

(variable separable form)

$$\frac{dt}{dx}=2\cos^2\frac{t}{2}$$

$$\int\frac{dt}{2\cos^2\frac{t}{2}}=\int dx$$

$$\frac{1}{2}\int\left(\sec^2\frac{t}{2}\right)dt=\int dx$$

$$\tan\frac{t}{2}=x+c$$

$$\tan\frac{(x+y)}{2}=x+c$$

76. Correct answer is [0.2592].

Probability of obtaining red colour on top face

$$p=\frac{2}{6}=\frac{1}{3}$$

Probability of not obtaining red colour on top face

$$\bar{p}=q=1-p=1-\frac{1}{3}=\frac{2}{3}$$

Apply binomial theorem for red color appearing at least twice

$$={}^3C_2\left(\frac{1}{3}\right)^2\frac{2}{3}+{}^3C_3\left(\frac{1}{3}\right)^3=3\left(\frac{1}{3}\right)^2\frac{2}{3}+\left(\frac{1}{3}\right)^3$$

$$=\frac{7}{27}=0.2592$$

77. Correct answer is [1.7533].

$$h=\frac{4-2.5}{5}=0.3$$

x	2.5	2.8	3.1	3.4	3.7	4
$y=\ln x$	0.9163 y_0	1.0296 y_1	1.1314 y_2	1.2237 y_3	1.3083 y_4	1.3863 y_5

Using trapezoidal rule

$$\int_{2.5}^{4}\ln(x)dx=\frac{h}{2}\left[(y_0+y_n)+2(y_0+y_1+y_2+...y_{n-1})\right]$$

$$=\frac{h}{2}\left[(y_0+y_5)+2(y_1+y_2+y_3+y_4)\right]$$

$$=\frac{0.3}{2}\left[(0.9163+1.3863)+2(1.0296+1.1314+1.2237+1.3083)\right]$$

$$=1.7533$$

78. Option (d) is correct.

According to property : For distinct eigen values of real symmetric matrix the eigen vectors will be orthogonal.

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}^T\cdot\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=x_1y_1+x_2y_2+x_3y_3=0$$

79. Option (d) is correct.

If a function $f(x)$ is said to be continuous at a point then the limit must exist at the point and value of limit should be same as the value of function at that point.

80. Option (c) is correct.

$$\text{Div}\left(x^2z\hat{i}+xy\hat{j}-z^2y\hat{k}\right)$$

$$=\frac{d}{dx}(x^2z)+\frac{d}{dy}(xy)+\frac{d}{dz}(-z^2y)=2xz+x-2zy$$

$$\text{Div}\left(x^2z\hat{i}+xy\hat{j}-x^2z\hat{k}\right)_{(1,-1,1)}$$

$$=2(1)(1)+(1)-2(1)(-1)=5$$

81. Correct answer is [0.65]

Probability of selected person being employed = (P) = Probability of selecting a woman × Probability of woman being employed + Probability of selecting a man × Probability of man being employed

$$=(0.5)\times(0.5)+(0.5)\times(0.8)$$

$$P=0.25+0.40$$

$$P=0.65$$

82. Correct answer is [1.165].

Given step size $h=1$

x	1	2	3
$y=\frac{1}{x}$	1 y_0	0.5 y_1	0.33 y_2

Applying Trapezoidal rule

$$\int_1^2\frac{1}{x}dx=\frac{h}{3}\left[(y_0+y_2)+2y_1\right]$$

$$=\frac{1}{2}\left[(1+0.33)+2(0.5)\right]=1.165$$

83. Option (c) is correct.

$f(z) = u(x,y) + iv(x,y)$

$u(x,y) = x^2 - y^2$

For analytic function

CR Equations will be satisfied

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$\frac{dv}{dy} = 2x$ and $-\frac{dv}{dx} = -2y$

$v = \int \frac{dv}{dy} dy = \int (2x) dy + \phi(x) + c$

$v = 2xy + c + \phi(x)$ ...(i)

$v = \int \frac{\partial v}{\partial x} \cdot dx$

$= \int 2y\, dx + \psi(y) + c$

$= 2xy + \psi(y) + c$ ...(ii)

