ELECTRONICS AND COMMUNICATION (EC) P1

Q. 1. Consider the 5 × 5 matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$$

It is given that A has only one real eigenvalue. Then the real eigenvalue of A is

(a) –2.5 **(b)** 0
(c) 15 **(d)** 25

Q. 2. The rank of the matrix $M = \begin{bmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{bmatrix}$ is

(a) 0 **(b)** 1
(c) 2 **(d)** 3

Q. 3. Consider the following statements about the linear dependence of the real valued functions $y_1 = 1$, $y_2 = x$ and $y_3 = x^2$ over the field or real numbers.

I. y_1, y_2 and y_3 are linearly independent on $-1 \le x \le 0$

II. y_1, y_2 and y_3 are linearly dependent on $0 \le x \le 1$

III. y_1, y_2 and y_3 are linearly independent on $0 \le x \le 1$

IV. y_1, y_2 and y_3 are linearly dependent on $-1 \le x \le 0$

Which one among the following is correct?

(a) Both I and II are true
(b) Both I and III are true
(c) Both II and IV are true
(d) Both III and IV are true

Q. 4. Three fair cubical dice are thrown simultaneously. The probability that all three dice have the same number of dots on the faces showing up is (up to third decimal place _____.

Q. 5. Let $f(x) = e^{x+x^2}$ for real x. From among the following, choose the Taylor series approximation of $f(x)$ around $x = 0$, which includes all powers of x less than or equal to 3.

(a) $1 + x + x^2 + x^3$

(b) $1 + x + \frac{3}{2}x^2 + x^3$

(c) $1 + x + \frac{3}{2}x^2 + \frac{7}{6}x^3$

(d) $1 + x + 3x^2 + 7x^3$

Q. 6. A three dimensional region R of finite volume is described by

$x^2 + y^2 \le z^3$; $0 \le z \le 1$,

Where x, y, z are real. The volume of R (up to two decimal places) is ______.

Q. 7. Let $I = \int_C (2z\,dx + 2y\,dy + 2x\,dz)$ where x y z are real, and let C be the straight line segment from point A: (0, 2, 1) to point B: (4, 1, –1). The value of I is ______.

Q. 8. Which one of the following is the general solution of the first order differential equation

$\frac{dy}{dx} = (x + y - 1)^2$,

Where x, y are real ?

(a) $y = 1 + x + \tan^{-1}(x + c)$, where c is a constant.

(b) $y = 1 + x + \tan(x + c)$, where c is a constant.

(c) $y = 1 - x + \tan^{-1}(x + c)$, where c is a constant.

(d) $y = 1 - x + \tan(x + c)$, where c is a constant.

Q. 9. Starting with $x = 1$, the solution of the equation $x^3 + x = 1$, after two iterations of Newton Raphson method (up to two decimal places) is ______.

ELECTRONICS AND COMMUNICATION (EC) P2

Q. 10. The rank of the matrix

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \text{ is } \underline{\hspace{1.5cm}}.$$

Q. 11. The general solution of the differential equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}-5y=0$$

In terms of arbitrary constant K_1 and K_2 is

(a) $K_1e^{(-1+\sqrt{6})x}+K_2e^{(-1-\sqrt{6})x}$

(b) $K_1e^{(-1+\sqrt{8})x}+K_2e^{(-1-\sqrt{8})x}$

(c) $K_1e^{(-2+\sqrt{6})x}+K_2e^{(-2-\sqrt{6})x}$

(d) $K_1e^{(-2+\sqrt{8})x}+K_2e^{(-2-\sqrt{8})x}$

Q. 12. The smaller angle (in degrees) between the planes $x + y + z = 1$ and $2x - y + 2z = 0$ is ______.

Q. 13. The residues of a function

$$f(z)=\frac{1}{(z-4)(z+1)^3} \text{ are}$$

(a) $\frac{-1}{27}$ and $\frac{-1}{125}$ **(b)** $\frac{1}{125}$ and $\frac{-1}{125}$

(c) $\frac{-1}{27}$ and $\frac{1}{5}$ **(d)** $\frac{1}{125}$ and $\frac{-1}{5}$

Q. 14. The values of the integrals

$$\int_0^1\left(\int_0^1\frac{x-y}{(x+y)^3}dy\right)dx \text{ and } \int_0^1\left(\int_0^1\frac{x-y}{(x+y)^3}dx\right)dy \text{ are}$$

(a) same and equal to 0.5

(b) same and equal to –0.5

(c) 0.5 and – 0.5 respectively

(d) –0.5 and 0.5 respectively

Q. 15. An integral I over a counterclockwise circle C is given by

$$I=\oint_C\frac{z^2-1}{z^2+1}e^z dz$$

If C is defined as $|z| = 3$, then the value of I is

(a) $-\pi i\sin(1)$ **(b)** $-2\pi i\sin(1)$

(c) $-3\pi i\sin(1)$ **(d)** $-4\pi i\sin(1)$

Q. 16. If the vector function

$$\vec{F}=\widehat{a_x}\left(3y-k_1z\right)+\widehat{a_y}\left(k_2x-2z\right)-\widehat{a_z}\left(k_3y+z\right)$$

is irrotational, then the values of the constants k_1, k_2 and k_3, respectively, are

(a) 0.3, –2.5, 0.5 **(b)** 0.0, 3.0, 2.0

(c) 0.3, 0.33, 0.5 **(d)** 4.0, 3.0, 2.0

Q. 17. Passengers try repeatedly to get a seat reservation in any train running between two stations until they are successful. If there is 40% chance of getting reservation in any attempt by a passenger. Then the average number of attempts that passengers need to make to get a seat reserved is ______.

Q. 18. The minimum value of the function $f(x)=\frac{1}{3}x(x^2-3)$ in the interval $-100 \le x \le 100$ occurs at $x =$ ______.

ELECTRICAL ENGINEERING (EE) P1

Q. 19. The matrix $\mathbf{A}=\begin{bmatrix} \frac{3}{2} & 0 & \frac{1}{2} \\ 0 & -1 & 0 \\ \frac{1}{2} & 0 & \frac{3}{2} \end{bmatrix}$ has three distinct eigenvalues and one of its eigenvectors is $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ which one of the following can be another eigenvector of **A**?

(a) $\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$ **(b)** $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

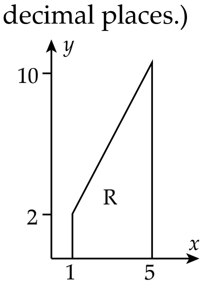
(c) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

Q. 20. For a complex number z, $\lim_{z\to i}\frac{z^2+1}{z^3+2z-i(z^2+2)}$ is

(a) $-2i$ **(b)** $-i$

(c) i **(d)** $2i$

Q. 21. Let $I = c\iint_R xy^2dxdy$ where R is the region shown in the figure and $c = 6 \times 10^{-4}$. The values of I equals ______. (Give the answer up to two decimal places.)



Q. 22. A function $f(x)$ is defined as $f(x) \begin{cases} e^x, & x < 1 \\ \ln x + ax^2 + bx, & x \geq 1 \end{cases}$, where $x \in \mathbb{R}$. Which one of the following statements is "TRUE"

(a) $f(x)$ is **NOT** differentiable at $x = 1$ for any values of a and b.

(b) $f(x)$ is differentiable at $x = 1$ for the unique values of a and b.

(c) $f(x)$ is differentiable at $x = 1$ for all values of a and b such that $a + b = e$.

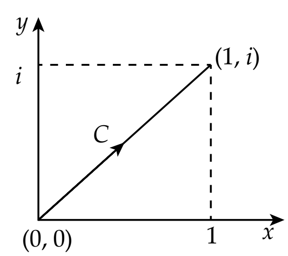
(d) $f(x)$ is differentiable at $x = 1$ for all values of a and b.

Q. 23. Consider the differential equation $(t^2 - 81)\frac{dy}{dt} + 5t\, y = \sin(t)$ with $y(1) = 2\pi$. There exists a unique solution for this differential equation when t belongs to the interval

(a) (–2, 2) **(b)** (–10, 10)

(c) (–10, 2) **(d)** (0, 10)

Q. 24. Consider the line integral $I = \int_C (x^2 + iy^2)dz$, where $z = x + iy$. The line C is shown in the figure below.



The value of I is

(a) $\frac{1}{2}i$ **(b)** $\frac{2}{3}i$

(c) $\frac{3}{4}i$ **(d)** $\frac{4}{5}i$

Q. 25. Only one of the real roots of $f(x) = x^6 - x - 1$ lies in the interval $1 \leq x \leq 2$ and bisection method is used to find its value. For achieving an accuracy of 0.001, the required minimum number of iterations is ______.

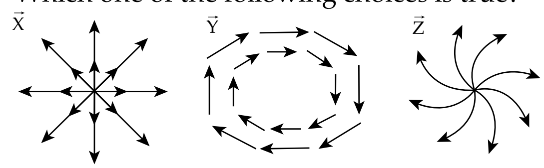
ELECTRICAL ENGINEERING (EE) P2

Q. 26. An urn contains 5 red balls and 5 black balls. In the first draw, one ball is picked at random and discarded without noticing its colour. The probability to get a red ball in the second draw is

(a) $\frac{1}{2}$ **(b)** $\frac{4}{9}$

(c) $\frac{5}{9}$ **(d)** $\frac{6}{9}$

Q. 27. The figures show diagramatic representation of vector fields $\vec{X}, \vec{Y}$ and $\vec{Z}$, respectively. Which one of the following choices is true?



(a) $\nabla \cdot \vec{X} = 0, \nabla \times \vec{Y} \neq 0, \nabla \times \vec{Z} = 0$

(b) $\nabla \cdot \vec{X} \neq 0, \nabla \times \vec{Y} = 0, \nabla \times \vec{Z} \neq 0$

(c) $\nabla \cdot \vec{X} \neq 0, \nabla \times \vec{Y} \neq 0, \nabla \times \vec{Z} \neq 0$

(d) $\nabla \cdot \vec{X} = 0, \nabla \times \vec{Y} = 0, \nabla \times \vec{Z} = 0$

Q. 28. Assume that in a traffic junction, the cycle of the traffic signal lights is 2 minutes of green (vehicle does not stop) and 3 minutes of red (vehicle stop). Consider that the arrival time of vehicles at the junction is uniformly distributed over 5 minute cycle. The expected waiting time (in minutes) for the vehicle at the junction is ______.

Q. 29. Consider a function $f(x, y, z)$ given by
$f(x, y, z) = (x^2 + y^2 - 2z^2)(y^2 + z^2)$
The partial derivative of this function with respect to x at the point, $x = 2$, $y = 1$ and $z = 3$ is ______.

Q. 30. Let x and y be integers satisfying the following equations
$2x^2 + y^2 = 34$
$x + 2y = 11$
The value of $(x+y)$ is ______.

Q. 31. Let $y^2 - 2y + 1 = x$ and $\sqrt{x} + y = 5$. The value of $x + \sqrt{y}$ equals ______. (Give the answer up to three decimal places) .

