ENGINEERING MATHEMATICS GATE Question Paper 2015

ELECTRONICS AND COMMUNICATION (EC) P1

Q. 1. Consider a system of linear equations :

$x - 2y + 3z = -1$

$x - 3y + 4z = 1$, and

$-2x + 4y - 6z = k$

The value of k for which the system has infinitely many solutions is ______.

Q. 2. A function $f(x) = 1 - x^2 + x^3$ is defined in the closed interval $[-1, 1]$. The value of x, in the open interval $(-1, 1)$ for which the mean value theorem is satisfied, is

(a) $\frac{-1}{2}$ **(b)** $\frac{-1}{3}$

(c) $\frac{1}{3}$ **(d)** $\frac{1}{2}$

Q. 3. Suppose A and B are two independent events with probabilities $P(A) \neq 0$ and $P(B) \neq 0$. Let $\overline{A}$ and $\overline{B}$ be their complements. Which one of the following statements is FALSE?

(a) $P(A \cap B) = P(A)\,P(B)$

(b) $P(A/B) = P(A)$

(c) $P(A \cup B) = P(A) + P(B)$

(d) $P\left(\overline{A} \cap \overline{B}\right) = P(\overline{A})P(\overline{B})$

Q. 4. Let $z = x + iy$ be a complex variable. Consider that contour integration is performed along the unit circle in anticlockwise direction. Which one of the following statements is NOT TRUE?

(a) The residue of $\frac{z}{z^2 - 1}$ at $z = 1$ is $\frac{1}{2}$

(b) $\oint_c z^2 dz = 0$

(c) $\frac{1}{2\pi i}\oint_c \frac{1}{z} dz = 1$

(d) $\bar{z}$ (complex conjugate of z) is an analytical function

Q. 5. The value of p such that the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix} 4 & 1 & 2 \\ p & 2 & 1 \\ 14 & -4 & 10 \end{bmatrix}$ is ______.

Q. 6. The solution of the differential equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0$ with $y(0) = y'(0) = 1$ is

(a) $(2 - t)e^t$ **(b)** $(1 + 2t)e^{-t}$

(c) $(2 + t)e^{-t}$ **(d)** $(1 - 2t)e^t$

Q. 7. A vector $\vec{P}$ is given by $\vec{P} = x^3 y\vec{a}_x - x^2y^2\vec{a}_y - x^2yz\vec{a}_z$. Which one of the following statements is **TRUE**?

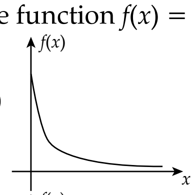
(a) $\vec{P}$ is solenoidal, but not irrotational

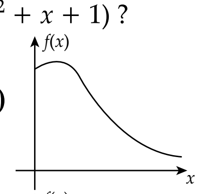
(b) $\vec{P}$ is irrotational, but not solenoidal

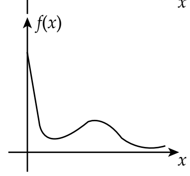
(c) $\vec{P}$ is neither solenoidal nor irrotational

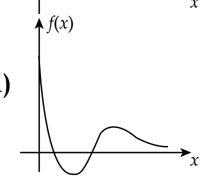
(d) $\vec{P}$ is both solenoidal and irrotational

Q. 8. Which one of the following graphs describes the function $f(x) = e^{-x}(x^2 + x + 1)$?

(a)


(b)


(c)


(d)


Q. 9. The maximum area (in square units) of a rectangle whose vertices lie on the ellipse $x^2 + 4y^2 = 1$ is ______.

ELECTRONICS AND COMMUNICATION (EC) P2

Q. 10. The bilateral Laplace transform of a function $f(t) = \begin{cases} 1 \text{ if } a \le t \le b \\ 0 \text{ otherwise} \end{cases}$ is

(a) $\frac{a-b}{s}$ **(b)** $\frac{e^s(a-b)}{s}$

(c) $\frac{e^{-as} - e^{-bs}}{s}$ **(d)** $\frac{e^{s(a-b)}}{s}$

Q. 11. The value of x for which all the eigen-values of the matrix given below are real is ______.

$$\begin{bmatrix} 10 & 5+j & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$$

(a) $5 + j$ **(b)** $5 - j$

(c) $1 - 5j$ **(d)** $1 + 5j$

Q. 12. Let $f(z) = \dfrac{az+b}{cz+d}$. If $f(z_1) = f(z_2)$ for all $z_1 \neq z_2$, $a = 2$, $b = 4$ and $c = 5$, then d should be equal to ______.

Q. 13. The general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{1+\cos 2y}{1-\cos 2x}$ is

(a) $\tan y - \cot x = c$ (c is a constant)
(b) $\tan x - \cot y = c$ (c is a constant)
(c) $\tan y + \cot x = c$ (c is a constant)
(d) $\tan x + \cot y = c$ (c is a constant)

Q. 14. Consider the differential equation $\dfrac{dx}{dt} = 10 - 0.2x$ with initial condition $x(0) = 1$. The response $x(t)$ for $t > 0$ is

(a) $2 - e^{-0.2t}$
(b) $2 - e^{0.2t}$
(c) $50 - 49e^{-0.2t}$
(d) $50 - 49e^{0.2t}$

Q. **15.** The value of the integral $\int_{-\infty}^{\infty} 12\cos(2\pi t)\dfrac{\sin(4\pi t)}{4\pi t}dt$ is ______.

Q. 16. If C denotes the counterclockwise unit circle, the value of the contour integral $\dfrac{1}{2\pi j}\oint_C \mathrm{Re}\{z\}dz$ is ______.

Q. 17. Let the random variable X-represent the number of times a fair coin needs to be tossed till two consecutive heads appear for the first time. The expectation of X is ______.

Q. 18. Let $X \in \{0,1\}$ and $Y \in \{0,1\}$ be two independent binary random variables. If $P(X = 0) = p$ and $P(Y = 0) = q$, then $P((X + Y \geq 1)$ is equal to

(a) $pq + (1-p)(1-q)$
(b) pq
(c) $p(1-q)$
(d) $1 - pq$

ELECTRONICS AND COMMUNICATION (EC) P3

Q. 19 For $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, the determinant of $A^T A^{-1}$ is

(a) $\sec^2 x$ **(b)** $\cos 4x$
(c) 1 **(d)** 0

Q. 20. The contour on the $x - y$ plane, where the partial derivative of $x^2 + y^2$ with respect to y is equal to the partial derivative of $6y + 4x$ with respect to x, is

(a) $y = 2$ **(b)** $x = 2$
(c) $x + y = 4$ **(d)** $x - y = 0$

Q. 21. If C is a circle of radius r with centre z_0, in the complex z–plane and if n is a non-zero integer, then $\oint_C \dfrac{dz}{(z-z_0)^{n+1}}$ equals

(a) $2\pi nj$ **(b)** 0
(c) $\dfrac{nj}{2\pi}$ **(d)** $2\pi n$

Q. 22. Consider the function $g(t) = e^{-t}\sin(2\pi t)\, u(t)$ where $u(t)$ is the unit step function. The area under $g(t)$ is ______.

Q. 23. The value of $\sum_{n=0}^{\infty} n\left(\dfrac{1}{2}\right)^n$ is ______.

Q. 24. The Newton-Raphson method is used to solve the equation $f(x) = x^3 - 5x^2 + 6x - 8 = 0$. Taking the initial guess as $x = 5$, the solution obtained at the end of the first iteration is ______.

Q. 25. A fair die with faces {1, 2, 3, 4, 5, 6} is thrown repeatedly till '3' is observed for the first time. Let X denote the number of times the die is thrown. The expected value of X is ______.

Q. 26. Consider the differential equation

$$\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 2x(t) = 0.$$

Given $x(0) = 20$ and $x(1) = 10/e$, where $e = 2.718$, the value of $x(2)$ is ______.

Q. 27. A vector field $D = 2\rho^2 a_\rho + z\, a_z$ exists inside a cylindrical region enclosed by the surfaces $\rho = 1$, $z = 0$ and $z = 5$. Let S be the surface bounding this cylindrical region. The surface integral of this field on S $\left(\oiint_S D \cdot ds\right)$ is ______.

ELECTRICAL ENGINEERING (EE) P1

Q. 28. A random variable X has probability density function $f(x)$ as given below :

$$f(x) = \begin{cases} a + bx & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

If the expected value $E[X] = \dfrac{2}{3}$, then $Pr\,[X < 0.5]$ is ______.

Q. 29. If a continuous function $f(x)$ does not have a root in the interval $[a, b]$, then which one of the following statements is **TRUE**?

(a) $f(a) \cdot f(b) = 0$ **(b)** $f(a) \cdot f(b) < 0$

(c) $f(a) \cdot f(b) > 0$ **(d)** $\frac{f(a)}{f(b)} \leq 0$

Q. 30. If the sum of the diagonal elements of a 2×2 matrix is -6, then the maximum possible value of determinant of the matrix is ______.

Q. 31. Consider a function $\vec{f} = \frac{1}{r^2}\hat{r}$, where r is the distance from the origin and $\hat{r}$ is the unit vector in the radial direction. The divergence of this function over a sphere of radius R, which includes the origin, is

(a) 0 **(b)** 2π

(c) 4π **(d)** $R\pi$

Q. 32. The maximum value of "a" such that the matrix $\begin{pmatrix} -3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & a & -2 \end{pmatrix}$ has three linearly independent real eigenvectors is

(a) $\frac{2}{3\sqrt{3}}$ **(b)** $\frac{1}{3\sqrt{3}}$

(c) $\frac{1+2\sqrt{3}}{3\sqrt{3}}$ **(d)** $\frac{1+\sqrt{3}}{3\sqrt{3}}$

Q. 33. A solution of the ordinary differential equation $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0$ is such that $y(0) = 2$ and $y(1) = -\frac{1-3e}{e^3}$. The value of $\frac{dy}{dt}(0)$ is ______.

Q. 34. Two players, A and B, alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is

(a) $\frac{5}{11}$ **(b)** $\frac{1}{2}$

(c) $\frac{7}{13}$ **(d)** $\frac{6}{11}$

ELECTRICAL ENGINEERING (EE) P2

Q. 35. Given $f(z) = g(z) + h(z)$, where f, g, h are complex valued functions of a complex variable z.

Which one of the following statements is **TRUE**?

(a) If $f(z)$ is differentiable at z_0, then $g(z)$ and $h(z)$ are also differentiable at z_0.

(b) If $g(z)$ and $h(z)$ are differentiable at z_0, then $f(z)$ is also differentiable at z_0.

(c) If $f(z)$ is continuous at z_0, then it is differentiable at z_0.

(d) If $f(z)$ is differentiable at z_0, then so are its real and imaginary parts.

Q. 36. We have a set of 3 linear equations in 3 unknowns. 'X ≡ Y' means X and Y are equivalent statements and 'X ≢ Y' means X and Y are not equivalent statements.

P: There is a unique solution.

Q: The equations are linearly independent.

R: All eigenvalues of the coefficient matrix are nonzero.

S: The determinant of the coefficient matrix is nonzero.

Which one of the following is **TRUE**?

(a) $P \equiv Q \equiv R \equiv S$ **(b)** $P \equiv R \not\equiv Q \equiv S$

(c) $P \equiv Q \not\equiv R \equiv S$ **(d)** $P \not\equiv Q \not\equiv R \not\equiv S$

Q. 37. Match the following.

P. Stokes's Theorem	1. $\oiint D \cdot ds = Q$
Q. Gauss's Theorem	2. $\oint f(z)dz = 0$
R. Divergence Theorem	3. $\iiint (\nabla.A)dv = \oiint A.ds$
S. Cauchy's Integral Theorem	4. $\iint (\nabla \times A).ds = \oint A.dl$

(a) P-2, Q-1, R-4, S-3

(b) P-4, Q-1, R-3, S-2

(c) P-4, Q-3, R-1, S-2

(d) P-3, Q-4, R-2, S-1

Q. 38. The volume enclosed by the surface $f(x, y) = e^x$ over the triangle bounded by the lines $x = y$; $x = 0$; $y = 1$ in the xy plane is ______.

Q. 39. Two coins R and S are tossed. The 4 joint events H_RH_S, T_RT_S, H_RT_S, T_RH_S have probabilities 0.28, 0.18, 0.30, 0.24, respectively, where H represents head and T represents tail. Which one of the following is TRUE?

(a) The coin tosses are independent.

(b) R is fair, S is not.

(c) S is fair, R is not.

(d) The coin tosses are dependent.

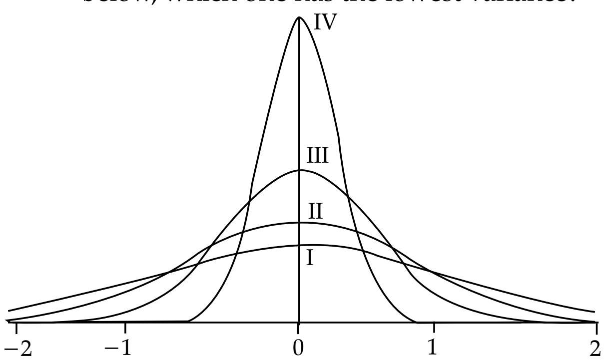
Q. 40. A differential equation $\frac{di}{dt} - 0.2i = 0$ is applicable over $-10 < t < 10$. If $i(4) = 10$, then $i(-5)$ is ______.

MECHANICAL ENGINEERING (ME) P1

Q. 41. If any two columns of a determinant $P = \begin{vmatrix} 4 & 7 & 8 \\ 3 & 1 & 5 \\ 9 & 6 & 2 \end{vmatrix}$ are interchanged, which one of the following statements regarding the value of the determinant is **CORRECT**?

(a) Absolute value remains unchanged but sign will change.
(b) Both absolute value and sign will change.
(c) Absolute value will change but sign will not change.
(d) Both absolute value and sign will remain unchanged.

Q. 42. Among the four normal distributions with probability density functions as shown below, which one has the lowest variance?



(a) I **(b)** II
(c) III **(d)** IV

Q. 43. Simpson's $\frac{1}{3}$ rule is used to integrate the function $f(x) = \frac{3}{5}x^2 + \frac{9}{5}$ between $x = 0$ and $x = 1$ using the least number of equal sub-intervals. The value of the integral is ______.

