ENGINEERING MATHEMATICS — GATE — Question Paper 2016

ELECTRONICS AND COMMUNICATION (EC) P1

Q. 1. Let $M^4 = I$, (where I denotes the identity matrix) and $M \neq I$, $M^2 \neq I$ and $M^3 \neq I$. Then, for any natural number k, M^{-1} equals:

(a) M^{4k+1} **(b)** M^{4k+2}
(c) M^{4k+3} **(d)** M^{4k}

Q. 2. The second moment of a Poisson-distributed random variable is 2. The mean of the random variable is _____.

Q. 3. Given the following statements about a function $f : \mathbb{R} \to \mathbb{R}$, select the right option:

P: If $f(x)$ is continuous at $x = x_0$, then it is also differentiable at $x = x_0$.

Q: If $f(x)$ is continuous at $x = x_0$, then it may not be differentiable at $x = x_0$.

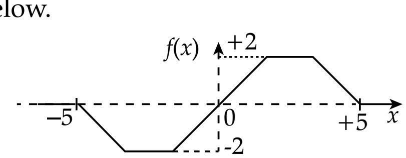
R: If $f(x)$ is differentiable at $x = x_0$, then it is also continuous at $x = x_0$.

(a) P is true, Q is false, R is false
(b) P is false, Q is true, R is true
(c) P is false, Q is true, R is false
(d) P is true, Q is false, R is true

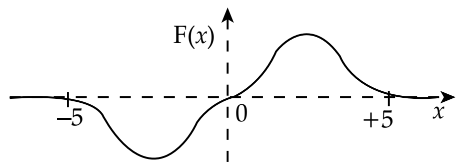
Q. 4. Which one of the following is a property of the solutions to the Laplace equation: $\nabla^2 f = 0$?

(a) The solutions have neither maxima nor minima anywhere except at the boundaries.
(b) The solutions are not separable in the coordinates.
(c) The solutions are not continuous.
(d) The solutions are not dependent on the boundary conditions.

Q. 5. Consider the plot of $f(x)$ versus x as shown below.



Suppose $F(x) = \int_{-5}^{x} f(y)dy$. Which one of the following is a graph of $F(x)$?

(a)


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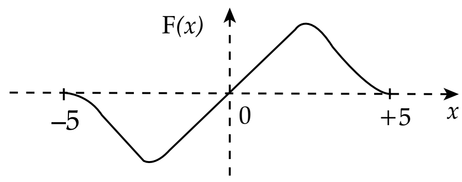

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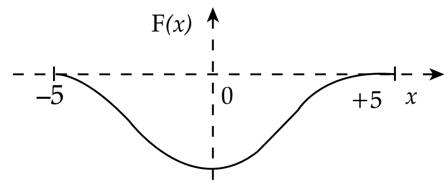

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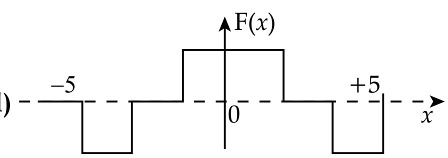


Q. 6. The integral $\frac{1}{2\pi}\iint_D (x+y+10)\,dx\,dy$, where D denotes the disc: $x^2 + y^2 \leq 4$, evaluates to_____

Q. 7. A sequence $x[n]$ is specified as

$$\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \text{for } n = \geq 2.$$

The initial conditions are $x[0] = 1$, $x[1] = 1$, and $x[n] = 0$ for $n < 0$. The value of $x[12]$ is ______.

Q. 8. In the following integral, the contour C encloses the points $2\pi i$ and $-2\pi i$ is

$$-\frac{1}{2\pi}\oint_c \frac{\sin z}{(z-2\pi i)^3}dz$$

The value of the integral is _____.

Q. 9. The region specified by $\{(\rho, \varphi, z): 3 \leq p \leq 5, \frac{\pi}{8} \leq \varphi \leq \frac{\pi}{4}, 3 \leq z \leq 4.5\}$ in cylindrical coordinates has volume of _____.

ELECTRONICS AND COMMUNICATION (EC) P2

Q. 10. The value of x for which the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 9 & 7 & 13 \\ -6 & -4 & -9+x \end{bmatrix}$ has zero as an eigen value is _____.

Q. 11. Consider the complex valued function $f(z) = 2z^3 + b\,|z|^3$ where z is a complex variable. The value of b for which the function $f(z)$ is analytic is _____.

Q. 12. As x varies from -1 to $+3$, which one of the following describes the behaviour of the function $f(x) = x^3 - 3x^2 + 1$?

(a) $f(x)$ increases monotonically.
(b) $f(x)$ increases, then decreases and increases again.
(c) $f(x)$ decreases, then increases and decreases again.
(d) $f(x)$ increases and then decreases.

Q. 13. How many distinct values of x satisfy the equation $\sin(x) = x/2$, where x is in radians?

(a) 1 **(b)** 2
(c) 3 **(d)** 4 or more

Q. 14. Consider the time-varying vector $I = \hat{x}\,15\cos(\omega t) + \hat{y}\,5\sin(\omega t)$ in Cartesian coordinates, where $\omega > 0$ is a constant. When the vector magnitude $|I|$ is at its minimum value, the angle θ that I makes with the x axis (in degrees, such that $0 \le \theta \le 180$) is _____.

Q. 15. The ordinary differential equation $\frac{dx}{dt} = -3x + 2$, with $x(0) = 1$ is to be solved using the forward Euler method. The largest time step that can be used to solve the equation without making the numerical solution unstable is _____.

Q. 16. Suppose C is the closed curve defined as the circle $x^2 + y^2 = 1$ with C oriented anti-clockwise. The value of $\oint (xy^2 dx + x^2 y\, dy)$ over the curve C equals_____.

Q. 17. Two random variables X and Y are distributed according to

$$f_{x,y}(x,y) = \begin{cases} (x+y), & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

The probability $P(X + Y \le 1)$ is _____.

Q. 18. The matrix $A = \begin{bmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{bmatrix}$ has det (A) = 100 and trace (A) = 14. The value of $|a - b|$ is _____.

ELECTRICAL ENGINEERING (EE) P1

Q. 19. The maximum value attained by the function $f(x) = x(x-1)(x-2)$ In the interval [1, 2] is _____.

Q. 20. Consider a 3×3 matrix with every element being equal to 1. Its only non-zero eigen value is ____.

Q. 21. The Laplace Transform of $f(t) = e^{2t}\sin(5t)\,u(t)$ is

(a) $\frac{5}{s^2 - 4s + 29}$ **(b)** $\frac{5}{s^2 + 5}$
(c) $\frac{s-2}{s^2 - 4s + 29}$ **(d)** $\frac{5}{s+5}$

Q. 22. A function $y(t)$, such that $y(0) = 1$ and $y(1) = 3e^{-1}$, is a solution of the differential equation

$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0$. Then $y(2)$ is

(a) $5e^{-1}$ **(b)** $5e^{-2}$
(c) $7e^{-1}$ **(d)** $7e^{-2}$

Q. 23. The value of the integral $\oint_c \frac{2z+5}{\left(z - \frac{1}{2}\right)(z^2 - 4z + 5)}\, dz$ over the contour $|z| = 1$, taken in the anti-clockwise direction, would be

(a) $\frac{24\pi i}{13}$ **(b)** $\frac{48\pi i}{13}$
(c) $\frac{24}{13}$ **(d)** $\frac{12}{13}$

Q. 24. Candidates were asked to come to an interview with 3 pens each. Black, blue, green and red were the permitted pen colours that the candidate could bring. The probability that a candidate comes with all 3 pens having the same colour is _____.

Q. 25. Let $S = \sum_{n=0}^{\infty} n\alpha^n$ where $|\alpha| < 1$. The value of α in the range $0 < \alpha < 1$, such that $S = 2\alpha$ is _____.

Q. 26. Let the eigen values of a 2 x 2 matrix A be 1, –2 with eigenvectors x_1 and x_2 respectively. Then the eigen values and eigenvectors of the matrix $A^2 - 3A + 4I$ would, respectively, be

(a) 2, 14; x_1, x_2 **(b)** 2, 14; $x_1 + x_2, x_1 - x_2$
(c) 2, 0; x_1, x_2 **(d)** 2, 0; $x_1 + x_2, x_1 - x_2$

Q. 27. Let A be a 4 × 3 real matrix with rank 2. Which one of the following statement is TRUE?

(a) Rank of A^TA is less than 2.
(b) Rank of A^TA is equal to 2.
(c) Rank of A^TA is greater than 2.
(d) Rank of A^TA can be any number between 1 and 3.

ELECTRICAL ENGINEERING (EE) P2

Q. 28. Consider the function $f(z) = z + z^*$ where z is a complex variable and z^* denotes its complex conjugate. Which one of the following is TRUE?

(a) $f(z)$ is both continuous and analytic
(b) $f(z)$ is continuous but not analytic
(c) $f(z)$ is not continuous but is analytic
(d) $f(z)$ is neither continuous nor analytic

Q. 29. A 3 × 3 matrix P is such that, $P^3 = P$. Then the eigen values of P are

(a) 1, 1, –1
(b) 1, 0.5 + j0.866, 0.5 –j0.866
(c) 1, –0.5 + j0.866, –0.5 –j0.866
(d) 0, 1, – 1

Q. 30. The solution of the differential equation for $t > 0$, $y''(t) + 2y'(t) + y(t) = 0$ with initial Conditions $y(0) = 0$ and $y'(0) = 1$, is ($u(t)$ denotes the unit step function).

(a) $te^{-t}u(t)$ **(b)** $(e^{-t} - te^{-t})u(t)$
(c) $(-e^{-t} + te^{-t})u(t)$ **(d)** $e^{-t}u(t)$

Q. 31. The value of the line integral $\int_c (2xy^2dx + 2x^2y\,dy + dz)$ along a path joining the origin (0, 0, 0) and the point (1, 1, 1) is

(a) 0 **(b)** 2
(c) 4 **(d)** 6

Q. 32. Let $f(x)$ be a real, periodic function satisfying $f(-x) = -f(x)$. The general form of its Fourier series representation would be

(a) $f(x) = a_0 + \sum_{k=1}^{\infty} a_k \cos(kx)$

(b) $f(x) = \sum_{k=1}^{\infty} b_k \sin(kx)$

(c) $f(x) = a_0 + \sum_{k=1}^{\infty} a_{2k} \cos(kx)$

(d) $f(x) = \sum_{k=0}^{\infty} a_{2k+1} \sin(2k+1)x$

Q. 33. The value of the integral $2\int_{-\infty}^{\infty}\left(\frac{\sin 2\pi t}{\pi t}\right) dt$ is equal to

(a) 0 **(b)** 0.5
(c) 1 **(d)** 2

Q. 34. Let $y(x)$ be the solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ with initial conditions $y(0) = 0$ and $\left.\frac{dy}{dx}\right|_{x=0} = 1$. Then the value of $y(1)$ is_____.

Q. 35. The line integral of the vector field $F = 5xz\,\hat{i} + (3x^2 + 2y)\hat{j} + x^2z\hat{k}$ along a path from (0, 0, 0) to (1, 1, 1) parametrized by (t, t^2, t) is_____.

Q. 36. Let $P = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Consider the set S of all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ such that $a^2 + b^2 = 1$, where $\begin{pmatrix} a \\ b \end{pmatrix} = P\begin{pmatrix} x \\ y \end{pmatrix}$. Then S is

(a) a circle of radius $\sqrt{10}$
(b) a circle of radius $\frac{1}{\sqrt{10}}$
(c) an ellipse with major axis along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
(d) an ellipse with major axis along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Q. 37. Let the probability density function of a random variable, X, be given as:

$$fx(x) = \frac{3}{2}e^{-3x}u(x) + ae^{4x}u(-x)$$

where $u(x)$ is the unit step function.
Then the value of 'a' and Prob {X ≤ 0}, respectively, are

(a) $2, \frac{1}{2}$ **(b)** $4, \frac{1}{2}$
(c) $2, \frac{1}{4}$ **(d)** $4, \frac{1}{4}$

MECHANICAL ENGINEERING (ME) P1

Q. 38. The solution to the system of equations $\begin{bmatrix} 2 & 5 \\ -4 & 3 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 2 \\ -30 \end{Bmatrix}$ is

(a) 6, 2 **(b)** –6, 2
(c) –6, –2 **(d)** 6, –2

Q. 39. If $f(t)$ is a function defined for all $t \geq 0$, its Laplace transform F(s) is defined as

(a) $\int_0^\infty e^{st} f(t)dt$ **(b)** $\int_0^\infty e^{-st} f(t)dt$

(c) $\int_0^\infty e^{ist} f(t)dt$ **(d)** $\int_0^\infty e^{-ist} f(t)dt$

Q. 40. $f(z) = u(x, y) + iv(x, y)$ is an analytic function of complex variable $z = x + iy$ where $i = \sqrt{-1}$. If $u(x, y) = 2xy$, then $v(x, y)$ may be expressed as

(a) $-x^2 + y^2$ + constant

(b) $x^2 - y^2$ + constant

(c) $x^2 + y^2$ + constant

(d) $-(x^2 + y^2)$ + constant

Q. 41. Consider a Poisson distribution for the tossing of a biased coin. The mean for this distribution is μ. The standard deviation for this distribution is given by

(a) $\sqrt{\mu}$ **(b)** μ^2

(c) μ **(d)** $1/\mu$

Q. 42. Solve the equation $x = 10 \cos(x)$ using the Newton-Raphson method. The initial guess is $x = \pi/4$. The value of the predicted root after the first iteration, up to second decimal, is _____.