Q. 32. Let $g(x) = \begin{cases} -x, & x \leq 1 \\ x+1, & x \geq 1 \end{cases}$ and

$$f(x) = \begin{cases} 1-x, & x \leq 0 \\ x^2, & x > 0 \end{cases}$$

Consider the composition of f and g, *i.e.*, $(f \circ g)(x) = f(g(x))$. The number of discontinuities in $(f \circ g)(x)$ present in the interval $(-\infty, 0)$ is :

(a) 0 **(b)** 1

(c) 2 **(d)** 4

Q. 33. The value of the contour integral in the complex –plane

$$\oint \frac{z^3 - 2z + 3}{z - 2} dz$$

along the contour $|z| = 3$, taken counter – clockwise is

(a) $-18\pi i$ **(b)** 0
(c) $14\pi i$ **(d)** $48\pi i$

Q. 34. The eigen values of the matrix given below are

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix}$$

(a) (0, –1, –3) **(b)** (0, –2, –3)
(c) (0, 2, 3) **(d)** (0, 1, 3)

Q. 35. A person decides to toss a fair coin repeatedly until he gets a head. He will make at most 3 tosses. Let the random variable Y denote the number of heads. The value of var {Y}, where var {.} denotes the variance, equals

(a) $\frac{7}{8}$ **(b)** $\frac{49}{64}$
(c) $\frac{7}{64}$ **(d)** $\frac{105}{64}$

MECHANICAL ENGINEERING (ME) P1

Q. 36. The product of eigenvalues of the matrix P is

$$P = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{bmatrix}$$

(a) –6 **(b)** 2
(c) 6 **(d)** –2

Q. 37. The value of $\lim_{x \to 0} \frac{x^3 - \sin(x)}{x}$ is

(a) 0 **(b)** 3
(c) 1 **(d)** –1

Q. 38. Consider the following partial differential equation for $u(x,y)$ with the constant $c > 1$:

$$\frac{\partial u}{\partial y} + c\frac{\partial u}{\partial x} = 0$$

Solution of this equation is

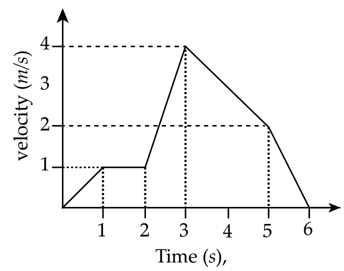
(a) $u(x,y) = f(x + cy)$
(b) $u(x,y) = f(x - cy)$
(c) $u(x,y) = f(cx + y)$
(d) $u(x,y) = f(cx - y)$

Q. 39. The differential equation $\frac{d^2y}{dx^2} + 16y = 0$ for $y(x)$ with the two boundary conditions $\left.\frac{dy}{dx}\right|_{x=0} = 1$ and $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} = -1$ has

(a) no solution
(b) exactly two solutions
(c) exactly one solution
(d) infinitely many solutions

Q. 40. A six-face fair dice is rolled a large number of times. The mean value of the outcomes is ____.

Q. 41. The following figure shows the velocity–time plot for a particle traveling along a straight line. The distance covered by the particle from $t = 0$ to $t = 5$ s is ______ m.



Q. 42. Consider the matrix $P = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$.

Which one of the following statements about P is INCORRECT?

(a) Determinant of P is equal to 1.
(b) P is orthogonal.
(c) Inverse of P is equal to its transpose.
(d) All eigenvalues of P are real numbers.

Q. 43. For the vector $\vec{V} = 2yz\hat{i} + 3xz\hat{j} + 4xy\hat{k}$, the value of $\nabla.(\nabla \times \vec{V})$ ______.

Q. 44. A parametric curve defined by $x = \cos\left(\frac{\pi u}{2}\right)$, $y = \sin\left(\frac{\pi u}{2}\right)$ in the range $0 \le u \le 1$ is rotated about the X-axis by 360 degrees. Area of the surface generated is

(a) $\frac{\pi}{2}$ **(b)** π
(c) 2π **(d)** 4π

Q. 45. P(0, 3), Q (0.5, 4), and R (1, 5) are three points on the curve defined by $f(x)$. Numerical integration is carried out using both Trapezoidal rule and Simpson's rule within limits $x = 0$ and $x = 1$ for the curve. The difference between the two results will be

(a) 0 **(b)** 0.25
(c) 0.5 **(d)** 1

MECHANICAL ENGINEERING (ME) P2

Q. 46. Two coins are tossed simultaneously. The probability (upto two decimal points accuracy) of getting at least one head is _____.

Q. 47. The divergence of the vector $-y\hat{i} + x\hat{j}$ is _____.

Q. 48. The determinant of a 2 × 2 matrix is 50. If one eigenvalue of the matrix is 10, the other eigenvalue is ______.

Q. 49. A sample of 15 data is as follows: 17, 18, 17, 17, 13, 18, 5, 5, 6, 7, 8, 9, 20, 17, 3. The mode of the data is

(a) 4 **(b)** 13
(c) 17 **(d)** 20

Q. 50. The Laplace transform te^t is

(a) $\frac{s}{(s+1)^2}$ **(b)** $\frac{1}{(s-1)^2}$

(c) $\frac{1}{(s+1)^2}$ **(d)** $\frac{s}{s-1}$

Q. 51. The surface integral $\iint_s F \cdot n\, ds$ over the surface S of the sphere of the $x^2 + y^2 + z^2 = 9$, where $F = (x + y)\boldsymbol{i} + (x + z)\boldsymbol{j} + (y + z)\boldsymbol{k}$ and n is the unit outward surface normal, yields______

Q. 52. Consider the differential equation $3y''(x) + 27y(x) = 0$ with initial conditions $y(0) = 0$ and $y'(0) = 2000$. The value of y at $x = 1$ is ______.

Q. 53. Consider the matrix $A = \begin{bmatrix} 50 & 70 \\ 70 & 80 \end{bmatrix}$ whose eigenvectors corresponding to eigenvalues λ_1 and λ_2 are $x_1 = \begin{bmatrix} 70 \\ \lambda_1 - 50 \end{bmatrix}$ and $x_2 = \begin{bmatrix} \lambda_2 - 80 \\ 70 \end{bmatrix}$, respectively. The value of $x_1^T x_2$ is ______.

Q. 54. If $f(z) = (x^2 + ay^2) + i\,bxy$ is a complex analytic function of $z = x + iy$, where $i = \sqrt{-1}$, then

(a) $a = -1, b = -1$ **(b)** $a = -1, b = 2$
(c) $a = 1, b = 2$ **(d)** $a = 2, b = 2$

CIVIL ENGINEERING (CE) P1

Q. 55. The matrix P is the inverse of a matrix Q. If I denotes the identity matrix, which one of the following options is correct?

(a) PQ = I but QP ≠ I
(b) QP = I but PQ ≠ I
(c) PQ = I and QP = I
(d) PQ – QP = I

Q. 56. The number of parameters in the univariate exponential and Gaussain distributions, respectively, are

(a) 2 and 2 **(b)** 1 and 2
(c) 2 and 1 **(d)** 1 and 1

Q. 57. Let x be a continuous variable defined over the interval $(-\infty, \infty)$, and $f(x) = e^{-x-e^{-x}}$.

The integral $g(x) = \int f(x)\,dx$ is equal to

(a) $e^{e^{-x}}$ **(b)** $e^{-e^{-x}}$
(c) $e^{-e^{x}}$ **(d)** e^{-x}

Q. 58. Consider the following partial differential equation:

$$3\frac{\partial^2 \phi}{\partial x^2} + B\frac{\partial^2 \phi}{\partial x \partial y} + 3\frac{\partial^2 \phi}{\partial y^2} + 4\phi = 0$$

For this equation to be classified as parabolic, the value B^2 must be ______.

Q. 59. $\lim_{x \to 0}\left(\frac{\tan x}{x^2 - x}\right)$ is equal to ______.

Q. 60. Vehicles arriving at an intersection from one of the approach roads follow the Poisson distribution. The mean rate of arrival is 900 vehicles per hour. If a gap is defined as the time difference between two successive vehicle arrivals (with vehicles assumed to be points), the probability (up to four decimal places) that the gap is greater than 8 seconds is ______.

Q. 61. For the function $f(x) = a + bx, 0 \le x \le 1$, to be a valid probability density function, which one of the following statements is correct?

(a) $a = 1, b = 4$ **(b)** $a = 0.5, b = 1$
(c) $a = 0, b = 1$ **(d)** $a = 1, b = -1$

Q. 62. The solution of the equation $\frac{dQ}{dt} + Q = 1$ with Q = 0 at $t = 0$ is

(a) $Q(t) = e^{-t} - 1$ **(b)** $Q(t) = 1 + e^{-t}$
(c) $Q(t) = 1 - e^{t}$ **(d)** $Q(t) = 1 - e^{-t}$

Q. 63. Consider the matrix $\begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix}$. Which one of the following statements is TRUE for the eigenvalues and eigenvectors of this matrix?

(a) Eigenvalue 3 has a multiplicity of 2, and only one independent eigenvector exists.
(b) Eigenvalue 3 has a multiplicity of 2, and two independent eigenvectors exist.
(c) Eigenvalue 3 has a multiplicity of 2, and no independent eigenvectors exists.
(d) Eigenvalues are 3 and –3 and two independent eigenvectors exists.

Q. 64. Consider the equation $\frac{du}{dt} = 3t^2 + 1$ with $u = 0$ at $t = 0$. This is numerically solved by using the forward Euler method with a step size, $\Delta t = 2$. The absolute error in the solution at the end of the first time step is ______.

CIVIL ENGINEERING (CE) P2

Q. 65. Consider the following simultaneous equations (with c_1 and c_2 being constants):

$3x_1 + 2x_2 = c_1$

$4x_1 + x_2 = c_2$

The characteristic equation for these simultaneous equation is

(a) $\lambda^2 - 4\lambda - 5 = 0$ **(b)** $\lambda^2 - 4\lambda + 5 = 0$
(c) $\lambda^2 + 4\lambda - 5 = 0$ **(d)** $\lambda^2 + 4\lambda + 5 = 0$

Q. 66. Let $w = f(x, y)$, where x and y are functions of t. Then according to the chain rule, $\frac{dw}{dt}$ is equal to

(a) $\frac{dw}{dx}\frac{dx}{dt} + \frac{dw}{dy}\frac{dt}{dt}$ **(b)** $\frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$

(c) $\frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}$ **(d)** $\frac{dw}{dx}\frac{\partial x}{\partial t} + \frac{dw}{dy}\frac{\partial y}{\partial t}$

Q. 67. The divergence of the vector field $V = x^2\boldsymbol{i} + 2y^3\boldsymbol{j} + z^4\boldsymbol{k}$ at $x = 1, y = 2, z = 3$ is ________

Q. 68. A two–faced fair coin has its faces designated as head (H) and tail (T). This coin is tossed three times in succession to record the following outcomes: H, H, H. If the coin is tossed one more time, the probability (up to one decimal place) of obtaining H again, given the previous realizations of H, H and H, would be_______

Q. 69. The tangent to the curve represented by $y = x \ln x$ is required to have 45° inclination with the x–axis. The coordinates of the tangent point would be

(a) (1, 0) **(b)** (0, 1)
(c) (1, 1) **(d)** $(\sqrt{2}, \sqrt{2})$

Q. 70. Consider the following definite integral:

$$I = \int_0^1 \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}} dx$$

The value of the integral is

(a) $\frac{\pi^3}{24}$ **(b)** $\frac{\pi^3}{12}$

(c) $\frac{\pi^3}{48}$ **(d)** $\frac{\pi^3}{64}$

Q. 71. If $A = \begin{bmatrix} 1 & 5 \\ 6 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 7 \\ 8 & 4 \end{bmatrix}$. AB^T is equal to

(a) $\begin{bmatrix} 38 & 28 \\ 32 & 56 \end{bmatrix}$ **(b)** $\begin{bmatrix} 3 & 40 \\ 42 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 43 & 27 \\ 34 & 50 \end{bmatrix}$ **(d)** $\begin{bmatrix} 38 & 32 \\ 28 & 56 \end{bmatrix}$

Q. 72. Consider the following second–order differential equation:

$y'' - 4y' + 3y = 2t - 3t^2$

The particular solution of the differential equation is

(a) $-2 - 2t - t^2$ **(b)** $-2t - t^2$
(c) $2t - 3t^2$ **(d)** $-2 - 2t - 3t^2$

COMPUTER SCIENCE (CS) P1

Q. 73. Consider the following functions from positive integers to real numbers:

$10, \sqrt{n}, n, \log_2 n, \frac{100}{n}$

The correct arrangement of the above function in increasing order of asymptotic complexity is:

(a) $\log_2 n, \frac{100}{n}, 10, \sqrt{n}, n$

(b) $\frac{100}{n}, 10, \log_2 n, \sqrt{n}, n$

(c) $10, \frac{100}{n}, \sqrt{n}, \log_2 n, n$

(d) $\frac{100}{n}, \log_2 n, 10, \sqrt{n}, n$

Q. 74. Let X be a Gaussian random variable with mean 0 and variance σ^2 . Let Y = max (X, 0) where max (a, b) is the maximum of a and b. The median of Y is ______.

