ENGINEERING MATHEMATICS GATE Question Paper 2018

ELECTRONICS AND COMMUNICATION (EC) P1

Q. 1. Consider $p(s) = s^3 + a_2 s^2 + a_1 s + a_0$ with all real coefficients. It is known that its derivative $p'(s)$ has no real roots. The number of real roots of $p(s)$ is

(a) 0 **(b)** 1
(c) 2 **(d)** 3

Q. 2. Let **M** be a real 4 × 4 matrix. Consider the following statements:

S1: M has 4 linearly independent eigenvectors.
S2: M has 4 distinct eigenvalues.
S3: M is non–singular (invertible).

Which one among the following is TRUE?

(a) S1 implies S2 **(b)** S1 implies S3
(c) S2 implies S1 **(d)** S3 implies S2

Q. 3. Let $f(x,y) = \frac{ax^2 + by^2}{xy}$, where a and b are constants. If $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$ at $x = 1$ and $y = 2$, then the relation between a and b is

(a) $a = \frac{b}{4}$ **(b)** $a = \frac{b}{2}$
(c) $a = 2b$ **(d)** $a = 4b$

Q. 4. Consider matrix $\mathbf{A} = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}$ and vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ The number of distinct real values of k for which the equation $Ax = 0$ has infinitely many solutions is ______.

Q. 5. Let X_1, X_2, X_3, and X_4, be independent normal random variables with zero mean and unit variance. The probability that X_4 is the smallest among the four is ______.

Q. 6. Taylor series expansion of $f(x) = \int_0^x e^{-\left(\frac{t^2}{2}\right)} dt$ around $x = 0$ has the form

$f(x) = a_0 + a_1 x + a_2 x^2 + \ldots$

The coefficient a_2 (correct to two decimal places) is equal to ______.

Q. 7. A curve passes through the point (x = 1, y = 0) and satisfies the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{2y} + \frac{y}{x}$. The equation that describes the curve is

(a) $\ln\left(1 + \frac{y^2}{x^2}\right) = x - 1$

(b) $\frac{1}{2}\ln\left(1 + \frac{y^2}{x^2}\right) = x - 1$

(c) $\ln\left(1 + \frac{y}{x}\right) = x - 1$

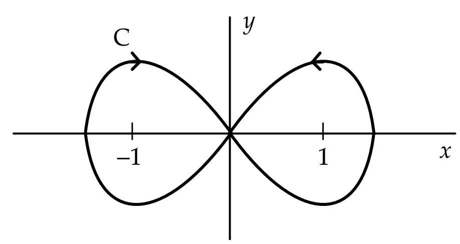
(d) $\frac{1}{2}\ln\left(1 + \frac{y}{x}\right) = x - 1$

Q. 8. The position of a particle $y(t)$ is described by the differential equation:

$$\frac{d^2y}{dt^2} = -\frac{dy}{dt} - \frac{5y}{4}.$$

The initial conditions are $y(0) = 1$ and $\left.\frac{dy}{dt}\right|_{t=0} = 0$. The position (accurate to two decimal places) of the particle at $t = \pi$ is ______.

Q. 9. The contour C given below is on the complex plane $z = x + iy$, where $i = \sqrt{-1}$.



The value of the integral $\frac{1}{\pi i}\oint_C \frac{dz}{z^2 - 1}$ is ______.

Q. 10. Let $r = x^2 + y - z$ and $z^3 - xy + yz + y^3 = 1$. Assume that x and y are independent variables. At $(x, y, z) = (2, -1, 1)$, the value (correct to two decimal places) of $\frac{\partial r}{\partial x}$ is ______.

ELECTRICAL ENGINEERING (EE) P1

Q. 11. Let f be a real–valued function of a real variable defined as $f(x) = x^2$ for $x \geq 0$, and $f(x) = -x^2$ for $x < 0$. Which one of the following statements is true?

(a) $f(x)$ is discontinuous at $x = 0$.

(b) $f(x)$ is continuous but not differentiable at $x = 0$.

(c) $f(x)$ is differentiable but its first derivative is not continuous at $x = 0$.

(d) $f(x)$ is differentiable but its first derivative is not differentiable at $x = 0$.

Q. 12. The value of the directional derivative of the function □ $(x, y, z) = xy^2 + yz^2 + zx^2$ at the point (2, –1, 1) in the direction of the vector $\vec{p} = \hat{i} + 2\hat{j} + 2\hat{k}$ is

(a) 1 **(b)** 0.95

(c) 0.93 **(d)** 0.9

Q. 13. The value of the integral $\oint_C \frac{z+1}{z^2-4}dz$ in counter clockwise direction around a circle C of radius 1 with center at the point $z = -2$ is

(a) $\frac{\pi i}{2}$ **(b)** 2□i

(c) $-\frac{\pi i}{2}$ **(d)** –2□i

Q. 14. Consider a non–singular 2 × 2 square matrix **A**. If trace **(A)** = 4 and trace $(\mathbf{A}^2)$ = 5, the determinant of the matrix **A** is _____(up to 1 decimal place).

Q. 15. Let f be a real–valued function of a real variable defined as $f(x) = x - [x]$, where $[x]$ denotes the largest integer less than or equal to x. The value of $\int_{0.25}^{1.25} f(x)dx$ is ______(up to 2 decimal places).

Q. 16. Consider a system governed by the following equations.

$$\frac{dx_1(t)}{dt} = x_2(t) - x_1(t)$$

$$\frac{dx_2(t)}{dt} = x_1(t) - x_2(t)$$

The initial conditions are such that $x_1(0) < x_2(0) < \infty$. Let $x_{1f} = \lim_{t\to\infty} x_1(t)$ and $x_{2f} = \lim_{t\to\infty} x_2(t)$. Which one of the following is true?

(a) $x_{1f} < x_{2f} < \infty$ **(b)** $x_{2f} < x_{1f} < \infty$

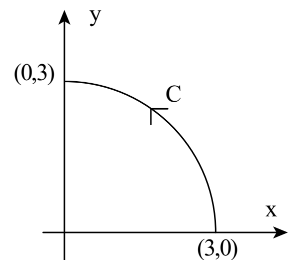
(c) $x_{1f} = x_{2f} < \infty$ **(d)** $x_{1f} = x_{2f} = \infty$

Q. 17. If C is a circle $|z| = 4$ and $f(z) = \frac{z^2}{(z^2-3z+2)^2}$, then $\oint_C f(z)dz$ is

(a) 1 **(b)** 0

(c) –1 **(d)** –2

Q. 18. As shown in the figure, C is the arc from the point (3, 0) to the point (0, 3) on the circle $x^2 + y^2 = 9$. The value of the integral $\int_C (y^2 + 2yx)dx + (2xy + x^2)dy$ is ______. (up to 2 decimal places).