$\therefore\ v = 2xy + c$ [from (i) and (ii)]

84. Option (a) is correct.

Given DE is $(D^2 + 1)x = 0$

$AE \rightarrow m^2 + 1 = 0$

$\Rightarrow\ m = \pm i$

Complex conjugate roots

$\therefore\ x = C_1 \cos t + C_2 \sin t$

Let $x_1(t) = \cos t$ and $x_2(t) = \sin t$

At $t = 0$ $x_1(t) = 1, \frac{dx_1}{dt} = 0; x_2(t) = 0, \frac{dx_2}{dt} = 1$

$$W(t) = \begin{vmatrix} x_1(t) & x_2(t) \\ \frac{dx_1}{dt} & \frac{dx_2}{dt} \end{vmatrix}$$

$$W(t) = \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix}$$

$W(t) = \cos^2 t + \sin^2 t$

$W(t) = 1$

85. Option (a) is correct.

x_i	0	1	2
P_i	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$

Mean $\mu = \sum x_i P_i = 0 \times \frac{1}{6} + 1 \times \frac{2}{3} + 2 \times \frac{1}{6}$

$= \frac{2}{3} + \frac{1}{3}$

$\mu = 1$

$\text{Var}(x) = E(x^2) - \mu^2$

$E(x^2) = 0^2 \times \frac{1}{6} + 1^2 \times \frac{2}{3} + 2^2 \times \frac{1}{6}$

$= \frac{2}{3} + \frac{4}{6} = \frac{4}{3}$

$\text{Var}(x) = \frac{4}{3} - 1^2$

$\text{Var}(x) = \frac{1}{3}$

86. Correct answer is [0.54].

Given $5x - 2\cos x - 1 = 0$

Let $f(x) = 5x - 2\cos x - 1$

$f'(x) = 5 + 2\sin x$

Assuming $x_0 = 1$

Applying Newton Raphson method.

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 1 - \frac{5 \times 1 - 2\cos[1] - 1}{5 + 2\sin[1]}$

$x_1 = 0.5632$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.5632 - \frac{5 \times 0.5632 - 2\cos[0.5632] - 1}{5 + 2\sin[0.5632]}$

$x_2 = 0.5425$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 0.5425 - \frac{5 \times 0.5425 - 2\cos(0.5425) - 1}{5 + 2\sin(0.5425)}$

$x_3 = 0.5425 = 0.54$

87. Option (d) is correct.

The valid identity for arbitrary real 3 × 3 matrix P, Q and R.

$(P + Q)^2 = P^2 + Q \cdot P + P \cdot Q + Q^2$

$(P + Q)^2 = P \cdot P + P \cdot Q + Q \cdot Q + Q \cdot P$

88. Option (b) is correct.

Given $\int_0^2 \frac{(x-1)^2 \sin(x-1)}{(x-1)^2 + \cos(x-1)} dx$

Putting $(x - 1) = z \Rightarrow dx = dz$

New limits of integration becomes

$\int_{-1}^{1} \frac{z^2 \sin z}{z^2 + \cos z} dz$

$$f(z) = \frac{z^2 \sin z}{z^2 + \cos z}$$

$$f(-z) = \frac{-z^2 \sin z}{z^2 + \cos z}$$

$\because \quad \sin(-z) = -\sin z$ and $\cos(-z) = \cos z$

$f(z) = f(-z)$ clearly the function is odd, and integration of odd function is zero integrating from $-a$ to a