Q. 75. Let $c_1,\ldots, c_n$ be scalars, not all zero, such that $\sum_{i=1}^{n} c_i a_i = 0$ where a_i are column vectors in R^n.

Consider the set of linear equations

$Ax = b$

Where $A = [a_1,\ldots,a_n]$ $b = \sum_{i=1}^{n} a_i$. The set of equations has

(a) a unique solution at $x = J_n$ where J_n denotes a n–dimensional vector of all 1

(b) no solution

(c) infinitely many solutions

(d) finitely many solutions

Q. 76. The value of $\lim_{x \to 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$

(a) is 0 **(b)** is –1

(c) is 1 **(d)** does not exist

Q. 77. Let u and v be two vectors in R^2 whose Euclidean norms satisfy $\|u\| = 2\|v\|$. What is the value of α such that $w = u + \alpha v$ bisects the angle between u and v?

(a) 2 **(b)** $\frac{1}{2}$

(c) 1 **(d)** $-\frac{1}{2}$

Q. 78. Let A be $n \times n$ real valued square symmetric matrix of rank 2 with $\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}^2 = 50$. Consider the following statements.

(I) One eigenvalue must be in [–5, 5]

(II) The eigenvalue with the largest magnitude must be strictly greater than 5

Which of the above statements about eigenvalues of A is/are necessarily **CORRECT**?

(a) Both (I) and (II) **(b)** (I) only

(c) (II) only **(d)** Neither (I) nor (II)

COMPUTER SCIENCE (CS) P2

Q. 79. If $f(x) = R\sin\left(\frac{\pi x}{2}\right) + S, f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)\,dx = \frac{2R}{\pi}$, then the constants R and S are respectively

(a) $\frac{2}{\pi}$ and $\frac{16}{\pi}$ **(b)** $\frac{2}{\pi}$ and 0

(c) $\frac{4}{\pi}$ and 0 **(d)** $\frac{4}{\pi}$ and $\frac{16}{\pi}$

Q. 80. Let $P = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$ be two matrices.

Then the rank of P + Q is ______.

Q. 81. P and Q are considering to apply for a job. The probability that P applies for the job is $\frac{1}{4}$, the probability that P applies for the job given that Q applies for the job is $\frac{1}{2}$, and the probability that Q applies for the job given that P applies for the job is $\frac{1}{3}$. Then the probability that P does not apply for the job given that Q does not apply for the job is

(a) $\frac{4}{5}$ **(b)** $\frac{5}{6}$

(c) $\frac{7}{8}$ **(d)** $\frac{11}{12}$

Q. 82. For any discrete random variable X, with probability mass function

$P(X = j) = p_j, p_j \geq 0, j \in \{0,\ldots,N\}$, and $\sum_{j=0}^{N} p_j = 1$,

define the polynomial function

$g_x(z) = \sum_{j=0}^{N} p_j z^j$. For a certain discrete random variable Y, there exists a scalar $\beta \in [0, 1]$ such that $g_y(z) = (1 - \beta + \beta z)^N$. The expectation of Y is

(a) $N\beta(1 - \beta)$

(b) $N\beta$

(c) $N(1 - \beta)$

(d) Not expressible in terms of N and β alone

Q. 83. If a random variable X has a Poisson distribution with mean 5, then the expectation $E[(X + 2)_2]$ equals ______.

Q. 84. If the characteristic polynomial of a 3 × 3 matrix M over $\mathbb{R}$ (the set of real numbers) is $\lambda^3 - 4\lambda^2 + a\lambda + 30$, $a \in \mathbb{R}$, and one eigenvalue of M is 2, then the largest among the absolute values of the eigenvalues of M is ______.

Answer Key			
Q. No.	**Answer**	**Topic Name**	**Chapter Name**
1	(c)	Eigen Values	Linear Algebra
2	(c)	Rank of Matrix	Linear Algebra
3	(b)	Linear Dependence of Function	Linear Algebra
4	0.028	Probability	Statistics and Probability
5	(c)	Taylor Series	Calculus
6	0.79	Volume Integral	Calculus
7	–11	Line Integral	Vector Calculus
8	(d)	First Order	Differential Equation
9	0.68	Newton Raphson Method	Numerical Analysis
10	(d)	Rank of Matrix	Linear Algebra
11	(a)	2nd Order	Differential Equation
12	54.73	Angle Between Planes	Vector Calculus
13	(b)	Residues	Complex Variable
14	(c)	Double Integral	Calculus
15	(d)	Cauchy's Residues Theorem	Complex Variable
16	(b)	Curl	Vector Calculus
17	2.5	Probability	Statistics and Probability
18	–100	Maximum/Minimum	Calculus
19	(c)	Eigen Values and Eigen Vectors	Linear Algebra
20	(d)	Limits	Complex Variable
21	1	Double Integral	Calculus
22	(b)	Differentiability	Calculus
23	(a)	1st Order Differential	Differential Equation
24	(b)	Line Integral	Vector Calculus
25	10	Bisection Method	Numerical Analysis
26	(a)	Basics Probability	Probability and Statistics
27	(c)	Curl and Divergence	Vector Analysis
28	0.9	Uniform Distribution Rv	Probability And Statistics
29	40	Partial Derivative	Calculus
30	7	Functions	Calculus
31	5.732	Functions	Calculus
32	(a)	Continuity	Calculus
33	(c)	Cauchy's Integral	Complex Variable
34	(a)	Eigen Values	Linear Algebra
35	(c)	Statistics	Statistics and Probability
36	(b)	Eigen Values	Linear Algebra
37	(d)	Limit	Calculus
38	(b)	Partial Differential Equation	Calculus
39	(a)	2nd Order Equation	Differential Equation
40	3.5	Statistics	Statistics and Probability
41	10	Integration	Calculus
42	(d)	Matrix and Determinant Properties	Linear Algebra

Q. No.	Answer	Topic Name	Chapter Name
43	0	Curl and Divergence Properties	Vector Analysis
44	(c)	Application of Integration	Calculus
45	(a)	Numerical Integration	Numerical Analysis
46	0.75	Probability	Statistics and Probability
47	0	Divergence	Vector Analysis
48	5	Eigen Values	Linear Algebra
49	(c)	Mode	Statistics and Probability
50	(b)	Laplace Transform	Transform Theory
51	226.2	Surface Integral	Vector Analysis
52	94.08	Higher Order DE	Differential Equation
53	0	Eigen Vector	Linear Algebra
54	(b)	Complex Variable Basics	Complex Variable
55	(c)	Inverse of Matrix	Linear Algebra
56	(b)	Normal Distribution	Probability and Statistics
57	(b)	Integration	Calculus
58	36	Partial Differential Equations	Differential Equations
59	–1	Limit	Calculus
60	0.1353	Poisson Distribution Probability	Probability and Statistics
61	(b)	PDF	Probability and Statistics
62	(d)	1st Order Differential Equation	Differential Equation
63	(a)	Eigen Values and Eigen Vector	Linear Algebra
64	8	Forward Eular Method	Numerical Analysis
65	(a)	Characteristic Equation	Linear Algebra
66	(c)	Total Derivative	Calculus
67	134	Divergence	Vector Analysis
68	0.5	Probability	Probability and Statistics
69	(a)	Application of Derivatives	Calculus
70	(a)	Definite Integral	Calculus
71	(a)	Matrix Multiplication	Linear Algebra
72	(a)	2nd Order Differential Equation	Differential Equation
73	(b)	Functions	Calculus
74	0	Normal Distribution	Probability and Statistics
75	(c)	Solving Linear Equations	Linear Algebra
76	(c)	Limit	Calculus
77	(a)	Vectors	Linear Algebra
78	(b)	Eigen Values	Linear Algebra
79	(c)	Functions	Calculus
80	2	Rank of Matrix	Matrix And Determinant
81	(a)	Conditional Probability	Probability and Statistics
82	(b)	Random Variable Distributions	Probability and Statistics
83	54	Distributions	Probability and Statistics
84	5	Eigen Values	Linear Algebra

ANSWERS WITH EXPLANATIONS

1. Option (c) is correct.

Given $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$R_1 \to R_1 + R_2 + R_3 + R_4 + R_5$

$$\Rightarrow \begin{vmatrix} 15-\lambda & 15-\lambda & 15-\lambda & 15-\lambda & 15-\lambda \\ 5 & 1-\lambda & 2 & 3 & 4 \\ 4 & 5 & 1-\lambda & 2 & 3 \\ 3 & 4 & 5 & 1-\lambda & 2 \\ 2 & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

gives $R_1 = 0$ and det = 0.

$\Rightarrow \quad 15 - \lambda = 0$

$\lambda = 15$ so, real eigen value of A is 15.

2. Option (c) is correct.

$$M = \begin{bmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{bmatrix}$$

Interchanging the Row 1 and Row 2

$$= \begin{bmatrix} 1 & 0 & 2 \\ 5 & 10 & 10 \\ 3 & 6 & 6 \end{bmatrix}$$

$R_2 \to R_2 - 5R_1$

$R_3 \to R_3 - 3R_1$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 10 & 0 \\ 0 & 6 & 0 \end{bmatrix}$$

$R_3 \to R_3 - \frac{6}{10} R_2$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 10 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

No. of non–zero row are 2. So, in row–echelon from rank is 2.

Shortcut Method:

$C_3 = 2C_1$

$|M| = 0$

So, rank < 3 hecking minor of order 2

$$\begin{vmatrix} 5 & 10 \\ 1 & 0 \end{vmatrix} = -10$$

$\neq 0$

So, rank M = 2.

3. Option (b) is correct.

Given ; $y_1 = 1$; $y_2 = 1$; $y_3 = 1$

$$W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y'_1 & y'_2 & y'_3 \\ y''_1 & y''_2 & y''_3 \end{vmatrix} = \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & 2x \\ 0 & 0 & 2 \end{vmatrix}$$

$$= 2(1-0) = 2 \neq 0$$

for all real value of x.

$\therefore$ y_1, y_2, y_3 are linearly independent, for all real values of x.

Statement I and III are correct.

4. Correct answer is [0.028].

Three dice are thrown.