Q. 43. Consider the function $f(x) = 2x^3 - 3x^2$ in the domain [–1, 2]. The global minimum of $f(x)$ is_____.

Q. 44. If $y = f(x)$ satisfies the boundary value problem $y'' + 9y = 0$, $y(0) = 0$, $y(\pi/2) = \sqrt{2}$, then $y(\pi/4)$ is _____.

Q. 45. The value of the integral

$$\int_{-\infty}^{\infty} \frac{\sin x}{x^2 + 2x + 2} dx$$ evaluated using contour integration and the residue theorem is

(a) $-\pi \sin(1)/e$ **(b)** $-\pi \cos(1)/e$

(c) $\sin(1)/e$ **(d)** $\cos(1)/e$

Q. 46. Gauss-Seidel method is used to solve the following equations (as per the given order):

$x_1 + 2x_2 + 3x_3 = 5$

$2x_1 + 3x_2 + x_3 = 1$

$3x_1 + 2x_2 + x_3 = 3$

Assuming initial guess as $x_1 = x_2 = x_3 = 0$, the value of x_3 after the first iteration is _____.

MECHANICAL ENGINEERING (ME) P2

Q. 47. The condition for which the eigen values of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$ are positive, is

(a) $k > 1/2$ **(b)** $k > -2$

(c) $k > 0$ **(d)** $k < -1/2$

Q. 48. The values of x for which the function $f(x) = \frac{x^2 - 3x - 4}{x^2 + 3x - 4}$ is NOT continuous are

(a) 4 and −1 **(b)** 4 and 1

(c) −4 and 1 **(d)** −4 and −1

Q. 49. Laplace transform of $\cos(\omega t)$ is

(a) $\frac{s}{s^2 + \omega^2}$ **(b)** $\frac{\omega}{s^2 + \omega^2}$

(c) $\frac{s}{s^2 - \omega^2}$ **(d)** $\frac{\omega}{s^2 - \omega^2}$

Q. 50. A function f of the complex variable $z = x + iy$, is given as $f(x, y) = u(x, y) + iv(x, y)$ where $u(x, y) = 2kxy$ and $v(x, y) = x^2 - y^2$. The value of k, for which the function is analytic, is _____.

Q. 51. Numerical integration using trapezoidal rule gives the best result for a single variable function, which is

(a) linear **(b)** parabolic

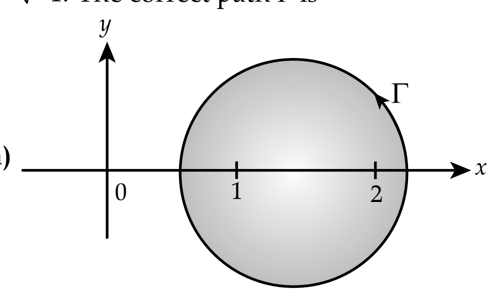
(c) logarithmic **(d)** hyperbolic

Q. 52. A scalar potential φ has the following gradient: $\nabla\varphi = yz\hat{i} + xz\hat{j} + xy\hat{k}$. Consider the integral $\int_C \nabla\varphi . d\vec{r}$ on the curve $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

The curve C is parameterized as follows: $\begin{cases} x = t \\ y = t^2 \\ z = 3t^2 \end{cases}$ and $1 \leq t \leq 3$. The value of the integral is _____.

Q. 53. The value of $\oint_\Gamma \frac{3z - 5}{(z-1)(z-2)} dz$ along a closed path Γ is equal to $(4\pi i)$, where $z = x + iy$ and $i = \sqrt{-1}$. The correct path Γ is

(a)



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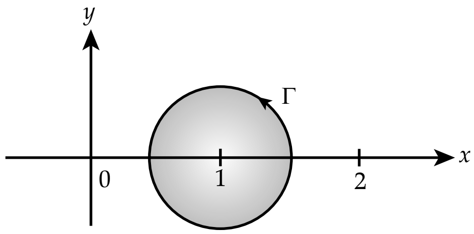

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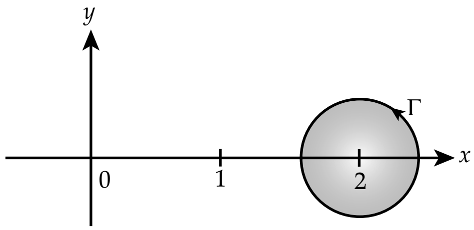

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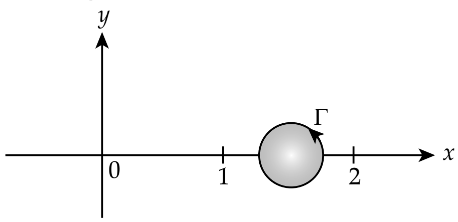


Q. 54. The probability that a screw manufactured by a company is defective is 0.1. The company sells screws in packets containing 5 screws and gives a guarantee of replacement if one or more screws in the packet are found to be defective. The probability that a packet would have to be replaced is _____.

Q. 55. The error in numerically computing the integral $\int_0^{\pi}(\sin x + \cos x)dx$ using the trapezoidal rule with three intervals of equal length between 0 and π is ______.

MECHANICAL ENGINEERING (ME) P3

Q. 56. A real square matrix A is called skew-symmetric if

(a) $A^T = A$ **(b)** $A^T = A^{-1}$

(c) $A^T = -A$ **(d)** $A^T = A + A^{-1}$

Q. 57. $\underset{x\to 0}{\text{Lt}} \dfrac{\log_e(1+4x)}{e^{3x}-1}$ is equal to

(a) 0 **(b)** $\dfrac{1}{12}$

(c) $\dfrac{4}{3}$ **(d)** 1

Q. 58. Solutions of Laplace's equation having continuous second-order partial derivatives are called

(a) biharmonic functions

(b) harmonic functions

(c) conjugate harmonic functions

(d) error functions

Q. 59. The area (in percentage) under standard normal distribution curve of random variable Z within limits from −3 to +3 is _____.

Q. 60. The root of the function $f(x) = x^3 + x - 1$ obtained after first iteration on application of Newton – Raphson scheme using an initial guess of $x_0 = 1$ is

(a) 0.682 **(b)** 0.686

(c) 0.750 **(d)** 1.000

Q. 61. The number of linearly independent eigenvectors of matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is____.

Q. 62. The value of the line integral $\oint_c \overline{F}.\overline{r'}\, ds$ where C is a circle of radius $\dfrac{4}{\sqrt{\pi}}$ units is _____. Here, $\overline{F}(x, y) = y\hat{i} + 2x\hat{j}$ and $\overline{r'}$ is the UNIT tangent vector on the curve C at an arc length s from a reference point on the curve. $\hat{i}$ and $\hat{j}$ are the basis vectors in the *x-y* Cartesian reference. In evaluating the line integral, the curve has to be traversed in the counter-clockwise direction.

Q. 63. $\lim_{x\to\infty} \sqrt{x^2 + x - 1} - x$ is

(a) 0 **(b)** ∞

(c) 1/2 **(d)** $-\infty$

Q. 64. Three cards were drawn from a pack of 52 cards. The probability that they are a king, a queen, and a jack is

(a) $\dfrac{16}{5525}$ **(b)** $\dfrac{64}{2197}$

(c) $\dfrac{3}{13}$ **(d)** $\dfrac{8}{16575}$

CIVIL ENGINEERING (CE) P1

Q. 65. **Newton-Raphson** method is to be used to find root of equation $3x - e^x + \sin x = 0$. If the initial trial value for the root is taken as 0.333, the next approximation for the root would be _____.

(**Note:** answer up to three decimal)

Q. 66. The type of partial differential equation $\dfrac{\partial^2 P}{\partial x^2} + \dfrac{\partial^2 P}{\partial y^2} + 3\dfrac{\partial^2 P}{\partial x \partial y} + 2\dfrac{\partial P}{\partial x} - \dfrac{\partial P}{\partial y} = 0$ is :

(a) elliptic **(b)** parabolic

(c) hyperbolic **(d)** none of these

Q. 67. If the entries in each column of a square matrix M add up to 1, then an eigen value of M is

(a) 4 **(b)** 3
(c) 2 **(d)** 1

Q. 68. Type II error in hypothesis testing is

(a) acceptance of the null hypothesis when it is false and should be rejected
(b) rejection of the null hypothesis when it is true and should be accepted
(c) rejection of the null hypothesis when it is false and should be rejected
(d) acceptance of the null hypothesis when it is true and should be accepted

Q. 69. The solution of the partial differential equation $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$ is of the form

(a) $C\cos(kt)\left[C_1 e^{(\sqrt{k/\alpha})x} + C_2 e^{-(\sqrt{k/\alpha})x}\right]$

(b) $Ce^{kt}\left[C_1 e^{(\sqrt{k/\alpha})x} + C_2 e^{-(\sqrt{k/\alpha})x}\right]$

(c) $Ce^{kt}\left[C_1 \cos(\sqrt{k/\alpha})x + C_2 \sin(-\sqrt{k/\alpha})x\right]$

(d) $C\sin(kt)\left[C_1 \cos(\sqrt{k/\alpha})x + C_2 \sin(-\sqrt{k/\alpha})x\right]$

Q. 70. Probability density function of a random variable X is given below $f(x) = \begin{cases} 0.25 & \text{if } 1 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$

$P(X \le 4)$ is _______.

(a) $\frac{3}{4}$ **(b)** $\frac{1}{2}$
(c) $\frac{1}{4}$ **(d)** $\frac{1}{8}$

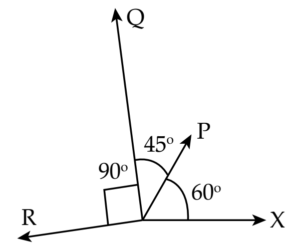
Q. 71. The value of $\int_0^\infty \frac{1}{1+x^2}dx + \int_0^\infty \frac{\sin x}{x}dx$ is

(a) $\frac{\pi}{2}$ **(b)** π
(c) $\frac{3\pi}{2}$ **(d)** 1

Q. 72. The area of the region bounded by the parabola $y = x^2 + 1$ and the straight line $x + y = 3$ is _______.

(a) $\frac{59}{6}$ **(b)** $\frac{9}{2}$
(c) $\frac{10}{3}$ **(d)** $\frac{7}{6}$

Q. 73. The magnitudes of vectors P, Q and R are 100 kN, 250 kN and 150 kN, respectively as shown in the figure.



The respective values of the magnitude (in kN) and the direction (with respect to the x-axis) of the resultant vector are

(a) 290.9 and 96.0° **(b)** 368.1 and 94.7°
(c) 330.4 and 118.9° **(d)** 400.1 and 113.5°

Q. 74. The respective expressions for complimentary function and particular integral part of the solution of the differential equation $\frac{d^4y}{dx^4} + 3\frac{d^2y}{dx^2} = 108x^2$ are

(a) $\left[c_1 + c_2 x + c_3 \sin\sqrt{3}x + c_4 \cos\sqrt{3}x\right]$ and $\left[3x^4 - 12x^2 + c\right]$

(b) $\left[c_2 x + c_3 \sin\sqrt{3}x + c_4 \cos\sqrt{3}x\right]$ and $\left[5x^4 - 12x^2 + c\right]$

(c) $\left[c_1 + c_3 \sin\sqrt{3}x + c_4 \cos\sqrt{3}x\right]$ and $\left[3x^4 - 12x^2 + c\right]$

(d) $\left[c_1 + c_3 x + c_3 \sin\sqrt{3}x + c_4 \cos\sqrt{3}x\right]$ and $\left[5x^4 - 12x^2 + c\right]$

CIVIL ENGINEERING (CE) P2

Q. 75. The spot speeds (expressed in km/hr) observed at a road section are 66, 62, 45, 79, 32, 51, 56, 60, 53, and 49. The median speed (expressed in km/hr) is _____
(**Note:** answer with one decimal accuracy)

Q. 76. The optimum value of the function $f(x) = x^2 - 4x + 2$ is

(a) 2 (maximum) **(b)** 2 (minimum)
(c) −2 (maximum) **(d)** −2 (minimum)

Q. 77. The Fourier series of the function,

$$f(x) = 0, \quad -\pi < x \le 0$$
$$= \pi - x, \quad 0 < x < \pi$$

In the interval $[-\pi, \pi]$ is

$$f(x) = \frac{\pi}{4} + \frac{2}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \dots\dots\dots\right] + \left[\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \dots\dots\right]$$

The convergence of the above Fourier series at $x = 0$ gives

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ **(b)** $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$

(c) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$ **(d)** $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = \frac{\pi}{4}$

Q. 78. X and Y are two random independent events. It is known that P(X) = 0.40 and $P(X \cup Y^c) = 0.7$.
Which one of the following is the value of $P(X \cup Y)$?