89. Option (b) is correct.

$$\frac{dy}{dx} + 2xy = 0$$

This is in the form of

$\frac{dy}{dx} + \text{Py} = \text{Q}$with P = $2x$, Q = 0

$$\text{IF} = e^{\int \text{P}dx} = e^{\int 2x dx} = e^{x^2}$$

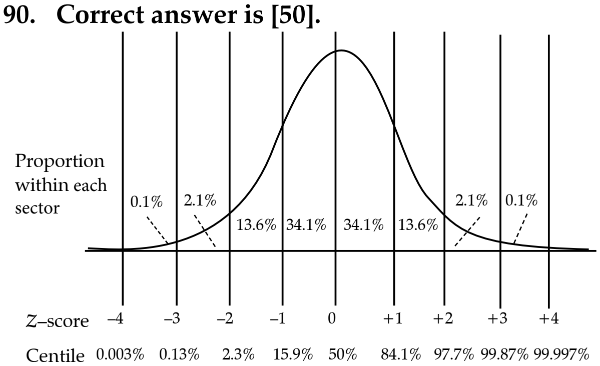
Solution of DE $\rightarrow y(\text{IF}) = \int \text{Q}(\text{IF})dx + \text{C}$

$$ye^{x^2} = 0 + \text{C} \Rightarrow y = \text{C}e^{-x^2}$$

$$at\ x = 0,\ y = 2 \Rightarrow \text{C} = 2$$

$$y = 2e^{-x^2}$$

90. Correct answer is [50].



In Normal distribution $z = \frac{x - \mu}{\sigma}$

Given $x = 500$, $\mu = 500$ and $\sigma = 50$

$$\therefore\ z = \frac{500 - 500}{50} = 0$$

$$z = 0 \Rightarrow \text{P}(\text{X} > 500) = \text{P}(\text{Z} > 50) = 50\%$$

91. Option (d) is correct.

For $\quad \text{L}\{e^{-2t} \cos 4t\}$

We know $\text{L}\{e^{-at} \cos bt\} = \frac{s + a}{(s + a)^2 + b^2}$

$$\therefore \text{L}\{e^{-2t} \cos 4t\} = \frac{s + 2}{(s + 2)^2 + 16}$$

92. Option (b) is correct.

$$\int_5^{3i} \frac{dz}{z} = [\ln z]_5^{3i} = \ln 3i - \ln 5$$

$$= \ln(0 + 3i) - \ln(5 - 0i)$$

$$\because \ln z = \frac{1}{2}\ln(x^2 + y^2) + i \tan^{-1}\left(\frac{y}{x}\right)$$

$$[\ln z]_5^{3i} = \frac{1}{2}\ln(0 + 9) + i \tan^{-1}\left(\frac{3}{0}\right)$$

$$- \frac{1}{2}\ln(25 + 0) - i \tan^{-1}\left(\frac{0}{5}\right)$$

$$= \frac{1}{2}\ln 9 + i \tan^{-1}(\infty) - \frac{1}{2}\ln 25 - 0$$

$$= \ln \frac{3}{5} + i \frac{\pi}{2}$$

$$= -0.511 + 1.57i$$

93. Option (b) is correct.

$$\int_0^2 \int_0^x e^{(x+y)} dy\, dx$$

$$= \int_0^2 \left[e^{(x+y)}\right]_0^x dx = \int_0^2 \left(e^{2x} - e^x\right) dx$$

$$= \left[\frac{e^{2x}}{2}\right]_0^2 - \left[e^x\right]_0^2$$

$$= \left[\frac{e^4 - 1}{2}\right] - \left[e^2 - 1\right]$$

$$= \frac{1}{2}e^4 - e^2 + \frac{1}{2}$$

$$= \frac{1}{2}\left(e^4 - 2e^2 + 1\right)$$

$$= \frac{1}{2}\left(e^2 - 1\right)^2$$

94. Option (b) is correct.

According to Poisson distribution probability of occurrence of an event is given by relation.

$$\text{P}(x) = \frac{\mu^x e^{-\mu}}{x!}$$

Here μ = mean

Probability of occurrence of less than two accidents.