Total possible cases = 6 x 6 x 6 = 216

No. of favourable case with all the dice having same number *i.e.*, {(1, 1, 1) (2, 2, 2) (3, 3, 3) (4, 4, 4) (5, 5, 5) (6, 6, 6)} = 6

$$P = \frac{6}{216} = \frac{1}{36} \approx 0.028$$

5. Option (c) is correct.

Around $x = 0$

$$f(x) = e^{x+x^2};\ f(x) = e^x . e^{x^2}$$

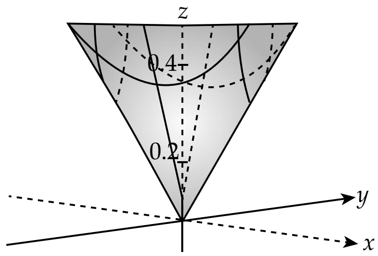
$$= \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\right)\left(1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \ldots\ldots\right)$$

$$= 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6} + \ldots\ x + x^3 + \frac{x^5}{2} + \frac{x^7}{6} + \ldots$$

$$+ \frac{x^2}{2} + \frac{x^4}{2} + \frac{x^6}{4} + \frac{x^8}{12} + \ldots \frac{x^3}{6} + \frac{x^5}{6} + \frac{x^7}{12} + \frac{x^9}{36} + \ldots$$

$$= 1 + x + \frac{3}{2}x^2 + \frac{7}{6}x^3$$

6. Correct answer is [0.79].



Given 3D region R $= x^2 + y^2 \leq z^3;\ 0 \leq z \leq 1$

$r = \sqrt{x^2+y^2}$ is the radius of variable circle at some z.

$(r)^2 = x^2 + y^2 = z^3$ (circle)

$\therefore$ volume of region R revolved around z–axis is

$$= \int_0^1 \pi (r)^2\, dz$$

$$= \int_0^1 \pi z^3 dz = \pi \left[\frac{z^4}{4}\right]_0^1 = \frac{\pi}{4} = 0.785$$

7. Correct answer is –11.

Given line segment C joining by points A (0, 2, 1) and B (4, 1, –1).

Equation of line segment is given by–

$$\frac{x-0}{4-0} = \frac{y-2}{1-2} = \frac{z-1}{-1-1} = t$$

$$x = 4t \Rightarrow dx = 4dt$$

$$y = -t + 2 \Rightarrow dy = -dt$$

$$z = -2t + 1 \Rightarrow dz = -2\,dt$$

$$\text{I} = \int_C (2zdx + 2ydy + 2xdz)$$

$$= \int_0^1 2(-2t+1)4dt + 2(-t+2)(-dt) + 2(4t)(-2dt)$$

$$= \int_0^1 -16tdt + 8dt + 2tdt - 4dt - 16tdt$$

$$= \int_0^1 (-30t+4)\, dt$$

$$= -30\left[\frac{t^2}{2}\right]_0^1 + 4[t]_0^1$$

$$\Rightarrow \quad \text{I} = -30\left(\frac{1}{2}\right) + 4(1) = -11$$

8. Option (d) is correct.

Given Differential equation

$$\frac{dy}{dx} = (x+y-1)^2 \quad \text{...(i)}$$

here we cannot directly apply variable separable form so putting $x+y-1=t$ and differentiating w.r.t. x we have.

$$\Rightarrow \quad 1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dt}{dx} - 1 \quad \text{...(ii)}$$

From (i) and (ii)

$$\frac{dt}{dx} - 1 = t^2$$

$$\Rightarrow \quad \frac{dt}{dx} = t^2 + 1$$

$$\Rightarrow \quad \frac{dt}{t^2+1} = dx$$

(variable separable form)

Integrating both sides

$$\Rightarrow \quad \int \frac{dt}{t^2+1} = \int dx + c$$

$$\Rightarrow \quad \tan^{-1}(t) = x + c$$

$$\Rightarrow \quad \tan^{-1}(x+y-1) = x + c$$

$$\Rightarrow \quad x + y - 1 = \tan(x+c)$$

$$\Rightarrow \quad y = 1 - x + \tan(x+c)$$

9. Correct answer is [0.68].

$$f(x) = x^3 + x - 1$$

$$f'(x) = 3x^2 + 1$$

$$f(1) = 1$$

$$f'(1) = 3(1)^2 + 1 = 4$$

$$x_0 = 1$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\Rightarrow \quad x_1 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{1}{4} = 1 - 0.25 = 0.75$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.75 - \frac{f(0.75)}{f'(0.75)}$$

$$= 0.75 - \frac{0.17187}{2.6865}$$

$$\Rightarrow \quad x_2 = 0.68$$

10. Option (d) is correct.

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_4 \to R_4 + R_1 + R_2 + R_3$

$$\Rightarrow \quad A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_5 \to R_4 + R_5$

$$\Rightarrow \quad A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

No. of non–zero rows = rank of matrix = 4

11. Option (a) is correct.

$AE \to D^2 + 2D - 5 = 0$, roots are $D = -1 \pm \sqrt{6}$

Therefore, the general solution is–

$$y = K_1 e^{(-1+\sqrt{6})x} + K_2 e^{(-1-\sqrt{6})x}$$

12. Correct answer is [54.73].

Given planes are $x + y + z = 1$ and $2x - y + 2z = 0$

For two planes $a_1 x + b_1 y + c_1 z = d_1$,
$a_2 x + b_2 y + c_2 z = d_2$

Angle is given by

$$\cos\theta = \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Angle between planes;

$$\cos\theta = \frac{|(1)(2) + (1)(-1) + (1)(2)|}{\sqrt{1^2 + 1^2 + 1^2}\sqrt{2^2 + (-1)^2 + (2)^2}}$$

$$\cos\theta = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.73^\circ$$

13. Option (b) is correct.

$$f(z) = \frac{1}{(z-4)(z+1)^3}$$

We have pole $z = 4$ of order 1, pole $z = -1$ of order 3

$$\operatorname*{Res}_{z=4} f(z) = \lim_{z\to 4}\left[(z-4)\cdot\frac{1}{(z-4)(z+1)^3}\right]$$

$$= \frac{1}{(4+1)^3} = \frac{1}{125}$$

$$\operatorname*{Res}_{z=-1} f(z) = \frac{1}{2!}\lim_{z\to -1}\frac{d^2}{dz^2}\left\{(z+1)^3\frac{1}{(z-4)(z+1)^3}\right\}$$

$$= \frac{1}{2}\lim_{z\to -1}\left\{\frac{2}{(z-4)^3}\right\} = \left\{\frac{1}{-125}\right\} = -\frac{1}{125}$$

14. Option (c) is correct.

Consider integral

$$I_1 = \int_0^1\int_0^1 \frac{x-y}{(x+y)^3}\,dy\,dx$$

$$= \int_0^1\int_0^1 \frac{x+x-x-y}{(x+y)^3}\,dy\,dx$$

[adding and subtracting x in numerator]

$$= \int_0^1\int_0^1 \frac{2x-(x+y)}{(x+y)^3}\,dy\,dx$$

$$= \int_0^1\left[\int_0^1\left(\frac{2x}{(x+y)^3} - \frac{1}{(x+y)^2}\right)dy\right]dx$$

$$= \int_0^1\left[\left(\frac{2x(x+y)^{-2}}{-2}\right)_0^1 - \left(\frac{(x+y)^{-1}}{-1}\right)_0^1\right]dx$$

$$= \int_0^1\left\{\left(\frac{-x}{(x+y)^2}\right)_0^1 + \left(\frac{1}{x+y}\right)_0^1\right\}$$

$$= \int_0^1\left[\left(\frac{-x}{(x+1)^2} + \frac{x}{(x+0)^2}\right) + \left(\frac{1}{x+1} - \frac{1}{x+0}\right)\right]dx$$

$$= \int_0^1\left(\frac{-x}{(x+1)^2} + \frac{1}{x} + \frac{1}{x+1} - \frac{1}{x}\right)dx$$

$$= \int_0^1\left(\frac{-x}{(x+1)^2} + \frac{1}{x+1}\right)dx$$

$$= \int_0^1 \left(\frac{-x+x+1}{(x+1)^2} \right) dx = \int_0^1 \frac{1}{(x+1)^2} dx = \left[\frac{-1}{x+1} \right]_0^1$$

$$= \left\{ \frac{-1}{1+1} - \frac{-1}{0+1} \right\} = \left\{ \frac{-1}{2} + 1 \right\} = \frac{1}{2} = 0.5$$

Now considering integral

$$I_2 = \int_0^1 \int_0^1 \left(\frac{x-y}{(x+y)^3} dx \right)$$

$$= \int_0^1 \int_0^1 \left(\frac{x+y-y-y}{(x+y)^3} dx \right) dy$$

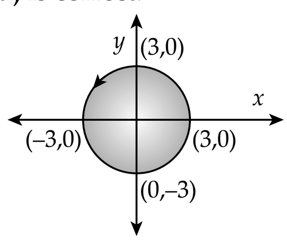
$$= \int_0^1 \int_0^1 \left(\frac{(x+y)-2y}{(x+y)^3} dx \right) dy$$

$$= \int_0^1 \left[\int_0^1 \left(\frac{1}{(x+y)^2} - \frac{2y}{(x+y)^3} \right) dx \right] dy$$

Similarly, $\int_0^1 \left(\frac{-1}{(x+y)} + \frac{2y}{2(x+y)} \right)_0^1 dy = \int_0^1 \frac{-1}{(1+y)^2} dy$

$$= -\left[\frac{-1}{(1+y)} \right]_0^1 \Bigg| = -0.5$$

15. Option (d) is correct.



Given Circle is defined as $|z| = 3$.

$$f(z) = \frac{z^2-1}{z^2+1} e^z = \frac{(z^2-1)e^z}{(z-i)(z+i)}$$

As $z = i, -i$ are the simple poles lies inside the circle.

$$\operatorname*{Res}_{z=i} f(z) = \frac{\left((i)^2-1\right)e^i}{(i+i)} = \frac{-2e^i}{2i} = ie^i$$

$$\operatorname*{Res}_{z=-i} f(z) = \frac{\left((-i)^2-1\right)e^i}{(-i-i)} = \frac{-2e^{-i}}{-2i} = \frac{e^{-i}}{i} = -ie^{-i}$$

By Cauchy's residues theorem.

$$I = 2\pi i(ie^i - ie^{-i})$$

$$= -2\pi(e^i - e^{-i})$$

$$= -2\pi(\cos 1 + i\sin 1 - \cos 1 + i \sin 1)$$

$$= -4\pi i \sin 1$$

16. Option (b) is correct.

$$\vec{F} = \hat{a}_x(3y - k_1 z) + \hat{a}_y(k_z x - 2z) - \hat{a}_z(k_3 y + z)$$

For irrotational field curl of vector = 0 or

$$\nabla \times \vec{F} = 0$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (3y-k_1z) & (k_2x-2z) & -(k_3y+z) \end{vmatrix} = 0$$

$$= \hat{a}_x \left[\frac{\partial}{\partial y}[-(k_3y+z)] - \frac{\partial}{\partial z}(k_2x-2z) \right] - \hat{a}_y$$

$$\left[\frac{\partial}{\partial x}[-(k_3y+z)] - \frac{\partial}{\partial z}(3y-k_1z) \right] + \hat{a}_z$$

$$\left[\frac{\partial}{\partial x}(k_2x-2z) - \frac{\partial}{\partial y}(3y-k_1z) \right] = 0$$

$$= \hat{a}_x(-k_3+2) - \hat{a}_y(k_1) + \hat{a}_z(k_2-3) = 0$$

$\therefore$ $-k_3 + 2 = 0 \Rightarrow k_3 = 2;$ $k_1 = 0;$

$k_2 - 3 = 0 \Rightarrow k_2 = 3$

$\therefore$ $k_1 = 0$, $k_2 = 3$ and $k_3 = 2$.