Q. 19. Let $f(x) = 3x^3 - 7x^2 + 5x + 6$. The maximum value of $f(x)$ over the interval [0, 2] is ______. (up to 1 decimal place).

Q. 20. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ and $B = A^3 - A^2 - 4A + 5I$, where I is the 3 × 3 identity matrix. The determinant of B is ______. (up to 1 decimal place).

MECHANICAL ENGINEERING (ME) P1

Q. 21. Four red balls, four green balls and four blue balls are put in a box. Three balls are pulled out of the box at random one after another without replacement. The probability that all the three balls are red is

(a) $\frac{1}{72}$ **(b)** $\frac{1}{55}$

(c) $\frac{1}{36}$ **(d)** $\frac{1}{27}$

Q. 22. The rank of the matrix $\begin{bmatrix} -4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1 \end{bmatrix}$ is

(a) 1 **(b)** 2

(c) 3 **(d)** 4

Q. 23. According to the Mean Value Theorem, for a continuous function $f(x)$ in the interval $[a, b]$, there exist a value ξ in the interval such that

$$\int_a^b f(x)dx =$$

(a) $f(\xi)(b-a)$ **(b)** $f(b)(\xi-a)$

(c) $f(a)(b-\xi)$ **(d)** 0

Q. 24. F(z) is a function of the complex variable $z = x + iy$ given by $F(z) = iz + k\,\mathrm{Re}(z) + i\,\mathrm{Im}(z)$. For what value of k will F(z) satisfy the Cauchy-Riemann equations?

(a) 0 **(b)** 1

(c) –1 **(d)** y

Q. 25. A six–faced fair dice is rolled five times. The probability (in %) of obtaining "ONE" at least four times is

(a) 33.3 **(b)** 3.33

(c) 0.33 **(d)** 0.0033

Q. 26. Let X_1, X_2 be two independent normal random variables with means μ_1, μ_2 and standard deviations σ_1, σ_2, respectively. Consider $Y = X_1 - X_2$, $\mu_1 = \mu_2 = 1$, $\sigma_1 = 1$, $\sigma_2 = 2$. Then,

(a) Y is normally distributed with mean 0 and variance 1

(b) Y is normally distributed with mean 0 and variance 5

(c) Y has mean 0 and variance 5, but is NOT normally distributed

(d) Y has mean 0 and variance 1, but is NOT normally distributed

Q. 27. The value of the integral

$$\oiint_S \vec{r}\cdot\vec{n}\cdot dS$$

over the closed surface S bounding a volume V, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector and $\vec{n}$ is the normal to the surface S, is

(a) V **(b)** 2V

(c) 3V **(d)** 4V

Q. 28. An explicit forward Euler method is used to numerically integrate the differential equation

$$\frac{dy}{dt} = y$$

using a time step of 0.1. With the initial condition $y(0) = 1$, the value of $y(1)$ computed by this method is ______. (correct to two decimal places).

Q. 29. F(s) is the Laplace transform of the function $f(t) = 2t^2e^{-t}$ F(1) is ______ (correct to two decimal places).

MECHANICAL ENGINEERING (ME) P2

Q. 30. The Fourier cosine series for an even function $f(x)$ is given by

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx)$$

The value of the coefficient a_2 for the function $f(x) = \cos^2(x)$ in $[0, \pi]$ is

(a) –0.5 **(b)** 0.0

(c) 0.5 **(d)** 1.0

Q. 31. The divergence of the vector field $\bar{u} = e^x\left(\cos y\hat{i} + \sin y\hat{j}\right)$ is.

(a) 0 **(b)** $e^x\cos y + e^x\sin y$

(c) $2e^x\cos y$ **(d)** $2e^x\sin y$

Q. 32. Consider a function u which depends on position x and time t. The partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

is known as the

(a) Wave eqaution

(b) Heat equation

(c) Laplace's equation

(d) Elasticity equation

Q. 33. If y is the solution of the differential equation $y^3\frac{dy}{dx} + x^3 = 0, y(0) = 1$, the value of $y(-1)$ is

(a) –2 **(b)** –1

(c) 0 **(d)** 1

Q. 34. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$, then $\det(A^{-1})$ is ______ (correct to two decimal places).

Q. 35. Let z be a complex variable. For a counter-clockwise integration around a unit circle C , centred at origin,

$$\oint_C \frac{1}{5z-4}dz = A\pi i,$$

The value of A is

(a) $\frac{2}{5}$ **(b)** $\frac{1}{2}$

(c) 2 **(d)** $\frac{4}{5}$

Q. 36. Let X_1 and X_2 be two independent exponentially distributed random variables with means 0.5 and 0.25, respectively. Then $Y = \min(X_1, X_2)$ is

(a) exponentially distributed with mean $\frac{1}{6}$

(b) exponentially distributed with mean 2

(c) normally distributed with mean $\frac{3}{4}$

(d) normally distributed with mean $\frac{1}{6}$

Q. 37. For a position vector $\vec{r} = x\hat{i} + y\hat{j} + zk$ the norm of the vector can be defined as $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$ Given a function $\phi = \ln|\vec{r}|$, its gradient $\nabla\phi$ is

(a) $\vec{r}$ **(b)** $\frac{\vec{r}}{|\vec{r}|}$

(c) $\frac{\vec{r}}{\vec{r}.\vec{r}}$ **(d)** $\frac{\vec{r}}{|\vec{r}|^3}$

Q. 38. Given the ordinary differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

With $y(0) = 0$ and $\frac{dy}{dx}(0) = 1$, the value of $y(1)$ is _____ (correct to two decimal places).