$\text{P}(x < 2) = \text{P}(x = 0) + \text{P}(x = 1)$

$$= \frac{\mu^0 e^{-\mu}}{0!} + \frac{\mu^1 e^{-\mu}}{1!}$$

$$= \frac{1}{e^\mu} + \frac{\mu}{e^\mu}$$

$$= \frac{1 + \mu}{e^\mu}$$

$$= \frac{1 + 5.2}{e^{5.2}} = 0.0342$$

95. Option (d) is correct.

Given DE is $\frac{dx}{dt} = 4t + 4 = f(t, x)$

According to Runge-Kutta 4^{th} order multi step method.

$$x_{n+1} = x_n + k$$

$$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$k_1 = h\text{F}(t_n, x_n)$$

$$k_2 = h\text{F}\left(t_n + \frac{h}{2},\ x_n + \frac{k_1}{2}\right)$$

$$k_3 = h\text{F}\left(t_n + \frac{h}{2},\ x_n + \frac{k_2}{2}\right)$$

$$k_4 = h\text{F}(t_{n+h},\ x_n + k_3)$$

$$\frac{dx}{dt} = 4t + 4 \text{ step size } h = 0.2 \text{ at } t = 0$$

$$k_1 = 0.2[4(0) + 4] = 0.8$$

$$k_2 = 0.2\left[4\left(0 + \frac{0.2}{2}\right) + 4\right] = 0.88$$

$$k_3 = 0.2\left[4\left(0 + \frac{0.2}{2}\right) + 4\right] = 0.88$$

$$k_4 = 0.2[4(0 + 0\cdot 2) + 4] = 0.96$$

$$k = \frac{1}{6}[0.8 + 2(0.88) + 2(0.88) + 0.96] = 0.88$$

$$x = x_0 + 0.88$$

Increment is 0.88

96. Option (c) is correct.

$$\lim_{x\to\infty}\left(\frac{x + \sin x}{x}\right) = \lim_{x\to\infty}\left(\frac{x}{x} + \frac{\sin x}{x}\right)$$

$$= \lim_{x\to\infty}\left(1 + \frac{\sin x}{x}\right)\left[\begin{array}{l}\sin x \text{ will have}\\ \text{a finite value}\end{array}\right]$$

$$= 1 + 0 = 1$$

97. Correct answer is [23].

$$\text{K}^\text{T}\text{JK} = [1 \quad 2 \quad -1]\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$= [6 \quad 8 \quad -1]\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$= 6 + 16 + 1 = 23$$

98. Correct answer is [0.4].

$$\text{Given, Pdf } f(\text{E}) = \begin{cases} \frac{1}{5} & 0 \le \text{E} \le 5 \\ 0 & \text{otherwise} \end{cases}$$

Required probability

$$p = \int_2^4 f(\text{E})\,d\text{E} = \int_2^4 \frac{1}{5}\,d\text{E} = \frac{1}{5}[\text{E}]_2^4 = \frac{4-2}{5} = 0.4$$

99. Option (a) is correct.

Sum of eigen values of matrix is = Sum of diagonal element of matrix = $215 + 150 + 550 = 915$

100. Option (a) is correct.

$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 2 & 4 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}|1(2-3) - 2(0-3) + 4(0-2)|$$

$$= \frac{1}{2}|-3| = \frac{3}{2}$$

101. Correct answer is [0.265].

$$\text{PDF P}(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \lambda = 5 \qquad \text{(Given)}$$

$$\text{P}(x < 4) = \text{P}(x = 0) + \text{P}(x = 1) + \text{P}(x = 2) + \text{P}(x = 3)$$

$$= \frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^2}{2!} + \frac{e^{-5}5^3}{3!}$$

$$= e^{-5}\left(1 + 5 + \frac{25}{2} + \frac{125}{6}\right) = 0.265$$

102. Option (b) is correct.

Each time when coin is tossed out comes is independent of previous outcomes. Favourable outcome is tail.

Total outcome = 2 (Head or Tail)

$$\text{Probability} = \frac{n(\text{E})}{n(\text{S})} = \frac{1}{2}$$

103. Correct answer is [88].

$$\text{Det} = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix}$$

Expanding along row 2

$$= -1\begin{vmatrix} 1 & 2 & 3 \\ 3 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix} - 3\begin{vmatrix} 0 & 1 & 3 \\ 2 & 3 & 1 \\ 3 & 0 & 2 \end{vmatrix}$$