Q. 44. The value of $\lim_{x\to 0} \frac{1-\cos(x^2)}{2x^4}$ is

(a) 0 **(b)** $\frac{1}{2}$
(c) $\frac{1}{4}$ **(d)** undefined

Q. 45. Given two complex numbers $z_1 = 5 + (5\sqrt{3})i$ and $z_2 = \frac{2}{\sqrt{3}} + 2i$, the argument of $\frac{z_1}{z_2}$ in degrees is

(a) 0 **(b)** 30
(c) 60 **(d)** 90

Q. 46. Consider a spatial curve in three-dimensional space given in parametric form by
$x(t) = \cos t,\ y(t) = \sin t,\ z(t) = \frac{2}{\pi}t,\ 0 \le t \le \frac{\pi}{2}$.
The length of the curve is ______.

Q. 47. Consider an ant crawling along the curve $(x-2)^2 + y^2 = 4$, where x and y are in meters. The ant starts at the point (4, 0) and moves counter-clockwise with a speed of 1.57 meters per second. The time taken by the ant to reach the point (2, 2) is (in seconds) ______.

Q. 48. Find the solution of $\frac{d^2y}{dx^2} = y$ which passes through the origin and the point $\left(\ln 2, \frac{3}{4}\right)$.

(a) $y = \frac{1}{2}e^x - e^{-x}$ **(b)** $y = \frac{1}{2}(e^x + e^{-x})$
(c) $y = \frac{1}{2}(e^x - e^{-x})$ **(d)** $y = \frac{1}{2}e^x + e^{-x}$

Q. 49. The probability of obtaining at least two "SIX" in throwing a fair dice 4 times is

(a) $\frac{425}{432}$ **(b)** $\frac{19}{144}$
(c) $\frac{13}{144}$ **(d)** $\frac{125}{432}$

MECHANICAL ENGINEERING (ME) P2

Q. 50. At least one eigenvalue of a singular matrix is

(a) positive **(b)** zero
(c) negative **(d)** imaginary

Q. 51. At $x = 0$, the function $f(x) = |x|$ has

(a) a minima
(b) a maxima
(c) a point of inflexion
(d) neither a maxima nor minima

Q. 52. Curl of vector $V(x, y, z) = 2x^2\hat{i} + 3z^2\hat{j} + y^3\hat{k}$ at $x = y = z = 1$ is

(a) $-3\hat{i}$ **(b)** $3\hat{i}$
(c) $3\hat{i} - 4\hat{j}$ **(d)** $3\hat{i} - 6\hat{k}$

Q. 53. The Laplace transform of e^{i5t} where $i = \sqrt{-1}$, is

(a) $\frac{s-5i}{s^2-25}$ **(b)** $\frac{s+5i}{s^2+25}$
(c) $\frac{s+5i}{s^2-25}$ **(d)** $\frac{s-5i}{s^2+25}$

Q. 54. Three vendors were asked to supply a very high precision component. The respective probabilities of their meeting the strict design specifications are 0.8, 0.7 and 0.5. Each vendor supplies one component. The probability that out of total three components supplied by the vendors, at least one will meet the design specification is ______.

Q. 55. The chance of a student passing an exam is 20%. The chance of a student passing the exam and getting above 90% marks in it is 5%. GIVEN that a student passes the examination, the probability that the student gets above 90% marks is

(a) $\frac{1}{18}$ **(b)** $\frac{1}{4}$

(c) $\frac{2}{9}$ **(d)** $\frac{5}{18}$

Q. 56. The surface integral $\iint_s \frac{1}{\pi}(9x\hat{i} - 3y\hat{j}).n\ ds$ over the sphere given by $x^2 + y^2 + z^2 = 9$ is ______

Q. 57. Consider the following differential equation:

$\frac{dy}{dt} = -5y$; initial condition: $y = 2$ at $t = 0$.

The value of y at $t = 3$ is

(a) $-5e^{-10}$ **(b)** $2e^{-10}$

(c) $2e^{-15}$ **(d)** $-15e^2$

Q. 58. The values of function $f(x)$ at 5 discrete points are given below:

x	0	0.1	0.2	0.3	0.4
$f(x)$	0	10	40	90	160

Using Trapezoidal rule with step size of 0.1, the value of $\int_0^{0.4} f(x)dx$ is ______.

Q. 59. The initial velocity of an object is 40 m/s. The acceleration a of the object is given by the following expression:

$a = -0.1\ v$,

where v is the instantaneous velocity of the object. The velocity of the object after 3 seconds will be ________

MECHANICAL ENGINEERING (ME) P3

Q. 60 Using a unit step size, the value of integral $\int_1^2 x \ln x\ dx$ by trapezoidal rule is ______.

Q. 61. If P(X) $=\frac{1}{4}$, P(Y) $= \frac{1}{3}$, and P(X $\cap$ Y) $= \frac{1}{12}$, the value of P(Y/X) is

(a) $\frac{1}{4}$ **(b)** $\frac{4}{25}$

(c) $\frac{1}{3}$ **(d)** $\frac{29}{50}$

Q. 62. The lowest eigenvalue of the 2 × 2 matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ is ______.

Q. 63. The value of $\lim_{x\to 0}\left(\frac{-\sin x}{2\sin x + x\cos x}\right)$ is ______.

Q. 64. Let ϕ be an arbitrary smooth real valued scalar function and $\vec{V}$ be an arbitrary smooth vector valued function in a three-dimensional space. Which one of the following is an identity?

(a) Curl $(\phi\vec{V}) = \nabla(\phi \text{Div}\vec{V})$

(b) Div $\vec{V} = 0$

(c) Div Curl $\vec{V} = 0$

(d) Div $(\phi\vec{V}) = \phi$ Div$\vec{V}$

Q. 65. The value of $\int_c [(3x - 8y^2)dx + (4y - 6xy)dy]$, (where C is the boundary of the region bounded by x = 0, y = 0 and x + y = 1) is ______.

Q. 66. For a given matrix P $= \begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$, where $i = \sqrt{-1}$, the inverse of matrix P is

(a) $\frac{1}{24}\begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}$

(b) $\frac{1}{25}\begin{bmatrix} i & 4-3i \\ 4+3i & -i \end{bmatrix}$

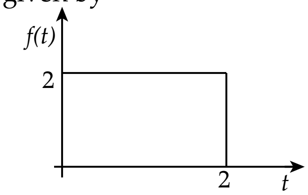
(c) $\frac{1}{24}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$

(d) $\frac{1}{25}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$

Q. 67. Newton-Raphson method is used to find the roots of the equation, $x^3 + 2x^2 + 3x - 1 = 0$. If the initial guess is $x_0 = 1$, then the value of x after 2^{nd} iteration is ______.

Q. 68. Laplace transform of the function $f(t)$ is given by $F(s) = L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt$.

Laplace transform of the function shown below is given by



(a) $\frac{1-e^{-2s}}{s}$ **(b)** $\frac{1-e^{-s}}{2s}$

(c) $\frac{2-2e^{-s}}{s}$ **(d)** $\frac{1-2e^{-s}}{s}$

CIVIL ENGINEERING (CE) P1

Q. 69. For what value of p the following set of equations will have no solution?

$2x + 3y = 5$

$3x + py = 10$

Q. 70. The integral $\int_{x_1}^{x_2} x^2 dx$ with $x_2 > x_1 > 0$ is evaluated analytically as well as numerically using a single application of the trapezoidal rule. If I is the exact value of the integral obtained analytically and J is the approximate value obtained using the trapezoidal rule, which of the following statements is correct about their relationship?

(a) J > I

(b) J < I

(c) J = I

(d) Insufficient data to determine the relationship

Q. 71. Consider the following probability mass function (p.m.f) of a random variable X:

$$p(x,q) = \begin{cases} q & \text{if } X = 0 \\ 1-q & \text{if } X = 1 \\ 0 & \text{otherwise} \end{cases}$$

If $q = 0.4$, the variance of X is ________.

Q. 72. The smallest and largest Eigen values of the following matrix are:

$$\begin{bmatrix} 3 & -2 & 2 \\ 4 & -4 & 6 \\ 2 & -3 & 5 \end{bmatrix}$$

(a) 1.5 and 2.5 **(b)** 0.5 and 2.5

(c) 1.0 and 3.0 **(d)** 1.0 and 2.0

Q. 73. The quadratic equation $x^2 - 4x + 4 = 0$ is to be solved numerically, starting with the initial guess $x_0 = 3$. The Newton-Raphson method is applied once to get a new estimate and then the Secant method is applied once using the initial guess and this new estimate. The estimated value of the root after the application of the Secant method is ________.

Q. 74. Consider the following differential equation:

$$x(y\,dx + x\,dy)\cos\frac{y}{x} = y(x\,dy - y\,dx)\sin\frac{y}{x}$$

Which of the following is the solution of the above equation (c is an arbitrary constant)?

(a) $\frac{x}{y}\cos\frac{y}{x} = c$ **(b)** $\frac{x}{y}\sin\frac{y}{x} = c$

(c) $xy\cos\frac{y}{x} = c$ **(d)** $xy\sin\frac{y}{x} = c$

Q. 75. Consider the following complex function:

$$f(z) = \frac{9}{(z-1)(z+2)^2}$$

Which of the following is one of the residues of the above function?

(a) –1 **(b)** $\frac{9}{16}$

(c) 2 **(d)** 9

Q. 76. The directional derivative of the field $u(x, y, z) = x^2 - 3yz$ in the direction of the vector $(\hat{i} + \hat{j} - 2\hat{k})$ at point (2, –1, 4) is _________.

CIVIL ENGINEERING (CE) P2

Q. 77. While minimizing the function $f(x)$, necessary and sufficient conditions for a point, x_0 to be a minima are :

(a) $f'(x_0) > 0$ and $f''(x_0) = 0$

(b) $f'(x_0) < 0$ and $f''(x_0) = 0$

(c) $f'(x_0) = 0$ and $f''(x_0) < 0$

(d) $f'(x_0) = 0$ and $f''(x_0) > 0$

Q. 78. In Newton-Raphson iterative method, the initial guess value (x_{ini}) is considered as zero while finding the roots of the equation: $f(x) = -2 + 6x - 4x^2 + 0.5x^3$. The correction, Δx, to be added to xin i in the first iteration is ______.

Q. 79. Given $i = \sqrt{-1}$, the value of the definite integral, $I = \int_0^{\pi/2} \frac{\cos x + i\sin x}{\cos x - i\sin x}dx$ is:

(a) 1 **(b)** –1

(c) i **(d)** $-i$

Q. 80. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x}$ is equal to

(a) e^{-2} (b) e

(c) 1 (d) e^2

Q. 81. Let A = $[a_{ij}]$, $1 \le i, j \le n$ with $n \ge 3$ and $a_{ij} = i.j$. The rank of A is :

(a) 0 (b) 1

(c) $n-1$ (d) n

Q. 82. The probability density function of a random variable, x is $f(x)=\frac{x}{4}(4-x^2)$ for $0 \le x \le 2$ = 0 otherwise the mean, μ_x of the random variable is ______.

Q. 83. Consider the following second order linear differential equation

$$\frac{d^2y}{dx^2}=-12x^2+24x-20$$

The boundary conditions are: at $x = 0$, $y = 5$ and at $x = 2$, $y = 21$

The value of y at $x = 1$ is _______.

Q. 84. The two Eigen values of the matrix $\begin{bmatrix}2 & 1\\ 1 & p\end{bmatrix}$ have a ratio of 3 : 1 for $p = 2$.

What is another value of p for which the Eigen values have the same ratio of 3 : 1?

(a) –2 (b) 1

(c) $\frac{7}{3}$ (d) $\frac{14}{3}$

Q. 85. For step-size, $\Delta x = 0.4$, the value of the following integral using Simpson's 1/3 rule is ________.

$$\int_0^{0.8}(0.2+25x-200x^2+675x^3-900x^4+400x^5)dx$$

COMPUTER SCIENCE (CS) P1

Q. 86. If $g(x) = 1 - x$ and $h(x)$ $\frac{x}{x-1}$, then $\frac{g(h(x))}{g(g(x))}$ is:

(a) $\frac{h(x)}{g(x)}$ (b) $\frac{-1}{x}$

(c) $\frac{g(x)}{h(x)}$ (d) $\frac{x}{(1-x)^2}$

Q. 87. $\lim_{x\to\infty} x^{\frac{1}{x}}$ is

(a) ∞ (b) 0

(c) 1 (d) Not defined

Q. 88. Which one of the following is NOT equivalent to $p \leftrightarrow q$?

(a) $(\neg p \vee q) \wedge (p \vee \neg q)$

(b) $(\neg p \vee q) \wedge (q \to p)$

(c) $(\neg p \wedge q) \vee (p \wedge \neg q)$

(d) $(\neg p \wedge \neg q) \vee (p \wedge q)$

Q. 89. For a set A, the power set of A is denoted by 2^A. If A={5,{6},{7}}, which of the following options are **TRUE**?

I. $\phi \in 2^A$ **II.** $\phi \subseteq 2^A$

III. $\{5,\{6\}\} \in 2^A$ **IV.** $\{5,\{6\}\} \subseteq 2^A$

(a) I and III only (b) II and III only

(c) I, II and III only (d) I, II and IV only

Q. 90. In the LU decomposition of the matrix $\begin{bmatrix}2 & 2\\ 4 & 9\end{bmatrix}$, if the diagonal elements of U are both 1, then the lower diagonal entry l_{22} of L is ______.

Q. 91. $\sum_{x=1}^{99}\frac{1}{x(x+1)}$ = ______.

Q. 92. Let G be a connected planar graph with 10 vertices. If the number of edges on each face is three, then the number of edges in G is ______.

Q. 93. Let a_n represent the number of bit strings of length n containing two consecutive 1 s. What is the recurrence relation for a_n?

(a) $a_{n-2}+a_{n-1}+2^{n-2}$ (b) $a_{n-2}+2a_{n-1}+2^{n-2}$

(c) $2a_{n-2}+a_{n-1}+2^{n-2}$ (d) $2a_{n-2}+2a_{n-1}+2^{n-2}$

Q. 94. $\int_{\frac{1}{\pi}}^{\frac{2}{\pi}}\frac{\cos\left(\frac{1}{x}\right)}{x^2}dx$ = ______.

Q. 95. Consider the following 2 × 2 matrix A where two elements are unknown and are marked by a and b. The eigenvalues of this matrix are –1 and 7. What are the values of a and b?

$$A=\begin{pmatrix}1 & 4\\ b & a\end{pmatrix}.$$

(a) $a = 6, b = 4$

(b) $a = 4, b = 6$

(c) $a = 3, b = 5$

(d) $a = 5, b = 3$

COMPUTER SCIENCE (CS) P2

Q. 96. Consider the following two statements.

S1: If a candidate is known to be corrupt, then he will not be elected

S2: If a candidate is kind, he will be elected

Which one of the following statements follows from S1 and S2 as per sound inference rules of logic?