CIVIL ENGINEERING (CE) P1

Q. 39. Which one of the following matrices is singular?

(a) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ **(b)** $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ **(d)** $\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$

Q. 40. For the given orthogonal matrix Q,

$$Q = \begin{bmatrix} \frac{3}{7} & \frac{2}{7} & \frac{6}{7} \\ -\frac{6}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{6}{7} & -\frac{3}{7} \end{bmatrix}$$

The inverse is

(a) $\begin{bmatrix} \frac{3}{7} & \frac{2}{7} & \frac{6}{7} \\ -\frac{6}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{6}{7} & -\frac{3}{7} \end{bmatrix}$ **(b)** $\begin{bmatrix} -\frac{3}{7} & -\frac{2}{7} & -\frac{6}{7} \\ \frac{6}{7} & -\frac{3}{7} & -\frac{2}{7} \\ -\frac{2}{7} & -\frac{6}{7} & \frac{3}{7} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{3}{7} & -\frac{6}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} & \frac{6}{7} \\ \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \end{bmatrix}$ **(d)** $\begin{bmatrix} -\frac{3}{7} & \frac{6}{7} & -\frac{2}{7} \\ -\frac{2}{7} & -\frac{3}{7} & -\frac{6}{7} \\ -\frac{6}{7} & -\frac{2}{7} & \frac{3}{7} \end{bmatrix}$

Q. 41. At the point $x = 0$, the function $f(x) = x^3$ has

(a) local maximum

(b) local minimum

(c) both local maximum and minimum

(d) neither local maximum nor local minimum

Q. 42. The value of the integral $\int_0^{\pi} x\cos^2 x\,dx$ is

(a) $\frac{\pi^2}{8}$ **(b)** $\frac{\pi^2}{4}$

(c) $\frac{\pi^2}{2}$ **(d)** π^2

Q. 43. The solution at $x = 1$, $t = 1$ of the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 25\frac{\partial^2 u}{\partial t^2}$ subject to initial conditions of $u(0) = 3x$ and $\frac{\partial u}{\partial t}(0) = 3$ is

(a) 1 **(b)** 2

(c) 4 **(d)** 6

Q. 44. The solution (up to three decimal places) at $x = 1$ of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ subject to boundary conditions $y(0) = 1$ and $\frac{dy}{dx}(0) = -1$ is ______.

Q. 45. Variation of water depth (y) in a gradually varied open channel flow is given by the first order differential equation

$$\frac{dy}{dx} = \frac{1 - e^{-\frac{10}{3}\ln(y)}}{250 - 45e^{-3\ln(y)}}$$

Given initial condition: $y\ (x = 0) = 0.8$ m. The depth (in m, up to three decimal places) of flow at a downstream section at $x = 1$ m from one calculation step of Single Step Euler Method is ______.

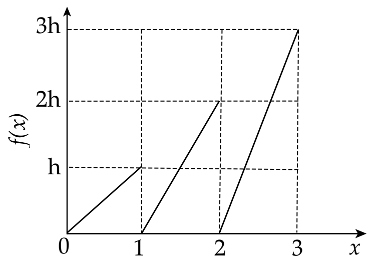
CIVIL ENGINEERING (CE) P2

Q. 46. The solution of the equation $x\frac{dy}{dx} + y = 0$ passing though the point (1, 1) is

(a) x **(b)** x^2

(c) x^{-1} **(d)** x^{-2}

Q. 47. The graph of a function $f(x)$ is shown in the figure.

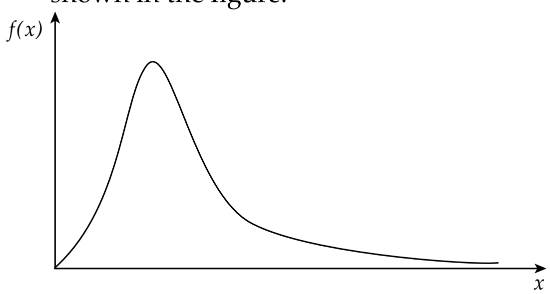


For $f(x)$ to be a valid probability density function, the value of h is

(a) $\frac{1}{3}$ **(b)** $\frac{2}{3}$

(c) 1 **(d)** 3

Q. 48. A probability distribution with right skew is shown in the figure.



The correct statement for the probability distribution is

(a) Mean is equal to mode

(b) Mean is greater than median but less than mode

(c) Mean is greater than median and mode

(d) Mode is greater than median

Q. 49. Probability (up to one decimal place) of consecutively picking 3 red balls without replacement from a box containing 5 red balls and 1 white ball is ______.

Q. 50. The quadratic equation $2x^2 - 3x + 3 = 0$ is to be solved numerically starting with an initial guess as $x_0 = 2$. The new estimate of x after the first iteration using Newton-Raphson method is ______.

Q. 51. A culvert is designed for a flood frequency of 100 years and a useful life of 20 years. The risk involved in the design of the culvert (in percentage, up to two decimal places) is ______.

Q. 52. The matrix $\begin{pmatrix} 2 & -4 \\ 4 & -2 \end{pmatrix}$ has

(a) real eigenvalues and eigenvectors

(b) real eigenvalues but complex eigenvectors

(c) complex eigenvalues but real eigenvectors

(d) complex eigenvalues and eigenvectors

Q. 53. The Laplace transform F(s) of the exponential function, $f(t) = e^{at}$ when $t \geq 0$, where a is a constant and $(s - a) > 0$, is

(a) $\frac{1}{s+a}$ **(b)** $\frac{1}{s-a}$

(c) $\frac{1}{a-s}$ **(d)** ∞

Q. 54. The rank of the following matrix is

$$\begin{bmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{bmatrix}$$

(a) 1 **(b)** 2

(c) 3 **(d)** 4

Q. 55. The value (up to two decimal places) of a line integral $\int_c \vec{F}(\vec{r}) \cdot d\vec{r}$, for $\vec{F}(\vec{r}) = x^2\hat{i} + y^2\hat{j}$ along C which is a straight line joining (0, 0) to (1, 1) is ______.

COMPUTER SCIENCE (CS) P1

Q. 56. Which one of the following is a closed form expression for the generating function of the sequence $\{a_n\}$, where $a_n = 2n + 3$ for all $n = 0, 1, 2, \ldots$?

(a) $\frac{3}{(1-x)^2}$ **(b)** $\frac{3x}{(1-x)^2}$

(c) $\frac{2-x}{(1-x)^2}$ **(d)** $\frac{3-x}{(1-x)^2}$

Q. 57. Two people, P and Q, decide to independently roll two identical dice, each with 6 faces, numbered 1 to 6. The person with the lower number wins. In case of a tie, they roll the dice repeatedly until there is no tie. Define a trial as a throw of the dice by P and Q. Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. The probability (rounded to 3 decimal places) that one of them wins on the third trial is ______.

Q. 58. The value of $\int_0^{\pi/4} x\cos(x^2)dx$ correct to three decimal places (assuming that $\pi = 3.14$) is ______.

Q. 59. Consider a matrix $A = uv^T$ where $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Note v^T denotes the transpose of v. The largest eigenvalue of A is ______.