$$= -[1(0-1) - 2(6-0) + 3(3-0)] - 3[0 - 1(4-3) + 3(0-9)]$$

$$= 88$$

104. Option (b) is correct.

Given, $z = \frac{2-3i}{-5+i}$

$= \frac{2-3i}{-5+i} \times \frac{-5-i}{-5-i}$

$= \frac{-10-2i+15i-3}{5^2-i^2}$

$= \frac{-13+13i}{26}$

$= \frac{13}{26}(i-1)$

$= -0.5+0.5i$

105. Option (d) is correct.

$\frac{dP}{dt} + k_2 P = k_1 L_0 e^{-k_1 t}$

This D.E. can be written in standard form as–

$\frac{dy}{dx} + Py = Q$ with $P = k_2$, $Q = k_1 L_0 e^{-k_1 t}$

$IF = e^{\int P dx}$

$IF = e^{\int k_2 dt} = e^{k_2 t}$

106. Option (b) is correct.

The given random variable has attributes of Normal RV with a as mean and b as standard deviation

$\therefore \int_{-\infty}^{\infty} f(x)dx = 1$

$\int_{-\infty}^{a} f(x)dx = 0.5$ as $P(x < a) = 0.5$

107. Option (a) is correct.

$\lim_{\alpha \to 0} \frac{x^\alpha - 1}{\alpha} \to \frac{0}{0}$ form

Applying L hospital rule.

Differentiating w.r.t. to α

$\lim_{\alpha \to 0} \frac{x^\alpha . \log x}{1} = \log x$

108. Correct answer is [0.27].

Given Average no. of vehicles/hr.

$\lambda = 240/hr$

$= \frac{240}{60} / \min 2/30 \sec$

$P(x=1) = \frac{e^{-2}(2)^1}{1!} = 0.270$

109. Correct answer is [2].

$$\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -10 \end{bmatrix}$$

$R_3 \to R_3 - 2R_1 + R_2$

$$\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

No. of non–zero rows = 2. So, rank = 2

110. Option (d) is correct.

The statement "Not all that glitters is gold" can be written as follows:

$\neg(\forall x (\text{glitters}(x) \Rightarrow \text{gold}(x))$ ...(i)

Where $\forall x$ (glitters(x) $\Rightarrow$ gold(x) means that all glitters is gold.

$\exists x \neg(\text{glitters}(x) \Rightarrow \text{gold}(x))$, ...(ii)

$[\because \neg\forall x\, p(x) \equiv \exists x \neg p(x)]$

$A \Rightarrow B = \neg A \vee B$ (negation A or B) ...(iii)

From equation (ii) and (iii), we have

$\exists x(\neg(\neg \text{glitters}(x) \vee \text{gold}(x)))$

$\Rightarrow \exists x(\text{glitters}(x) \wedge \neg \text{gold}(x))$ ...(iv)

Negation cancellation

$\neg(\neg) = ()$: and $\neg(() \vee ()) = (\neg() \wedge \neg())$.

So, option (d) is correct.

111. Correct answer is [0.25].

The length of the shorter stick can be from 0 to 0.5 for a stick of unit length.

The random variable L follows a uniform distribution,

$\therefore$ Pdf of L is $P(L) = \frac{1}{0.5-0} = 2$; for all lengths in range 0 to 0.5.

$\therefore E(L) = \int_0^{0.5} L * P\{L\} dL = \int_0^{0.5} L * 2dL = 2 * \left[\frac{L^2}{2}\right]_0^{0.5}$

$= 0.25$

112. Correct answer is [1].

Here,

$$[A|B] = \begin{bmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 7 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & -2 & 7 & 0 \end{bmatrix}$$

$R_4 \to R_4 - (R_2 - R_1)$

$$[A|B] = \begin{bmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 7 & 1 \\ 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$[A|B] = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 4 & 0 & 7 & 1 \\ 3 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 4R_1,\ R_3 \to R_3 - 3R_1$

$$[A|B] = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -4 & 3 & -11 \\ 0 & -1 & -3 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 - \frac{1}{4}R_2$

$$[A|B] = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -4 & 3 & -11 \\ 0 & 0 & \frac{-15}{4} & \frac{-21}{4} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of A = Rank of [A|B] = no. of unknowns = 3

So, system of linear equation has unique solution

113. Correct answer is [0].

Since, the eigen vectors corresponding to different eigen values of a real symmetric matrix are orthogonal to each other. Therefore, dot product of orthogonal vectors is zero.