(a) If a person is known to be corrupt, he is kind

(b) If a person is not known to be corrupt, he is not kind

(c) If a person is kind, he is not known to be corrupt

(d) If a person is not kind, he is not known to be corrupt

Q. 97. The cardinality of the power set of $\{0, 1, 2, \ldots, 10\}$ is ______.

Q. 98. Let R be the relation on the set of positive integers such that aRb if and only if a and b are distinct and have a common divisor other than 1. Which one of the following statements about R is true?

(a) R is symmetric and reflexive but not transitive

(b) R is reflexive but not symmetric and not transitive

(c) R is transitive but not reflexive and not symmetric

(d) R is symmetric but not reflexive and not transitive

Q. 99. The number of divisors of 2100 is

Q. 100. The larger of the two eigenvalues of the matrix $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ is ______.

Q. 101. Let $f(x) = x^{-(1/3)}$ and A denote the area of the region bounded by $f(x)$ and the X-axis, when x varies from -1 to 1. Which of the following statements is/are TRUE?

(I) f is continuous in $[-1, 1]$

(II) f is not bounded in $[-1, 1]$

(III) A is nonzero and finite

(a) II only **(b)** III only

(c) II and III only **(d)** I, II and III

Q. 102. Perform the following operations on the matrix $\begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix}$.

(I) Add the third row to the second row

(II) Subtract the third column from the first column.

The determinant of the resultant matrix is ______.

Q. 103. The number of onto functions (surjective functions) from set X = $\{1,2,3,4\}$ to set Y = $\{a, b, c\}$ is ______.

Q. 104 Let X and Y denote the sets containing 2 and 20 distinct objects respectively and F denote the set of all possible functions defined from X to Y. Let f be randomly chosen from F. The probability of f being one-to-one is ______.

Q. 105. In a connected graph, a bridge is an edge whose removal disconnects a graph. Which one of the following statements is true?

(a) A tree has no bridges

(b) A bridge cannot be part of a simple cycle

(c) Every edge of a clique with size ≥ 3 is a bridge (A clique is any complete subgraph of a graph)

(d) A graph with bridges cannot have a cycle

COMPUTER SCIENCE (CS) P3

Q. 106. Suppose U is the power set of the set S = $\{1, 2, 3, 4, 5, 6\}$. For any T $\in$ U, let $|T|$ denote the number of elements in T and T′ denote the complement of T. For any T, R $\in$ U, let T/R be the set of all elements in T which are not in R. Which one of the following is true?

(a) $\forall X \in U\ (|X| = |X'|)$

(b) $\exists X \in U\ \exists Y \in U\ (|X| = 5, |Y| = 5$ *and* $X \cap Y = \varnothing)$

(c) $\forall X \in U\ \forall Y \in U\ (|X| = 2, |Y| = 3$ and $X \setminus Y = \varnothing)$

(d) $\forall X \in U\ \forall Y \in U\ (X \setminus Y = Y' \setminus X')$

Q. 107. In the given matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$, one of the eigenvalues is 1. The eigenvectors corresponding to the eigenvalue 1 are

(a) $\{\alpha(4,2,1) \mid \alpha \neq 0, \alpha \in \mathbb{R}\}$

(b) $\{\alpha(-4,2,1) \mid \alpha \neq 0, \alpha \in \mathbb{R}\}$

(c) $\{\alpha(\sqrt{2},0,1) \mid \alpha \neq 0, \alpha \in \mathbb{R}\}$

(d) $\{\alpha(-\sqrt{2},0,1) \mid \alpha \neq 0, \alpha \in \mathbb{R}\}$

Q. 108. The value of $\lim_{x\to\infty}\left(1+x^2\right)^{e^{-x}}$ is

(a) 0 **(b)** $\frac{1}{2}$

(c) 1 **(d)** ∞

Q. 109. The number of 4 digit numbers having their digits in non-decreasing order (from left to right) constructed by using the digits belonging to the set {1, 2, 3} is ______.

Q. 110. In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You give a fair coin to a person in that room, without knowing which type he is from and tell him to toss it and hide the result from you till you ask for it. Upon asking, the person replies the following

"The result of the toss is head if and only if I am telling the truth."

Which of the following options is correct?

(a) The result is head

(b) The result is tail

(c) If the person is of Type 2, then the result is tail

(d) If the person is of Type 1, then the result is tail

Q. 111. The velocity v (in kilometer/minute) of a motorbike which starts from rest, is given at fixed intervals of time t (in minutes) as follows:

t	2	4	6	8	10	12	14	16	18	20
v	10	18	25	29	32	20	11	5	2	0

The approximate distance (in kilometers) rounded to two places of decimals covered in 20 minutes using Simpson's 1/3rd rule is ______.

Q. 112. If the following system has non-trivial solution,

$px + qy + rz = 0$

$qx + ry + pz = 0$

$rx + py + qz = 0,$

then which one of the following options is TRUE?

(a) $p - q + r = 0$ or $p = q = -r$

(b) $p + q - r = 0$ or $p = -q = r$

(c) $p + q + r = 0$ or $p = q = r$

(d) $p - q + r = 0$ or $p = -q = -r$

Q. 113. If for non-zero x, $af(x)+bf\left(\frac{1}{x}\right)=\frac{1}{x}-25$ where $a \neq b$ then $\int_1^2 f(x)dx$ is

(a) $\frac{1}{a^2-b^2}\left[a(\ln 2-25)+\frac{47b}{2}\right]$

(b) $\frac{1}{a^2-b^2}\left[a(2\ln 2-25)-\frac{47b}{2}\right]$

(c) $\frac{1}{a^2-b^2}\left[a(2\ln 2-25)+\frac{47b}{2}\right]$

(d) $\frac{1}{a^2-b^2}\left[a(\ln 2-25)-\frac{47b}{2}\right]$

Q. 114. Let R be a relation on the set of ordered pairs of positive integers such that $((p, q), (r, s)) \in$ R if and only if $p - s = q - r$. Which one of the following is true about R?

(a) Both reflexive and symmetric

(b) Reflexive but not symmetric

(c) Not reflexive but symmetric

(d) Neither reflexive nor symmetric

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	2	System of Equation	Linear Algebra
2	(b)	Mean Value Theorem	Calculus
3	(c)	Proability	Probability and Statistics
4	(d)	Properties	Complex Variable
5	17	Eigen Vector	Matrix and Determinant
6	(b)	2nd Order DE	Differential Equation
7	(a)	Curl and Divergence	Vector Analysis
8	(b)	Maxima and Minima	Calculus
9	1	Maxima and Minima	Calculus
10	(c)	Laplace Transform	Transform Theory
11	(b)	Eigen Values	Linear Algebra

12	10	Complex Functions	Complex Variable
13	(c)	1st Order DE	Differential Equation
14	(c)	1st Order DE	Differential Equation
15	3	Integral	Calculus
16	0.5	Contor Integral	Complex Variable
17	1.5	Mean	Probability and Statistics
18	(d)	Random Variable	Probability and Statistics
19	(a)	Determinant	Linear Algebra
20	(a)	Partial Derivative	Calculus
21	(b)	Contor Integral	Complex Variable
22	0.155	Laplace Transform	Transform Theory
23	2	Series	Calculus
24	4.2903	Newton Raphson	Numerical Integration
25	6	Mean	Probability and Statistics
26	0.8556	2nd Order DE	Differential Equation
27	78.52	Surface Integral	Vector Analysis
28	0.25	Mean	Probability and Statistics
29	(c)	Function	Calculus
30	9	Determinant	Linear Algebra
31	(a)	Divergence	Vector Analysis
32	(b)	Eigen Vector	Linear Algebra
33	–3	2nd Order DE	Differential Equation
34	(d)	Probability	Probability and Statistics
35	(b)	Complex Functions	Complex Variable
36	(a)	Properties	Linear Algebra
37	(b)	Theorems	Vector Analysis
38	0.71	Volume	Vector Analysis
39	(d)	Probability	Probability and Statistics
40	1.65	1st Order	Differential Equation
41	(a)	Determinant	Linear Algebra
42	(d)	Normal Distribution	Probability and Statistics
43	2	Simpson's Rule	Numerical Analysis
44	(c)	Limit	Calculus
45	(a)	Basics of Complex Variable	Complex Variable
46	1.86	Line Integral	Vector Analysis
47	2	Line Integral	Calculus
48	(c)	2nd Order DE	Differential Equation
49	(b)	Probability	Probability and Statistics
50	(b)	Eigen Values	Linear Algebra
51	(a)	Maximum and Minimum	Calculus

52	(a)	Curl	Vector Analysis
53	(b)	Laplace Transform	Transform Theory
54	0.97	Probability	Probability and Statistics
55	(b)	Conditional Probability	Probability and Statistics
56	216	Surface Integral	Vector Analysis
57	(c)	1st Order	Differentail Equation
58	22	Trapezoidal Rule	Numerical Analysis
59	29.6	Application of Integration	Calculus
60	0.693	Trapezoidal Rule	Numerical Analysis
61	(c)	Probability	Probability and Statistics
62	2	Eigen Values	Linear Algebra
63	–0.33	Limit	Calculus
64	(c)	Curl and Divergence	Vector Analysis
65	1.66	Line Integral	Vector Analysis
66	(a)	Inverse	Linear Algebra
67	0.30	Newton Raphson	Numerical Analysis
68	(c)	Laplace Transform	Transform Theory
69	4.5	System of Equations	Linear Algebra
70	(a)	Trapezoidal Rule	Numerical Analysis
71	0.24	PDF	Probability and Statistics
72	(d)	Eigen Vlaues	Linear Algebra
73	2.3333	Newton Raphson	Numerical Analysis
74	(c)	1st Order	Differential Equation
75	(a)	Residues	Complex Variable
76	–5.715	Directional Derivative	Vector Analysis
77	(d)	Maxima and Minima	Calculus
78	0.333	Newton Raphson	Numerical Analysis
79	(c)	Definite Integral	Calculus
80	(d)	Limit	Calculus
81	(b)	Rank	Linear Algebra
82	1.0667	Mean	Probability and Statistics
83	18	2nd Order DE	Differential Equation
84	(d)	Eigen Values	Linear Algebra
85	–3.8293	Simpson's Rule	Numerical Analysis
86	(a)	Functions	Calculus
87	(c)	Limit	Calculus
88	(c)	Recurrence Relations	Discrete Mathematics
89	(c)	Sets	Discrete Mathematics
90	5	Lu Decomposition	Matrix and Determinant
91	0.99	Series Sum	Calculus

92	24	Graph Theoy	Discrete Mathematics
93	(a)	Recurrence Relations	Discrete Mathematics
94	–1	Definite Integral	Calculus
95	(d)	Eigen Value	Linear Algebra
96	(c)	Logic	Discrete Mathematics
97	2048	Sets	Discrete Mathematics
98	(d)	Recurrence Relations	Discrete Mathematics
99	36	Graph Theory	Discrete Mathematics
100	6	Eigen Values	Linear Algebra
101	(c)	Continuity and Differentiability	Calculus
102	0	Determinant	Linear Algebra
103	36	Functions	Discrete Mathematics
104	0.95	Probability	Probability and Statistics
105	(b)	Graph Theory	Discrete Mathematics
106	(d)	Sets	Discrete Mathematics
107	(b)	Eigen Vectors	Linear Algebra
108	(c)	Limit	Calculus
109	15	Sets	Discrete Mathematics
110	(a)	Logic	Discrete Mathematics
111	309.33	Simpson's Rule	Numerical Analysis
112	(c)	System of Linear Equations	Linear Algebra
113	(a)	Functions	Calculus
114	(c)	Recurrence Relations	Discrete Mathematics

ANSWERS WITH EXPLANATIONS

1. Correct answer is [2].

This can be penned in the form of AX = B

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & -3 & 4 \\ -2 & 4 & -6 \end{bmatrix};\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix};\quad B = \begin{bmatrix} -1 \\ 1 \\ k \end{bmatrix}$$

Augmented matrix

$$\left[\frac{A}{B}\right] = \begin{bmatrix} 1 & -2 & 3 & -1 \\ 1 & -3 & 4 & 1 \\ -2 & 4 & -6 & k \end{bmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 + 2R_1$

$$\therefore \quad \left[\frac{A}{B}\right] = \begin{bmatrix} 1 & -2 & 3 & -1 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & k-2 \end{bmatrix}$$

For system has infinitely many solution

$$\rho(A) = \rho\left(\left[\frac{A}{B}\right]\right) < 3 = \text{(No. of unknown)}$$

Making R_3 completely zero.

$\Rightarrow \quad k - 2 = 0$

$\Rightarrow \quad k = 2$

2. Option (b) is correct.

By Lagrange's mean value theorem

$$f'(x) = \frac{f(1) - f(-1)}{1-(-1)} = \frac{2}{2} = 1$$

$\Rightarrow \quad f'(x) = -2x + 3x^2 = 1$

$\Rightarrow \quad 3x^2 - 2x - 1 = 0$

$\Rightarrow \quad x = 1, \frac{-1}{3}$

$x = \frac{-1}{3};$

is the only value of x that lies (–1, 1)

3. Option (c) is correct.

Given A and B are two independent events with probabilities P(A) and P(B).

From option (a) $P(A \cap B) = P(A)P(B)$ true;

[according to the rule of independent events]

From option (b)

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A) \text{true}$$

From option (c)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

So, given statement is false.

From option (d) $P(\overline{A} \cap \overline{B}) = P(\overline{A})P(\overline{B})$ true.

With A and B independent $\overline{A}$ and $\overline{B}$ will also be independent.

4. Option (d) is correct.

We know complex variable is defined as $z = x + iy$

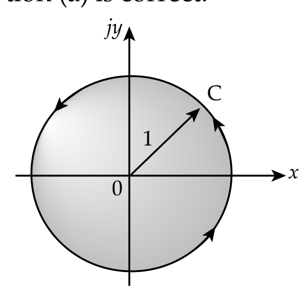
Where contour is unit circle in a anti clockwise direction.