(a) 0.7 **(b)** 0.5
(c) 0.4 **(d)** 0.3

Q. 79. What is the value of $\lim_{\substack{x \to 0 \\ y \to 0}} \frac{xy}{x^2 + y^2}$?

(a) 1 **(b)** –1
(c) 0 **(d)** Limit does not exist

Q. 80. Consider the following linear system.

$x + 2y - 3z = a$
$2x + 3y + 3z = b$
$5x + 9y - 6z = c$

This system is consistent if a, b and c satisfy the equation

(a) $7a - b - c = 0$ **(b)** $3a + b - c = 0$
(c) $3a - b + c = 0$ **(d)** $7a - b + c = 0$

Q. 81. If $f(x)$ and $g(x)$ are two probability density functions,

$$f(x) = \begin{cases} \frac{x}{a} + 1 & : \quad -a \le x < 0 \\ -\frac{x}{a} + 1 & : \quad 0 \le x \le a \\ 0 & : \quad \text{otherwise} \end{cases}$$

$$g(x) = \begin{cases} -\frac{x}{a} & : \quad -a \le x < 0 \\ \frac{x}{a} & : \quad 0 \le x \le a \\ 0 & : \quad \text{otherwise} \end{cases}$$

Which one of the following statements is true?

(a) Mean of $f(x)$ and $g(x)$ are same; Variance of $f(x)$ and $g(x)$ are same
(b) Mean of $f(x)$ and $g(x)$ are same; Variance of $f(x)$ and $g(x)$ are different
(c) Mean of $f(x)$ and $g(x)$ are different; Variance of $f(x)$ and $g(x)$ are same
(d) Mean of $f(x)$ and $g(x)$ are different; Variance of $f(x)$ and $g(x)$ are different

Q. 82. The angle of intersection of the curves $x^2 = 4y$ and $y^2 = 4x$ at point (0, 0) is

(a) 0° **(b)** 30°
(c) 45° **(d)** 90°

Q. 83. The area between the parabola $x^2 = 8y$ and the straight line $y = 8$ is ________.

Q. 84. The quadratic approximation of $f(x) = x^3 - 3x^2 - 5$ at the point $x = 0$ is

(a) $3x^2 - 6x - 5$ **(b)** $-3x^2 - 5$
(c) $-3x^2 + 6x - 5$ **(d)** $3x^2 - 5$

COMPUTER SCIENCE (CS) P1

Q. 85. Let p, q, r, s represent the following propositions.

$p : x \in \{8, 9, 10, 11, 12\}$
q : x is a composite number
r : x is a perfect square
s : x is a prime number

The integer $x \ge 2$ which satisfies : $\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s))$ is ____________.

Q. 86. Let a_n be the number of n-bit strings that do NOT contain two consecutive 1s.Which one of the following is the recurrence relation for a_n?

(a) $a_n = a_{n-1} + 2a_{n-2}$ **(b)** $a_n = a_{n-1} + a_{n-2}$
(c) $a_n = 2a_{n-1} + a_{n-2}$ **(d)** $a_n = 2a_{n-1} + 2a_{n-2}$

Q. 87. $\lim_{x \to 4} \frac{\sin(x-4)}{x-4} =$ ____.

Q. 88. A probability density function on the interval $[a, 1]$ is given by $1/x^2$ and outside this interval the value of the function is zero .The value of a is _____.

Q. 89. Two eigen values of a 3 × 3 real matrix P are $(2 + \sqrt{-1})$ and 3. The determinant of P is_____

Q. 90. The coefficient of x^{12} in $(x^3 + x^4 + x^5 + x^6 + ...)^3$ is_____.

Q. 91. Consider the recurrence relation $a_1 = 8$, $a_n = 6n^2 + 2n + a_{n-1}$. Let $a_{99} = K \times 10^4$. The value of K is_____

Q. 92. A function $f : N^+ \to N^+$, defined on the set of positive integers N^+, satisfies the following properties:

$f(n) = f(n/2)$ if n is even
$f(n) = f(n+5)$ if n is odd

Let $R = \{i \mid \exists j : f(j) = i\}$ be the set of distinct values that f takes. The maximum possible size of R is______.

Q. 93. Consider the following experiment.

Step 1: Flip a fair coin twice.

Step 2: If the outcomes are (TAILS, HEADS) then output Y and stop.

Step 3: If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.

Step 4: If the outcomes are (TAILS, TAILS), then go to Step 1.

The probability that the output of the experiment is Y is (up to two decimal places)_____.

COMPUTER SCIENCE (CS) P2

Q. 94. Consider the following expressions:

(i) false **(ii)** Q

(iii) true **(iv)** $P \vee Q$

(v) $\neg Q \vee P$

The number of expressions given above that are logically implied by $P \wedge (P \Rightarrow Q)$ is _____.

Q. 95. Let $f(x)$ be a polynomial and $g(x) = f'(x)$ be its derivative. If the degree of $(f(x) + f(-x))$ is 10, then the degree of $(g(x) - g(-x))$ is_____.

Q. 96. The minimum number of colours that is sufficient to vertex - colour any planar graph is_____.

Q. 97. Consider the systems, each consisting of m linear equations in n variables.

I. If $m < n$, then all such systems have a solution

II. If $m > n$, then none of these systems has a solution

III. If $m = n$, then there exists a system which has a solution

Which one of the following is CORRECT?

(a) I, II and III are true

(b) Only II and III are true

(c) Only III is true

(d) None of them is true

Q. 98. Suppose that a shop has an equal number of LED bulbs of two different types. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100 hours is _____.

Q. 99. Suppose that the eigen values of matrix A are 1, 2, 4. The determinant of $(A^{-1})^T$ is_____

Q. 100. A binary relation R on $\mathbb{N} \times \mathbb{N}$ is defined as follows: (a, b) R (c, d) if $a \le c$ or $b \le d$. Consider the following propositions:

P: R is reflexive

Q: R is transitive

Which one of the following statements is **TRUE**?

(a) Both P and Q are true

(b) P is true and Q is false

(c) P is false and Q is true

(d) Both P and Q are false

Q. 101. Which one of the following well-formed formulae in predicate calculus is **NOT** valid?

(a) $(\forall x\, p(x)) \Rightarrow \forall x\, q(x)) \Rightarrow (\exists x \neg p(x \vee \forall x\, q(x))$

(b) $(\exists x\, p(x) \vee \exists x\, q(x)) \Rightarrow \exists x\, (p(x) \vee q(x))$

(c) $\exists x\, (p(x) \wedge q(x)) \Rightarrow (\exists x\, p(x) \wedge \exists x\, q(x))$

(d) $\forall x\, (p(x) \vee q(x)) \Rightarrow (\forall x\, p(x) \vee \forall x\, q(x))$

Q. 102. Consider a set U of 23 different compounds in a Chemistry lab. There is a subset S of U of 9 compounds, each of which reacts with exactly 3 compounds of U. Consider the following statements:

I. Each compound in U \ S reacts with an odd number of compounds.

II. At least one compound in U \ S reacts with an odd number of compounds.

III. Each compound in U \ *S* reacts with an even number of compounds.

Which one of the above statements is **ALWAYS TRUE**?

(a) Only I **(b)** Only II

(c) Only III **(d)** None

Q. 103. The value of the expression 13^{99}(mod 17), in the range 0 to 16, is_____.

Answer Key			
Q. No.	Answer	Topic Name	Chapter Name
1	(c)	Matrix	Linear Algebra
2	1	Poisson Distribution	Probability and Statistics
3	(b)	Continuity and Differentiability	Calculus
4	(a)	2nd Order Equation Solution	Differential Equations
5	(c)	Functions	Calculus
6	20	Double Integration	Calculus
7	233	Recursive Relation	Numerical Analysis
8	–133.87	Cauchy's Integral	Complex Variable
9	4.712	Volume Intergral	Vector Calculus
10	1	Eigen Value	Linear Algebra
11	0	Analytic Function	Complex Variable
12	(b)	Increasing/Decreasing Functions	Calculus
13	(c)	Functions	Calculus
14	90°	Basics	Vector Analysis
15	0.66	Euler Method	Numerical Analysis
16	0	Green's Theorem	Vector Calculus
17	0.33	Probability of RV	Probability and Statistics
18	3	Determinant	Linear Algebra
19	0	Functions - Maximum Value	Calculus
20	3	Eigen Value	Linear Algebra
21	(a)	Lapalce Transform	Transform Theory
22	(b)	2^{nd} Order Equation Solution	Differential Equations
23	(b)	Cauchy's Integral	Complex Variable
24	0.2	Probability	Probability and Statistics
25	0.293	Sequence and Series	Calculus
26	(a)	Eigen Value	Linear Algebra
27	(b)	Rank of Matrix	Linear Algebra
28	(b)	Analytic Function	Complex Variable
29	(d)	Eigen Value	Linear Algebra
30	(a)	2^{nd} Order Equation Solution	Differential Equations
31	(b)	Line Integral	Vector Calculus
32	(b)	Fourier Series	Transform Theory
33	(d)	Definite Integral	Calculus
34	7.4	2^{nd} Order Equation Solution	Differential Equations
35	4.42	Line Integral	Vector Calculus
36	(d)	Vector Space	Vector Analysis

Q. No.	Answer	Topic Name	Chapter Name
37	(a)	Probability Density Function	Probability and Statistics
38	(d)	System of Linear Equations	Linear Algebra
39	(b)	Laplace Transform	Transform Theory
40	(a)	Analytic Function	Complex Variable
41	(a)	Poisson Distribution	Probability and Statistics
42	1.56	Newton Raphson Method	Numerical Analysis
43	–5	Global Maximum/Minimum	Calculus
44	–1	2^{nd} Order Equation Solution	Differential Equations
45	(a)	Residue Theorem	Complex Variable
46	–6	Gauss-Seidel Method	Numerical Analysis
47	(a)	Eigen Value	Linear Algebra
48	(c)	Functions	Calculus
49	(a)	Laplace Transform	Transform Theory
50	–1	Analytic Function	Complex Variable
51	(a)	Trapezoidal Rule	Numerical Analysis
52	726	Line Integral	Vector Analysis
53	(b)	Cauchy's Integral	Complex Variable
54	0.41	Probability	Probability and Statistics
55	0.1862	Trapezoidal Rule	Numerical Analysis
56	(c)	Properties	Linear Algebra
57	(c)	Limit	Calculus
58	(b)	Harmonic Functions	Complex Variable
59	99.7	Normal Distribution	Probability and Statistics
60	(c)	Newton Raphson Method	Numerical Analysis
61	2	Eigen Vector	Linear Algebra
62	16	Line Integral	Vector Analysis
63	(c)	Limit	Calculus
64	(a)	Probability	Probability and Statistics
65	0.360	Newton Raphson Method	Numerical Analysis
66	(c)	Partial Differential Equation	Differential Equation
67	(d)	Eigen Value	Linear Algebra
68	(a)	Error	Numerical Analysis
69	(b)	Partial Differential Equation	Differential Equation
70	(a)	Probability Density Function	Probability and Statistics
71	(b)	Definite Integral	Calculus
72	(b)	Application of Integration	Calculus
73	(c)	Vector Algebra	Vector Analysis

Q. No.	Answer	Topic Name	Chapter Name
74	(a)	Higher Order Differential Equations	Differential Equation
75	54.5	Median	Probability and Statistics
76	(d)	Global Maximum/Minimum	Calculus
77	(c)	Fourier Series	Transform Theory
78	(a)	Random Variable Probability	Probability and Statistics
79	(d)	Two Variable Functions	Calculus
80	(b)	System of Linear Equation	Linear Algebra
81	(b)	Mean and Variance	Probability and Statistics
82	(d)	Vector Algebra	Vector Analysis
83	85.33	Application of Integration	Calculus
84	(b)	Taylor Series	Calculus
85	11	Sets	Discrete Mathematics
86	(b)	Recurrence Relation	Combinatrics
87	1	Limit	Calculus
88	0.5	Probability Density Function	Probability and Statistics
89	15	Determinant	Linear Algebra
90	10	Sum of Series	Calculus
91	198	Recurrence Relation	Combinatrics
92	2	Group	Discrete Mathematics
93	0.33	Probability	Probability and Statistics
94	4	Prepositional	Discrete Mathematics
95	9	Functions	Calculus
96	4	Matching and Coloring	Graph Theory
97	(c)	System of Linear Equations	Linear Algebra
98	0.55	Probability	Probability and Statistics
99	0.125	Determinant	Linear Algebra
100	(b)	Prepositional	Discrete Mathematics
100	(d)	Recurrence Relations	Discrete Mathematics
102	(b)	Sets	Discrete Mathematics
103	4	Powers	Basic Mathematics

ANSWERS WITH EXPLANATIONS

1. Option (c) is correct.

Given
$$M^4 = I$$
$$M^{4k} = I$$
$$M^4.M^{-1} = I.M^{-1}$$
$$M^3 = M^{-1}$$
$$M^3 I = M^{-1} I$$
$$M^{3+4k} = M^{-1}$$

Now, from option (c) M^{4k+3} put K = 0, 1, 2…. where k is a natural number.