114. Correct answer is [–2].

For $f(x) = x\sin x$

$f'(x) = x\cos x + \sin x$

$f''(x) = x(-\sin x) + \cos x + \cos x$

Given $f''(x) + f(x) + t\cos x = 0$

$\Rightarrow x(-\sin x) + \cos x + \cos x + x\sin x + t\cos x = 0$

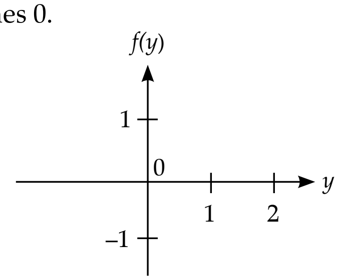
$\Rightarrow 2\cos x + t\cos x = 0$

$\Rightarrow (t+2)\cos x = 0$

$\Rightarrow t + 2 = 0, t = -2$

115. Option (a) is correct.

The function is continuous, there must be point between 0 and 1 where it becomes 0 and there must also be a point between 1 and 2 where it becomes 0.



116. Correct answer is [10].

There can be two possible sets whose elements sum is 22 : $\{6,6,6,4\}, \{6,6,5,5\}$

Arrangements from the first set can be done in

$= \frac{4!}{3!}$ = 4 ways

Arrangements from the second set can be done in $= \frac{4!}{2! \times 2!} = 6$ ways

So, total no. of ways of sum being 22 = 10

So, Probability $= \frac{10}{6^4} = \frac{10}{1296} = \frac{x}{1296}$

$\therefore \quad x = 10$

117. Correct answer is [89].

Let n pennants are represented by $f(n)$. so, $f(10)$ is number of 10 pennants.

So, 10 pennants can be formed by 8–pennants and 9 pennants by adding 1 or 2 respectively.

$f(10) = f(9) + f(8)$

So, the observation is a series, in which $f(1) = 1, f(2) = 2$ So, this becomes.

$1, 2, 3, 5, 8, 13, 21, 34, 55, 89 \ldots$

So, $f(10) = 89$

118. Correct answer is [16].

For set $f : \{0, 1\}^4 \to \{0, 1\}$

Size of domain D = 2^4 = 16

Size of range R = 2

$|S| = (2)^{2^4} = 2^{16}$

Total number of functions will be $N = 2^{|S|}$

$\therefore \log_2 \log_2 N = \log_2 \log_2 2^{|S|} = \log_2 \log 2^{2^{2^4}}$

$= 2^4 = 16$

119. Correct answer is [11.90].

Total no. of computers n = 10

No. of working computer r = 4

No. of non–working computers = 6

Probability system is deemed functional $p = p$ (at least working computer inspected)

$p = p$ (three working) + p (four working)

$= \frac{{}^4C_3 \cdot {}^6C_1}{{}^{10}C_4} + \frac{{}^4C_4 \cdot {}^6C_0}{{}^{10}C_4}$

$= \frac{24+1}{210} = 0.119$

$\therefore \quad 100\,p = 11.9$

120. Correct answer is [3.9].

From total nine words

No. of words with length three is 4

No. of words with length four is 2

No. of words length five is 3

x : length of word

x	3	4	5
$p(x)$	4/9	2/9	3/9

[$\because$ total words in 9 and each word is equally likely to be choosen]

$\therefore E(x) = \sum x_i \, p(x_i)$

$= 3 \cdot \frac{4}{9} + 4 \cdot \frac{2}{9} + 5 \cdot \frac{3}{9} = \frac{35}{9} = 3.888$

121. Correct answer is [36].

Maximum number of edges occur in a complete bipartite graph *i.e.* when every vertex of the first set is connected every vertex of the second set.

No. of edges in a complete bipartite graph

$\frac{n^2}{4} = \frac{12^2}{4} = 36$

122. Correct answer is [0].

Matrix will be a 3×3 matrix where 1^{st} row will be $2[1 \quad 9 \quad 5]$, 2^{nd} row will be $-4[1 \quad 9 \quad 5]$ and third row will be $7[1 \quad 9 \quad 5]$

With linearly dependent rows the determinant value will be zero.