From option (a) $f(z) = \dfrac{z}{(z^2 - 1)} = \dfrac{z}{(z+1)(z-1)}$

$$\text{Residue} = \frac{1}{n!}\lim_{z \to 1} \frac{d^{n-1}}{dz^{n-1}}(z-1)^n f(z)$$

$$n = 1 \quad \lim_{z \to 1}(z-1)\frac{z}{(z+1)(z-1)} = \frac{1}{2}$$

$\therefore$ option (a) is correct.



From option (b) $f(z) = z^2$

$$\oint_C f(z)\,dz = \oint_C z^2\,dz$$

And we can see here $f(z) = z^2$ there is no pole.

So, $f(z)$ is analytic anywhere, $\oint_C z^2 dz = 0$

[Cauchy's integral formula]

Option (b) is correct.

From option (c) $\dfrac{1}{2\pi i}\oint_C \dfrac{1}{z}\,dz$

$z = 0$ simple pole and lies inside the circle.

Residues at $z = 0$; $\lim_{z\to 0} zf(z) = \lim_{z\to 0} 1 = 1$

So, $\oint_C f(z)dz = 2\pi i.1$

$\frac{1}{2\pi i}\oint_C f(z)dz = 1$ so, option (c) is correct.

Now $\bar{z} = \overline{x+iy} = x - iy$

$$u(x,y) = x$$

$$v(x,y) = -y$$

$$u_x = \frac{\partial u}{\partial x} = 1, \qquad u_y = \frac{\partial u}{\partial y} = 0$$

$$v_x = \frac{\partial v}{\partial x} = 0, \qquad v_y = \frac{\partial v}{\partial y} = -1$$

$$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$$

CR equation is not satisfied. Hence it is not analytic function. Therefore (d) is false.

5. **Correct answer is [17].**

We know $AX = \lambda X$

$$\therefore \begin{bmatrix} 4 & 1 & 2 \\ p & 2 & 1 \\ 14 & -4 & 10 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 12 \\ p+7 \\ 36 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow 12 = \lambda \times 1, \qquad p+7 = \lambda \times 2, \qquad 36 = \lambda \times 3$$

$$\lambda = 12$$

$$p+7 = \lambda \times 2 \Rightarrow p+7 = 12 \times 2$$

$$\Rightarrow p+7 = 24$$

$$\Rightarrow p = 17$$

6. **Option (b) is correct.**

Given DE is

$$(D^2 + 2D + 1)y = 0$$

$$AE \to m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0$$

$$m = -1, -1$$

$$y = C_1 e^{-t} + C_2 te^{-t}$$

$$t = 0, y = 1$$

$$C_1 = 1$$

$$y' = -C_1 e^{-t} + C_2\left(-te^{-t} + e^{-t}\right)$$

$$y' = -C_1 e^{-t} + C_2 e^{-t}(-t+1)$$

$$t = 0, y' = 1$$

$$\Rightarrow 1 = -1 + C_2$$

$$\Rightarrow C_2 = 2$$

$$\Rightarrow y = e^{-t} + 2te^{-t} = e^{-t}(1+2t)$$

7. **Option (a) is correct.**

$\vec{P} = x^3 y\vec{a_x} - x^2y^2\vec{a_y} - x^2yz\vec{a_z}$

For vector to be solenoidal $\nabla.\vec{P} = 0$

$$\therefore \nabla.\vec{P} = \frac{\partial}{\partial x}(x^3y) + \frac{\partial}{\partial y}(-x^2y^2) + \frac{\partial}{\partial z}(-x^2yz)$$

$$= 3x^2y - 2x^2y - x^2y$$

$$= 0$$

And for irrotational field

$\nabla \times \vec{P} = 0$

$$\nabla \times \vec{P} = \begin{vmatrix} \widehat{a_x} & \widehat{a_y} & \widehat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x^3y) & (-x^2y^2) & (-x^2yz) \end{vmatrix}$$

$$= (-x^2z - 0)\hat{a}_x - (-2xyz - 0)\hat{a}_y + (-2xy^2 - x^3)\hat{a}_z$$

$$\nabla \times \vec{P} \neq 0$$

8. **Option (b) is correct.**

$$f(x) = e^{-x}(x^2 + x + 1)$$

$$f'(x) = e^{-x}(2x+1) - e^{-x}(x^2 + x + 1)$$

$$= e^{-x}(2x + 1 - x^2 - x - 1)$$

$$= e^{-x}(x - x^2)$$

$$f'(x) = 0 \Rightarrow x - x^2 = 0 \Rightarrow x = 0, 1$$

$$f''(x) = e^{-x}(1-2x) - e^{-x}(x - x^2)$$

$$= e^{-x}(1 - 2x - x + x^2) = e^{-x}(1 - 3x + x^2)$$

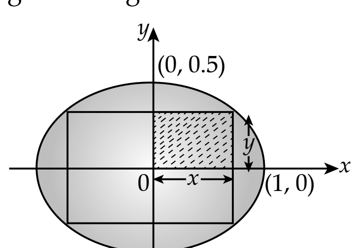
At $x = 0 \Rightarrow f''(x) = 1 + ve$ so, minima here.

$x = 1 \Rightarrow f''(x) = -\frac{1}{e}, -ve$ maxima

Option (b) shows the local minima at $x = 0$ and maxima at $x = 1$.

9. **Correct answer is [1].**

Let x and y be the breadth and length of the rectangle as in figure.



Rectangle's four corners are touching the ellipes and consider a point P (x, y) on the ellipse.

The area of the shaded rectangle = xy.
Therefore area of rectangle is

$$A = 4xy = 4x\sqrt{\frac{1-x^2}{4}} = 2x\sqrt{1-x^2}$$

For area to be maximum $\frac{dA}{dx} = 0$

$$\Rightarrow \quad \frac{dA}{dx} = 2\left[1\sqrt{1-x^2} + x\frac{1}{2}\left(\sqrt{1-x^2}\right)^{-\frac{1}{2}}(-2x)\right]$$

$$= 0$$

$$\sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}} = 0$$

$$\Rightarrow \quad \left(1-x^2\right) - x^2 = 0$$

$$\Rightarrow \quad 1-2x^2 = 0$$

$$\Rightarrow \quad x = \frac{1}{\sqrt{2}}$$

Maximum area $= 2x\sqrt{1-x^2}$

$$= 2\left(\frac{1}{\sqrt{2}}\right)\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= 1$$

10. Option (c) is correct.

$$f(t) = \begin{cases} 1 & a \le t \le b \\ 0 & \text{otherwise} \end{cases}$$

$$F(s) = \int_{-\infty}^{+\infty} f(t)e^{-st}dt$$

$$= \int_a^b 1.e^{-st}dt = \frac{e^{-as} - e^{-bs}}{s}$$

11. Option (b) is correct.

$$A = \begin{bmatrix} 10 & 5+j & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$$

Eigen values of A are real therefore A is a Hermitian matrix.

$$\Rightarrow \quad \left(\overline{A}\right)^T = A$$

$$\Rightarrow \quad \begin{bmatrix} 10 & \overline{x} & 4 \\ 5-j & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix} = \begin{bmatrix} 10 & 5+j & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$$

$$\therefore \quad x = 5-j$$

12. Correct answer is [10].

Given $\quad f(z_1) = f(z_2)$

$$f(z) = \frac{az+b}{cz+d}$$

$$\frac{az_1+b}{cz_1+d} = \frac{az_2+b}{cz_2+d}$$

$$\Rightarrow \quad \frac{2z_1+4}{5z_1+d} = \frac{2z_2+4}{5z_2+d}$$

$$\Rightarrow \quad (2z_1+4)(5z_2+d) = (2z_2+4)(5z_1+d)$$

$$\Rightarrow \quad 10z_1z_2 + 2z_1d + 20z_2 + 4d = 10z_1z_2 + 20z_1 + 2dz_2 + 4d$$

$$\Rightarrow \quad 2z_1d + 20z_2 = 20z_1 + 2dz_2$$

$$\Rightarrow \quad 2d(z_1 - z_2) = 20(z_1 - z_2)$$

$$\therefore \quad d = 10$$

13. Option (c) is correct.

Given DE $\quad \frac{dy}{dx} = \frac{1+\cos 2y}{1-\cos 2x}$

$$\frac{dy}{1+\cos 2y} = \frac{dx}{1-\cos 2x}$$

$$\frac{dy}{2\cos^2 y} = \frac{dx}{2\sin^2 x}$$

$$\int \sec^2 y dy = \int \text{cosec}^2 x\ dx$$

$$\tan y + k = -\cot x$$

$$\tan y + \cot x = c$$

14. Option (c) is correct.

$$\frac{dx}{dt} = 10 - 0.2x$$

$$\frac{dx}{dt} + 0.2x = 10$$

$$\text{IF} = e^{\int 0.2dt} = e^{0.2t}$$

$$xe^{0.2t} = \int 10e^{0.2t}dt + c$$

$$xe^{0.2t} = 10\left[\frac{e^{0.2t}}{0.2}\right] + c$$

$$xe^{0.2t} = \frac{10}{2} \times 10e^{0.2t} + c$$

$$x(t) = 50 + ce^{-0.2t}$$

At $\quad t = 0$

$$x(0) = 50 + c = 1$$

$$50 + c = 1$$

$$c = -49$$

$$x(t) = 50 - 49e^{-0.2t}$$

15. Correct answer is [3].

$$I = \int_{-\infty}^{\infty} 12\cos(2\pi t)\frac{\sin(4\pi t)}{(4\pi t)}dt$$

$$= \frac{12}{4\pi}\int_0^{\infty} \frac{2\cos(2\pi t)\sin(4\pi t)}{t}dt$$

$$= \frac{3}{\pi}\left[\int_0^{\infty} \frac{\sin 6\pi t}{t}dt + \int_0^{\infty} \frac{\sin 2\pi t}{t}dt\right]$$

$\because \left[2\sin A \cdot \cos B = \sin(A+B) + \sin(A-B)\right]$

$$I = \frac{3}{\pi}\left[\int_0^{\infty} e^{st}\frac{\sin 6\pi t}{t}dt + \int_0^{\infty} e^{st}\frac{\sin 2\pi t}{t}dt\right]$$

Take the laplace transform with $s = 0$

$$= \frac{3}{\pi}\left[L\left\{\frac{\sin 6\pi t}{t}\right\} + L\left\{\frac{\sin 2\pi t}{t}\right\}\right]$$

$$= \frac{3}{\pi}\left[\int_s^{\infty}\left\{\frac{6\pi}{s^2+36\pi^2}\right\}ds + \int_s^{\infty}\left\{\frac{2\pi}{s^2+4\pi^2}\right\}ds\right]$$

$$= \frac{3}{\pi}\left[6\pi\frac{1}{6\pi}\tan^{-1}\left(\frac{s}{6\pi}\right) + 2\pi\frac{1}{2\pi}\tan^{-1}\left(\frac{s}{2\pi}\right)\Bigg|_s^{\infty}\right]$$

$$= \frac{3}{\pi}\left[\tan^{-1}(\infty) - \tan^{-1}\left(\frac{s}{6\pi}\right) + \tan^{-1}(\infty) - \tan^{-1}\left(\frac{s}{2\pi}\right)\right]$$

$$= \frac{3}{\pi}\left[\frac{\pi}{2} - \tan^{-1}(0) + \frac{\pi}{2} - \tan^{-1}(0)\right] \text{ for } s = 0$$

$$= \frac{3}{\pi}\left[\frac{\pi}{2} - 0 + \frac{\pi}{2} - 0\right] = 3$$

16. Correct answer is [0.5].

Given $\frac{1}{2\pi j}\oint_C \text{Re}\{z\}\,dz$

$z = e^{j\theta}$

$dz = je^{j\theta}d\theta$

Limit of integration from $0 \to 2\pi$

$$= \frac{1}{2\pi j}\int_0^{2\pi} \text{Re}\left(e^{j\theta}\right)je^{j\theta}d\theta$$

$$= i\frac{1}{2\pi j}\int_0^{2\pi} j\left(\cos^2\theta + j\sin\theta\cos\theta\right)d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{2\pi}\left(\cos^2\theta\right)d\theta + j\int_0^{2\pi}(\sin\theta\cos\theta)d\theta\right]$$

$$= \frac{1}{2\pi}[\pi - 0]$$

$$= \frac{1}{2} = 0.5$$

17. Correct answer is [1.5].

Let the number of tosses to get two head be denoted by random variable X.

$$P(X=2) = HH = \frac{1}{2}\times\frac{1}{2}$$

$$P(X=3) = THH = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$

$$P(X=4) = TTHH = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$

$$E(X) = \sum XP(X) = 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^4 \ldots$$

$$s = 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^4 \ldots \qquad \ldots(i)$$

$$\frac{s}{2} = 2\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^5 \ldots \qquad \ldots(ii)$$

Subtract equation (ii) from (i)

$$\frac{s}{2} = 2\left(\frac{1}{2}\right)^2 + (3-2)\left(\frac{1}{2}\right)^3 + (4-3)\left(\frac{1}{2}\right)^4 \ldots$$

$$\frac{s}{2} = 2\left(\frac{1}{2}\right)^2 + 1\left(\frac{1}{2}\right)^3 + 1\left(\frac{1}{2}\right)^4 \ldots$$

$$\frac{s}{2} - \frac{1}{2} = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^5 + \ldots$$

$$\frac{s}{2} - \frac{1}{2} = \frac{\left(\frac{1}{2}\right)^3}{1-\frac{1}{2}} = \left(\frac{1}{2}\right)^2$$

$$\frac{s}{2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$E(X) = \frac{3}{2} = 1.5$$

18. Option (d) is correct.

Given $\quad P(x=0) = p$

$\therefore \quad P(x=1) = 1-p$

$\quad P(y=0) = q$

$\therefore \quad P(y=1) = 1-q$

Let $Z = X + Y$

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	2

$P(z \geq 1) = P(z = 1) + P(z = 2)$

$$\Rightarrow P(z \geq 1) = P(x=0 \text{ and } y=1) + P(x=1 \text{ and } y=0) + P(x=1 \text{ and } y=1)$$

$$= 1 - P(x=0 \text{ and } y=0)$$

$$= 1 - pq$$

19. Option (c) is correct.

$$|A^T A^{-1}| = |A^T|\cdot|A^{-1}|$$

$$= |A|\cdot\frac{1}{|A|} \qquad \left[\begin{array}{l}\because |A^T| = |A| \\ |A^{-1}| = \frac{1}{|A|}\end{array}\right]$$

$$= 1$$

20. Option (a) is correct.

Partial derivative of (x^2+y^2) wrt y = Partial derivative of $(6y+4x)$ wrt x

So, $0+2y = 0+4$

$\Rightarrow \quad 2y = 4$

$\Rightarrow \quad y = 2$

21. Option (b) is correct.

Using residue theorem

$$\oint_C \frac{f(z)}{(z-z_0)^{n+1}}dz = 2\pi j \times (\text{Residue at } z_0)$$