2. Correct answer is [1].

For Poisson Distribution second moment
$$E(X^2) = 2$$

Variance $V(X) = E(X^2) - [E(X)]^2$,

Let mean = x

$$x = 2 - x^2 \qquad [\because 6^2 = \mu = \lambda]$$

$\Rightarrow \quad x^2 + x - 2 = 0$

$\Rightarrow \quad x = 1, -2$, ignoring negative value

$\therefore \quad$ mean = x = 1

3. Option (b) is correct.

It is not necessary that continuous function is differentiable but differentiable function is always continuous.

Therefore, statement P is false, and Q and R, are true.

4. Option (a) is correct.

The solutions have neither maxima nor minima anywhere except at the boundaries.

5. Option (c) is correct.

$$F(x) = \int_{-5}^{x} f(y)\, dy$$

Differentiating on both sides

$$\frac{dF(x)}{dx} = \frac{d}{dx}\int_{-5}^{x} f(y)dy$$
$$= f(x).x' - f(-5).(-5)' = f(x)$$

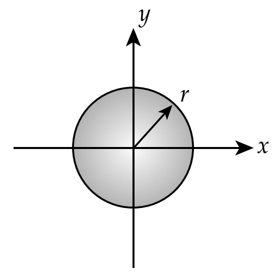
$\therefore \quad F'(x) = f(x)$,

now, $f(x)$ is $\begin{cases} -ve & \text{for } x \quad -5 \text{ to } 0 \\ +ve & \text{for } x \quad 0 \text{ to } 5 \end{cases}$

So, $F(x)$ is $\begin{cases} \text{increasing} & \text{for } x \quad -5 \text{ to } 0 \\ \text{decreasing} & \text{for } x \quad 0 \text{ to } 5 \end{cases}$

6. Correct answer is [20].

Converting to polar coordinate.



DISC: $x^2 + y^2 \le 4$

$$= \frac{1}{2\pi}\iint_D (x + y + 10)\, dxdy$$

$$= \frac{1}{2\pi}\int_{r=0}^{2}\int_{\theta=0}^{2\pi} (r\cos\theta + r\sin\theta + 10)\, rdrd\theta$$

$$= \frac{1}{2\pi}\int_{r=0}^{2}\int_{\theta=0}^{2\pi} (r^2\cos\theta + r^2\sin\theta + 10r)\, drd\theta$$

$$= \frac{1}{2\pi}\int_{\theta=0}^{2\pi}\left(\frac{r^3}{3}\cos\theta + \frac{r^3}{3}\sin\theta + 10\frac{r^2}{2}\right)_0^2 d\theta$$

$$= \frac{1}{2\pi}\int_{\theta=0}^{2\pi}\left(\frac{8}{3}\cos\theta + \frac{8}{3}\sin\theta + 20\right)_0^2 d\theta$$

$$= \frac{1}{2\pi}\left(\frac{8}{3}\sin\theta - \frac{8}{3}\cos\theta + 20\theta\right)_0^{2\pi}$$

$$= \frac{1}{2\pi}(20(2\pi) - 0)$$

since integration of sin or cos function for one complete cycle is 0

$$= \frac{1}{2\pi} \times 40\pi = 20$$

7. Correct answer is [233].

$$\begin{bmatrix} x(n) \\ x(n-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}, n \geq 2$$

$n = 2$

$$\begin{bmatrix} x(2) \\ x(1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$x(2) = 2, x(1) = 1$ ...(i)

$$\begin{bmatrix} x(3) \\ x(2) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$x(3) = 3, x(2) = 2$ ...(ii)

From the above observation we get

$x(n) = x(n-1) + x(n-2)$
$x(4) = x(3) + x(2) = 3 + 2 = 5$
$x(5) = x(4) + x(3) = 5 + 3 = 8$
$x(6) = x(5) + x(4) = 8 + 5 = 13$
$x(7) = x(6) + x(5) = 13 + 8 = 21$
$x(8) = x(7) + x(6) = 21 + 13 = 34$
$x(9) = x(8) + x(7) = 34 + 21 = 55$
$x(10) = 89$
$x(11) = 144$
$x(12) = 233$

8. Correct answer is [–133.87].

By Cauchy's Integral formula for derivatives

$$f^n(z_0) = \frac{n!}{2\pi i} \oint \frac{f(z)}{(z-z_0)^{n+1}} dz$$

$$\therefore \frac{1}{-2\pi} \oint \frac{\sin z}{(z-2\pi j)^3} dz = -i \frac{f''(z_0)}{2!},$$

for $f(z) = \sin z, \quad z_0 = 2\pi i$

$f'(z) = \cos z, \quad f''(z) = -\sin z$

$$= \frac{i}{-2}\{-\sin(2\pi i)\} = \frac{i}{-2}\{i \sin h(2\pi)\}$$

$$[\because \sin(ix) = i \sin h(x)]$$

$$= \frac{1}{-2} \sin h(2\pi) = -133.87$$

9. Correct answer is [4.712].

Volume of cylinder in cylindrical coordinates will given by.

$$V = \int_{z=3}^{4.5} \int_{\phi=\frac{\pi}{8}}^{\frac{\pi}{4}} \int_{\rho=3}^{5} \rho \, d\rho \, d\phi \, dz \qquad [\because dv = \rho d\rho d\phi dz]$$

$$= \frac{\rho^2}{2}\Big|_3^5 \times \phi\Big|_{\frac{\pi}{8}}^{\frac{\pi}{4}} \times z\Big|_3^{4.5}$$

$$= \frac{(5)^2-(3)^2}{2} \times \left(\frac{\pi}{4} - \frac{\pi}{8}\right) \times (4.5-3)$$

$$= 8 \times \frac{\pi}{8} \times 1.5 = 4.712 \text{ m}^3$$

10. Correct answer is [1].

Given one of the eigen value is zero and product of eigen values of a matrix = determinant of matrix.

$\therefore \quad |A| = 0$

$$|A| = \begin{vmatrix} 3 & 2 & 4 \\ 9 & 7 & 13 \\ -6 & -4 & -9+x \end{vmatrix} = 0$$

From the property of determinant when two rows or columns have linear dependence then determinant is zero.

If $x = 1$, then we have $R_3 = -2R_1$

So, $|A| = 0$ for $x = 1$

11. Correct answer is [0].

Given $f(z) = 2z^3 + b|z|^3$ is an analytic function.

This is possible when $b = 0$

$\because |z|^3$ is differentiable at origin but not analytic,

$2z^3$ is analytic everywhere.

$\therefore \quad b = 0$ for function to be analytic.

12. Option (b) is correct.

Given

$f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x = 3x(x-2)$

$f'(x) = 0 \Rightarrow x = 0, x = 2$

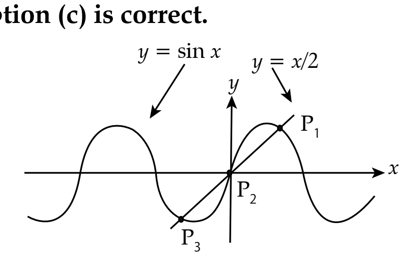
$\Rightarrow \quad f'(x) = 3x^2 - 6x$

being quadratic so on number line

$+ve \;\; 0 \;\; -ve \;\; 2 \;\; +ve$

Therefore, $f(x)$ is increasing from $x = -1$ to 0, decreasing from $x = 0$ to 2 then increasing from $x = 2$ to 3.

13. Option (c) is correct.



$$\sin x = \frac{x}{2}$$

From the graph clearly, there are 3 distinct points of intersection on graph.

14. Correct answer is [90°].

The time varying vector

$I = \hat{x}15\cos(\omega t) + \hat{y}5\sin(\omega t)$

When $\theta = 0 \Rightarrow |I| = 15$ $\quad [\because I = \hat{x}15]$

When $\theta = \frac{\pi}{2} \Rightarrow |I| = 5$ $\quad [\because I = \hat{y}5]$

$\therefore$ the magnitude of I lies between 5 to 15 for any value of θ.

So, the value of $\theta = \frac{\pi}{2}$ or 90° that makes the $|I|$ value minimum.

15. Correct answer is [0.66].

Given $\frac{dx}{dt} = -3x + 2$; initial condition $x(0) = 1$

Solution of given differential equation is stable, if $|1 - 3h| < 1$

$-1 < 1 - 3h < 1$

$-2 < -3h < 0$

$0 < h < \frac{2}{3}$

$h = 0.66$

So, maximum value of $h = 0.66$.

16. Correct answer is [0].

Given C is a circle $x^2 + y^2 = 1$

Let $\quad \phi(x, y) = xy^2 \Rightarrow \frac{d\phi}{dy} = 2xy$

$\psi(x, y) = x^2 y \Rightarrow \frac{d\psi}{dx} = 2xy$

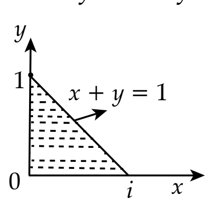
$$\oint_C (xy^2 dx + yx^2 dy) = \oint_C (\phi(x, y)dx + \psi(x, y)dy)$$

$$= \iint_R \left(\frac{d\psi}{dx} - \frac{d\phi}{dy}\right) dxdy = \iint_R (2xy - 2xy) = 0$$

[By Green's theorem]

17. Correct answer is [0.33].

The joint probability Density function is–



$$f_{X,Y}(x, y) = \begin{cases} (x + y) & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$P(X + Y \le 1) = \int_{y=0}^{1} \left(\int_{x=0}^{1-y} (x + y) dx \right) dy$$

$$= \int_{y=0}^{1} \left(\frac{x^2}{2} + yx \right)_0^{1-y} dy = \int_{y=0}^{1} \left(\frac{(1-y)^2}{2} + y(1-y) \right) dy$$

$$= \int_{y=0}^{1} \left(\frac{1 - y^2}{2} \right) dy = \frac{1}{2}\left(y - \frac{y^3}{3} \right)_0^1 = \frac{1}{2}\left(1 - \frac{1}{3}\right) = 0.33$$

18. Correct answer is [3].

Given trace (A) = 14 = sum of diagonal elements of matrix.

$\Rightarrow \quad a + 5 + 2 + b = 14$

$\Rightarrow \quad a + b = 7$ ...(i)

Given, $\quad |A| = 100$

$$\Rightarrow \begin{vmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{vmatrix} = 100$$

$$\Rightarrow b \begin{vmatrix} a & 0 & 3 \\ 2 & 5 & 1 \\ 0 & 0 & 2 \end{vmatrix} = 100$$

$$\Rightarrow b.2 \begin{vmatrix} a & 0 \\ 2 & 5 \end{vmatrix} = 100$$

$10\,ab = 100$ ...(ii)

$ab = 10$

and $\quad (a - b)^2 = (a + b)^2 - 4ab$

$\Rightarrow \quad (a - b)^2 = (7)^2 - 4 \times 10$

$\Rightarrow \quad (a - b)^2 = 9$

$\Rightarrow \quad a - b = \pm 3$

$\Rightarrow \quad |a - b| = 3$

19. Correct answer is [0].

$f(x) = x(x - 1)(x - 2) = x(x^2 - 3x + 2)$

$= x^3 - 3x^2 + 2x$

$f'(x) = 3x^2 - 6x + 2$

$f'(x) = 0 \Rightarrow x = 1 \pm \frac{1}{\sqrt{3}}$

But the given interval is [1, 2] So, $x = 1 + \frac{1}{\sqrt{3}}$.