123. Option (a) is correct.

The roots are $x = 1, x = 2$ and $x = 3$.

So, polynomial is

$$f(x) = k(x-1)(x-2)(x-3)$$

$$f(0) = -6k, f(4) = 6k$$

$$\therefore f(0) \cdot f(4) = (-6k)(6k) = -36k^2 < 0$$

124. Option (a) is correct.

$$f(x) = 0.75x^3 - 2x^2 - 2x + 4$$

$$f'(x) = 2.25x^2 - 4x - 2$$

$$x_0 = 2$$

$$f(x_0) = 0.75(2)^3 - 2(2)^2 - 2(2) + 4 = -2$$

$$f'(x_0) = 2.25(2)^2 - 4(2) - 2 = -1$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{(-2)}{(-1)} = 0$$

$$f(x_1) = 0.75(0)^3 - 2(0)^2 - 2(0) + 4 = 4$$

$$f'(x_1) = 2.25(0)^2 - 4(0) - 2 = -2$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0 - \frac{4}{-2} = 2$$

$$f(x_2) = 0.75(2)^3 - 2(2)^2 - 2(2) + 4 = -2$$

$$f'(x_2) = 2.25(2)^2 - 4(2) - 2 = -1$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2 - \frac{-2}{-1} = 0$$

So, Newton-Raphson method Diverges if $x_0 = 2$

$\therefore$ Only statement 1 is true.

125. Correct answer is [6].

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

According to definition of eigen vectors

$$AX = \lambda X$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$x_1 + x_5 = \lambda x_1$$

$$x_2 + x_3 + x_4 = \lambda x_2$$

$$x_2 + x_3 + x_4 = \lambda x_3$$

$$x_2 + x_3 + x_4 = \lambda x_4$$

$$x_1 + x_5 = \lambda x_5 \Rightarrow x_1 + x_5 = \lambda x_5 = \lambda x_1$$

$$x_2 + x_3 + x_4 = \lambda x_2 = \lambda x_3 = \lambda x_4$$

if $\lambda \neq 0$ say $x_1 = x_5 = a$

$$x_2 = x_3 = x_4 = b$$

$$\Rightarrow x_1 + x_5 = \lambda a \Rightarrow 2a = \lambda a \Rightarrow \lambda = 2$$

$$x_2 + x_3 + x_4 = \lambda a \Rightarrow 3a = \lambda a \Rightarrow \lambda = 3$$

$\therefore$ Three different eigen values are 0, 2, 3 and the product of non–zero eigen values = $2 \times 3 = 6$.

126. Correct answer is [0.26].

Number of integers divisible by 2 = 50.
Number of integers divisible by 3 = 33
Number of integers divisible by 5 = 20.
Number of integers divisible by 2 and 3 = 16
Number of integers divisible by 2 and 5 = 10.
Number of integers divisible by 3 and 5 = 6.
Number of integers divisible by 2 and 3 and 5 = 3
Total numbers divisible by =
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
2 or 3 or 5 = 50 + 33 + 20 – 16 – 10 – 6 + 3 = 74
Total number not divisible 2 or 3 or 5
= 100 – 74 = 26

Probability $= \frac{26}{100} = 0.26$

127. Correct answer is [8].

Prime factorisation of = 2014 = 2 x 19 x 53

Selection of these factors can be done in two ways each, So, total no. of positive integral factors are

$= 2 \times 2 \times 2 = 8$

128. Option (d) is correct.

Breaking the compound statements into atomic statements.

A : Good mobile phones.

B : Cheap mobile phones.

$P : (A \to \neg B) \equiv (\neg A \vee \neg B)$

$Q : (B \to \neg A) \equiv ((\neg B \vee \neg A) \equiv \neg A \vee \neg B))$

Hence, $(P \equiv Q)$ it means $(P \to Q)$ and $(Q \to P)$.