17. Correct answer is [2.5].

Let X be the No. of attempts to reserve the seat.

x	1	2	3	4	5	...
F(x)	$\frac{2}{5}$	$\frac{3}{5}.\frac{2}{5}$	$\left(\frac{3}{5}\right)^2 . \left(\frac{2}{5}\right)$	$\left(\frac{3}{5}\right)^3 . \left(\frac{2}{5}\right)$	$\left(\frac{3}{5}\right)^4 . \left(\frac{2}{5}\right)$	...

$p = 40\% = 2/5$

$q = 1 - p = 3/5$

$$E(x) = \sum x f(x)$$

$$\Rightarrow E(x) = 1 \times \frac{2}{5} + 2 \times \left(\frac{3}{5}\right).\left(\frac{2}{5}\right) + 3\left(\frac{3}{5}\right)^2.\left(\frac{2}{5}\right)$$

$$+4\left(\frac{3}{5}\right)^3.\left(\frac{2}{5}\right) + ...$$

$$= \frac{2}{5}\left\{ 1 + 2\left(\frac{3}{5}\right) + 3\left(\frac{3}{5}\right)^2 + ... \right\}$$

$$\left(\because \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + ... \right)$$

$$= \frac{2}{5}\left\{ 1 - \frac{3}{5} \right\}^{-2} = \frac{2}{5}\left\{ \frac{2}{5} \right\}^{-2} = \left\{ \frac{5}{2} \right\} = 2.5$$

18. Correct answer is [–100].

$$f(x)=\frac{1}{3}x\left(x^2-3\right)$$

$$\Rightarrow \quad f(x)=\frac{1}{3}\left(x^3-3x\right)$$

$$f'(x)=\frac{1}{3}\left(3x^2-3\right)=x^2-1$$

$$f'(x)=0$$

$x^2-1=0 \Rightarrow x=\pm 1$, critical points

Minimum value of $f(x)$ in the given interval $-100 \le x \le 100$·

$\text{Min}\{f(-100), f(100), f(1), f(-1)\}$

$\text{Min}\{-33323.3, 333233.3, -0.666, 0.666\}$

So, minimum value occurs at $x = -100$.

19. Option (c) is correct.

$$A=\begin{bmatrix} \frac{3}{2} & 0 & \frac{1}{2} \\ 0 & -1 & 0 \\ \frac{3}{2} & 0 & \frac{3}{2} \end{bmatrix}$$

is real symmetric matrix (*i.e.* $A^T=A$)

$\because$ A has three distinct eigen values.

$\therefore$ the corresponding eigen vectors are orthogonal vectors. *i.e.* dot product = 0.

$\because$ one eigen vector is $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

So, the dot product of $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and vector $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Let, $X=\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$; $Y=\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ from the options

$X^TY = (1)(1) + (0)(0) + (1)(-1) = 0$

20. Option (d) is correct.

$\lim\limits_{z\to i}\dfrac{z^2+1}{z^3+2z-i\left(z^2+2\right)}$ is $\dfrac{0}{0}$ form.

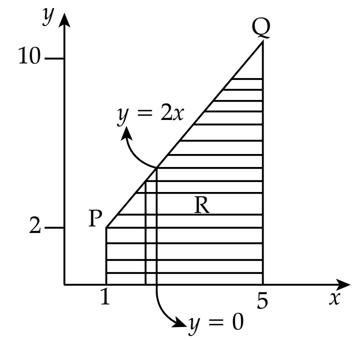
$\left[\because i^2=-1 \quad i^3=-i\right]$

Now applying L Hospital rule.

$$\lim_{z\to i}\frac{2z}{3z^2+2-i(2z)}$$

$$=\frac{2i}{3(i)^2+2-i(2i)}=\frac{2i}{-3+2+2}=2i$$

21. Correct answer is [1].



From figure coordinates of P (1, 2) and Q (5, 10).

Equation of straight–line PQ $\Rightarrow y = 2x$.

$\Rightarrow$ y limit is 0 to $2x$ and x limit is 1 to 5.

$$\therefore \quad I=c\iint_R xy^2\,dx\,dy$$

$$=c\int_{x=0}^{5}\left(\int_{y=0}^{2x} xy^2\,dy\right)dx$$

$$=c\int_1^5 x\left[\frac{y^3}{3}\right]_0^{2x}dx=\frac{8c}{3}\int_1^5 x^4\,dx=\frac{8c}{3}\left[\frac{x^5}{5}\right]_1^5$$

$$=\frac{8c}{15}[3125-1]=\frac{8c}{15}[3124]$$

Put $\quad c=6\times10^{-4}$

$\quad I=0.99968\simeq 1$

22. Option (b) is correct.

$$f(x)=\begin{cases} e^x & , x<1 \\ \ln x+ax^2+bx & , x\ge 1 \end{cases} \quad , x\in R$$

at $\qquad x = 1$

$$\text{LHD}=\lim_{x\to1^-}\frac{f(x)-f(1)}{x-1}$$

$$=\lim_{x\to1^-}\left\{\frac{e^x-(\ln 1+a+b)}{x-1}\right\}$$

$$=\lim_{x\to1}\frac{e^x-(a+b)}{x-1}$$

for limit to exist $e^x = a + b$ to take $\dfrac{0}{0}$ then 8y LH rule $\lim\limits_{x\to1}\dfrac{e^x}{1}=e$ ...(i)

$$\text{RHD}=\lim_{x\to1^+}\frac{f(x)-f(1)}{x-1}$$

$$=\lim_{x\to1^+}\left\{\frac{\left(\ln x+ax^2+bx\right)-(\ln 1+a+b)}{x-1}\right\}$$

$$i\left\{\frac{0}{0}\text{form}\right\}$$

By LH Rule $\lim_{x\to 1^+} \dfrac{\frac{1}{x}+2ax+b}{1} = 1+2a+b$ ...(ii)

For LHD = RHD $\Rightarrow$ $e = 1 + 2a + b$ ...(iii)

and if $f(x)$ is differentiable at $x = 1$, it must be continuous also

LHL $\lim_{x\to 1^-} f(x) = \lim_{x\to 1}\left(e^x\right) = e$ ...(iv)

RHL $\lim_{x\to 1^+} f(x) = \lim_{x\to 1}\left(\ln x + ax^2 + bx\right) = a+b$ ...(v)

for LHL = RHL $\Rightarrow e = a + b$ ...(vi)

using (vi) in (iii)

$\Rightarrow a = -1$

then in (vi)

$b = e + 1$

So, $f(x)$ is differentiable at $x = 1$ for unique value of a, b.

23. Option (a) is correct.

Given DE $\left(t^2-81\right)\dfrac{dy}{dt}+5ty = \sin t$

Dividing by $(t^2 - 81)$ we have

$$\frac{dy}{dt}+\underbrace{\left(\frac{5t}{\left(t^2-81\right)}\right)}_{P}y = \underbrace{\frac{\sin t}{\left(t^2-81\right)}}_{Q}$$

This is 1st order differential equation in y.

$$\text{IF} = e^{\int P dt} = e^{\int\left(\frac{5t}{(t^2-81)}\right)dt}$$

$$= e^{\frac{5}{2}\int\frac{2t}{(t^2-81)}dt} = e^{\frac{5}{2}\ln\left(t^2-81\right)}$$

$$= (t^2-81)^{5/2} \qquad \left[\because e^{\ln f(x)} = f(x)\right]$$

Therefore, the general solution is

$$y\left(t^2-81\right)^{\frac{5}{2}} = \int \frac{\sin t}{\left(t^2-81\right)}\times\left(t^2-81\right)^{\frac{5}{2}}dt + c$$

$$\Rightarrow y = \frac{\int \sin t \times \left(t^2-81\right)^{\frac{3}{2}}dt}{\left(t^2-81\right)^{\frac{5}{2}}} + \frac{c}{\left(t^2-81\right)^{\frac{5}{2}}}$$

$t^2-81 \neq 0 \Rightarrow t \neq \pm 9$, gives a unique solution.

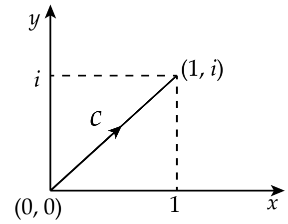
$t = -9, 9 \notin (-2, 2)$ as other intervals solution.

Other intervals (–10, 10) (–10, 2) (0, 10) contain t = –9 or t = 9 or both.

$\therefore$ When $t \in (-2, 2)$ there exists a unique solution.

24. Option (b) is correct.

Equation of straight line joining (0, 0) and (1, i) is



$y = x \Rightarrow dy = dx$

$\therefore \quad \text{I} = \int_{C}\left(x^2+iy^2\right)\left(dx+idy\right)$

$[\because z = x+iy]$

$$\text{I} = \int_{x=0}^{1}\left(x^2+ix^2\right)\left(dx+idx\right)$$

$$= \left(1+i\right)^2\int_0^1 x^2 dx$$

$$= \left(1-1+2i\right)\left(\frac{x^3}{3}\right)_0^1 = \frac{2i}{3}$$

25. Correct answer is [10].

$f(x) = x^6 - x - 1$

$\because$ $1 \le x \le 2$; here a = 1, b = 2, n is number of iterations.

$\in$ = 0.001 is an accuracy, n = ?

By bisection method

$$\frac{|b-a|}{2^n} \le \in$$

$\Rightarrow \quad 2^n \ge \dfrac{1}{0.001}$

$\Rightarrow \quad 2^n \ge 1000$

$\Rightarrow \quad n = 10$

$\therefore$ Minimum number of iterations are 10.

26. Option (a) is correct.

Total balls = 5 red balls +5 black balls = 10 balls.

$\therefore$ The probability to get a red ball in 2nd draw

P = Probability first ball is red and second ball is red + probability first ball is black and second ball is red

$$\text{P} = \frac{{}^5C_1}{{}^{10}C_1}\times\frac{{}^4C_1}{{}^9C_1}+\frac{{}^5C_1}{{}^{10}C_1}\times\frac{{}^5C_1}{{}^9C_1} = \frac{20+25}{90} = \frac{1}{2}$$

27. Option (c) is correct.

$\text{Div}\left(\vec{X}\right) \neq 0 \Rightarrow \nabla.\ \vec{X} \neq 0$; $\vec{X}$ is source vector.

$\vec{Y}, \vec{Z}$ are rotational vectors.

$\nabla \times \vec{Y} \neq 0$ and $\nabla \times \vec{Z} \neq 0$

28. Correct answer is [0.9].

Let X be a RV denote the arrival time of vehicle at junction.

$$\Rightarrow \quad X \cup \Big(\underset{a}{\underset{\downarrow}{0}}, \underset{b}{\underset{\downarrow}{5}}\Big)$$

$$\Rightarrow \quad \text{pdf of X is } f(x) = \begin{cases} \dfrac{1}{5-0}; & 0 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

Let Y be a RV denote the waiting time of vehicle at junction.

$$\Rightarrow \quad Y = \begin{cases} 0 & x \le 2 \\ 5-x & 2 < x \le 5 \end{cases}$$

Y is a continuous RV function of x.