$x = 1 + \frac{1}{\sqrt{3}}$ is a stationary point.

$x = 1, 2$ are end point.

$f\left(1 + \frac{1}{\sqrt{3}}\right) = -0.3848$

$f(1) = 0, f(2) = 0$

$\therefore$ maximum value of given function is 0.

20. Correct answer is [3].

$Ax = \lambda x$

$$\text{For } A = \begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}, \begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \lambda\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

$$\left.\begin{aligned}&\Rightarrow x_1+x_2+x_3=\lambda x_1\\&\Rightarrow x_1+x_2+x_3=\lambda x_2\\&\Rightarrow x_1+x_2+x_3=\lambda x_3\end{aligned}\right\}\text{is satisfied if } \lambda = 0$$

or if $x_1 = x_2 = x_3$

giving $\lambda = 3$

21. Option (a) is correct.

$$f(t) = e^{2t}\sin(5t)u(t)$$

$$L\{\sin(5t)u(t)\} = \frac{5}{s^2+25}$$

Using frequency shifting property at $s_0 = 2$

$$L\left\{e^{2t}\sin(5t)u(t)\right\} = \frac{5}{(s-2)^2+25}$$

$$= \frac{5}{s^2-2s+29}$$

22. Option (b) is correct.

Given DE is

$$\left(D^2+2D+1\right)y = 0$$

$$AE \to m^2+2m+1=0$$

$$\Rightarrow \quad m = -1, -1$$

$$y = e^{-t}\left(C_1 + C_2 t\right)$$

$$at\ t = 0,\ y = 1$$

$$\Rightarrow \quad C_1 = 1$$

$$at\ t = 1,\ y = 3e^{-1}$$

$$\Rightarrow \quad e^{-1}(1+C_2) = 3e^{-1}$$

$$\Rightarrow \quad C_2 = 2$$

$$y = e^{-t}(1+2t)$$

$$y(2) = e^{-2}(1+2(2))$$

$$\therefore \quad y(2) = 5e^{-2}$$

23. Option (b) is correct.

Singular points are

$$\left(z-\frac{1}{2}\right)\left(z^2-4z+5\right) = 0 \Rightarrow z = \frac{1}{2},\ z = 2 \pm i$$

Given that $|z| \le 1$ so only $z = \frac{1}{2}$ lies inside the contour

Using Cauchy's integral formula

$$\therefore \quad \oint_C \frac{2z+5}{\left(z-\frac{1}{2}\right)\left(z^2-4z+5\right)}dz$$

$$= \oint_C \frac{\dfrac{2z+5}{\left(z^2-4z+5\right)}}{\left(z-\dfrac{1}{2}\right)}$$

$$\text{for } f(z) = \frac{2z+5}{z^2-4z+5}$$

$$z_0 = \frac{1}{2}$$

$$= 2\pi i\left[\frac{2\left(\frac{1}{2}\right)+5}{\left(\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right)+5\right)}\right] = \frac{48\pi i}{13}$$

24. Correct answer is [0.2].

All 3 pens of same colour can be done in 4 ways (all red, all green, all black, all blue) 2 pens of same colour can be selected in 4 × 3 ways (4 ways of selecting 2 pens of same colour and the other 1 pen can be picked in 3 ways.)

No pen of same colour can be choose in C (4, 3) ways

∴ Probability of all 3 pens of same Colour

$$= \frac{\text{No. of ways of selecting 3 pens of same colour}}{\text{Total number of ways of selecting 3 pens}}$$

$$= \frac{4}{\left[4+(4*3)+C(4,3)\right]}$$

$$= \frac{4}{20} = 0.2$$

25. Correct answer is [0.293].

$$\text{Given} \quad S = \sum_{n=0}^{\infty} n\alpha^n$$

$$\Rightarrow \quad 2\alpha = \alpha + 2\alpha^2 + 3\alpha^3 + \ldots\ldots$$

$$\Rightarrow \quad 2\alpha = \alpha\left[1+\alpha+2\alpha+3\alpha^2+\ldots\ldots\right]$$

$$\Rightarrow \quad 2\alpha = \frac{\alpha}{[1-\alpha]^2} \text{ if } |\alpha| < 1$$

$$\Rightarrow \quad [1-\alpha]^{-2} = 2$$

$$\Rightarrow \quad (1-\alpha)^2 = \frac{1}{2}$$

$$\Rightarrow \quad \alpha = 1 - \frac{1}{\sqrt{2}} \text{ for } 0 < \alpha < 1$$

$$\Rightarrow \quad \alpha = 0.293$$

26. Option (a) is correct.

Given eigen values of the matrix are 1, –2.

Eigen values for

$A^2 - 3A + 4I$ will be given by

$$\lambda_1 = (1)^2 - 3(1) + 4 = 2$$

$$\lambda_2 = (-2)^2 - 3(-2) + 4 = 14$$

$\therefore$ Eigen values are 2, 14.

$\therefore$ $A^2 - 3A + 4I$ will have the same eigen vectors x_1 and x_2.

27. Option (b) is correct.

Given $[A]_{4\times 3}$ matrix having rank is 2.

Using the properties of rank of matrix

$$\rho(AA^T) = \rho(A)$$

$$\Rightarrow \quad \rho(A^TA) = \rho(AA^T) = \rho(A) = 2$$

28. Option (b) is correct.

Given $\quad f(z) = z + z^*$

$$z = x + iy,\ z^* = x - iy$$

$$z + z^* = x + iy + x - iy = 2x$$

$$u = 2x,\ v = 0$$

$$u_x = 2,\ u_y = 0$$

$$v_x = 0,\ v_y = 0$$

Cauchy Riemann equations is not satisfied.

$\therefore$ $f(z)$ is continuous but not analytic.

29. Option (d) is correct.

Using Cayley Hamilton theorem.

$$\lambda^3 = \lambda$$

$$\Rightarrow \quad \lambda^3 - \lambda = 0$$

$$\Rightarrow \quad \lambda(\lambda^2 - 1) = 0$$

$$\Rightarrow \quad \lambda = 0,\ \text{and } \lambda^2 - 1 = 0$$

$$\Rightarrow \quad \lambda = \pm 1$$

$$\therefore \quad \lambda = \pm 1,\ 0$$

30. Option (a) is correct.

DE is

$$y''(t) + 2y'(t) + y(t) = 0$$

Taking laplace transform

$$\left[s^2Y(s) - sy(0) - y'(0)\right] + 2\left[sY(s) - y(0)\right] + Y(s) = 0$$

$$\Rightarrow \quad Y(s) = \frac{sy(0) + y'(0) + 2y(0)}{s^2 + 2s + 1}$$

Given $y'(0) = 1,\ y(0) = 0$

$$Y(s) = \frac{1}{s^2 + 2s + 1} = \frac{1}{(s+1)^2}$$

$$\Rightarrow \quad y(t) = te^{-t}u(t)$$

31. Option (b) is correct.

$$\vec{F} = 2xy^2\hat{i} + 2x^2y\hat{j} + \hat{k}$$

Equation of straight line passing through the given coordinates (0, 0, 0) and (1, 1, 1).

$$\frac{x}{1} = \frac{y}{1} = \frac{z}{1} = t \Rightarrow \quad x = y = z = t$$

$$\Rightarrow dx = dy = dz = dt$$

$$\int_C xy^2dx + 2x^2ydy + dz = \int_{t=0}^{1} 2t^3dt + 2t^3dt + dt$$

$$= \int_{t=0}^{1} (4t^3 + 1)dt = \left[t^4 + t\right]_0^1$$

$$= 2$$

32. Option (b) is correct.

$f(x)$ is periodic function and for odd periodic function, Fourier series contains only harmonics of sine this is satisfied by relation

$$f(x) = \sum_{k=1}^{\infty} b_k \sin(kx)$$

33. Option (d) is correct.

Fourier transform of

$$\frac{2\sin(t\tau/2)}{t} \to 2\pi rect\left(\frac{\omega}{\tau}\right)$$

$$\frac{\sin(2\pi t)}{\pi t} \to rect\left(\frac{\omega}{4\pi}\right)$$

$$\therefore \int_{-\infty}^{\infty} \frac{\sin(2\pi t)}{\pi t} e^{-j\omega t} dt = rect\left(\frac{\omega}{4\pi}\right) \qquad \text{...(i)}$$

Put $\omega = 0$ in (i)

$$\int_{-\infty}^{\infty} \frac{\sin(2\pi t)}{\pi t} dt = 1$$

$$\Rightarrow \quad 2\int_{-\infty}^{\infty} \frac{\sin(2\pi)}{\pi t} dt = 2$$

34. Correct answer is [7.4].

Given DE is $(D^2 - 4D + 4)y = 0$

$$AE \to m^2 - 4m + 4 = 0$$

$$(m-2)^2 = 0 \Rightarrow m = 2, 2$$

$$\therefore \quad \text{solution } y = e^{2x}(C_1 + C_2x) \qquad \text{...(i)}$$

At $\quad y(0) = 0 \Rightarrow C_1 = 0$

$$y'(x) = e^{2x}\{2(C_1 + C_2x) + C_2\}$$

At $y'(0) = 1 \Rightarrow 1 = C_2$

Putting the value of $C_1 = 0$ and $C_2 = 1$ in (i)

$$y = xe^{2x}$$

$$y(1) = e^2 = 7.389 = 7.4$$

35. Correct answer is [4.42].

$$\vec{F} = 5xz\hat{i} + \left(3x^2 + 2y\right)\hat{j} + x^2 z\hat{k}$$

$$x = t; y = t^2; z = t \Rightarrow dx = dt; dy = 2tdt; dz = dt$$

$$\therefore \int_C \vec{F}.\vec{dr} = \int 5xzdx + \left(3x^2 + 2y\right)dy + x^2 zdz$$

$$\Rightarrow \int_0^1 5t^2 dt + \left(3t^2 + 2t^2\right)2tdt + t^3 dt$$

$$\Rightarrow \int_0^1 5t^2 dt + 10t^3 dt + t^3 dt$$

$$\Rightarrow \int_0^1 5t^2 dt + 11t^3 dt$$

$$5\left(\frac{t^3}{3}\right)_0^1 + 11\left(\frac{t^4}{4}\right)_0^1 = \frac{5}{3} + \frac{11}{4} = \frac{20+33}{12} = \frac{53}{12}$$

$$= 4.4167 = 4.42$$

36. Option (d) is correct.

Given $P = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x + y \\ x + 3y \end{bmatrix}$$

$$\Rightarrow \quad a = 3x + y; \quad b = x + 3y$$

$$a^2 + b^2 = 1$$

$$\Rightarrow \quad (3x + y)^2 + (x + 3y)^2 = 1$$

$$\Rightarrow \quad 9x^2 + 6xy + y^2 + x^2 + 6xy + 9y^2 = 1$$

$$\Rightarrow \quad 10x^2 + 10y^2 + 12xy = 1$$

$$\Rightarrow \quad x^2 + y^2 + 1.2xy = 0.1$$

$$\Rightarrow \quad x^2 + y^2 + 1.2xy - 0.1 = 0$$

$$\Rightarrow \quad \frac{(x+y)^2}{\frac{1}{8}} + \frac{(x-y)^2}{\frac{1}{2}} = 1$$

With $a < b$, it represents an ellipse.

Major Axis = $x + y = 0$

Minor Axis = $x - y = 0$

So, minor axis is along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\therefore$ vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ lies on $y = x$ line.