Here Residue = $\lim_{z\to z_0}\frac{1}{n!}\frac{d^n f(z)}{dz^n}$, z_0 is pole of order n

Given $f(z) = 1$ so, derivative of $f(z) = 0$

$\therefore$ residue = 0

$$\oint_C \frac{dz}{(z-z_0)^{n+1}} = 0$$

22. Correct answer is [0.155].

Given $g(t) = e^{-t}\sin 2\pi t u(t)$

Taking laplace transform

$$G(s) = \frac{2\pi}{(s+1)^2+(2\pi)^2}, \text{ First shifting property}$$

$$G(s) = \int_{-\infty}^{\infty} g(t)e^{-st}dt$$

$$G(0) = \int_{-\infty}^{\infty} g(t)\,dt = \text{area under the curve } g(t)$$

Putting ($s = 0$)

$$G(0) = \frac{2\pi}{(0+1)^2+(2\pi)^2} = \frac{2\pi}{1+(2\pi)^2} = 0.155$$

23. Correct answer is [2].

$$\sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n$$

$$\text{Let S} = 0+1\left(\frac{1}{2}\right)^1+2\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)^3+\ldots \qquad \ldots\text{(i)}$$

Multiplying equation (1) by $\frac{1}{2}$

$$\frac{S}{2} = \left(\frac{1}{2}\right)^2+2\left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^4+\ldots \qquad \ldots\text{(ii)}$$

Subtracting equation (ii) from (i)

$$S-\frac{S}{2} = \frac{1}{2}+\left(\frac{1}{2}\right)^2(2-1)+\left(\frac{1}{2}\right)^3(3-2)+\left(\frac{1}{2}\right)^4(4-3)+\ldots\text{GP}$$

$$\Rightarrow \quad \frac{S}{2} = \frac{1}{2}+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^4+\ldots$$

$$\Rightarrow \quad \frac{S}{2} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

$\therefore \quad S = 2$

24. Correct answer is [4.2903].

$$f(x) = x^3-5x^2+6x-8$$

$$x_0 = 5$$

$$f'(x) = 3x^2-10x+6$$

According to Newton Raphson

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$f(5) = (5)^3-5(5)^2+6(5)-8 = 22$$

$$f'(5) = 3(5)^2-10(5)+6 = 31$$

$$x_1 = 5-\frac{f(5)}{f'(5)} = 5-\frac{22}{31} = 4.29$$

25. Correct answer is [6].

Probability of getting 3 = $\frac{1}{6}$

Probability of not getting 3 = $1-\frac{1}{6} = \frac{5}{6}$

Given that the random variable X represent the no. of throws required for getting 3.

$$X = 1, P(x=1) = \frac{1}{6}$$

$$X = 2, P(x=2) = \frac{5}{6}\times\frac{1}{6}$$

$$X = 3, P(x=3) = \frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}$$

$$X = 4, P(x=4) = \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}$$

$$E[X] = \sum xP(x)$$

$$= (1)\left(\frac{1}{6}\right)+2\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)+3\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)+4\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)\ldots$$

$$= \left(\frac{1}{6}\right)\left[1+2\left(\frac{5}{6}\right)+3\left(\frac{5}{6}\right)^2+4\left(\frac{5}{6}\right)^3 \ldots\right]$$

$$\because \left(1+2x+3x^2+4x^3+\ldots\right) = \frac{1}{(1-x)^2}$$

$$\therefore \ \mathrm{E}(x) = \frac{1}{6}\cdot\frac{1}{\left(1-\frac{5}{6}\right)^2}$$

$$\Rightarrow \mathrm{E}(x) = 6$$

26. Correct answer is [0.8556].

Given DE is $\frac{d^2x(t)}{dt^2}+3\frac{dx(t)}{dt}+2x(t)=0$

AE $\rightarrow m^2+3m+2=0$

$\Rightarrow \quad m = -1, -2$

$x(t) = \mathrm{A}e^{-t}+\mathrm{B}e^{-2t}$

at $t=0$, $x(0)=20$

$\Rightarrow \quad 20 = \mathrm{A} + \mathrm{B}$

at $t=1$, $x(1) = \frac{10}{e}$

$\Rightarrow \quad \frac{10}{e} = \mathrm{A}e^{-1}+\mathrm{B}e^{-2}$

$$\mathrm{A} = \left(\frac{10e-20}{e-1}\right), \quad \mathrm{B} = \left(\frac{10e}{e-1}\right)$$

$$\therefore \quad x(t) = \frac{(10e-20)}{(e-1)}\cdot e^{-t}+\frac{10e}{(e-1)}\cdot e^{-2t}$$

at $t = 2$

$$x(t) = \left(\frac{10e-20}{e-1}\right)e^{-2}+\left(\frac{10e}{e-1}\right)e^{-4}$$

$$= 0.8556$$

27. Correct answer is [78.52].

$$\mathrm{D} = 2\rho^2 a_\rho + z a_z$$

Using divergence theorem

$$\oiiint_{\mathrm{V}}(\nabla.\mathrm{D})d\mathrm{V} = \oiint_{\mathrm{S}}\mathrm{D}.ds$$

$$\nabla.\mathrm{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho \mathrm{D}_\rho\right)+\frac{1}{\rho}\frac{\partial \mathrm{D}\phi}{\partial\phi}+\frac{\partial \mathrm{D}_z}{\partial z}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho 2\rho^2\right)+0+\frac{\partial z}{\partial z}$$

$$= \frac{1}{\rho}(2)(3)\rho^2+1 = 6\rho+1$$

$$\oiiint_{\mathrm{V}}(\nabla.\mathrm{D})d\mathrm{V} = \int_{\rho=0}^{1}\int_{\phi=0}^{2\pi}\int_{z=0}^{5}(6\rho+1)(\rho)\,d\rho\, d\phi\, dz$$

$$\left(2\rho^3+\frac{\rho^2}{2}\right)_0^1(\phi)_0^{2\pi}(2)_0^5 = 78.52$$

28. Correct answer is [0.25].

Given, $\quad f(x) = \begin{cases} a+bx & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$

$$\because \quad \int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \quad \int_0^1 (a+bx)dx = 1$$

$$\Rightarrow \quad \left[ax+\frac{bx^2}{2}\right]_0^1 = 1$$

$$\Rightarrow \quad \left[a+\frac{b}{2}\right] = 1 \qquad \ldots(\mathrm{i})$$

Given, $\quad \mathrm{E}[\mathrm{X}] = \frac{2}{3}$

$$\therefore \quad \int_{-\infty}^{\infty} xf(x)dx = \frac{2}{3}$$

$$\Rightarrow \quad \int_0^1 x(a+bx)dx = \frac{2}{3}$$

$$\Rightarrow \quad \left(\frac{ax^2}{2}+\frac{bx^3}{3}\right)_0^1 = \frac{2}{3}$$

$$\Rightarrow \quad \left(\frac{a}{2}+\frac{b}{3}\right) = \frac{2}{3}$$

$$\Rightarrow \quad \frac{3a+2b}{6} = \frac{2}{3} \Rightarrow 3a+2b = 4 \qquad \ldots(\mathrm{ii})$$

Solving Equation (i) and (ii)

$a = 0$, $b = 2$

$$\Pr(\mathrm{X}<0.5) = \int_{x=-\infty}^{0.5} f(x)dx$$

$$\int_0^{0.5} 2x\,dx = \left[x^2\right]_0^{0.5}$$

$$= 0.25$$

29. Option (c) is correct.

$f(x)$ is continuous function and does not have any root in $[a, b]$. This means function is either positive or negative in $[a, b]$.

There can be two cases.

For function > 0

$f(a)\cdot f(b) \equiv (+ve)\,(+ve)$

$\equiv (+ve)$

For function < 0

$f(a)\cdot f(b) \equiv (-ve)\,(-ve)$

$\equiv (+ve)$

So, in both case $f(a), f(b) > 0$

30. Correct answer is [9].

For $A_{(2\times2)}$ Trace $= -6$

Let λ_1, λ_2 be eigen value of A

Then $\lambda_1 + \lambda_2 = -6$

also $|A| = \lambda_1 \lambda_2$

$\Rightarrow$ $|A| = \lambda_1(-6-\lambda_1)$

$\Rightarrow$ $|A| = -\lambda_1^2 - 6\lambda_1$

For maximum $|A|$

$$\frac{d|A|}{d\lambda_1} = 0$$

$\Rightarrow$ $-2\lambda_1 - 6 = 0$

$\Rightarrow$ $\lambda_1 = \frac{6}{-2} = -3$

So, $\lambda_2 = -6-(-3) = -3$

$|A| = (-3)(-3) = 9$

31. Option (a) is correct.

$$\vec{f} = \frac{1}{r^2}\vec{r}$$

$\vec{r}$ is unit vector in the radial direction.

$$\nabla,\vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta F_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\left(F_\phi\right)$$

$$= \frac{1^2}{r^2}\frac{\partial}{\partial r}\left(r^2 + \frac{1}{r^2}\right) + 0$$

$$\nabla.\vec{F} = 0$$

32. Option (b) is correct.

$$A = \begin{bmatrix} -3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & a & -2 \end{bmatrix}$$

Characteristic equation

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -3-\lambda & 0 & -2 \\ 1 & -1-\lambda & 0 \\ 0 & a & -2-\lambda \end{vmatrix} = 0$$

$$(-3-\lambda)(-1-\lambda)(-2-\lambda) - 2a = 0$$

$$(3+\lambda)(1+\lambda)(2+\lambda) + 2a = 0$$

$$\lambda^3 + 6\lambda^2 + 11\lambda + 6 + 2a = 0$$

$$a = -\frac{\lambda^3}{2} - 3\lambda^2 - \frac{11}{2}\lambda - 3 \quad \text{...(i)}$$

For maximum value of a,

$$\frac{da}{d\lambda} = -\frac{3}{2}\lambda^2 - 6\lambda - \frac{11}{2} = 0$$

$$3\lambda^2 + 12\lambda + 11 = 0$$

$$\lambda = \frac{-12 \pm \sqrt{144 - 4(3)(11)}}{2\times3} = -2 \pm \frac{1}{\sqrt{3}}$$

a will be maximum for

$$\frac{d^2a}{d\lambda^2} = -3\lambda - 6$$

$$= -3\left(-2+\frac{1}{\sqrt{3}}\right) - 6 = -\sqrt{3} < 0$$

$$\left(\text{for } \lambda = -2+\frac{1}{\sqrt{3}}\right)$$

$$= -3\left(-2-\frac{1}{\sqrt{3}}\right) - 6 = \sqrt{3} > 0$$

$$\left(\text{for } \lambda = -2-\frac{1}{\sqrt{3}}\right)$$

a is maximum for $\lambda = -2 + \frac{1}{\sqrt{3}}$

$$a = -\frac{1}{2}\left(-2+\frac{1}{\sqrt{3}}\right)^3 - 3\left(-2+\frac{1}{\sqrt{3}}\right)^2 - \frac{11}{2}\left(-2+\frac{1}{\sqrt{3}}\right) - 3$$

$$= 0.1924 \approx \frac{1}{3\sqrt{3}}$$

$$a = \frac{1}{3\sqrt{3}}$$

33. Correct answer is [–3].

Given DE can be written in the form

$$\left(D^2 + 5D + 6\right)y = 0$$

$$AE \rightarrow m^2 + 5m + 6 = 0$$

$$m = -2, -3$$

$$y(t) = C_1e^{-2t} + C_2e^{-3t} \quad \text{...(i)}$$

at $t = 0$, $y(0) = 2 \Rightarrow 2 = C_1 + C_2$ ...(ii)

at $t = 1$, $y(1) = -\frac{(1-3e)}{e^3}$ ...(iii)

$$\Rightarrow -\frac{1-3e}{e^3} = \frac{C_1e + C_2}{e^3} \Rightarrow -e^{-3} + 3e^{-2} = C_1e^{-2} + C_2e^{-3}$$

Solving (ii) and (iii)

$$C_1 = 3, C_2 = -1 \quad \text{...(iv)}$$

Putting the value of equation (iv) in (i)

$$y(t) = 3e^{-2t} - e^{-3t}$$

$$\therefore \quad \frac{dy(t)}{dt} = 3(-2)\,e^{-2t} + 3e^{-3t} = -be^{-2t} + 3e^{-3t}$$

$$\frac{dy(0)}{dt} = -6 + 3 = -3$$

34. Option (d) is correct.

Probability of getting 6= $\frac{1}{6}$

Probability of not getting 6 $=1-\frac{1}{6}=\frac{5}{6}$

Probability that A wins the game in first chance $=\frac{1}{6}$

The probability that A wins the game in 2^{nd} chance is possible if 6 does not appear in 1^{st} chance of A. Again 6 does not appear in 1^{st} chance of B and then 6 comes in 2^{nd} chance of A.

$\therefore$ Probability is $\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}$

So, the total probability that A wins the game

$$= P(A)+P(\overline{A})P(\overline{B})P(A)+P(\overline{A})P(\overline{B})$$
$$P(\overline{A})P(\overline{B})P(A)+.....$$

$$=\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+......$$

$$=\frac{1}{6}+\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^4\cdot\frac{1}{6}+......\text{is a GP series}$$

$$=\frac{\frac{1}{6}}{1-\left(\frac{5}{6}\right)^2}=\frac{1}{6}\times\frac{36}{11}=\frac{6}{11}$$

35. Option (b) is correct.

Given $f(z)=g(z)+h(z)$

$f(z), g(z), h(z)$ are complex variable functions.

Option (i) is not correct $g(z)$ and $h(z)$may individually be indifferentiable in split of their sum being differentiable at a point.

Option (ii) is correct, if both the functions are individually differentiable then their sum is also differentiable.

It is not necessary that every continuous function is differentiable.