$\therefore$ Expected waiting time

$$= E(Y) = \int_x y f(x)\,dx$$

$$= \int_0^2 (0)\frac{1}{5} + \int_2^5 (5-x)\frac{1}{5}dx$$

$$= 0 + \frac{1}{5}\left(5x - \frac{x^2}{2}\right)_2^5$$

$$= \frac{1}{5}\left[\left(25 - \frac{25}{2}\right) - (10-2)\right]$$

$$= \frac{1}{5}\left(\frac{9}{2}\right) = 0.9$$

29. Correct answer is [40].

$$f = \left(x^2 + \underbrace{y^2 - 2z^2}_{\text{constant}}\right)\underbrace{\left(y^2 + z^2\right)}_{\text{constant}}$$

$$\left(\frac{\partial f}{\partial x}\right) = \left(y^2 + z^2\right)(2x + 0)$$

$$\left(\frac{\partial f}{\partial x}\right)_{x=2, y=1, z=3} = (1+9)(4) = 40$$

30. Correct answer is [7].

$$2x^2 + y^2 = 34 \qquad \text{...(i)}$$

$$x + 2y = 11 \qquad \text{...(ii)}$$

From (ii)

$$y = \frac{11-x}{2} \text{ putting in (i)} \qquad \text{...(iii)}$$

$$2x^2 + \left(\frac{11-x}{2}\right)^2 = 34$$

$$\Rightarrow 8x^2 + 121 - 22x + x^2 = 136$$

$$\Rightarrow \quad 9x^2 - 22x - 15 = 0$$

$$\Rightarrow \quad 9x^2 - 27x + 5x - 15 = 0$$

$$\Rightarrow 9x(x-3) + 5(x-3) = 0$$

$$\Rightarrow \quad (x-3)(9x+5) = 0$$

$\therefore \quad x = 3$ [$\because$ x and y are integers.]

Putting x value in (iii)

$$y = \frac{11-3}{2} = \frac{8}{2} = 4$$

$$\therefore \quad x + y = 7$$

31. Correct answer is [5.732].

$$y^2 - 2y + 1 = x \qquad \text{...(i)}$$

$$\sqrt{x} + y = 5 \qquad \text{...(ii)}$$

From (i)

$$(y-1)^2 = x$$

$$\Rightarrow \quad y - 1 = \sqrt{x} \qquad \text{...(iii)}$$

$$\Rightarrow \quad y = \sqrt{x} + 1$$

Putting value from (iii) in (ii)

$$y - 1 + y = 5$$

$$\Rightarrow \quad 2y = 6$$

$$\therefore \quad y = 3$$

From (i) $\quad x = (3-1)^2 = 4$

$$\therefore \quad x + \sqrt{y} = 4 + \sqrt{3} = 5.732$$

32. Option (a) is correct.

$$g(x) = \begin{cases} -x, & x \le 1 \\ x+1, & x \ge 1 \end{cases} \text{ and } f(x) = \begin{cases} 1-x, & x \le 0 \\ x^2 & x > 0 \end{cases}$$

Clearly $g(x) = -x$ for $x \le 1$ and $f(x) = 1 - x$ for $x \le 0$ are polynomial function.

$\therefore$ $g(x)$ and $f(x)$ are continuous in the interval $(-\infty, 0)$.

$(f \circ g)(x) = f[g(x)]$ is also continuous in $(-\infty, 0)$,

[$\because$ composite function of two continuous function is also continuous.]

$\therefore$ the number of discontinuous of $(f \circ g)(x)$ present in the interval $(-\infty, 0)$ is 0.

33. Option (c) is correct.

C is $|z| = 3$ (Counter Clockwise)

$z - 2 = 0 \Rightarrow z = 2$ is a singular point lying inside C

$\therefore \oint \dfrac{z^3 - 2z + 3}{z-2}dz = 2\pi i\, f(z)$, by Cauchy's integral formula. ...(i)

where $z_0 = 2$ and $f(z) = z^3\, 2z + 3$

$$f(2) = 2^2 - 2(2) + 3 = 7$$

$$I = 2\pi i(7) = 14\pi i$$

34. Option (a) is correct.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix}$$

So, characteristics equation is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 0 & -3 & -4-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-\lambda)\{-\lambda(-4-\lambda)+3\} = 0$$

$\Rightarrow \lambda = 0$ and $\lambda^2 + 4\lambda + 3 = 0$

$$(\lambda+1)(\lambda+3) = 0$$

$$\lambda = 0, -1, -3$$

35. Option (c) is correct.

Let Y is discrete RV denote the number of head.

Y	0	1
P(Y)	$\frac{1}{8}$	$\frac{7}{8}$

$$E(Y) = \sum_{Y=0}^{1} YP(Y) = 0 \times \frac{1}{8} + 1 \times \frac{7}{8} = \frac{7}{8}$$

$$E(Y^2) = \sum_{Y=0}^{1} Y^2P(Y) = 0^2 \times \frac{1}{8} + 1^2 \times \frac{7}{8} = \frac{7}{8}$$

$$\therefore \quad Var(V) = E(Y^2) - [E(Y)]^2$$

$$= \frac{7}{8} - \frac{49}{64} = \frac{56-49}{64} = \frac{7}{64}$$

36. Option (b) is correct.

$$P = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{bmatrix}$$

$$|P| = 2(3-6) + 1(8-0) = -6 + 8 = 2$$

Product of eigen values = determinant of matrix = 2.

37. Option (d) is correct.

$$\lim_{x \to 0} \frac{x^3 - \sin(x)}{x} \to \frac{0}{0}$$

Applying L' Hospital Rule.

$$\lim_{x \to 0} \frac{3x^2 - \cos x}{1} = \frac{0-1}{1} = -1$$

38. Option (b) is correct.

$$u = f(x - cy)$$

Taking the partial differentiation of u w.r.t. x and then y.

$$\frac{\partial u}{\partial x} = f'(x-cy)(1)$$

$$\Rightarrow \quad \frac{\partial u}{\partial y} = f'(x-cy)(-c) = -cf'(x-cy) = -c\frac{\partial u}{\partial x}$$

$$\therefore \quad \frac{\partial u}{\partial y} + c\frac{\partial u}{\partial x} = 0$$

39. Option (a) is correct.

Given $\frac{d^2y}{dx^2} + 16y = 0$

$$\Rightarrow \quad (d^2 + 16)y = 0$$

$$AE \to m^2 + 16 = 0$$

$$\Rightarrow \quad m = \pm 4i$$

Solution is $y = c_1 \cos 4x + c_2 \sin 4x$

$$\frac{dy}{dx} = -4c_1 \sin 4x + 4c_2 \cos 4x$$

Given,

$$\left.\frac{dy}{dx}\right|_{x=0} = 1 \Rightarrow -4c_1 \sin 4(0) + 4c_2 \cos 4(0) = 1$$

$$\Rightarrow {}^4c_2 = 1 \qquad \left[\therefore \ c_2 = \frac{1}{4}\right]$$

And

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} = -1 \Rightarrow -4c_1 \sin 4\left(\frac{\pi}{2}\right) + 4c_2 \cos 4\left(\frac{\pi}{2}\right) = -1$$

$$\Rightarrow \quad -4c_1 \sin 2\pi + 4c_2 \cos 2\pi = -1$$

$$\Rightarrow \quad 0 + 4c_2 = -1$$

$$\Rightarrow \quad c_2 = -\frac{1}{4}$$

$\therefore$ no solution is existing for DE.

40. Correct answer is [3.5].

Probability of each outcome from (number 1 to 6) = $\frac{1}{6}$.

Probability distribution table

x	1	2	3	4	5	6
P (x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

So, mean =

$$E[x] = \sum x.P(x) = 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right)$$

$$= \left(\frac{1}{6}\right)(1+2+3+4+5+6) = \frac{21}{6} = 3.5$$

41. Correct answer is [10].

Distance travelled from t = 0 to 5 sec.

$$V = \frac{ds}{dt}$$

$$ds = V dt$$

$$S = \int_0^5 V dt$$

S = Area under V–T graph

$$= \left(\frac{1}{2}\times 1\times 1\right)+(1\times 1)+\frac{(1+4)}{2}\times 1+\frac{(4+2)}{2}\times 2 = 10$$

42. Option (d) is correct.

$$|P| = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}-0\right)+0+\frac{1}{\sqrt{2}}\left(0+\frac{1}{\sqrt{2}}\right) = 1$$

Also, P is orthogonal since $PP^T = I$

From here $P^T = \frac{I}{P} = P^{-1}$

Eigen values of an orthogonal matrix of magnitude one can be all real or be one real and a complex conjugate pair. Hence, (d) is incorrect.

43. Correct answer is [0].

According to vector identities $\nabla\circ\left(\nabla\times\vec{V}\right) = 0$.

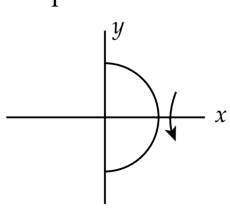
44. Option (c) is correct.

Given $x = \cos\left(\frac{\pi u}{2}\right)$ and $x = \cos\left(\frac{\pi u}{2}\right)$

Range is $0 \le u \le 1$

$0 \le x \le 1$, $0 \le y \le 1$

$x^2 + y^2 = 1 \rightarrow$ represents a circle.



When $0 \le \theta \le \frac{\pi}{2} \rightarrow$ it is quarter circle.

when rotated by 360 it forms a hemisphere

$\therefore$ area $= 2\pi(r)^2 = 2\pi(1)^2 = 2\pi$

45. Option (a) is correct.

x	0	0.5	1
y	3	4	5
	y_0	y_1	y_2

Using Trapezoidal rule

$$\int_0^1 f(x)dx = \frac{h}{2}\left[(y_0+y_2)+2y_1\right]$$

$$= \frac{0.5}{2}\left[(3+5)+2(4)\right] = 4$$

Using Simpsons rule

$$\int_0^1 f(x)dx = \frac{h}{3}\left[(y_0+y_2)+4y_1\right]$$

$$= \frac{0.5}{3}\left[(3+5)+4(4)\right] = 4$$

Difference of results = 0

46. Correct answer is [0.75].

For Sample space S

$n(s)$ or $|s| = 2^2 = 4$

$$s = \left\{\underbrace{\text{HH,HT,TH,}}_{\text{atleast one Head}}\ \text{TT}\right\}$$

Probability of getting at least one head = 1 – Prob. (all tails)

$= 1 - P_r(TT)$

$= 1 - \frac{1}{4}$

$= 0.75$

47. Correct answer is [0].

$$\vec{F} = \underbrace{-y}_{f_1}\hat{i} + \underbrace{x}_{f_2}\hat{j}$$

$\Rightarrow \quad \text{div}\left(\vec{F}\right) = \frac{\partial}{\partial x}(-y)+\frac{\partial}{\partial y}(x)$

$= 0 + 0 = 0$

48. Correct answer is [5].

Let λ_1 and λ_2 be two eigen values of given matrix.

$|A| = 50$; $\quad \lambda_1 = 10$; $\quad \lambda_2 = ?$

We know product of eigen values = determinant of matrix.

i.e. $\quad \lambda_1.\lambda_2 = 50$

$\Rightarrow \quad 10\lambda_2 = 50$

$\Rightarrow \quad \lambda_2 = 5$

49. Option (c) is correct.

$\because$ 17 occurs maximum times.

$\therefore$ mode is 17.

50. Option (b) is correct.

$f(t) = te^t$

$[f(t)] = L\left[te^t\right]$

$\Rightarrow \quad F(S) = \frac{1}{(s-1)^2}$ where F(s)= L {f (t)}

$$\left[\because L[t] = \frac{1}{S^2}\right]$$

[using first shifting property]

51. Correct answer is [226.2].

S is closed surface, $\vec{F}$ is 3D vector.