37. Option (a) is correct.

Given PDF $f_x(x) = \begin{cases} ae^{4x} & x < 0 \\ \frac{3}{2}e^{-3x} & x \geq 0 \end{cases}$

We know $\int_{-\infty}^{\infty} f_x(x)dx = 1$

$$\int_{-\infty}^{0} ae^{4x}dx + \int_0^{\infty} \frac{3}{2}e^{-3x} = 1$$

$$a\left(\frac{e^{4x}}{4}\right)_{-\infty}^0 + \left(\frac{\frac{3}{2}e^{-3x}}{-3}\right)_0^{\infty} = 1$$

$$\Rightarrow \quad \frac{a}{4} + \frac{3}{6} = 1$$

$$\Rightarrow \quad a = 2$$

$$P[x \leq 0] = \frac{a}{4} = \frac{1}{2}$$

38. Option (d) is correct.

By verification method,

$$\begin{bmatrix} 2 & 5 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 6 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -30 \end{bmatrix}$$

39. Option (b) is correct.

According to definition of Laplace transform

$$F(S) = \int_0^{\infty} e^{-st} f(t)dt; \quad t \geq 0$$

40. Option (a) is correct.

Given $u(x, y) = 2xy$

From the definition of total derivative

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$$

$$= -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy \quad \text{...(i)}$$

[from CR equations]

$$\therefore \quad u(x, y) = 2xy$$

$$\Rightarrow \quad \frac{\partial u}{\partial x} = 2y; \frac{\partial u}{\partial y} = 2x$$

From equation (i)

$$dv = -2xdx + 2ydy \quad \text{...(ii)}$$

Integrating (ii) we have

$$v = -2\left(\frac{x^2}{2}\right) + 2\left(\frac{y^2}{2}\right) + \text{constant}$$

$$\Rightarrow \quad v = -x^2 + y^2 + \text{constant}$$

41. Option (a) is correct.

We know for Poisson Distribution mean = variance = μ

$\therefore$ standard deviation $= \sqrt{\text{variance}} = \sqrt{\mu}$

42. Correct answer is [1.56].

Given, $f(x) = x - 10\cos(x)$ and $x_o = \left(\frac{\pi}{4}\right)$

$$f'(x) = 1 + 10\sin(x)$$

$$x_1 = x_o - \frac{f(x_o)}{f'(x_o)} = \frac{\pi}{4} - \frac{\left(\frac{\pi}{4} - \frac{10}{\sqrt{2}}\right)}{\left(1 + \frac{10}{\sqrt{2}}\right)} = \frac{\pi}{4} + \frac{6.2857}{8.0711}$$

$$x_1 = 1.5641 \approx 1.56$$

43. Correct answer is [–5].

$$f(x) = 2x^3 - 3x^2 \qquad \text{in } [-1, 2]$$

$$f'(x) = 6x^2 - 6x$$

$$f'(x) = 0 \Rightarrow 6x^2 - 6x = 0$$

$$\Rightarrow 6x(x-1) = 0 \Rightarrow x = 0, x = 1$$

Function value at end points and stationary points.

$$f(0) = 0$$
$$f(-1) = -5$$
$$f(1) = -1$$
$$f(2) = 4$$

$\therefore$ global minimum is –5 at $x = -1$.

44. Correct answer is [–1].

Given Differential equation is

$$y'' + 9y = 0$$

$$\Rightarrow \quad (D^2 + 9)y = 0$$

$$AE \to m^2 + 9 = 0$$

$$m = \pm 3i$$

$\therefore$ solution

$$y = C_1 \cos 3x + C_2 \sin 3x \qquad \text{...(i)}$$

$$at\ x = 0; y = 0$$

$$\Rightarrow \quad 0 = C_1 \cos 0 + C_2 \sin 0$$

$$\Rightarrow \quad C_1 = 0$$

at $\quad x = \frac{\pi}{2}; y = \sqrt{2}$

$$\Rightarrow \quad \sqrt{2} = C_2 \sin 3\frac{\pi}{2}$$

$$\Rightarrow \quad -\sqrt{2} = C_2$$

From (i)

$$\Rightarrow \quad y = (0)\cos 3x - \sqrt{2}\sin 3x$$

$$\Rightarrow \quad y = -\sqrt{2}\sin 3x$$

$$y\left(\frac{\pi}{4}\right) = -\sqrt{2}\sin 3\left(\frac{\pi}{4}\right) = -\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = -1$$

45. Option (a) is correct.

$$\sin x = I_m\left(e^{ix}\right)$$

$$\therefore \quad f(z) = \frac{e^{iz}}{z^2 + 2z + 2}$$

$$\therefore \text{ poles of } f(z) = \frac{-2 \pm \sqrt{4 - 4(2)}}{2} = \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm 2i}{2} = -1 \pm i$$

$z = -1 + i$ is the pole lying in upper half of z plane.

$$\therefore \quad \underset{z=-1+i}{\text{Res}}\, f(z) = \lim_{z \to -1+i} \{z - (-1+i)\}$$

$$\frac{e^{iz}}{\{z - (-1+i)\}.\{z - (-1-i)\}}$$

$$\lim_{z \to -1+i} \frac{e^{iz}}{\{z + (1+i)\}} = \frac{e^{i(-1+i)}}{\{(-1+i) + (1+i)\}}$$

$$= \frac{e^{-i-1}}{2i} = \frac{e^{-i-1}}{2i} = \frac{e^{-i}}{2ie}$$

$$\int_C \frac{e^{iz}}{z^2 + 2z + 2} dz = 2\pi i\left[\frac{e^{-i}}{2ie}\right] = \frac{\pi e^{-i}}{e}$$

$$\therefore \qquad = \frac{\pi}{e}(\cos 1 - i\sin 1)$$

$$\int_C \frac{\sin z}{z^2 + 2z + 2} dz = \frac{\pi(-\sin(1))}{e} = -\frac{\pi\sin(1)}{e}$$

46. Correct answer is [–6].

Let the equation are

$$x + 2y + 3z = 5$$

$$2x + 3y + z = 1$$

$$3x + 2y + z = 3$$

And $x_o = 0, y_o = 0, z_o = 0$

Then first iteration –

$$x_1 = 5 - 2y_o - 3z_o = 5 - 0 - 0 = 5$$

$$y_1 = \frac{1}{3}(1 - 2x_1 - z_o) = \frac{1}{3}(1 - 2(5) - 0) = -3$$

$$z_1 = 3 - 3x_1 - 2y_1 = 3 - 3(5) - 2(-3) = -6$$

So, $x_3 = z_1 = -6$

47. Option (a) is correct.

$$A = \begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$$

Let λ_1, λ_2 are the eigen values of $[A]_{2\times 2}$

We know $|A| = \lambda_1.\lambda_2$

$$|A| = 2k - 1$$

$$\lambda_1.\lambda_2 > 0$$

$\Rightarrow \quad 2k - 1 > 0$

$\Rightarrow \quad k > \frac{1}{2}$ ...(i)

And $\lambda_1 + \lambda_2 = > 0$

$\Rightarrow \quad$ Trace of A > 0

$\Rightarrow \quad 2 + k > 0$

$\Rightarrow \quad k > -2$ ...(ii)

From (i) and (ii)

$$\left(k > \frac{1}{2}\right) \cap (k > -2) = k > \frac{1}{2}$$

48. Option (c) is correct.

$$f(x) = \frac{x^2 - 3x - 4}{x^2 + 3x - 4}$$

at $x = 1$ and $x = -4$ denominator becomes zero. Therefore, function is not defined and hence is not continuous at $x = 1$ and $x = -4$.

49. Option (a) is correct.

$$L\{\cos(\omega t)\} = \frac{s}{s^2 + \omega^2}$$

50. Correct answer is [–1].

$$u(x, y) = 2kxy$$

$$v(x, y) = x^2 - y^2$$

For analytic function CR equation is satisfied.

$$\therefore \quad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$\frac{\partial u}{\partial x} = 2ky$ ...(i)

$\frac{\partial v}{\partial y} = -2y$ ...(ii)

From (i) and (ii)

$$2ky = -2y \Rightarrow k = -1$$

51. Option (a) is correct.

When $f(x)$ is linear function numerical integration using trapezoidal gives best result for a single variable function. It will have error for second or higher order polynomials.

52. Correct answer is [726].

For ϕ,

$$\nabla\phi = \frac{d\phi}{dx}\hat{i} + \frac{d\phi}{dy}\hat{j} + \frac{d\phi}{dz}\hat{k}$$

$$= yz\hat{i} + yz\hat{i} + xy\hat{k}$$

$$\frac{d\phi}{dx} = yz, \frac{d\phi}{dy} = xz, \text{ and } \frac{d\phi}{dz} = xy$$

on integration we get $\phi = xyz$ this potential function makes line integral path independent.

at $t = 1 \quad x_0 = 1, \quad y_0 = 1, \quad z_0 = 3$

$t = 3 \quad x_1 = 3, \quad y_1 = 9, \quad z_1 = 27$

$$\int_C \nabla\phi.d\vec{r} = \phi\int_{(x_0, y_0, z_0)}^{(x_1, y_1, z_1)}$$

$$= (xyz)\int_{(1,1,3)}^{(3,9,27)}$$

$$= 3 * 9 * 27 - 1 * 1 * 3$$

$$= 726$$

53. Option (b) is correct.

The correct path – By Cauchy's integral theorem

$$\int_C \frac{\left(\frac{3z-5}{z-2}\right)}{z-1} dz = 2\pi i \left[\frac{3z-5}{z-2}\right]_{z=1}$$

$$= 2\pi i \left[\frac{3(1)-5}{(1)-2}\right] = 2\pi i\left(\frac{-2}{-1}\right) = 4\pi i$$

54. Correct answer is [0.41].

For x = number of defective screws found

Required Probability

$$= P[x \geq 1] = 1 - P[x = 0]$$

$= 1 - {}^5C_0\,(0.1)^0\,(0.9)^5$ with $n = 5, p = 0.1$

$$= 0.4095 \simeq 0.41$$

55. Correct answer is [0.1862].

Given $I = \int_0^{\pi} (\sin x + \cos x)\, dx$

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π
$f(x)$	1	$\frac{\sqrt{3}}{2}+1$	$\frac{\sqrt{3}}{2}-1$	-1

$$h = \frac{b-a}{n} = \frac{\pi - 0}{3} = \frac{\pi}{3}$$

Approximate value by Trapezoidal rule.

$$I = \int_0^{\pi} f(x)\, dx$$

$$= \frac{h}{2}\left[(y_0 + y_3) + 2(y_1 + y_2)\right]$$

$$= \frac{\pi}{6}\left[(1-1) + 2\left(\frac{\sqrt{3}}{2} + 1 + \frac{\sqrt{3}}{2} - 1\right)\right] = \frac{\pi}{6}\left(2\sqrt{3}\right)$$

$$= \frac{\pi}{\sqrt{3}} = 1.8138$$

(approximate value)

And by exact integration

$$\int_0^{\pi}(\sin x + \cos x)\,dx = \left[(-\cos x) + \sin x\right]_0^{\pi}$$

$$= \left[-(-1-1) + (0-0)\right] = 2$$

(exact value)

∴ error = exact value – approximate value
= 2 – 1.8138
= 0.1862

56. Option (c) is correct.

A real square matrix A is skew symmetric if $A^T = -A$

57. Option (c) is correct.

$$\underset{x\to 0}{\text{Lt}} \frac{\log(1+4x)}{e^{3x}-1} \to \frac{0}{0} \text{ form}$$

Applying L' Hospital Rule.

$$\underset{x\to 0}{\text{Lt}} \frac{\frac{1}{(1+4x)}(4)}{3e^{3x}} = \frac{4}{3}$$

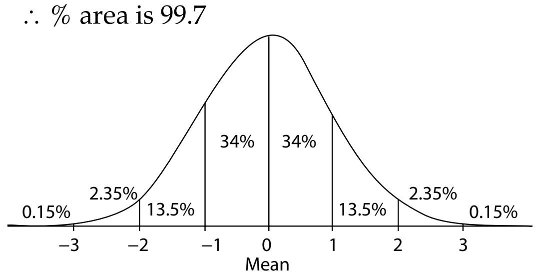
58. Option (b) is correct.

Solution of Laplace's equation having continuous function of 2nd order partial derivatives is known as Harmonic functions.

59. Correct answer is [99.7].

The area under standard normal distribution curve between –3 and 3 is 0.9973.

∴ % area is 99.7



60. Option (c) is correct.

$$f(x) = x^3 + x - 1$$

$$f'(x) = 3x^2 + 1$$

$$x_0 = 1$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{1+1-1}{3+1} = \frac{1}{4} = 0.75$$

61. Correct answer is [2].

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

⇒ $\lambda = 2, 2, 3$

$\lambda = 2$

∴ $(A - \lambda I)$

$$\begin{bmatrix} 2-\lambda & 1 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

rank of $(A - \lambda I) = 2$

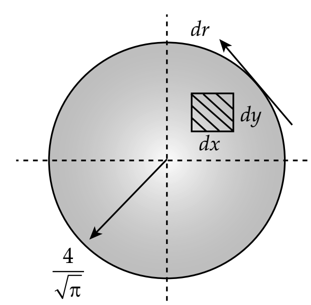
$n = 3$,

∵ So, $n - r = 3 - 2 = 1$, number of linearly independent eigen vectors for $\lambda = 2$ will be one.

∴ No. of linearly independent eigen vectors for $\lambda = 3$ is one.

So, number of linearly independent eigen vectors of matrix is 2.