Q. 60. Let G be a finite group on 84 elements. The size of a largest possible proper subgroup of G is ______.

Q. 61. Consider a matrix **P** whose only eigenvectors are the multiples of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Consider the following statements.

(I) **P** does not have an inverse

(II) **P** has a repeated eigenvalue

(III) **P** cannot be diagonalized

Which one of the following options is correct?

(a) Only I and III are necessarily true

(b) Only II is necessarily true

(c) Only I and II are necessarily true

(d) Only II and III are necessarily true

Q. 62. Let N be the set of natural numbers. Consider the following sets.

P: Set of Rational numbers (positive and negative)

Q: Set of functions from {0, 1} to N

R: Set of functions from N to {0, 1}

S: Set of finite subsets of N.

Which of the sets above are countable?

(a) Q and S only **(b)** P and S only

(c) P and R only **(d)** P, Q and S only

Q. 63. Consider the first–order logic sentence

$\varphi \equiv \exists s \exists t\ \exists u \forall v \forall w \forall x \forall y\ \psi(s, t, u, v, w, x, y)$

Where ψ (s, t, u, v, w, x, y) is a quantifier-free first–order logic formula using only predicate symbols, and possibly equality, but no function symbols. Suppose φ has a model with a universe containing 7 elements.

Which one of the following statements is necessarily true?

(a) There exists at least one model of ϕ with universe of size less than or equal to 3.

(b) There exists no model of ϕ with universe of size less than or equal to 3.

(c) There exists no model of ϕ with universe of size greater than 7.

(d) Every model of ϕ has a universe of size equal to 7.

CHEMICAL ENGINEERING (CH) P1

Q. 64. Consider the following two equations:

$$\frac{dx}{dt} + x + y = 0$$

$$\frac{dy}{dt} - x = 0$$

The above set of equations is represented by

(a) $\frac{d^2y}{dt^2} - \frac{dy}{dt} - y = 0$ **(b)** $\frac{d^2y}{dt^2} - \frac{dx}{dt} - y = 0$

(c) $\frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 0$ **(d)** $\frac{d^2x}{dt^2} + \frac{dx}{dt} + y = 0$

Q. 65. The fourth order Runge-Kutta (RK4) method to solve an ordinary differential equation $\frac{dy}{dx} = f(x, y)$ is given as

$$y(x+h) = y(x) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x, y)$$

$$k_2 = hf\left(x + \frac{h}{2}, y + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x + \frac{h}{2}, y + \frac{k_2}{2}\right)$$

$$k_4 = hf(x + h, y + k_3)$$

For a special case when the function f depends solely on x, the above RK4 method reduces. To

(a) Euler's explicit method

(b) Trapezoidal rule

(c) Euler's implicit method

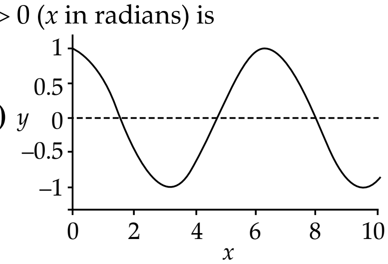
(d) Simpson's 1/3 rule

Q. 66. A watch uses two electronic circuits (ECs). Each EC has a failure probability of 0.1 in one year of operation. Both ECs are required for functioning of the watch. The probability of the watch functioning for one year without failure is

(a) 0.99 **(b)** 0.90

(c) 0.81 **(d)** 0.80

Q. 67. The figure which represents $y = \frac{\sin x}{x}$ for $x > 0$ (x in radians) is

(a)


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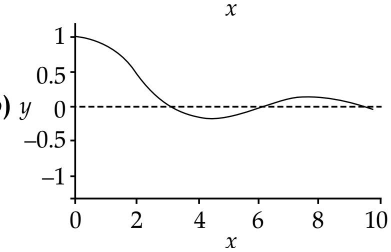
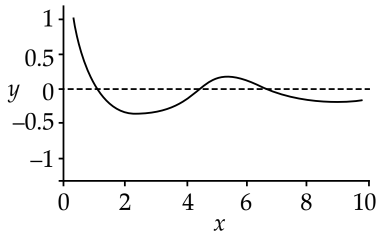
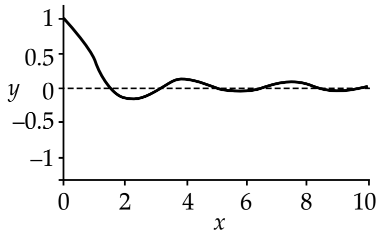


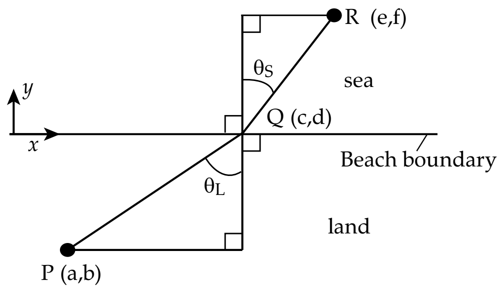
(c) 

(d) 

Q. 68. The initial water level in a tank is 4 m. When the valve located at the bottom is opened, the rate of change of water level (L) with respect to time (t) is, $\frac{dL}{dt} = -k\sqrt{t}$, where $k = 0.6\ \text{ms}^{-3/2}$.

The level of water (in m) in the tank at time 0.5 s after opening the valve is ______. (rounded off to second decimal place.)

Q. 69. A person is drowning in sea at location R and the lifeguard is standing at location P. The beach boundary is straight and horizontal, as shown in the figure.



The lifeguard runs at a speed of V_L and swims at a speed of V_S. In order to reach to the drowning person in optimum time, the lifeguard should choose point Q such that

(a) $\frac{\sin^2\theta_L}{\sin^2\theta_S} = \frac{V_S}{V_L}$ **(b)** $\frac{\sin\theta_L}{\sin\theta_S} = \frac{V_S}{V_L}$

(c) $\frac{\sin^2\theta_L}{\sin^2\theta_S} = \frac{V_L}{V_S}$ **(d)** $\frac{\sin\theta_L}{\sin\theta_S} = \frac{V_L}{V_S}$

Q. 70. If $y = e^{-x^2}$, then the value of $\lim_{x\to\infty} \frac{1}{x}\frac{dy}{dx}$ is ______.

Q. 71. For the matrix A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ if **det** stands for the determinant and A^T is the transpose of A then the value of **det**(A^T A) is ______.