$\Rightarrow$ Gauss Divergence theorem.

$$\iint_S \vec{F}.\hat{n}ds = \iiint_V \text{div}.\vec{F}\, dv$$

$$= \iiint_V \left\{\frac{\partial}{\partial x}(x+y)+\frac{\partial}{\partial y}(x+z)+\frac{\partial}{\partial z}(y+z)\right\} dv$$

$$= \iiint_V (1+0+1)dv = 2\iiint dv = 2v$$

Where V is volume of sphere with radius =3.

$$\therefore\ V = 2.\frac{4}{3}\pi(r)^3 = \frac{8}{3}\pi(3)^3 = 72\pi = 226.19 = 226.2$$

52. Correct answer is [94.08].

DE is $\left(3D^2+27\right)y(x)=0;$

where $\frac{d}{dx} = D$

$$AE \rightarrow 3m^2+27=0$$

$$m^2 = -9 \Rightarrow m = \pm 3i$$

$$\alpha + i\beta \Rightarrow \alpha = 0; \beta = 3$$

$\therefore$ general solution is

$$y = CF = e^{0x}\left(c_1\cos 3x + c_2\sin 3x\right)$$

$$= c_1\cos 3x + c_2 \sin 3x \quad ...(i)$$

$$y'(x) = -3c_1\sin 3x + 3c_2\cos 3x \quad ...(ii)$$

Using $x = 0, y = 0$ in (i)

$$0 = c_1\cos 3(0) + c_2\sin 3(0) \Rightarrow c_1 = 0$$

Using $x = 0, y'(x) = 2000$

$$\Rightarrow \quad 2000 = -3c_1\sin 3(0) + 3c_2\cos 3(0)$$

$$\Rightarrow c_2 = \frac{2000}{3}$$

Put the value of c_1 and c_2 in (i)

$$y = (0)\cos 3x + \left(\frac{2000}{3}\right)\sin 3x$$

$$\Rightarrow y = \left(\frac{2000}{3}\right)\sin 3x$$

At $x = 1$, $y = \frac{2000}{3}.\sin(3) = 94.08$

53. Correct answer is [0].

$$X_1 = \begin{bmatrix} 70 \\ \lambda_1 - 50\end{bmatrix} \quad \text{and} \quad X_2 = \begin{bmatrix} \lambda_2 - 80 \\ 70\end{bmatrix}$$

$X^T{}_1.X_2$

$$= (70)(\lambda_2 - 80) + (\lambda_1 - 50)(70)$$

$$= (70)\{\lambda_2 - 80 + \lambda_1 - 50\}$$

$$= (70)\{\lambda_1 + \lambda_2 - 80 - 50\}$$

[$\lambda_1 + \lambda_2$ is trace of matrix A = sum of diagonal matrix *i. e.* = 50 + 80]

$$= (70)\{50 + 80 - 80 - 50\}$$

$$= 0$$

54. Option (b) is correct.

$f(z) = \left(\underbrace{x^2+ay^2}_{u}\right) + i\left(\underbrace{bxy}_{v}\right)$ is complex analytic function.

$\therefore$ Cauchy's Riemann equation is satisfied.

i.e. $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$2x = bx$ and $2ay = -by$

$\therefore$ $b = 2$ and $\Rightarrow 2a = -2 \Rightarrow a = -1$

55. Option (c) is correct.

Given $P = Q^{-1}$

Post multiplying by Q,

$$PQ = Q^{-1}Q$$

$$PQ = I$$

$$P = Q^{-1}$$

Pre multiplying by Q,

$$QP = QQ^{-1}$$

$$QP = I$$

From (i) and (ii)

$$PQ = I \quad \text{and} \quad QP = I$$

56. Option (b) is correct.

PDF of exponential distribution is given by relation.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise}\end{cases}$$

λ is the parameter.

For Gaussian distribution PDF is given by relation.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; \qquad -\infty < x < \infty$$

σ and μ are parameters.

So, for exponential distribution parameter is one and for Gaussian distribution parameter is 2.

57. Option (b) is correct.

$$f(x) = e^{-x-e^{-x}} = e^{-x}.e^{-e^{-x}}$$

$$g(x) = \int f(x)\,dx = \int e^{-x}.e^{-e^{-x}}\,dx$$

Putting $e^{-x} = t \Rightarrow -e^{-x}dx = dt$

$$\int f(x)\,dx = \int e^{-t}(-dt) = e^{-t} = e^{-e^{-x}}$$

58. Correct answer is [36].

Given partial differential equation is parabolic with A = 3 and C = 3.

then $\quad B^2 - 4\,AC = 0$

$\Rightarrow \quad B^2 - 4(3)(3) = 0$

$\Rightarrow \quad B^2 = 36$

59. Correct answer is [–1].

$\lim\limits_{x\to 0}\left(\dfrac{\tan x}{x^2 - x}\right) \to \dfrac{0}{0}$ form

Appling L' Hospital rule.

$$\lim_{x\to 0}\left[\frac{\sec^2 x}{2x-1}\right] = \frac{1}{-1} = -1$$

60. Correct answer is [0.1353].

Given distribution is Poisson distribution.

Where $\lambda = 900\,\text{veh/hr} = \dfrac{900}{3600} = \dfrac{1}{4}\text{veh/sec.}$

The probability of gap is greater 8 sec.

$$P[t \geq 8\,\text{sec}] = e^{-\lambda t} = e^{-\left(\frac{1}{4}\right)8} = e^{-2} = 0.1353$$

61. Option (b) is correct.

$f(x) = a + bx$

For a valid Probability density function

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$\Rightarrow \quad \displaystyle\int_0^1 (a+bx)dx = 1$

$\Rightarrow \quad \left[ax + \dfrac{bx^2}{2}\right]_0^1 = 1$

$\Rightarrow \quad a + \dfrac{b}{2} = 1$

$a = 0.5, b = 1$ satisfies above condition.

62. Option (d) is correct.

Given $\quad \dfrac{dQ}{dt} + Q = 1 \quad$ ...(i)

Comparing (i) with standard form of first order DE.

$$IF = e^{\int 1dt} = e^t$$

Solution is $\quad Q.e^t = \int 1.e^t dt + C$

$\Rightarrow \quad Q.e^t = e^t + C$

$$Q = 1 + Ce^{-t} \quad \text{...(ii)}$$

At $\quad t = 0,\ Q = 0$

$$0 = 1 + Ce^{-(0)} \Rightarrow C = -1$$

Putting value of C = –1

$$Q = 1 - e^{-t}$$

63. Option (a) is correct.

$$A = \begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix}$$

Characteristic equation

$$|A - \lambda I| = 0$$

$\Rightarrow \quad \begin{vmatrix} 5-\lambda & -1 \\ 4 & 1-\lambda \end{vmatrix} = 0$

$\Rightarrow \quad (5-\lambda)(1-\lambda) + 4 = 0$

$\Rightarrow \quad 5 - 5\lambda - \lambda + \lambda^2 + 4 = 0$

$\Rightarrow \quad \lambda^2 - 6\lambda + 9 = 0$

$\Rightarrow \quad \lambda = 3, 3$

Rank of $(A - \lambda I)$ for $\lambda = 3$

$$\begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix} = 0$$

So, rank = 1 Number of independent eigen vector = $n - r = 2 - 1 = 1$

$\therefore$ eigen value 3 has a multiplicity of 2 and only one independent vector exists.

64. Correct answer is [8].

$\dfrac{du}{dt} = 3t^2 + 1$ so $f(u,t) = 3t^2 + 1$

Given $u_0 = 0, t_0 = 0$ and $\Delta t = h$

Approximation value by Euler Method

$u_1 = u_0 + hf(u_0, t_0)$

$= 0 + 2\left(3t_0^2 + 1\right)$

$= 2\left(3(0)^2 + 1\right) = 2$

$u_1 = 2$

Exact value

$$du = \left(3t^2 + 1\right)dt$$

(separating the variables)

Integrating both sides.

$$u = t^3 + t + c \quad \text{...(i)}$$

Putting $u = 0, t = 0 \Rightarrow c = 0$

Putting value of c in (i)

$$u = t^3 + t$$

$$u(2) = (2)^3 + (2) = 10$$

$\therefore$ absolute error = exact value – approximate value

Absolute value = 10 – 2 = 8

65. Option (a) is correct.

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$

Characteristics equation

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & 2 \\ 4 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad (3-\lambda)(1-\lambda) - 8 = 0$$

$$\Rightarrow \quad 3 - 3\lambda - \lambda + \lambda^2 - 8 = 0$$

$$\Rightarrow \quad \lambda^2 - 4\lambda - 5 = 0$$

66. Option (c) is correct.

$w = f(x, y)$ here x and y are function of t and w is function of t alone.

$\therefore$ according to chain rule of differentiation.

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}$$

67. Correct answer is [134].

$$\vec{V} = x^2\hat{i} + 2y^3\hat{j} + z^4\hat{k}$$

$$f_1 = x^2; \qquad f_2 = 2y^3; \qquad f_3 = z^4$$

$$\nabla.\vec{V} = \frac{\partial}{\partial x}(f_1) + \frac{\partial}{\partial y}(f_2) + \frac{\partial}{\partial z}(f_3)$$

$$= 2x + 6y^2 + 4z^3$$

$$(\nabla.\vec{V})_{(1,2,3)} = 2(1) + 6(2)^2 + 4(3)^3 = 134$$

68. Correct answer is [0.5].

1st three tosses one after one, outcome is = H, H, H. As the outcome of fourth toss will be independent from the first three tosses.

$\therefore$ Probability $= \frac{1}{2} = 0.5$

69. Option (a) is correct.

Let P (x, y) be the coordinates of the tangent to the curve $y = x\ln x$ with inclination of 45° with x–axis.

Slope of the tangent $= \frac{dy}{dx} = \tan\theta$

$$y = x\ln x \qquad \text{...(i)}$$

$$\frac{dy}{dx} = x.\frac{1}{x} + \ln x$$

$$\frac{dy}{dx} = 1 + \ln x$$

$$1 + \ln x = \tan 45°$$

$$1 + \ln x = 1$$

$$\ln x = 0$$

$$x = 1$$

Putting value of x in (i) we have $y = 0$.

$\therefore$ Coordinates of P is (1, 0).

70. Option (a) is correct.

$$I = \int_0^1 \frac{(\sin^{-1}x)^2}{\sqrt{1-x^2}}dx$$

Put $\sin^{-1}x = t$

$$\frac{1}{\sqrt{1-x^2}}dx = dt$$

$$\because x = 0 \Rightarrow t = 0$$

$$x = 1 \Rightarrow t = \frac{\pi}{2}$$

$$I = \int_0^{\frac{\pi}{2}} t^2 dt = \left(\frac{t^3}{3}\right)_0^{\frac{\pi}{2}} = \frac{1}{3}\left(\frac{\pi^3}{8} - 0\right) = \frac{\pi^3}{24}$$

71. Option (a) is correct.

$$AB^T = \begin{bmatrix} 1 & 5 \\ 6 & 2 \end{bmatrix}\begin{bmatrix} 3 & 8 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 3+35 & 8+20 \\ 18+14 & 48+8 \end{bmatrix}$$

$$= \begin{bmatrix} 38 & 28 \\ 32 & 56 \end{bmatrix}$$

72. Option (a) is correct.

DE is $\Big(\underbrace{D^2 - 4D + 3}_{f(D)}\Big)y = 2t - 3t^2$

Particular solution is

$$= \frac{1}{D^2 - 4D + 3}(2t - 3t^2)$$

$$= \frac{1}{(D-1)(D-3)}(2t - 3t^2)$$

$$= \left(\frac{-1}{2}.\frac{1}{(D-1)} + \frac{1}{2}.\frac{1}{(D-3)}\right)(2t - 3t^2)$$

$$= \left(\frac{-1}{2}.\frac{1}{-(1-D)} + \frac{1}{2}.\frac{1}{(-3)\left(1-\frac{D}{3}\right)}\right)(2t - 3t^2)$$

$$= \left\{\frac{1}{2}.(1-D)^{-1} - \frac{1}{6}\left(1 - \frac{D}{3}\right)^{-1}\right\}(2t - 3t^2)$$

$$=\left\{\frac{1}{2}.\left(1+D+D^2.....\right)-\frac{1}{6}\left(1+\frac{D}{3}+\frac{D^2}{9}+....\right)\right\}\left(2t-3t^2\right)$$

$\because$ short method procedure for PI. Here $m=2$

$$=\left\{\frac{1}{3}+\frac{4}{9}D+\frac{13}{27}D^2.....\right\}\left(2t-3t^2\right)$$

$$=\left\{\left(\frac{2}{3}t-t^2\right)+\frac{4}{9}(2-6t)+\frac{13}{27}(-6)\right\}$$

$$=-2-2t-t^2$$

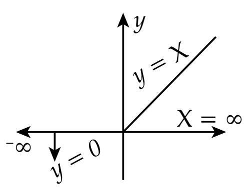
73. Option (b) is correct.