62. Correct answer is [16].



$$d\vec{r} = \vec{r}ds$$

$$\vec{F}(x, y) = \left(y\hat{i} + 2x\hat{j}\right)$$

$$\oint_C \vec{F}.d\vec{r} = \oint_C \left(y\hat{i} + 2x\hat{j}\right).d\vec{r}$$

Applying Green's Theorem

$$\oint_C \left(y\hat{i} + 2x\hat{j}\right).d\vec{r} = \iint_A \left(\frac{\partial}{\partial x}(2x) - \frac{\partial}{\partial y}(y)\right) dxdy$$

$$= \iint_A dxdy$$

$$= \{\text{area of circle of radius } \frac{4}{\sqrt{\pi}} \text{units}\}$$

$$= \pi\left(\frac{4}{\sqrt{\pi}}\right)^2 = 16$$

63. Option (c) is correct.

$$\lim_{x\to\infty}\sqrt{x^2+x-1}-x$$

$$=\lim_{x\to\infty}\left(\sqrt{x^2+x-1}-x\right)\times\frac{\left(\sqrt{x^2+x-1}+x\right)}{\left(\sqrt{x^2+x-1}+x\right)}$$

$$=\lim_{x\to\infty}\frac{\left(x^2+x-1-x^2\right)}{\left(\sqrt{x^2+x-1}+x\right)}=\lim_{x\to\infty}\frac{(x-1)}{\left(\sqrt{x^2+x-1}+x\right)}$$

$$=\lim_{x\to\infty}\frac{x\left(1-\frac{1}{x}\right)}{x\left(\sqrt{1+\frac{1}{x}-\frac{1}{x^2}}+1\right)}=\frac{1}{2}\quad\left[\because\lim_{x\to\infty}\left(\frac{1}{x}\right)=0\right]$$

64. Option (a) is correct.

No. of cards drawn = 3

$$\text{Required Probability}=\frac{{}^4C_1\times{}^4C_1\times{}^4C_1}{{}^{52}C_3}=\frac{16}{5525}$$

65. Correct answer is [0.360].

$$f(x)=3x-e^x+\sin x$$

$$x_0=0.333$$

$$f'(x)=3-e^x+\cos x$$

$$f(x_0)=3(0.333)-e^{(0.333)}+\sin(0.333)=-0.069$$

$$f'(x_0)=3-e^{(0.333)}+\cos(0.333)=2.549$$

According to newton Raphson Method

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=0.333-\frac{(-0.069)}{2.549}=0.360$$

66. Option (c) is correct.

From given PDE B = 3, A = 1, C = 1

$\therefore B^2-4AC=3^2-4(1)(1)=9-4$
$=5>0$

So, given PDE is Hyperbolic.

67. Option (d) is correct.

For m order matrix A $\begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1m}\\ a_{21} & a_{22} & \cdots & a_{2m}\\ a_{31} & a_{32} & \cdots & a_{3m}\\ \vdots & & & \\ a_{m_1} & a_{m_2} & \cdots & a_{mm}\end{bmatrix}$,

$$\sum_{i=1}^{m}a_{ij}=1\quad\text{for all }1\le j\le m$$

For eigen value AX = λX

$$\therefore\quad\begin{aligned}a_{11}x_1+a_{12}x_2+\cdots a_{1m}x_m&=\lambda x_1\\ a_{21}x_1+a_{22}x_2+\cdots a_{2m}x_m&=\lambda x_2\\ \vdots\qquad\vdots\qquad\vdots\qquad&\vdots\\ a_{m_1}x_1+a_{m_2}x_2+\cdots a_{mm}x_m&=\lambda x_m\end{aligned}$$

$$x_1+x_2+\cdots x_m=\lambda(x_1+x_2+\cdots x_m)$$

$$\left[\begin{array}{l}\text{Adding all equation}\\ \because\sum_{i=1}^{m}a_{ij}=1\ \text{ for all } j\end{array}\right]$$

So, λ = 1 Satisfies this condition.

68. Option (a) is correct.

Type II error ⇒ acceptance of the null hypothesis when it is false and should be rejected.

69. Option (b) is correct.

For 1 – Dim Heat equation

$$\frac{du}{dt}=\alpha\frac{d^2u}{dx^2}\qquad[\text{For } k=\alpha p^2\ k \text{ be positive}]$$

$$u(x,t)=\left\{C_1e^{px}+C_2e^{-px}\right\}C_3e^{p^2\alpha t}\qquad\left[\therefore\sqrt{\frac{k}{\alpha}}=p\right]$$

$$\left\{C_1e^{\sqrt{k/\alpha}\,x}+C_2e^{-\sqrt{k/\alpha}\,x}\right\}C_3e^{kt}$$

70. Option (a) is correct.

$$\text{Given }f(x)=\begin{cases}0.25 & \text{if } 1\le x\le 5\\ 0 & \text{otherwise}\end{cases}$$

$$P[x\le 4]=\int_{-\infty}^{4}f(x)dx=\int_{-\infty}^{1}0dx+\int_{1}^{4}0.25\,dx$$

$$=\frac{1}{4}[x]_1^4=\frac{1}{4}[4-1]=\frac{3}{4}$$

71. Option (b) is correct.

$$I_1=\int_0^\infty\frac{1}{x^2+1}dx=\left[\tan^{-1}x\right]_0^\infty$$

$$=\tan^{-1}\infty-\tan^{-1}0=\frac{\pi}{2}$$

$$\therefore\quad I_1=\frac{\pi}{2}$$

We know

$$L(\sin x)=\frac{1}{s^2+1}\Rightarrow L\left(\frac{\sin x}{x}\right)=\int_s^\infty\left(\frac{1}{s^2+1}\right)dx$$

[From division property of Laplace Transform]

$$=\left[\tan^{-1}s\right]_s^\infty=\tan^{-1}\infty-\tan^{-1}(s)=\cot^{-1}(s)$$

$$\Rightarrow\quad\int_0^\infty e^{-sx}.\frac{\sin x}{x}dx=\cot^{-1}(s)$$

putting $s = 0$

$$I_2 = \int_0^\infty \frac{\sin x}{x} dx = \cot^{-1}(0) = \frac{\pi}{2}$$

$\therefore \quad I = I_1 + I_2 = \pi$

72. Option (b) is correct.

Given Curves are $y = x^2 + 1$ and

$x + y = 3 \Rightarrow y = 3 - x$

Point of intersection of the curves

$\Rightarrow x^2 + 1 = 3 - x$

$\Rightarrow x^2 + x - 2 = 0$

$\Rightarrow \quad x = -2, 1$

and $3 - x \geq x^2 + 1$

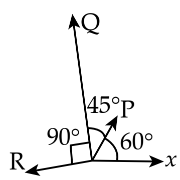
$\therefore \quad \text{Area} = \iint_R dydx$

$$= \int_{x=-2}^{1}\left[\int_{y=x^2+1}^{3-x} dy\right]dx = \int_{x=-2}^{1}\left[(3-x)-(x^2+1)\right]dx$$

$$= \int_{x=-2}^{1}\left[-x^2 - x + 2\right]dx = \left[-\frac{x^3}{3} - \frac{x^2}{2} + 2x\right]_{-2}^{1} = \frac{9}{2}$$

73. Option (c) is correct.

Resolving the given vector components w.r.t. x-axis



$$\sum F_X = P\cos 60° + Q\cos(60° + 45°)Q + R\cos(90° + 45° + 60°)$$

$$\sum F_X = 100\cos 60° + 250\cos(105°) + 150\cos(195°)$$

$$F_X = -159.6kN$$

$$\sum F_Y = P\sin 60° + Q\sin(60° + 45°)Q + R\sin(90° + 45° + 60°)$$

$$\Rightarrow \quad \sum F_Y = 100\sin 60° + 250\sin(105°) + 150\sin(195°)$$

$$F_Y = 289.3kN$$

$$|F| = \sqrt{(-159.6)^2 + (289.3)^2} = 330.4kN$$

$$\tan\theta = \frac{F_Y}{F_X} = \frac{289.3}{-159.6}$$

$\therefore \quad$ w.r.t x–axis $180° + (-61.1°) = 118.9°$

74. Option (a) is correct.

Given DE is

$$(D^4 + 3D^2)y = 108x^2$$

Let $\quad D = \frac{d}{dx}$

$\therefore \quad AE \rightarrow m^4 + 3m^2 = 0$

$\Rightarrow \quad m^2(m^2 + 3) = 0$

$\Rightarrow \quad m = 0, 0, \pm\sqrt{3}i$

$\therefore \quad CF = (C_1 + C_2x) + C_3\sin(\sqrt{3}x) + C_4\cos(\sqrt{3}x)$

and $PI = \frac{1}{D^4 + 3D^2}(108x^2)$

$$= \frac{1}{3D^2\left[1 + \frac{D^2}{3}\right]}(108x^2)$$

$$= \frac{36}{D^2}\left[1 + \frac{D^2}{3}\right]^{-1}(x^2)$$

$$= \frac{36}{D^2}\left[1 - \frac{D^2}{3} + ...\right](x^2)$$

$$= \frac{36}{D^2}\left[x^2 - \frac{1}{3}(2) + 0\right]$$

$$\iint\left[36x^2 - \frac{2}{3}\right]dxdx = 36\left(\frac{x^4}{4\times 3} - \frac{2}{3}\frac{x^2}{2\times 1}\right)$$

$$= 3x^4 - 12x^2 + c$$

75. Correct answer is [54.5].

Median value is the middle value when arranged in ascending order.

Ascending order of speeds– 32, 45, 49, 51, 53, 56, 60, 62, 66, 79

$$\text{Median} = \frac{5^{th}\text{ term} + 6^{th}\text{ term}}{2}$$

$\Rightarrow \quad \text{Median Speed} = \frac{53 + 56}{2} = 54.5$ km/h

76. Option (d) is correct.

$f(x) = x^2 - 4x + 2$

$f'(x) = 2x - 4$

$f'(x) = 0$

$\Rightarrow \quad x = 2 \rightarrow$ Stationary Point

$f''(x) = 2 > 0$

$\Rightarrow \quad f(x)$ is minimum at $x = 2$.

Min value at $x = 2$

$f(2) = (2)^2 - 4(2) + 2 = -2$

$\therefore \quad$ optimum value of given function is –2 (Minimum).

77. Option (c) is correct.

Given

$f(x) = 0, -\pi < x \le 0$

$$f(x) = \frac{\pi}{4} + \frac{2}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2}...\right] + \left[\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + ...\right]$$

At $x = 0$; a point of discontinuity,

The Fourier series converges

$$\rightarrow \quad \frac{1}{2}\left[f(0^-) + f(0^+)\right]$$

$$f(0^-) = \lim_{x\to 0}(0) = 0 \text{ and}$$

$$f(0^+) = \lim_{x\to 0}(\pi - x) = \pi$$

$$\therefore \quad \frac{\pi}{2} = \frac{\pi}{4} + \frac{2}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...\right]$$

$$\Rightarrow \quad \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$$

$$\Rightarrow \quad \sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$

78. Option (a) is correct.

$$P(X \cup Y^c) = 0.7 = P(x) + P(y^c) - P(x).P(y^c) = 0.7$$

$\because$ x and y are independent events.

$$\Rightarrow \quad P(x) + 1 - P(y) - P(x)\{1 - P(y)\} = 0.7$$

$$\Rightarrow \quad -P(y) + P(x).\,P(y) = 0.7 - 1$$

$$\Rightarrow \quad P(y) - P(x \cap y) = 0.3$$

$$\because \quad P(x \cup y) = P(x) + P(y) - P(x \cap y)$$

$$= 0.4 + 0.3 = 0.7$$

79. Option (d) is correct.

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{xy}{x^2 + y^2}$$

[Finding limit along $y = mx$, $x \to 0, y \to 0$]

$$= \lim_{x\to 0}\left(\frac{x.mx}{x^2 + m^2x^2}\right) = \lim_{x\to 0}\frac{m}{1+m^2},$$ limit will vary with different value of m.