Therefore option (iii) is not correct, and option (iv) is also not correct.

36. Option (a) is correct.

Given a set of 3 linear equations in 3 unknowns.

If the linear equations is having a unique solution, then the equations are linearly independent.

For system of equation to have unique solution it has to be a non singular matrix i.e. determinat will be non-zero.

For non-zero determinant as it will be product of it's eigen value so eigen values will be non-zero.

$\therefore$ $P \equiv Q \equiv R \equiv S$ is correct.

37. Option (b) is correct.

According to vector calculus

Stoke's theorem $\iint(\nabla\times A)ds=\oint A.dl$

Divergence theorem $\iiint(\nabla.A)dv=\oiint A.ds$

Cauchy's integral theorem

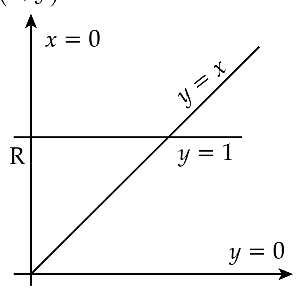
$\oint f(z)dz=0$

Gauss Theorem

$\oiint D.ds=Q$

38. Correct answer is [0.71].

Given $f(x,y)=e^x$



$y = x$ line is shown in figure and required area between $x = 0$, $x = y$ and $y = 1$.

$$=\int_{y=0}^{y=1}\int_{x=0}^{x=y}e^x dxdy=\int_{y=0}^{y=1}\left[e^x\right]_0^y dy$$

$$\int_{y=0}^{y=1}\left[e^y-1\right]dy=\left[e^y-y\right]_0^1=\left[e^1-1\right]-\left[e^0-0\right]$$

$$=e-2=0.718$$

39. Option (d) is correct.

Given 4 joint events are mutually exclusive then $P(H_R)=P(H_RH_S)+P(H_RT_S)$

$$=0.28+0.30=0.58$$
$$P(H_S)=P(H_RH_S)+P(T_RH_S)$$
$$=0.28+0.24=0.52$$

$\therefore$ $P(T_R) = 1- P(H_R) = 0.42,\quad P(T_R) \neq P(H_R)$

$P(T_S) = 1- P(H_S) = 0.48,\quad P(T_S) \neq P(H_S)$

So, R is not fair, S is also not fair.

Also $P(H_R\,H_S) = 0.28 \neq P(H_R)\cdot P(H_S) = 0.58 * 0.52$

So, Coin tosses are dependent.

40. Correct answer is [1.65].

Given DE $\frac{di}{dt}-\frac{1}{5}i=0\quad$ for $-10<t<10$

Separating the variables

$$\frac{di}{i}=\frac{1}{5}dt$$

Integrating both sides

$$\ln i=\frac{1}{5}t+C$$

$$i = e^{\frac{1}{5}t+C}$$

$$i = Ae^{\frac{1}{5}t} \quad ...(i)$$

$$i(4) = 10$$

$$\Rightarrow \quad 10 = Ae^{\frac{4}{5}} \Rightarrow A = \frac{10}{e^{\frac{4}{5}}}$$

$$\therefore \quad i = \frac{10}{e^{\frac{4}{5}}} . e^{\frac{1}{5}t}$$

$$i(-5) = \frac{10}{e^{\frac{4}{5}}} . e^{\frac{1}{5}(-5)} = \frac{10}{e^{\frac{4}{5}}} . e^{-1} = 10e^{-1-\frac{4}{5}}$$

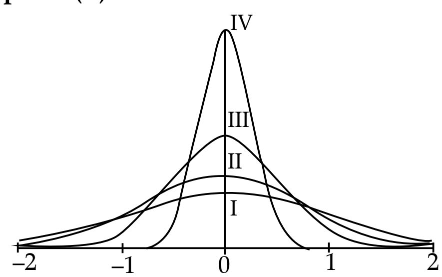
$$= 10e^{-\frac{9}{5}} = 1.65$$

41. Option (a) is correct.

According to properties of determinant.

Property if two rows or columns of a matrix are interchanged, the determinant sign changes but the absolute value remains same.

42. Option (d) is correct.



As for IV case the density is clustered around the mean tightly, the measure of variance will be low.

43. Correct answer is [2].

$$f(x) = \frac{3}{5}x^2 + \frac{9}{5}$$

Least number of interval needs to be even for Simpson's rule.

So, $n = 2$

$$h = \frac{1-0}{2} = 0.5$$

x	0	0.5	1
$y = f(x)$	1.8	1.95	2.4
	y_0	y_1	y_2

According to Simpson's 1/3rd rule.

$$\int_0^1 f(x)dx = \frac{h}{3}[y_0 + 4y_1 + y_2]$$

$$= \frac{0.5}{3}\left[1.8 + 4(1.95) + (2.4)\right] = 2$$

44. Option (c) is correct.

$$\lim_{x\to 0} \frac{1-\cos x^2}{2x^4} \to \frac{0}{0} \text{ form.}$$

Applying L' Hospital Rule.

$$= \lim_{x\to 0} \frac{2x\sin x^2}{8x^3}$$

$$= \frac{1}{4}\lim_{x^2\to 0} \frac{\sin x^2}{x^2} \quad \begin{bmatrix} \text{for } x \to 0 \\ x^2 \to 0 \end{bmatrix}$$

$$= \frac{1}{4} \times 1 = \frac{1}{4}$$

45. Option (a) is correct.

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$$

$$\arg(z_1) = \theta_1 = \tan^{-1} = \left(\frac{5\sqrt{3}}{5}\right) = \sqrt{3}$$

$$\arg(z_1) = \theta_1 = 60^\circ$$

$$\arg(z_2) = \theta_2 = \tan^{-1} = \left(\frac{2}{\frac{2}{\sqrt{3}}}\right) = \sqrt{3}$$

$$\arg(z_2) = \theta_2 = 60^\circ$$

$$\therefore \quad \arg\left(\frac{z_1}{z_2}\right) = 60^\circ - 60^\circ = 0^\circ$$

46. Correct answer is [1.86].

Given $x(t) = \cos t$, $y(t) = \sin t$, $z(t) = \frac{2}{\pi}t$ and

$$0 \le t \le \frac{\pi}{2}$$

Length of the curve =

$$L = \int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{(\cos t)^2 + (-\sin t)^2 + \left(\frac{2}{\pi}\right)^2}\, dt$$

$$= \sqrt{1 + \frac{4}{\pi^2}}\,(t)_0^{\frac{\pi}{2}}$$

$$= \sqrt{1 + \frac{4}{\pi^2}}\left(\frac{\pi}{2}\right)$$

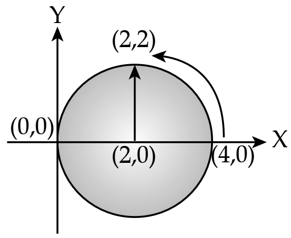
$$= 1.185 \times \left(\frac{\pi}{2}\right) = 1.86$$

47. Correct answer is [2].

Distance travelled by ant is $\frac{1}{4}$th fraction of circumference of circle with centre (2,0) and radius is 2m.

Now time taken by ant to reach the point (2,2) is in sec.

$$t = \frac{1}{4} \times \frac{(2\pi r)}{1.57} = 2\sec \quad \left[\text{time} = \frac{\text{distance}}{\text{speed}}\right]$$



48. Option (c) is correct.

∵ The solution of differential equation passes through origin, only option (c) passes through point (0,0) and no remaining option satisfies it.

49. Option (b) is correct.

Probability of getting 6 $\rightarrow \frac{1}{6}$

Probability of not getting 6 $\rightarrow 1 - \frac{1}{6} = \frac{5}{6}$

Required Probability = P$\{x \geq 2\}$ = 1– P (x < 2)

$$= 1 - \{P(X=0) + P(X=1)\}$$

$$= 1 - \{{}^4C_0 p^0 q^4 + {}^4C_1 p^1 q^3\}$$

$$= 1 - \left\{\left(\frac{5}{6}\right)^4 + 4\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3\right\}$$

$$= 1 - \frac{125}{144} = \frac{19}{144}$$

50. Option (b) is correct.

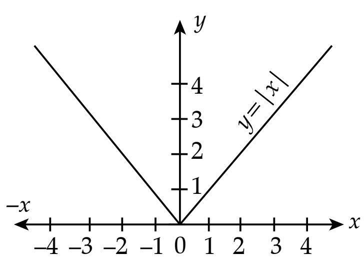
For a singular matrix A, $|A| = 0$

And we know product of eigen values = |A|

Therefore, one eigen values must be zero for the product of all the eigen values to be zero.

51. Option (a) is correct.

At x = 0, function has a minima.



52. Option (a) is correct.

$$\nabla \times V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2 & 3z^2 & y^3 \end{vmatrix}$$

$$= \hat{i}(3y^2 - 6z) - \hat{j}(0) + \hat{k}(0-0)$$

$$(\nabla \times V)_{(1,1,1)} = \hat{i}\left(3(1)^2 - 6(1)\right) = -3\hat{i}$$

53. Option (b) is correct.

$$e^{i5t} = \cos 5t + i\sin 5t$$

$$L\{e^{i5t}\} = L\{\cos 5t\} + L\{i\sin 5t\}$$

$$L\{e^{i5t}\} = \frac{s}{s^2+25} + i\frac{5}{s^2+25} = \frac{s+5i}{s^2+25}$$

54. Correct answer is [0.97].

Probability that none will satisfy the design specification –

P = 0.2 × 0.3 × 0.5

P = 0.03

Probability that at least one will satisfy the design specification

= 1 – Probability (none will meet the specifications)

= 1 – P = 1 – 0.03 = 0.97

55. Option (b) is correct.

Event A – Student passes the exam

Event B – Student will get above 90% marks

P(A) = 20%; $P(A \cap B) = 5\%$

Required Probability

$$= P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{0.05}{0.20} = \frac{1}{4}$$

56. Correct answer is [216].

According to Gauss Divergence theorem

$$\iint_s \vec{F}.\vec{n}.ds = \iiint_v \text{Div}\vec{F}\, dv$$

Given $\vec{F} = 9x\hat{i} - 3y\hat{j}$

$$\text{Div}\vec{F} = 9 - 3 = 6$$

∴ Surface integral = $\frac{1}{\pi}\iiint 6\, dv$

∵ $r = 3$

Surface integral = $\frac{1}{\pi} 6 \times \frac{4}{3}\pi(3)^3 = 216$

57. Option (c) is correct.

$$\frac{dy}{dt} = -5y$$

Separating the variables

$$\int \frac{dy}{y} = -5\int dt$$

$$\log y = -5t + c$$

$$at\ t = 0,\ y = 2$$

$$\ln 2 = c$$

$$\log y = -5t + \ln 2$$

For $t = 3 \log \frac{y}{2} = -5\times 3 = -15$

$$\frac{y}{2} = e^{-15} \quad y = 2e^{-15}$$

58. Correct answer is [22].

According to Trapezoidal rule

x	0	0.1	0.2	0.3	0.4
$f(x)$	0 y_0	10 y_1	40 y_2	90 y_3	160 y_4

$$I = \int_0^x f(x)dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3) + y_4\right]$$

$$= \frac{0.1}{2}\left[0 + 2(10 + 40 + 90) + 160\right] = 22$$

59. Correct answer is [29.6].

Given $a = \frac{dv}{dt} = -0.1\,v$

$$\int_{40}^{v} \frac{dv}{v} = -\int_0^3 0.1dt$$

$$\Rightarrow \quad \ln[v]_{40}^{v} = -0.1[t]_0^3$$

$$\Rightarrow \quad \ln v - \ln 40 = -0.1\times 3$$

$$\Rightarrow \quad \ln v - 3.6888 = -0.3$$

$$\Rightarrow \quad \ln v = 3.3888$$

$$\Rightarrow \quad v = 29.6\ \text{m/sec}$$

60. Correct answer is [0.693].

x	1	2
$y = x\ln x$	0	2ln2

According to Trapezoidal rule

$$\int_1^2 x\ln x dx = \frac{h}{2}\left[y_0 + y_1\right]$$

$$= \frac{1}{2}[0 + 2\ln 2]$$

$$= \ln 2 = 0.693$$

61. Option (c) is correct.

Given $P(X) = \frac{1}{4}$, $P(Y) = \frac{1}{3}$ and

$$P(X\cap Y) = \frac{1}{12}$$

We know $P\left(\frac{Y}{X}\right) = \frac{P(X\cap Y)}{P(X)} = \frac{\frac{1}{12}}{\frac{1}{4}} = \frac{4}{12} = \frac{1}{3}$

62. Correct answer is [2].

We know $|A - \lambda I| = 0$

Characteristics equation

$$\begin{vmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad (4-\lambda)(3-\lambda) - 2 = 0$$

$$\Rightarrow \quad \lambda^2 - 7\lambda + 10 = 0$$

$$\Rightarrow \quad \lambda = 5,\ 2$$

Therefore, lowest eigen value is 2.

63. Correct answer is [– 0.33].

$$\lim_{x\to 0}\left(\frac{-\sin x}{2\sin x + x\cos x}\right) \to \frac{0}{0}$$

Applying L Hospital rule

$$\lim_{x\to 0}\left(\frac{-\cos x}{2\cos x - x\sin x + \cos x}\right) = -\frac{1}{3} = -0.333$$

64. Option (c) is correct.

$$\text{Div}\left(\text{Curl}\vec{V}\right) = 0$$

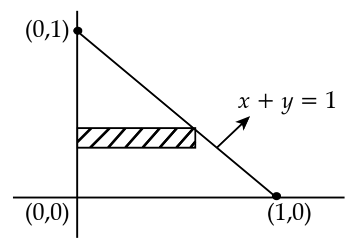
65. Correct answer is [1.66].

Considering the limit, for $x = 0$ to $1 - y$

And for $y = 0$ to 1.