Increasing order of given functions is.

$$\frac{100}{n}<10<\log_2 n<\sqrt{n}<n$$

74. Correct answer is [0].

Given X is Random variable.



$E(X)=0$, $Var(X)=\sigma^2$

$$Y=\max(X,0)$$

$$\begin{cases} X \text{ if} & X\ge 0\\ 0 & X<0\end{cases}$$

Since median is positional average value, therefore median of Y = 0.

75. Option (c) is correct.

$c_1a_1+c_2a_2+c_3a_3+......c_na_n=0$

where $c_1, c_2, c_3.....c_n$ are scalars and $a_1, a_2, a_3........a_n$ are vectors.

Now if $c_1, c_2, c_3.....c_n$ are not all zero then the vectors $a_1, a_2, a_3........a_n$ are linearly dependent vectors.

Therefore $a_1, a_2, a_3.....a_n$ are linearly dependent column vectors.

Ax = b is the system of linear equations.

Where $A=[a_1, a_2, a_3..... a_n]$

There for rank of (A) < n or |A| = 0

$\because$ $b=\sum_{i=0}^{n}a_i=0$; b is also linearly dependent on $a_1, a_2, a_3.....a_n$.

$\rho(A)=\rho(C)<n;$

where C = [A|B] is augmented matrix.

Therefore, linear equations AX = b has infinitely many solutions.

76. Option (c) is correct.

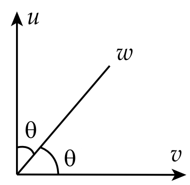
$$\lim_{x\to 1}\left[\frac{x^7-2x^5+1}{x^3-3x^2+2}\right]\to\frac{0}{0}\text{ form}$$

Using L' Hospital rule.

$$\lim_{x\to 1}\left[\frac{7x^6-10x^4}{3x^2-6x}\right]=\frac{7-10}{3-6}=1$$

77. Option (a) is correct.

Given, $u, v\in R^2$



Let $u=\begin{bmatrix}2\\0\end{bmatrix}$ and $v=\begin{bmatrix}0\\1\end{bmatrix}$ are two column vectors $\in R^2$.

$\|u\|=\sqrt{2^2+0^2}=2$ and $\|v\|=\sqrt{0^2+1^2}=1$

$\therefore$ $\|u\|=2\|v\|$

$w=u+\alpha v$

$$=\begin{bmatrix}2\\0\end{bmatrix}+\alpha\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}2\\0\end{bmatrix}+\begin{bmatrix}0\\\alpha\end{bmatrix}=\begin{bmatrix}2\\\alpha\end{bmatrix}$$

w bisects the angle between u and v.

from figure $\cos\left(\underbrace{u,w}_{\theta}\right)=\cos\left(\underbrace{w,v}_{\theta}\right)$

$$\frac{(2)(2)+(0)(\alpha)}{\sqrt{2^2+0^2}\sqrt{2^2+\alpha^2}}=\frac{(0)(2)+(1)(\alpha)}{\sqrt{0^2+1^2}\sqrt{2^2+\alpha^2}}$$

$\Rightarrow$ $\frac{4}{2}=\alpha$

$\Rightarrow$ $\alpha=2$

78. Option (b) is correct.

Given $\rho(A)=2$, $\sum_{i=1}^{n}\sum_{j=1}^{n}A^2_{ij}=50$ here A_{ij} is the element of the matrix.

If $\rho(A)=2<n\Rightarrow|A|=0\Rightarrow$ one eigen value is $0\in[-5,5]$, therefore statement I is correct.

Let n = 2 $\Rightarrow$ $A=\begin{bmatrix}5&0\\0&5\end{bmatrix}$, as A is a diagonal matrix.

$$\sum_{i=1}^{2}\sum_{j=1}^{2} A^2_{ij} = 5^2 + 0^2 + 5^2 + 0^2 = 50$$

And $\lambda = 5, 5$ are eigen values $\in [-5, 5]$. Magnitude of λ is 5 is not greater than 5.

Let $n = 3 \Rightarrow A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, is a diagonal matrix. $\rho(A) = 2$

$$\sum_{i=1}^{3}\sum_{j=1}^{3} A^2_{ij} = 5^2 + 0^2 + 0^2 + ... + (-5)^2 + ...0^2 = 50$$

$\lambda = 5, -5, 0$ are eigen values $\in [-5, 5]$ and magnitude of each eigen value is not greater than 5. Therefore, statement II is not correct.

79. Option (c) is correct.

$$f(x) = R\sin\left(\frac{\pi x}{2}\right) + S$$

$$f'(x) = R\frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)$$

At $\quad x = \frac{1}{2}$

$$f'\left(\frac{1}{2}\right) = R\frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right) = R\frac{\pi}{2}.\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sqrt{2} = R\frac{\pi}{2}.\frac{1}{\sqrt{2}}$$

$$\therefore \quad R = \frac{4}{\pi}$$

$$\int_0^1 f(x)dx = \frac{2R}{\pi}$$

$$\int_0^1 \left(R\sin\left(\frac{\pi x}{2}\right) + S\right) dx = \frac{2R}{\pi}$$

$$\left[-R\frac{\cos\left(\frac{\pi x}{2}\right)}{\left(\frac{\pi}{2}\right)}\right]_0^1 + S[x]_0^1 = \frac{2R}{\pi}$$

$$\Rightarrow \quad -\frac{2R}{\pi}\left[\cos\left(\frac{\pi}{2}\right) - \cos 0\right] + S = \frac{2R}{\pi}$$

$$\Rightarrow \quad \frac{2R}{\pi} + S = \frac{2R}{\pi}$$

$$\Rightarrow \quad S = 0$$

80. Correct answer is [2].

$$P + Q = \begin{bmatrix} 0 & -1 & -2 \\ 8 & 9 & 10 \\ 8 & 8 & 8 \end{bmatrix}$$

$$R_1 \leftrightarrow R_2 \sim \begin{bmatrix} 8 & 9 & 10 \\ 0 & -1 & -2 \\ 8 & 8 & 8 \end{bmatrix}$$

$$R_3 \to R_3 - R_1 \sim \begin{bmatrix} 8 & 9 & 10 \\ 0 & -1 & -2 \\ 0 & -1 & -2 \end{bmatrix}$$

$$R_3 \to R_3 - R_2 \sim \begin{bmatrix} 8 & 9 & 10 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore \quad$ Rank of $(P + Q) = 2$

81. Option (a) is correct.

Let A denotes P applies for job and B denotes Q applies for job. Where A and B are events.

$$P(A) = \frac{1}{4},$$

$$P\left(\frac{A}{B}\right) = \frac{1}{2} \qquad ...(i)$$

$$P\left(\frac{B}{A}\right) = \frac{1}{3} \qquad ...(ii)$$

$$P\left(\frac{A'}{B'}\right) = ?$$

Now from (ii)

$$\frac{P(B \cap A)}{P(A)} = \frac{1}{3} \Rightarrow \frac{P(B \cap A)}{\frac{1}{4}} = \frac{1}{3}$$

$$\Rightarrow P(B \cap A) = \frac{1}{12}$$

From (i)

$$\frac{P(A \cap B)}{P(B)} = \frac{1}{2} \Rightarrow \frac{\frac{1}{12}}{P(B)} = \frac{1}{2} \Rightarrow P(B) = \frac{1}{6}$$

$$P(A'|B') = \frac{P(A' \cap B')}{P(\vec{B})} \Rightarrow P(A'|B') = \frac{1 - P(A \cup B)}{1 - P(B)}$$

$$= \frac{1 - \{P(A) + P(B) - P(A \cap B)\}}{1 - P(B)} \qquad ...(iii)$$

[by Addition theorem of probability $A' \cap B' = (A \cup B)'$ and $P(A') = 1 - P(A)$]

By substituting all values in (iii).

$$= \frac{1 - \frac{1}{4} - \frac{1}{6} + \frac{1}{12}}{1 - \frac{1}{6}} = \frac{\frac{12 - 3 - 2 + 1}{12}}{\frac{5}{6}} = \frac{\frac{8}{12}}{\frac{5}{6}} = \frac{4}{5}$$

82. Option (b) is correct.

The Cumulative generating function

$$K_y(z) = \log_e\left(g_y(z)\right)$$
$$= \log_e(1-\beta+\beta z)^N$$
$$= N.\log_e(1-\beta+\beta z)$$
$$E(Y) = K_1 = \frac{d}{dz}\left(K_y(z)\right)_{z=1}$$
$$= \left(N \times \frac{1}{1-\beta+\beta z} \times \beta\right)_{z=1} = N\beta$$

83. Correct answer is [54].

X is Poisson Distribution with mean 5.

i. e. $\lambda = 5$ is Poisson parameter.

$$X \sim P(\lambda)$$
$$\therefore \quad E\left[(X+2)^2\right] = E\left[\left(X^2+4X+2\right)\right]$$
$$= E\left(X^2\right) + 4E(X) + 4 \qquad \text{...(i)}$$

Since mean = variance in Poisson distribution.

$$E(X) = V(X) = 5$$
$$\Rightarrow \quad E\left(X^2\right) - \left(E(X)\right)^2 = 5$$
$$\Rightarrow \quad E\left(X^2\right) - (5)^2 = 5$$
$$\Rightarrow \quad E\left(X^2\right) = 30 \text{ putting in (i)}$$
$$\therefore \quad E\left[(X+2)^2\right] = 30 + 20 + 4 = 54$$

84. Correct answer is [5].

Given one eigen value *i.e.* $\lambda = 2$ this value also satisfies the characteristics equation.

$$\therefore \quad \lambda^3 - 4\lambda^2 + a\lambda + 30 = 0 \qquad \text{...(i)}$$
$$\Rightarrow \quad (2)^3 - 4(2)^2 + a(2) + 30 = 0$$
$$\Rightarrow \quad 8 - 16 + 2a + 30 = 0$$
$$\Rightarrow \quad a = -11$$

From (i), $\lambda^3 - 4\lambda^2 - 11\lambda + 30 = 0$

$$\Rightarrow \quad (\lambda-2)\left(\lambda^2 - 2\lambda - 15\right) = 0$$
$$\Rightarrow \quad \lambda = 2, 5, -3$$

And the absolute largest eigen value of M is 5.