80. Option (b) is correct.

$$[A\,|\,B] = \begin{bmatrix} 1 & 2 & -3 & a \\ 2 & 3 & 3 & b \\ 5 & 9 & -6 & c \end{bmatrix}$$

$$R_2 \to R_2 - 2R_1, R_3 \to R_3 - 5R_1$$

$$\begin{bmatrix} 1 & 2 & -3 & a \\ 0 & -1 & -a & b-2a \\ 0 & -1 & -a & c-5a \end{bmatrix}$$

With $R_2 = R_3$, rank A = rank (A|B)

$$\Rightarrow \quad b - 2a = c - 5a \Rightarrow 3a + b - c = 0$$

81. Option (b) is correct.

Mean of $f(x)$

$$= E[X] = \int_{-a}^{0} x.\left(1 + \frac{x}{a}\right)dx + \int_{0}^{a} x.\left(-\frac{x}{a} + 1\right)dx$$

$$= \left(\frac{x^3}{3a} + \frac{x^2}{2}\right)_{-a}^{0} + \left(-\frac{x^3}{3a} + \frac{x^2}{2}\right)_{0}^{a} = 0$$

Variance of $f(x) = E(X^2) - \{E[X]\}^2$

$$E[X^2] = \int_{-a}^{0} x^2.\left(1 + \frac{x}{a}\right)dx + \int_{0}^{a} x^2.\left(-\frac{x}{a} + 1\right)dx$$

$$= \left(\frac{x^4}{4a} + \frac{x^3}{3}\right)_{-a}^{0} + \left(-\frac{x^4}{4a} + \frac{x^3}{3}\right)_{0}^{a} = \frac{a^3}{6}$$

$$\therefore \quad \text{Var} = \frac{a^3}{6}$$

Mean of $g(x)$ $$= E[X] = \int_{-a}^{0} x.\left(-\frac{x}{a}\right)dx + \int_{0}^{a} x.\left(\frac{x}{a}\right)dx$$

$$= \left(-\frac{x^3}{3a}\right)_{-a}^{0} + \left(\frac{x^3}{3a}\right)_{0}^{a} = 0$$

Variance of $g(x) = E(X^2) - \{E[X]\}^2$

$$E[X^2] = \int_{-a}^{0} x^2.\left(-\frac{x}{a}\right)dx + \int_{0}^{a} x^2.\left(\frac{x}{a}\right)dx$$

$$= \left(-\frac{x^4}{4a}\right)_{-a}^{0} + \left(\frac{x^4}{4a}\right)_{0}^{a} = \frac{a^3}{2}$$

$$\therefore \quad \text{Var} = \frac{a^3}{2}$$

$\therefore$ Mean of $f(x) = g(x) = 0$ and variance of $f(x)$ and $g(x)$ are different.

82. Option (d) is correct.

Curves are $x^2 = 4y$ ...(i)

$y^2 = 4x$ ...(ii)

Differentiating both curves w.r.t. to x,

$$2x = 4\frac{dy}{dx} \quad \Rightarrow \left(\frac{dy}{dx}\right)_{(0,0)} = 0 = m_1$$

$$2y\frac{dy}{dx} = 4 \quad \Rightarrow \left(\frac{dy}{dx}\right)_{(0,0)} = \infty = m_2$$

With one slope parallel to x axis and another parallel to y-axis, the angle between the curve at (0, 0)

$$\theta = \frac{\pi}{2} = 90^\circ$$

83. Correct answer is [85.33].

Given equation of parabola is

$$x^2 = 8y \Rightarrow y = \frac{x^2}{8}$$

And straight–line $y = 8$.

Point of intersection $x = 8, -8$

Required area $\int_{-8}^{8}(y_1 - y_2)\, dx$

$$= \int_{x=-8}^{8}\left(8 - \frac{x^2}{8}\right)dx = 2\int_0^8\left(8 - \frac{x^2}{8}\right)dx$$

$$= 2\left(8x - \frac{x^3}{24}\right)_0^8 = \frac{256}{3} = 85.33 \text{ sq units}$$

84. Option (b) is correct.

Quadratic approximation of $f(x)$ at $x = 0$ is

$$f(x) = x^3 - 3x^2 - 5$$

$$f'(x) = 3x^2 - 6x \qquad \Rightarrow f'(0) = 0$$

$$f''(x) = 6x - 6 \qquad \Rightarrow f''(0) = -6$$

$$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) \quad \text{[Taylor's series]}$$

$$= -5 + x.0 + \frac{x^2}{2!}(-6)$$

$$= -5 - 3x^2$$

85. Correct answer is [11].

$$\sim((p \Rightarrow q) \wedge (\sim r \vee \sim s))$$

$$= \sim(p \Rightarrow q) \vee \sim(\sim r \vee \sim s)$$

$$= \sim(p \Rightarrow q) \vee (r \wedge s)$$

$$= \sim(\sim p \vee q) \vee (r \wedge s)$$

$$= (p \wedge \sim q) \vee (r \wedge s)$$

$$p \wedge \sim q = 11$$

$$r \wedge s = \phi$$

For $x = 11$ the given compound preposition is true.

86. Option (b) is correct.

There can be two cases.

Case I first bit is '0' $\underbrace{0------}_{a_{n-1}}$

a_{n-1}

Must be zero.

Case II first bit is '1' $\underbrace{10-----}_{a_{n-2}}$

$\therefore \quad a_n = a_{n-1} + a_{n-2}$

87. Correct answer is [1].

$$\lim_{x \to 4} \frac{\sin(x-4)}{(x-4)}$$

$$= \lim_{x-4 \to 0} \frac{\sin(x-4)}{(x-4)}$$

$$= \lim_{y \to 0} \frac{\sin y}{y} \qquad (\text{put } y = x - 4)$$

$$= 1$$

88. Correct answer is [0.5].

$$f(x) = \begin{cases} \dfrac{1}{x^2} & a \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \quad \int_a^1 \frac{1}{x^2}dx = 1$$

$$\Rightarrow \quad \left[-\frac{1}{x}\right]_a^1 = 1$$

$$\Rightarrow \quad -1 + \frac{1}{a} = 1$$

$$\Rightarrow \quad a = 0.5$$

89. Correct answer is [15].

$[P]_{3\times3}$ is a real matrix.

Given Eigen values are $2 + \sqrt{-1}, 3$ or $2 + i, 3$

Complex eigen values occur in pair so $2 - i$ is also an eigen value.

We know $|P|$ = Product of eigen vlaues

$\Rightarrow \quad |P| = (2+i)(2-i)3 = 5 \times 3 = 15$

90. Correct answer is [10].

Given expression is

$$(x^3 + x^4 + x^5 + x^6 + ...)^3$$

$$= x^9(1 + x + x^2 + x^3 + ...)^3$$

$$= x^9\left((1-x)^{-1}\right)^3$$

$$= x^9(1-x)^{-3}$$

$$= x^9 \sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} x^n$$

For coefficient of x^{12}, put $n = 3$; $\frac{4 \times 5}{2} = 10$

91. Correct answer is [198].

Recurrence relation $= a_n - a_{n-1} = 6n^2 + 2n$ ...(i)

Characteristic equation is $m - 1 = 0 \Rightarrow m = 1$

Complementary solution

$= a^{(1)}_n = C_1(1)^n = C_1$

Let particular solution

$= a^{(b)}_n = \left(An^2 + Bn + C\right)n$ ...(ii)

Putting $a_n = \left(An^2 + Bn + C\right)n$ in equation (i)

we get

A = 2, B = 4, C = 2

General solution

$= a^{(c)}_n + a^{(b)}_n = C_1 + \left(2n^2 + 4n + 2\right)n$

As $a_1 = B \Rightarrow B = C_1 + B \Rightarrow C_1 = 0$

Also $a_{99} = k \times 10^4$

$\Rightarrow \left[2(99)^2 + 4(99) + 2\right]99 = k \times 10^4$

$\Rightarrow 198 \times 10^4 = k \times 10^4$

$\Rightarrow \quad k = 198$

92. Correct answer is [2].

$$f(n) = \begin{cases} f\left(\frac{n}{2}\right) & \text{if } n \text{ is even} \\ f(n+5) & \text{if } n \text{ is odd} \end{cases}$$

We can see that

$f(1) = f(2) = f(3) = f(4) = f(6) = f(7)...$

and $f(5) = f(10) = f(15)...$

$\therefore$ range of function contain two distinct elements only.

93. Correct answer is [0.33].

We can observe that the chances of y are

$$\frac{1}{4}, \left(\frac{1}{4}\right)\left(\frac{1}{4}\right), \left(\frac{1}{4}\right)^2\left(\frac{1}{4}\right)...$$

$$\therefore \quad P = \frac{1}{4} + \left(\frac{1}{4} \times \frac{1}{4}\right) + \left(\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)\right)\left(\frac{1}{4}\right) + ...$$

$$= \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + ...$$

$$= \frac{1}{4}\left(1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + ...\right)$$

$$= \frac{1}{4}\left(\frac{1}{1 - \frac{1}{4}}\right) = \frac{1}{4} \times \frac{4}{3} = \frac{1}{3} = 0.33$$

94. Correct answer is [4].

P	Q	$P \Rightarrow Q$	$P \wedge (P \Rightarrow Q)$
F	F	T	F
F	T	T	F
T	F	F	F
T	T	T	T

Let $\left(P \wedge (P \Rightarrow Q)\right) \Leftrightarrow$ A(for notation only)

For options (i), (ii), (iii), (iv), (v)

If $(A \Rightarrow \text{option } x)$ is a tautology.

Then $P \wedge (P \Rightarrow Q)$ is logically implies option x.

Else $P \wedge (P \Rightarrow Q)$ does not logically implies option x.

			Options				
			(i)	(ii)	(iii)	(iv)	(v)
P	Q	A	False $A \Rightarrow F$	Q $A \Rightarrow Q$	True $A \Rightarrow$ true	$P \vee Q$ $A \Rightarrow$ $(P \vee Q)$	$\neg Q \vee P$ $A \Rightarrow$ $(\neg P \vee Q)$
F	F	F	T	T	T	T	T
F	T	F	T	T	T	T	T
T	F	F	T	T	T	T	T
T	T	T	F F	T T	T T	T T	T T

Blank entries in the truth table are don't care conditions because in these rows the value of A is set to false. Therefore (A anything) would be set to true. Four of the expressions are logically implied.

95. Correct answer is [9].

Let the degree of polynomial $f(x)$ is n.

And $g(x) = f'(x)$ has also degree $n - 1$.

Given $f(x) + f(-x)$ has degree is 10.

$\Rightarrow \quad g(x) + g(-x)$ degree = 9

96. Correct answer is [4].

Any planar graph is four colourable.

97. Option (c) is correct.

Statement I is incorrect.

$x + y + z = 1$

$x + y + z = 0$

Has no solution when No. of equations < NO. of variables.

Statement II is incorrect.

Eg. $x + 3y = 6$

$2x + 8y = 14$

$x + y = 4$

Has a solution $y = 1, x = 3$

Statement III is correct.

$x + y = 4$

$x + 2y = 0$

Has a solution $x = 8, y = -4$.

98. Correct answer is 0.55.

E1 = event of selecting type 1 bulb.

E2 = event of selecting type 2 bulb.

X– event of selecting a bulb lasts more than 100 hours.

Given $P(E1) = 0.5$ and $P(E2) = 0.5$

$P\left(\frac{X}{E1}\right) = 0.7, P\left(\frac{X}{E2}\right) = 0.4$

Required Probability

$= P(X) = P(E1)P\left(\frac{X}{E1}\right) + P(E2)P\left(\frac{X}{E2}\right)$

$= 0.5 \times 0.7 + 0.5 \times 0.4$

$= 0.55$

99. Correct answer is [0.125].

Given Eigen values of A are 1, 2 and 4.

$|A| = 8 =$ product of eigen values.

$|A^{-1}| = \frac{1}{|A|} = \frac{1}{8}$

$|A^{-1}|^T = |A^{-1}| = \frac{1}{8} = 0.125$

100. Option (b) is correct.

As every ordered pair is related to itself so, it is reflexive.

$(a, b)R(a, b)$ $[\because a \le a \text{ or } b \le b]$

As (2, 4) R (3, 2) $[\because 2 < 3 \{a < c\}]$

and (3, 2) R (1, 3) $[\because 2 < 3 \{b < d\}]$

but (2, 4) R (1, 3) $\begin{bmatrix} \because \text{ neither } & 2 < 1\{a < c\} \\ \text{nor} & 4 < 3\{b < d\} \end{bmatrix}$

So, it is not transitive.

101. Option (d) is correct.

If every x is either even or odd, then every x must be even or must be odd.

If the domain is the set of natural numbers left hand side is true but right–hand side is not true as not all–natural numbers are even or not all natural numbers are odd.

102. Option (b) is correct.

Considering an undirected graph with 23 vertices and 9 of these with 3 degree.

Assume that if two compounds react with each other, their there exists an edge between the vertices.

Given 9 vertices of degree 3 (odd).

According to degree of vertices theorem atleast one of the remaining vertices must have odd degree. Since number of vertices of odd degree is always even.

103. Correct answer is [4].

By Fermat's Little Theorem, If P is prime, then

$a^{p-1} \equiv 1 \mod p$

$13^{16} \equiv 1 \mod 17$

$13^{96} = 13^{16 \times 6} \equiv 1 \mod 17$

$13^{99} = 13^{96} \times 13^3 \equiv 13^3 \mod 17 \equiv 2197 \mod 17$

which is 4.

(By remainder theorem)