Now according to Green's theorem.

$$\int_C P dx + Q dy = \iint_R \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dA$$



$$\text{Line integral} = \int_{y=0}^{1}\int_{x=0}^{1-y}\left[-6y - (-16y)\right]$$

with $P = 3x - 8y^2$

$Q = 4y - 6xy$

$$= \int_{y=0}^{1}\left[10xy\right]_0^{1-y} dy$$

$$= \int_{y=0}^{1}\left[10(1-y)y\right] dy$$

$$= 10\left[\frac{y^2}{2} - \frac{y^3}{3}\right]_0^1$$

$$= 1.66$$

66. Option (a) is correct.

$$|A| = \begin{vmatrix} 4+3i & -i \\ i & 4-3i \end{vmatrix}$$

$$= (4+3i)\times(4-3i) - (-i)\times i$$
$$= 25 - 1$$
$$= 24$$

$$\text{Adj}(A) = \text{Adj}\begin{bmatrix} 4-3i & -i \\ i & 4-3i \end{bmatrix}$$

$$= \begin{bmatrix} +(4-3i) & -(i) \\ -(-i) & +(4+3i) \end{bmatrix}^T$$

$$= \begin{bmatrix} 4-3i & -i \\ i & 4+3i \end{bmatrix}^T$$

$$= \begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}^T$$

Now, $A^{-1} = \dfrac{1}{|A|}\times \text{Adj}(A)$

$$= \frac{1}{24}\times\begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}$$

67. Correct answer is [0.30].

$$f(x) = x^3 + 2x^2 + 3x - 1$$
$$f'(x) = 3x^2 + 4x + 3$$
$$x_0 = 1$$
$$f(x_0) = 5 \text{ and } f'(x_0) = 10$$

By Newton Raphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{5}{10} = 0.5$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$f(x_1) = 1.125 \text{ and } f'(x_1) = 5.75$$

$$x_2 = 0.5 - \frac{1.125}{5.75}$$

$\Rightarrow \quad x_2 = 0.304$

68. Option (c) is correct.

$$F(s) = \int_0^\infty f(t)e^{-st}dt$$

$$F(s) = \int_0^1 (2)e^{-st}dt + \int_1^\infty (0)e^{-st}dt$$

$$F(s) = 2\left[\frac{e^{-st}}{-s}\right]_0^1$$

$$F(s) = \frac{2-2e^{-s}}{s}$$

69. Correct answer is [4.5].

Given, $2x + 3y = 5$

$$3x + py = 10$$

Above equations can be written in the form of AX = B as

$$\Rightarrow \begin{bmatrix} 2 & 3 \\ 3 & p \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

Augmented matrix

$$\left[\frac{A}{B}\right] = \begin{bmatrix} 2 & 3 & 5 \\ 3 & p & 10 \end{bmatrix}$$

$$R_2 \to R_2 - \frac{3}{2}R_1 \begin{bmatrix} 2 & 3 & 5 \\ 0 & p-\frac{9}{2} & \frac{5}{2} \end{bmatrix}$$

System will have no solution when

$$\rho(A/B) \neq \rho(A)$$

$$\Rightarrow \quad p - \frac{9}{2} = 0 \qquad \Rightarrow 2p - 9 = 0$$
$$p = 4.5$$

70. Option (a) is correct.

As the integrand *i.e.* x^2 is concave up the error $e = I - J$ will be negative

$\therefore$ J > I; Here J is approximated value, and I is analytical value.

71. Correct answer is [0.24].

For $q = 0.4$

$$p(x, q) = \begin{cases} 0.4 & \text{if } X = 0 \\ 0.6 & \text{if } X = 1 \\ 0 & \text{otherwise} \end{cases}$$

X	0	1
$p(x)$	0.4	0.6

$$V(X) = E(X^2) - \{E(X)\}^2$$

$$E(X) = \sum x_i p_i = 0(0.4) + 1(0.6) = 0.6$$

$$E(X^2) = \sum x_i^2 p_i = 0^2(0.4) + 1^2(0.6) = 0.6$$

$$V(X) = 0.6 - (0.6)^2 = 0.24$$

72. Option (d) is correct.

Let $A = \begin{bmatrix} 3 & -2 & 2 \\ 4 & -4 & 6 \\ 2 & -3 & 5 \end{bmatrix}$

Characteristic equation

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 3-\lambda & -2 & 2 \\ 4 & -4-\lambda & 6 \\ 2 & -3 & 5-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad \lambda^3 - 4\lambda^2 + 5\lambda - 2 = 0$

$\Rightarrow \quad (\lambda - 1)(\lambda^2 - 3\lambda + 2) = 0$

$\Rightarrow \quad (\lambda-1)(\lambda-1)(\lambda-2) = 0 \Rightarrow \lambda = 1, 1, 2$

Smallest eigen value 1, largest eigen value 2.

73. Correct answer is [2.333].

$f(x) = x^2 - 4x + 4$

$f'(x) = 2x - 4$

$x_0 = 3$

$f(x_0) = 1$

$f'(x_0) = 2$

According to Newton Raphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{1}{2} = 2.5$$

Now Secant method

$x_0 = 2.5$

$x_1 = 3$

$$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_1)$$

$$= 3 - \frac{3 - 2.5}{f(3) - f(2.5)} = 3 - \frac{0.5}{1 - 0.25} \times 1$$

$$= 3 - \frac{0.5}{0.75} = 2.333$$

74. Option (c) is correct.

$$x(ydx + xdy)\cos\frac{y}{x} = y(xdy - ydx)\sin\frac{y}{x}$$

$$x(ydx + xdy)\cos\frac{y}{x} + y(ydx - xdy)\sin\frac{y}{x} = 0$$

$$\Rightarrow \quad ydx\left(x\cos\frac{y}{x} + y\sin\frac{y}{x}\right) + xdy\left(x\cos\frac{y}{x} - y\sin\frac{y}{x}\right) = 0$$

$$\Rightarrow \quad ydx\left(\cos\frac{y}{x} + \frac{y}{x}\sin\frac{y}{x}\right) + dy\left(x\cos\frac{y}{x} - y\sin\frac{y}{x}\right) = 0$$

dividing by x

$$\Rightarrow \quad \left(y\cos\frac{y}{x} + \frac{y^2}{x}\sin\frac{y}{x}\right)dx + \left(x\cos\frac{y}{x} - y\sin\frac{y}{x}\right)dy = 0$$

$$\Rightarrow \quad \partial\left\{xy\cos\frac{y}{x}\right\} = 0$$

integrating

$$\Rightarrow \quad xy\cos\frac{y}{x} = C$$

75. Option (a) is correct.

$$f(z) = \frac{9}{(z-1)(z+2)^2}$$

$z = 1$ is a simple pole.

$z = -2$ is a pole of order 2.

By residue theorem

$$\text{Res } f(z)_{z=1} = \lim_{z\to 1}\left\{(z-1)\frac{9}{(z-1)(z+2)^2}\right\} = 1$$

$$\text{Res } f(z)_{z=-2} = \frac{1}{1!}\lim_{z\to -2}\left\{\frac{d}{dz}\left[(z+2)^2\frac{9}{(z-1)(z+2)^2}\right]\right\}$$

$$= \lim_{z\to -2}\frac{-9}{(z-1)^2} = \frac{-9}{9} = -1$$

76. [–5.715]

Given $u(x,y,z) = x^2 - 3yz$

$\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and a point A$= (2, -1, 4)$

$$\nabla u = \hat{i}\frac{\partial u}{\partial x} + \hat{j}\frac{\partial u}{\partial y} + \hat{k}\frac{\partial u}{\partial z}$$

$$= \hat{i}\,2x + \hat{j}(-3z) + \hat{k}(-3y)$$

$$(\nabla u)_{(2,-1,4)} = 4\hat{i} - 12\hat{j} + 3\hat{k}$$

$$|\vec{a}| = \sqrt{1^2 + 1^2 + (-2)^2} = \sqrt{6}$$

$$\nabla u.\hat{a} = (4i - 12j + 3k)\frac{(\hat{i} + \hat{j} - 2\hat{k})}{\sqrt{6}}$$

$$= \frac{4 - 12 - 6}{\sqrt{6}} = \frac{-14}{\sqrt{6}} = -5.715$$

77. Option (d) is correct.

For a point x_0 to be a minima $f'(x_0) = 0$ and $f''(x_0) > 0$

78. Correct answer is [0.333].

$f(x) = -2 + 6x - 4x^2 + (0.5)x^3$

$f'(x) = 6 - 8x + 1.5x^2$

$x_0 = 0$

$f(0) = -2$ and $f'(0) = 6$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{(-2)}{6} = \frac{1}{3} = 0.333$$

$$\Delta x = x_1 - x_0 = 0.333 - 0 = 0.333$$

79. Option (c) is correct.

$$I = \int_0^{\pi} \frac{\cos x + i\sin x}{\cos x - i\sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{e^{ix}}{e^{-ix}} dx = \int_0^{\frac{\pi}{2}} e^{2ix} dx = \left(\frac{e^{2ix}}{2i}\right)_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2i}\left[e^{\frac{2i\pi}{2}} - e^0\right] = \frac{1}{2i}\left[e^{i\pi} - 1\right]$$

$$= \frac{1}{2i}[\cos\pi + i\sin\pi - 1] = \frac{1}{2i}(-1+0-1) = \frac{-1}{i} = i$$

80. Option (d) is correct.

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x}$$

$$= \left(\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x\right)^2 \qquad \left[\because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e\right]$$

$$= e^2$$

81. Option (b) is correct.

Given $A = [a_{ij}]\ 1 \le i, j \le n, n \ge 3$ and $a_{ij} = i.j$

$$A = \begin{Bmatrix} 1 & 2 & 3 & --- \\ 2 & 4 & 6 & --- \\ 3 & 6 & 9 & --- \\ - & - & - & --- \end{Bmatrix}$$

We can observe

$R_2 = 2R_1$

$R_3 = 3R_1$

$\vdots$

$R_n = nR_1$

So, number of independent rows $= 1$

$\therefore \quad \rho(A) = 1$

82. Correct answer is [1.0667].

$$f(x) = \begin{cases} \frac{x}{4}(4-x^2) & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_x = \int_0^2 x f(x)\, dx$$

$$= \int_0^2 x\frac{x}{4}(4-x^2)dx = \frac{1}{4}\int_0^2 (4x^2 - x^4)dx$$

$$= \frac{1}{4}\left[4\frac{x^3}{3} - \frac{x^5}{5}\right]_0^2 = \frac{1}{4}\left\{\frac{32}{3} - \frac{32}{5}\right\}$$

$$= 8\left[\frac{2}{15}\right] = \frac{16}{15} = 1.0667$$

83. [18]

$$\frac{d^2y}{dx^2} = -12x^2 + 24x - 20$$

Integrating both sides

$$\frac{dy}{dx} = -4x^3 + 12x^2 - 20x + c_1$$

Integrating again on both sides

$$y = -x^4 + 4x^3 - 10x^2 + c_1x + c_2 \qquad \text{(i)}$$

Given $y = 5$ for $x = 0$

In (i) $\quad 5 = c_2$

Also $\quad y = 21$ for $x = 2$

In (i) $\quad 21 = -2^4 + 4(2^3) - 10(2)^2 + 2c_1 + 5$

$\Rightarrow \quad 40 = 2c_1$

$\therefore \quad c_1 = 20$

So, $\quad y = -x^4 + 4x^3 - 10x^2 + 20x + 5$

$y(x = 1) = -1 + 4 - 10 + 20 + 5 = 18$

84. Option (d) is correct.

Let $A = \begin{bmatrix} 2 & 1 \\ 1 & p \end{bmatrix}$ have eigen value $\lambda, 3\lambda$

$|A| = 2p - 1 = \lambda(3\lambda) = 3\lambda^2$

$\Rightarrow \quad \frac{2p-1}{3} = \lambda^2$

Trace (A) $= 2 + p = \lambda + 3\lambda = 4\lambda$

$\Rightarrow \quad \frac{2+p}{4} = \lambda$

for $\quad \lambda^2 = \lambda^2$

$$\left(\frac{2+p}{4}\right)^2 = \frac{2p-1}{3}$$

$\Rightarrow \quad \frac{4+p^2+4p}{16} = \frac{2p-1}{3}$

$\Rightarrow \quad 12 + 3p^2 + 12p = 32p - 16$

$\Rightarrow \quad 3p^2 - 20p + 28 = 0$

Solving we get $\lambda = 2$ and $\lambda = \frac{14}{3}$

85. Correct answer is [–3.8293].

$h = \Delta x = 0.4$

$$f(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5$$

$$x_0 = 0, x_n = 0.8 \Rightarrow n = \frac{0.8-0}{0.4} = 2$$

x	0	0.4	0.8
$y = f(x)$	0.2 y_0	2.456 y_1	0.232 y_2

According to Simpson's rule

$$= \frac{h}{3}[y_0 + 4y_1 + y_2]$$

$$= \frac{0.4}{3}\{0.2 + 4(2.456) + 0.232\}$$

$$= 1.367$$

86. Option (a) is correct.

$$g(h(x)) = g\left(\frac{x}{x-1}\right)$$
$$= 1 - \frac{x}{x-1}$$
$$= \frac{-1}{x-1}$$
$$h(g(x)) = h(1-x) = \frac{1-x}{(1-x)-1}$$
$$= \frac{1-x}{-x}$$
$$\frac{g(h(x))}{h(g(x))} = \frac{x}{(1-x)(x-1)} = \frac{h(x)}{g(x)}$$

87. Option (c) is correct.

By application of exponential of algorithm, we have the expression–

$$\lim_{x\to\infty} x^{\frac{1}{x}} = \lim_{x\to\infty} \exp\left(\log\left(x^{\frac{1}{x}}\right)\right)$$
$$= \lim_{x\to\infty} \exp\left(\frac{\log(x)}{x}\right)$$

$\because$ exponential function is continuous, so we can factor it out as

$$= \lim_{x\to\infty} \exp\left(\frac{\log(x)}{x}\right) = \exp \lim_{x\to\infty}\left(\frac{\log(x)}{x}\right)$$

And exponential functions grow asymptotically slower than polynomials.

$\because$ the $\log(x)$ grows asymptotically slower than polynomial x as $x \to \infty$

$$\lim_{x\to\infty}\left(\frac{\log(x)}{x}\right) = 0$$

$\Rightarrow$ $e^0 = 1$

88. Option (c) is correct.

$p \leftrightarrow q$ this means both $p \to q$ *and* $q \to p$, $q \to p$ *and* $p \to q$ is equivalent to ($\neg q \vee p$) so, option (a) and (b) are correct.