Answer Key			
Q. No.	**Answer**	**Topic Name**	**Chapter Name**
1	(b)	Functions	Calculus
2	(c)	Eigen Values	Linear Algebra
3	(d)	Partial Derivative	Calculus
4	2	System of Linear Equations	Linear Algebra
5	0.25	Normal Distributions	Probability and Statistics
6	0	Taylor Series	Calculus
7	(a)	Differential Equation 1st Order	Differential Equation
8	–0.21	Differential Equation 2nd Order	Differential Equation
9	2	Contour Integral	Complex Analysis
10	4.5	Partial Derivative	Calculus
11	(d)	Continuity and Differentiability	Calculus
12	(a)	Directional Derivative	Vector Calculus
13	(a)	Cauchy's Integral	Complex Variables
14	5.5	Determinant	Linear Algebra

Q. No.	Answer	Topic Name	Chapter Name
15	0.50	Definite Integral	Calculus
16	(c)	Application of Derivatives	Calculus
17	(b)	Residue Theorem	Complex Variables
18	0	Line Integral	Calculus
19	12	Maximum / Minimum	Calculus
20	1	Cayley Hamilton Theorem	Linear Algebra
21	(b)	Probability	Probability and Statistics
22	(b)	Rank	Linear Algebra
23	(a)	Mean Values Theorem	Calculus
24	(b)	CR Equation	Complex Variable
25	(c)	Probability	Probability and Statistics
26	(b)	Normal Distributions	Probability and Statistics
27	(c)	Gauss Divergence Theorem	Vector Calculus
28	2.59	Euler Method For ODE	Numerical Methods
29	0.50	Laplace Transform	Transform Theory
30	(c)	Fourier Series	Calculus
31	(c)	Divergence	Vector Calculus
32	(b)	PDE	Differential Equations
33	(c)	1st Order DE Solution	Differential Equation
34	0.25	Determinant	Linear Algebra
35	(a)	Cauchy's Integral Formula	Complex Variables
36	(a)	Exponential Distribution	Probability and Statistics
37	(c)	Gradient	Vector Calculus
38	1.46	2nd Order DE Solution	Differential Equation
39	(c)	Singular Matrix	Linear Algebra
40	(c)	Inverse of Matrix	Linear Algebra
41	(d)	Maxima, Minima	Calculus
42	(b)	Definite Integral	Calculus
43	(d)	Partial Differentiation	Partial Differential Equations
44	0.36	Euler Method For ODE	Numerical Methods
45	0.79	Solution of 1st Order DE	Differential Equation
46	(c)	1st Order DE Solution	Differential Equation
47	(a)	PDF	Statistics and Probability
48	(c)	Skewness	Statistics and Probability

Q. No.	Answer	Topic Name	Chapter Name
49	0.5	Probability	Statistics and Probability
50	1	Newton Raphson Methods	Numerical Methods
51	18.20%	Probability	Statistics and Probability
52	(d)	Eigen Values and Eigen Vectors	Linear Algebra
53	(b)	Laplace Transform	Transform Theory
54	(b)	Rank of Matrix	Linear Algebra
55	0.67	Line Integral	Vector Calculus
56	(d)	Combinators	Discrete Mathematics
56	0.023	Probability	Statistics and Probability
58	0.28	Definite Integral	Calculus
59	3	Eigen Values	Linear Algebra
60	42	Sub Group	Discrete Mathematics
61	(d)	Eigen Values	Linear Algebra
62	(d)	Sets	Discrete Mathematics
63	(a)	1st Order Logic	Discrete Mathematics
64	(c)	Differential Equation	Differential Equation
65	(d)	Runge-Kutta Method	Numerical Methods
66	(c)	Probability	Statistics and Probability
67	(b)	Functions	Calculus
68	3.85	1st Order Equation	Differential Equation
69	(d)	Application of Derivatives	Calculus
70	0	1st Order DE Solution	Differential Equation
71	1	Determinant	Linear Algebra

ANSWERS WITH EXPLANATIONS

1. Option (b) is correct.

Given, $p(s)$ has all coefficients real.

"If a polynomial has r real roots then its derivative has at least $r-1$ real roots."

Here it is given that derivative has no real roots it means polynomial has 1 real root.

2. Option (c) is correct.

For M to be real 4 × 4 matrix then eigen vector corresponding to 4 distinct eigen values are linearly independent.

∴ S2 implies S1.

3. Option (d) is correct.

Given, $f(x, y) = \dfrac{ax^2 + by^2}{xy}$

$$= a\left(\frac{x}{y}\right) + b\left(\frac{y}{x}\right)$$

$$\frac{\partial f}{\partial x} = \frac{a}{y} - b\frac{y}{x^2}$$

$$\left.\frac{\partial f}{\partial x}\right|_{(1,2)} = \frac{a}{2} - 2b \quad \text{...(i)}$$

$$\frac{\partial f}{\partial y} = -a\frac{x}{y^2} + \frac{b}{x}$$

$$\left.\frac{\partial f}{\partial y}\right|_{(1,2)} = -a\frac{1}{4} + b = \frac{-a}{4} + b \quad \text{...(ii)}$$

From (i) and (ii)

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$$

$$\Rightarrow \quad \frac{a}{2} - 2b = \frac{-a}{4} + b$$

$$\Rightarrow \quad \frac{a}{2} + \frac{a}{4} = 3b$$

$$\Rightarrow \quad 3\frac{a}{4} = 3b$$

$$\Rightarrow \quad a = 4b$$

4. Correct answer is [2].

Given, $A = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}$ $\quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

As AX = 0 has infinitely many solutions.

So, $\rho(A) < 2$ means $|A| = 0$

$$\Rightarrow \quad k^3 - 2k(k^2 - k) = 0$$

$$\Rightarrow \quad k^3 - 2k^3 + 2k^2 = 0$$

$$\Rightarrow \quad -k^3 + 2k^2 = 0$$

$$\Rightarrow \quad k^2(k-2) = 0$$

$$\Rightarrow \quad k = 0, 0, 2$$

∴ the number of distinct real values of k = 2. *i.e.* 0 and 2

5. Correct answer is [0.25].

Given, random variables has E[X] = 0 and variance = 1.