129. Option (d) is correct.

Let $X = \{a, b, c\}$, $Y = \{1, 2\}$

As each element of X will be mapped to exactly one element of Y for $f : X \geq Y$

Let $A = \{a, c\}$, $B = \{b, c\}$ be subset of X.

with $f(a) = 1$, $f(b) = 1$ and $f(c) = 2$

Then $|f(A \cup B)| = 2; |f(A)| = 2; |f(B)| = 2$

$$|f(A \cup B) \neq |f(A)| + |f(B)|$$

$f(A \cap B) = \{2\}; f(A) = \{1, 2\}; f(B) = \{1, 2\}$

$f(A) \cap f(B) = \{1, 2\}$

$f(A \cap B) \neq f(A) \cap f(B)$

$|f(A \cap B)| = 1, |f(A \cap B)| \neq \min\{|f(A)|, |f(B)|\}$

$\therefore$ Option (a), (b), (c) are incorrect. Only (d) is correct.

130. Correct answer is [5].

Lagrange's theorem states that the order of subgrouo should divide the order of group.

G has 15 elements.

Factors of 15 are 1,3,5, and 15.

$\because$ the size of L is atleast 4 (1 and 3 are eliminated) and not equal to G (15 is eliminate), only size left is 5.

131. Option (a) is correct.

When the determinant of the matrix is negative then at least one of its eigen values is negative.

As determinant = product of eigen values.

132. Option (a) is correct.

$$\int_0^{2\pi} |x \sin x| dx = k\pi, \text{ given}$$

$$\Rightarrow \int_0^{\pi} x \sin x dx + \int_{\pi}^{2\pi} -(x \sin x) dx = k\pi$$

$$\begin{bmatrix} \because \sin x < 0, \ \pi < x < 2\pi \\ \sin > 0, \ 0 < x < \pi \end{bmatrix}$$

$$\Rightarrow \left[x(-\cos x) - 1(-\sin x)\right]_0^{\pi}$$

$$-[-x \cos x + \sin x]_{\pi}^{2\pi} = k\pi$$

$$\Rightarrow \pi + 0 - [-2\pi + 0 - (\pi + 0)] = k\pi$$

$$\Rightarrow 4\pi = k\pi \Rightarrow k = 4$$

133. Option (c) is correct.

If the integrand is concave up then the error is negative and the trapezoidal rule overestimates the true value. Since the given function is a polynomial of degree 2 with positive coefficient the error = True value = Approx. value will be less than zero.

If the degree of the polynomial is < = 3 Simpson's will provide the exact value, Since the given function is a polynomial of degree 2 so Simpson will give the exact value.

134. Option (a) is correct.

$$\int_0^{\pi} x^2 \cos x dx$$

$$= \left[x^2 (\sin x) - 2x(-\cos x) + 2(-\sin x)\right]_0^{\pi}$$

$$= \left(\pi^2 (\sin \pi) - 2\pi(-\cos \pi) + 2(-\sin \pi)\right) - (0 + 0 + 0)$$

$$= -2\pi$$

135. Correct answer is [0.25].

Given, $A \cup B = S$

$P(A \cup B) = P(S) = 1$

$P(A) + P(B) = 1 \quad \begin{bmatrix} \because \text{A and B are mutually} \\ \text{exclusive } P(A \cap B) = \phi \end{bmatrix}$

$\Rightarrow P(B) = 1 - P(A)$

Let $P(A) = x$, $f(x) = P(A) \cdot P(B)$

Let $f(x) = x(1 - x) = x - x^2$

For maximum $f'(x) = 0 \Rightarrow 1 - 2x = 0 \Rightarrow x = \frac{1}{2}$

At $x = \frac{1}{2}$ the maximum value

$$f\left(\frac{1}{2}\right) = \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{4} = 0.25$$

136. Option (b) is correct.

Consider the function as

$f(0) = 1$, $f(1) = 0$, $f(2) = 3$

$f(3) = 2$ ....$f(2012) = 2013$

For $f(i) = k, f(f(i)) = f(k)$

For some cases we may have $f(k) = i$ and this may not be a case for all i. As the domain and co-domain are same, the function will be onto function.

Therefore, only Q and R are true.