Option (d) is different way to writing option (a)

$p \leftrightarrow q = (p \to q) \wedge (q \to p)$

$= (\neg p \vee q) \wedge (q \to p)$ $\quad [\because p \to q = \neg p \vee q]$

$= (\neg p \vee q) \wedge (\neg q \vee p)$

$= (\neg p \wedge p) \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee (q \wedge \neg q)$

(According to Distributive law)

[As $((\neg p \wedge p) = 0, (q \wedge \neg q) = 0)$ (complementation)]

$(\neg p \wedge \neg q) \vee (p \wedge q)$ which is option (d)

Only option which is not equivalent to $p \leftrightarrow q$ is option (c)

89. Option (c) is correct.

Power set of A consists of all sets of A and from the definition of subset, ϕ is subset of any set. So, I and II are true.

5 and {6} are element of A and hence {5,{6}} is subset of A and hence an element of 2^A. An element of a set is never subset of the set. For the element must be inside a set– that is singleton set containing the element is a subset of the set, but the element itself is not. So, option IV is false. To make this option correct we write it as: {5,{6}}is an element of 2^A. So,$\{\{5,\{6\}\}\} \subseteq 2^A$

90. Correct answer is [5].

Given $\begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} * \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$

$$\begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} * \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$$

$$l_{11} = 2$$
$$l_{11} \times u_{12} = 2$$
$$\Rightarrow \quad u_{12} = 1$$
$$l_{21} = 4$$
$$l_{21} \times u_{12} + l_{22} = 9$$
$$4 \times 1 + l_{22} = 9 \Rightarrow l_{22} = 5$$

91. Correct answer is [0.99].

Given expression $\sum_{x=1}^{99} \frac{1}{x(x+1)}$

$$= \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots.. + \frac{1}{99.100}$$
$$= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots\ldots + \left(\frac{1}{99} - \frac{1}{100}\right)$$
$$= 1 - \frac{1}{100} = 0.99$$

92. Correct answer is [24].

By Euler formula of connected planner graph

$n - e + f = 2$

$10 - 3\frac{f}{2} + f = 2$ $\quad \left[\begin{array}{l}\because 2e = 3f\text{, as every edge} \\ \text{will be shared by 2 faces}\end{array}\right]$

$$\frac{f}{2} = 8 \Rightarrow f = 16$$
$$e = 3\frac{f}{2} = 24$$

93. Option (a) is correct.

Counting the no. of bits string not containing two consecutive 1's.

01

00 01 10 (append both 0 and 1 to any string ending in 0 and append 0 to any string putting ending in 1)

000 001 010 100 101(all strings ending in 0 give two in strings and those ending 1 give 1 string)
0000 0001 0010 0100 0101 1000 1001 1010 –8
⋮
$a'_n = a'_{n-1} + a'_{n-2}$ (where $a_n$denote the No. of bit strings of length n containing two consecutive 1′s)

$2^n - a_n = (2^{n-1} - a_{n-1}) + (2^{n-2} - a_{n-2})$

$-a_n = 2^{n-1} - 2^n + 2^{n-2} - a_{n-1} - a_{n-2}$

$a_n = a_{n-1} + a_{n-2} + 2^{n-2}$

94. Correct answer is –1.

Let $I = \int_{\frac{1}{\pi}}^{\frac{2}{\pi}} \frac{\cos\left(\frac{1}{x}\right)}{x^2} dx$

Putting $t = \frac{1}{x} \Rightarrow dt = -\frac{1}{x^2} dx$

New limits becomes $t = \frac{1}{\frac{1}{\pi}} = \pi$ to $t = \frac{1}{\frac{2}{\pi}} = \frac{\pi}{2}$

$$I = -\int_{\pi}^{\frac{\pi}{2}} \cos(u) du = \int_{\frac{\pi}{2}}^{\pi} \cos(u) du$$

$$= [\sin u]_{\frac{\pi}{2}}^{\pi} = \sin \pi - \sin\frac{\pi}{2} = 0 - 1 = -1$$

95. Option (d) is correct.

$$A = \begin{bmatrix} 1 & 4 \\ b & a \end{bmatrix}$$

Characteristic equation of matrix A.

$$|A - \lambda I| = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1-\lambda & 4 \\ b & a-\lambda \end{vmatrix} = 0$$

$\Rightarrow$ $(1 - \lambda)(a - \lambda) - 4b = 0$

Putting $\lambda = -1$

$(1-(-1))(a-(-1)) - 4b = 0$

$\Rightarrow$ $2(a+1) - 4b = 0$

$\Rightarrow$ $2a + 2 - 4b = 0$

$2b - a = 1$ ...(i)

$\Rightarrow$ $\lambda = 7$

$\Rightarrow$ $(1-7)(a-7) - 4b = 0$

$\Rightarrow$ $-6a + 42 - 4b = 0$

$\Rightarrow$ $2b + 3a = 21$ ...(ii)

From equation (i) and (ii) we get $a = 5, b = 3$

96. Option (c) is correct.

C – Candidate is corrupt.
K – Candidate is kind.
E – Candidate is elected.

$S_1 = C \rightarrow \neg E$

$S_2 = K \rightarrow E$

So, using primary operations

$S_1 = \neg C \vee \neg E$

$S_2 = \neg K \vee E$

Using resolution principal.

$\neg E$ and Ecancels each other out.

$\therefore \quad \neg C \vee \neg K$

This can also be written as $K \rightarrow \neg C$, therefore option (c) is correct.

97. Correct answer is [2048].

The no. of elements in set = 11

$\therefore$ cardinality of power set $= 2^{11} = 2048$

98. Option (d) is correct.

Considering (3, 6) and (6, 2) elements of R. For transitivity (3, 2) must be element of R but 3 and 2 does not have a common divisor and hence not in R.

R will be symmetric as if (3, 6) is element of R then (6, 3) will also be element of R.

For any positive integer n (n, n) is not element of R as only distinct m and n are allowed for (m, n) in R. so, not reflexive.

99. Correct answer is [36].

$2100 = 7 \times 3 \times 2^2 \times 5^2$

Total factors are–

$= (1+1) \times (1+1) \times (2+1) \times (2+1)$

$= 2 \times 2 \times 3 \times 3 = 36$

As any factor is obtained by multiplying the prime factors zero or more times.

So, there are three choice for 2 and 5 both and two choices for 3 and 7 both.

100. Correct answer is [6].

$$A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$$

Characteristic equation is

$$|A - \lambda I| = 0$$

$$\Rightarrow \quad \begin{vmatrix} 4-\lambda & 5 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow$ $(4-\lambda)(1-\lambda) - 10 = 0$

$\Rightarrow$ $4 - 5\lambda + \lambda^2 = 10$

$\Rightarrow$ $\lambda^2 - 5\lambda - 6 = 0$

$\Rightarrow$ $(\lambda - 6)(\lambda + 1) = 0$

$\Rightarrow$ $\lambda = -1, 6$

Therefore, larger eigen value is 6.

101. Option (c) is correct.

$$f(x) = x^{-1/3} = \frac{1}{\sqrt[3]{x}}$$

will tends to infinity as $x \to 0$

So, $f(x)$ is not continuous at $x = 0$

Also $f(x)$ is not bounded at $x = 0$.

$$\text{Area} = \int_{-1}^{1} f(x)dx = \int_{-1}^{0} x^{-1/3}dx + \int_{0}^{1} x^{-1/3}dx$$

$$= \left.\frac{x^{2/3}}{\frac{2}{3}}\right|_{-1}^{0} + \left.\frac{x^{2/3}}{\frac{2}{3}}\right|_{0}^{1}$$

$$= \frac{3}{2} + \frac{3}{2} = 3 \text{ is non zero and finite}$$

I – False, II – True, III – True

102. Correct answer is [0].

The elementary transformation in a matrix will not effect the determinant value. The observation is $C_3 = 15\ C_1$.

So, rank < 3. Therefore, determinant must be 0.

103. Correct answer is [36].

A function is onto from X to Y if for every element of Y there is an element in X So, this problem reduces to distributing 4 distinct elements ($r = 4$) among 3 distinct bins ($n = 3$) in such a way that no bean is empty. It is given by $n!$ S (r, n), where S (r, n) is Stirling's No. of second kind. So, we need S (4, 3).

$$S(r+1, n) = n * S(r, n) + S(r, n-1)$$

So, S (4, 3) = 6 and 3! = 6 this gives the number of surjective functions = 6 × 6 = 36

104. Correct answer is [0.95].

With X having 2 elements and Y having 20 elements, there can be 20*20 functions from X to Y.

With first element of X having 20 choices. For a one–to–one function, the 2nd element has only 19 choices left after 1 being taken by the first. So, required probability.

$$= \frac{20 \times 19}{20 \times 20} = 0.95$$

105. Option (b) is correct.

As we know a bridge cannot be a part of a simple cycle. Ans as every vertex of cycle will be connected with every other vertex in two ways.

106. Option (d) is correct.

Option (a) is wrong let X = {1, 2}

$\therefore X' = \{3, 4, 5, 6\}, |X| = 2$ and $|X'| = 4$.

Option (b) is incorrect as any two possible subset of U with 5 elements should have at least 4 elements in common.

Option (c) is also incorrect, X and Y can have any 2 and 3 elements respectively elements from 0 to 5. Let X = {1, 2}, Y = {3, 4, 5} and X/Y = {1, 2} which is not null.

107. Option (b) is correct.

$$(A - \lambda I) = \begin{bmatrix} 1-\lambda & -1 & 2 \\ 0 & 1-\lambda & 0 \\ 1 & 2 & 1-\lambda \end{bmatrix}$$

for $\lambda = 1$

$$(A - \lambda I)x = 0 \Rightarrow \begin{bmatrix} 0 & -1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-y + 2z = 0 \Rightarrow y = 2z$$

$$x + 2y = 0 \Rightarrow x = -2y$$

For $y = k$

$\therefore$ $z = \frac{1}{2}y = \frac{k}{2}$

$x = -2y = -2k$

So, $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2k \\ k \\ \frac{k}{2} \end{bmatrix} = k\begin{bmatrix} -2 \\ 1 \\ \frac{1}{2} \end{bmatrix} = \frac{k}{2}\begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix}$$

or $$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = k'\begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix} \quad k' \neq 0 \text{ and } k \in R$$

108. Option (c) is correct.

Using application of an exponential form of logarithm to the expression.

$$\lim_{x\to\infty} (x^2 + 1)e^{-x}$$

$$= \lim_{x\to\infty} \exp\left(\log(x^2+1)e^{-x}\right)$$

$$= \lim_{x\to\infty} \exp\left(\frac{\log(x^2+1)}{e^x}\right)$$

$\because$ for exponential function is continuous we can factor it out of the limit.

$$\exp\left\{\lim_{x\to\infty}\left(\frac{\log(x^2+1)}{e^x}\right)\right\}$$

The numerator of $\log(x^2+1)/e^x$ grows asymptotically slower than its denominator as $x \to \infty$

$$\lim_{x\to\infty} \frac{\log(x^2+1)}{e^x} = 0$$

So, $e^0 = 1$

109. Correct answer is [15].

Digits in non-decreasing order

{1, 1, 1, 1}, {1, 1, 1, 2}, {1, 1, 1, 3}, {1, 1, 2, 2}, {1, 1, 2, 3}, {1, 2, 2, 2}, {1, 2, 2, 3}, {1, 2, 3, 2}, {1, 2, 3, 3}, {1, 3, 3, 3}

{2, 2, 2, 2}, {2, 2, 2, 3}, {2, 2, 3, 3}, {2, 3, 3, 3}

{3, 3, 3, 3}

So, 15 4 - digit number is possible.

110. Option (a) is correct.

For the given reply,

If the person is from Type 1 he must be saying truth and if he is saying truth it must be head.

If the person is from Type 2 he must be lying so what he is saying is also a lie that it will be a head only if he is saying truth. So it's ahead only.

111. Correct answer is [309.33].

t	2	4	6	8	10	12	14	16	18	20
v	10	18	25	29	32	20	11	5	2	0

h = step size =2

$$S = \frac{h}{3}\begin{bmatrix} f(0) + 4(f(2)+f(6)+f(10)+f(14)+f(18)) \\ +2(f(4)+f(8)+f(12)+f(16)+f(20)) \end{bmatrix}$$

$$= \frac{2}{3}[0 + 4(10+25+32+11+2)$$

$$+2(18+29+20+5)]$$

$$= 309.33$$

112. Option (c) is correct.

Coefficient matrix $A = \begin{bmatrix} p & q & r \\ q & r & p \\ r & p & q \end{bmatrix}$,

for non–trivial solution rank (A) < 3

$\therefore \quad |A| = 0$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} p+q+r & p+q+r & p+q+r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

$$(p+q+r)\begin{vmatrix} 1 & 1 & 1 \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

if $p + q + r = 0$ then $|A| = 0$

also if $p = q = r$ then $R_2 = R_3$ then $|A| = 0$

113. Option (a) is correct.

$$a f(x) + b f\left(\frac{1}{x}\right) = \frac{1}{x} - 25... \quad ...(i)$$

Integrating both sides

$$a\int_1^2 f(x)dx + b\int_1^2 f\left(\frac{1}{x}\right)dx = [\log(x) - 25x]_1^2 \quad ...(ii)$$

$$= \log 2 - 25$$

Replacing x *by* $\frac{1}{x}$ in (i)

$$a\int_1^2 f\left(\frac{1}{x}\right)dx + b\int_1^2 f(x)dx = x - 25$$

$$= \left[\frac{x^2}{2} - 25x\right]_1^2 = \frac{-47}{2} \quad ...(iii)$$

Eliminate $\int_1^2 f\left(\frac{1}{x}\right)$ between (ii) and (iii)

$$(a^2 - b^2)\int_1^2 f(x)dx = a(log 2 - 25) + b \times \frac{47}{2}$$

$$\therefore \quad \int_1^2 f(x)dx = \frac{1}{(a^2 - b^2)}\left[a(log 2 - 25) + \frac{47b}{2}\right]$$

114. Option (c) is correct.

For reflexive

$\forall a, b$ if$((a,b),(a,b)) \in R$

then $((a,b),(a,b)) \in R \leftrightarrow (a - b = b - a)$

(not possible for any positive integers b and a)

So it is not reflexive

For symmetric

$((a,b),(c,d)) \in R \to ((c,d),(a,b)) \in R$

then $(a - d = b - c)$

if $((c,d),(a,b)) \in R$ then $c - b = d - a$

$$\because (c - b = d - a) \leftrightarrow (d - a = c - b)$$

$$\leftrightarrow (-(a-d) = -(b-c))$$

$$\leftrightarrow (a - d = b - c)$$

So, symmetric.