So, probability for X_4 is smallest

$$P = \frac{3!}{4!} = \frac{1}{4} = 0.25$$

6. Correct answer is [0].

Taylor series expansion of $f(x)$ around $x = 0$ is

$$f(x) = f(0) + (x-0)f'(0) + \frac{(x-0)^2}{2!} f''(0) + \frac{(x-0)^3}{3!} f'''(0) + \ldots$$

Coefficient $a_2 = \dfrac{f''(0)}{2!}$

For $\quad f(x) = \int_0^x e^{-\left(\frac{t^2}{2}\right)} dt$ then $f'(x) = e^{-\left(\frac{x^2}{2}\right)}$

$$f''(x) = -x \cdot e^{-x^2/2}$$

$$\therefore \quad f''(0) = -0 \cdot e^0 = 0$$

$$\therefore \quad a_2 = \frac{0}{2!} = 0$$

7. Option (a) is correct.

Given, $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{2y} + \dfrac{y}{x}$

$$\frac{dy}{dx} = \frac{x^2}{2y} + \frac{y}{2} + \frac{y}{x} \quad \text{...(i)}$$

Putting $\quad \dfrac{y}{x} = t$

$$\Rightarrow \quad y = xt$$

$$\frac{dy}{dx} = x\frac{dt}{dx} + t$$

Putting value of $\frac{dy}{dx}$ in (i) we have,

$$x\frac{dt}{dx}+t=\frac{x}{2t}+\frac{xt}{2}+t$$

$$\Rightarrow \quad x\frac{dt}{dx}=\frac{x}{2t}+\frac{xt}{2}$$

$$\Rightarrow \quad x\frac{dt}{dx}=x\left(\frac{1}{2t}+\frac{t}{2}\right)$$

$$\Rightarrow \quad \frac{dt}{dx}=\left(\frac{1}{2t}+\frac{t}{2}\right)$$

$$\Rightarrow \quad \frac{dt}{dx}=\frac{t^2+1}{2t}$$

$$\Rightarrow \quad \int\frac{2t}{t^2+1}dt=\int dx+c$$

On integration

$$\ln\left(t^2+1\right)=x+c$$

$\because$ we have $t=\frac{y}{x}$

$$\ln\left(\frac{y^2}{x^2}+1\right)=x+c \quad \text{...(ii)}$$

Now at $x = 1, y = 0$

$$\Rightarrow \quad \ln(0+1)=1+c$$

$$\Rightarrow \quad c=-1$$

Put value of c in (ii)

$$\ln\left(\frac{y^2}{x^2}+1\right)=x-1$$

$$\Rightarrow \quad \ln\left(1+\frac{y^2}{x^2}\right)=x-1$$

8. Correct answer is [–0.21].

Given, $\frac{d^2y}{dt^2}=-\frac{dy}{dt}-\frac{5y}{4}$

$$\frac{d^2y}{dt^2}+\frac{dy}{dt}+\frac{5y}{4}=0$$

By applying Laplace transform

$$S^2Y(s)-sy(0)-y'(0)+SY(s)-y(0)+\frac{5}{4}Y(s)=0$$

$$S^2Y(s)+-s(1)+SY(s)-1+\frac{5}{4}Y(s)=0$$

$$\Rightarrow Y(s)=\frac{s+1}{S^2+S+\frac{5}{4}}=\frac{s+1}{\left(S+\frac{1}{2}\right)^2+1}=\frac{s+\frac{1}{2}+\frac{1}{2}}{\left(S+\frac{1}{2}\right)^2+1}$$

$$=\frac{\left(s+\frac{1}{2}\right)}{\left(S+\frac{1}{2}\right)^2+1}+\frac{\frac{1}{2}}{\left(S+\frac{1}{2}\right)^2+1}$$

By applying inverse Laplace transform, we have

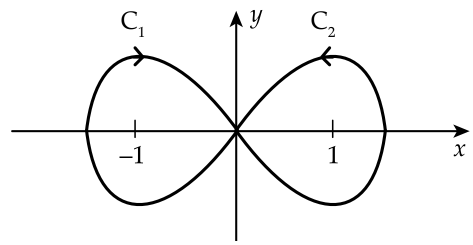
$$y(t)=e^{-\frac{t}{2}}\left[\cos(t)+\frac{1}{2}\sin(t)\right];\ t>0$$

The position of particle at $t = \pi$

$$y(\pi)=e^{-\frac{\pi}{2}}\left[\cos(\pi)+\frac{1}{2}\sin(\pi)\right]$$

$$\Rightarrow y(\pi)=e^{-\frac{\pi}{2}}[-1+0]=-e^{-\frac{\pi}{2}}=-0.2078\approx-0.21$$

9. Correct answer is [2].



For C_1 : singularity $z = -1$

For C_2 : singularity $z = 1$

$$\frac{1}{\pi i}\oint_C\frac{dz}{z^2-1}$$

$$=\frac{1}{\pi i}\oint_C\frac{dz}{(z-1)(z+1)}$$

$$=\frac{1}{\pi i}\left[-\oint_{C_1}\frac{dz}{(z+1)(z-1)}+\oint_{C_2}\frac{dz}{(z+1)(z-1)}\right]$$

$$=\frac{1}{\pi i}\left[-\oint_{C_2}\frac{\left(\frac{1}{z-1}\right)dz}{(z+1)}+\oint_{C_2}\frac{\left(\frac{1}{z+1}\right)dz}{(z-1)}\right]$$

By Cauchy's Integral formula

$$=\frac{1}{\pi i}\left[-2\pi i\left(\frac{1}{z-1}\right)\Big|_{z=-1}+2\pi i\left(\frac{1}{z+1}\right)\Big|_{z=1}\right]$$

$$=2\left[\frac{-1}{-1-1}+\frac{1}{1+1}\right]=2$$

10. Correct answer is [4.5].

By application of Partial Derivatives

For $r = x^2 + y - z$ ...(i)

$$\frac{\partial r}{\partial x}=2x-\frac{\partial z}{\partial x} \quad \text{...(ii)}$$

For $z^3-xy+yz+y^3+1$

$$3z^2\frac{dz}{dx}-y+y\frac{dz}{dx}+0=0$$

$$\Rightarrow \quad \frac{\partial z}{\partial x}\left(3z^2+y\right)=y$$

$$\Rightarrow \quad \frac{\partial z}{\partial x}=\frac{y}{\left(3z^2+y\right)}$$

Using $\frac{\partial z}{\partial x}$ in (ii)

$$\frac{\partial r}{\partial x} = 2x - \frac{y}{(3z^2 + y)}$$

$$\left[\frac{\partial r}{\partial x}\right]_{(2,-1,1)} = 2(2) - \frac{(-1)}{(3(1)^2 - 1)} = 4.5$$

11. Option (d) is correct.

Given, $f(x) = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$

$$f'(x) = \begin{cases} 2x & x \geq 0 \\ -2x & x < 0 \end{cases}$$

$$f''(x) = \begin{cases} 2 & x \geq 0 \\ -2 & x < 0 \end{cases}$$

LHD = RHD for $f(x)$ at $x = 0$ but

LHD ≠ RHD for $f'(x)$ at $x = 0$

This means $f(x)$ is differentiable at $x = 0$ but its first derivative is not differentiable at $x = 0$.

12. Option (a) is correct.

Given, $\phi(x, y, z) = xy^2 + yz^2 + zx^2$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$= (y^2 + 2xz)\hat{i} + (2xy + z^2)\hat{j} + (2yz + x^2)\hat{k}$$

$$(\nabla\phi)_{(2,-1,1)} = ((-1)^2 + 2(2)\cdot 1)\hat{i} + (2(2)(-1) + (1)^2)\hat{j} + (2(-1)(1) + (2)^2)\hat{k}$$

$$= 5\hat{i} - 3\hat{j} + 2\hat{k}$$

$$= \hat{i} + 2\hat{j} + 2\hat{k}$$

The D.D. of $\phi(x, y, z)$ at given point (2, –1, 1) in the direction of $\vec{p}$ is

$$(\nabla\phi)_{(2,-1,1)} \cdot \frac{\vec{p}}{|\vec{p}|}$$

$$= \frac{(5\hat{i} - 3\hat{j} + 2\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{1^2 + 2^2 + 2^2}}$$

$$= \frac{5 - 6 + 4}{3} = 1$$

13. Option (a) is correct.

Given, Radius = 1 unit; centre at $z = -2$, circle C

$$\oint_C \frac{z+1}{z^2-4} dz$$

Here $z^2 - 4 = 0$

$\Rightarrow$ $z = \pm 2$

$\Rightarrow$ $z = -2$ lies inside and $z = 2$ lies outside the circle C.

$\therefore$ By applying Cauchy's integral formula

$$\oint_C \frac{z+1}{z^2-4} dz = \oint_C \frac{z+1}{(z-2)(z+2)} dz = \oint_C \frac{\frac{z+1}{(z-2)}}{(z+2)} dz$$

$$= 2\pi \, if(-2) \quad \text{...(i)}$$

with $f(z) = \frac{z+1}{(z-2)}$

$$f(-2) = \frac{-2+1}{(-2-2)} = \frac{-1}{-4}$$

Putting $f(-2)$ in (i)

$$2\pi i\left(\frac{-1}{-4}\right) = \frac{\pi i}{2}$$

14. Correct answer is [5.5].

Given, A is non-singular 2 × 2 matrix.

For λ_1, λ_2 be eigen values of A

$\text{trace}(A) = 4$

$$\lambda_1 + \lambda_2 = 4 \quad \text{...(i)}$$

$$\text{trace}(A^2) = 5$$

$$\lambda^2_1 + \lambda^2_2 = 5 \quad \text{...(ii)}$$

Squaring both side of equation (i) we have

$\lambda^2_1 + \lambda^2_2 + 2\lambda_1\lambda_2 = 16$

Using equation (ii)

$\Rightarrow$ $5 + 2\lambda_1\lambda_2 = 16$

$\Rightarrow$ $\lambda_1\lambda_2 = \frac{11}{2}$

$\therefore$ $|A| = \frac{11}{2} = 5.5$

15. Correct answer is [0.50].

$f(x) = x - [x]$

$$\int_{0.25}^{1.25} (x - [x]) dx$$

$$= \int_{0.25}^{1} (x - [x]) dx + \int_{1}^{1.25} (x - [x]) dx$$

$$= \int_{0.25}^{1.25} (x) dx - \left[\int_{0.25}^{1} [x] dx + \int_{1}^{1.25} [x] dx\right]$$

$$\left[\because \begin{array}{l} [x] = 0, \ 0 \leq x < 1 \\ [x] = 1, \ 1 \leq x \leq 1.25 \end{array}\right]$$

$$= \left[\frac{x^2}{2}\right]_{0.25}^{1.25} - \left[\int_{0.25}^{1} 0 \, dx + \int_{1}^{1.25} 1 \, dx\right]$$

$$= \frac{(1.25)^2}{2} - \frac{(0.25)^2}{2} - [x]_1^{1.25}$$

$$= \frac{1}{2}[1.5625 - 0.0625] - [1.25 - 1]$$

$$= 0.75 - 0.25 = 0.50$$

16. Option (c) is correct.

Given, $x_1(0) < x_2(0) < \infty$

At $t = 0$

$\therefore \frac{dx_1(t)}{dt} = x_2(t) - x_1(t) > 0$ so $x_1(t)$ is increasing function

$\frac{dx_2(t)}{dt} = x_1(t) - x_2(t) < 0$ so $x_2(t)$ is decreasing function

At some point of t x_1 increases and x_2 decreases and both become equal. At that t

$$\frac{dx_1(t)}{dt} = \frac{dx_2(t)}{dt} = 0$$

from this t on wards $x_1(t) = x_2(t)$ and will be finite

$x_{1f} = x_{2f} < \infty$

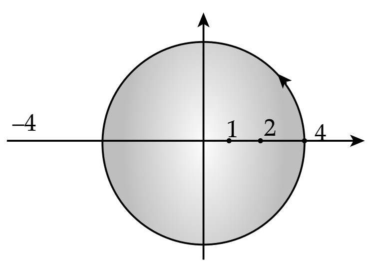
17. Option (b) is correct.

Given, $f(z) = \frac{z^2}{(z^2 - 3z + 2)^2}$

$C : |z| = 4$

$$I = \oint_C f(z)\,dz$$

$$= \oint_C \frac{z^2}{(z^2 - 3z + 2)^2}\,dz$$



$$= \oint_C \frac{z^2}{(z-1)^2 (z-2)^2}\,dz$$

$$\operatorname*{Res}_{z=1} f(z) = \lim_{z\to1} \frac{1}{1!}\frac{d}{dz}\left((z-1)^2 \cdot \frac{z^2}{(z-1)^2 (z-2)^2}\right)$$

$$= \lim_{z\to1}\left(\frac{(z-2)^2 .(2z) - 2z^2 (z-2)}{(z-2)^4}\right)$$

$$= \lim_{z\to1}\left(\frac{(z-2).(2z) - 2z^2}{(z-2)^3}\right) = \frac{-4}{-1} = 4$$

$$\operatorname*{Res}_{z=2} f(z) = \lim_{z\to2} \frac{1}{1!}\frac{d}{dz}\left((z-2)^2 \cdot \frac{z^2}{(z-1)^2 (z-2)^2}\right)$$

$$= \lim_{z\to2}\left(\frac{(z-1)^2 .(2z) - 2z^2 (z-1)}{(z-1)^4}\right)$$

$$= \lim_{z\to2}\left(\frac{(z-1).(2z) - 2z^2}{(z-1)^3}\right) = \frac{4-8}{1} = -4$$

$\therefore \quad I = 2\pi i\left[\operatorname*{Res}_{z=1} f(z) + \operatorname*{Res}_{z=2} f(z)\right]$

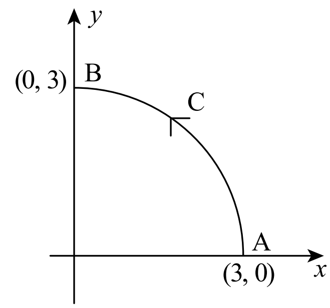
$\Rightarrow \quad I = 2\pi i(4 - 4) = 0$

18. Correct answer is [0].

Here, $I = \int_c (y^2 + 2yx)dx + (2xy + x^2)\,dy$

So, $\vec{F} = (y^2 + 2yx)\hat{i} + (2xy + z^2)\,j$

$= f_1\hat{i} + f_2 j$



Then $\vec{F}$ is a gradient function of potential function.

$f = x^2y + xy^2$ such $f_x = f_1$ and $f_y = f_2$

This makes $\vec{F}$ line integral path independent

and $\int_A^B \vec{F}\cdot d\vec{R} = f(B) - f(A)$

$$= \left[x^2y + xy^2\right]_{(3,0)}^{(0,3)}$$

$= (0 - 0) = 0$

19. Correct answer is [12].

Given, $f(x) = 3x^3 - 7x^2 + 5x + 6$

$f'(x) = 9x^2 - 14x + 5$

For $f'(x) = 0$

$\Rightarrow \quad 9x^2 - 14x + 5 = 0$

$\Rightarrow \quad 9x^2 - 9x - 5x + 5 = 0$

$\Rightarrow \quad (9x - 5)(x - 1) = 0$

$\therefore \quad x = 1, \frac{5}{9}$

$f(0) = 6$

$f(1) = 3 - 7 + 5 + 6 = 7$

$f\left(\frac{5}{9}\right) = 7.13$

$\Rightarrow \quad f(2) = 12$

So, maximum value of $f(x)$ is 12.

20. Correct answer is [1].

Given, $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & -1 \\ -1 & 2-\lambda & 0 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0$$

$\Rightarrow (1-\lambda)\{(2-\lambda)(-2-\lambda)-0\}-0-1(0-0)=0$

CE: $-\lambda^3 + \lambda^2 - 4\lambda - 4 = 0$

$\therefore \lambda^3 - \lambda^2 + 4\lambda + 4 = 0$

By Cayley Hamilton Theorem

$A^3 - A^2 + 4A + 4I = 0$

$\therefore A^3 - A^2 + 4A + 5I = I$

So, $|B| = |I| = 1$

21. Option (b) is correct.

The probability that all the three balls are red.

$$P = \frac{{}^4C_3}{{}^{12}C_3} = \frac{\frac{4}{1}}{\frac{12 \times 11 \times 10}{3!}} = \frac{4}{220} = \frac{1}{55}$$

22. Option (b) is correct.

Let $A = \begin{bmatrix} -4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1 \end{bmatrix}$

$R_1 \to R_1 - 4R_2$

$$R_3 \to R_3 + 7R_2 = \begin{bmatrix} 0 & 5 & 3 \\ -1 & -1 & -1 \\ 0 & -10 & -6 \end{bmatrix}$$

$$R_3 \to R_3 + 2R_1 = \begin{bmatrix} 0 & 5 & 3 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 5 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

And the number of non–zero row are two. So, rank of matrix is 2.

23. Option (a) is correct.

Given : $f(x)$ is continuous on $[a, b]$

We know,

If $F(x) = \int_a^x f(t)dt$ ...(i)

then, $F'(x) = f(x)$ ...(ii)

Using mean value theorem, we have

$$f'(\xi) = \frac{f(b) - f(a)}{b-a},$$

where $\xi \in (a, b)$ ...(iii)

from (i),

$$f(b) = \int_a^b f(t)\,dt,\ f(a) = \int_a^a f(t)dt = 0$$

from (iii),

$$f'(\xi) = \frac{\int_a^b f(t)\,dt - 0}{b-a}$$

$$\Rightarrow \quad f'(\xi) = \frac{\int_a^b f(t)\,dt}{b-a}$$

$$\Rightarrow \quad (b-a)\,f'(\xi) = \int_a^b f(t)\,dt$$

$$\therefore \quad (b-a)\,f'(\xi) = \int_a^b f(x)\,dx \text{ (replacing } t \text{ by } x\text{)}$$

24. Option (b) is correct.

Given, $F(z) = iz + k\,\text{Re}(z) + i\,\text{Im}(z)$

$= i(x+iy) + kx + iy$

$= kx - y + i(x+y)$, for $F(z) = u + iv$

$u = kx - y; \quad v = x + y$

$u_x = k; \quad u_y = -1$

$v_x = 1; \quad v_y = 1$

For CR eqn $u_x = v_y \Rightarrow k = 1$

25. Option (c) is correct.

Given, $n = 5$

p = (probability of getting 1) $= \frac{1}{6}$

$$q = 1 - \frac{1}{6} = \frac{5}{6}$$

Probability of getting 1 at least 4 times out of 5 times.

$$P(x \geq 4) = P(x = 4) + P(x = 5)$$

Using binomial distribution

$$P(x \geq 4) = P(x = 4) + P(x = 5)$$

$$= 5C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right) + 5C_5\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^0$$

$$= 5\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right) + 1\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^0$$

$$= \frac{25}{(6)^5} + \frac{1}{(6)^5} = \frac{26}{(6)^5} = 0.0033$$

% probability = 0.33%

26. Option (b) is correct.

Given, X_1 and X_2 are two independent RV.

$$Y = X_1 - X_2 \quad ...(i)$$

$$\mu_1 = \mu_2 = 1$$

$$\sigma_1 = 1, \sigma_2 = 2$$

$$E[Y] = E[X_1 - X_2] = E[X_1] - E[X_2]$$

$$\mu = \mu_1 - \mu_2 = 1 - 1$$

$$\Rightarrow \quad \mu = 0$$

Now take variance of equation (i) we have.

$$\text{Var}(Y) = \text{Var}(X_1 - X_2)$$

$\because$ X_1 and X_2 are two normal independent random variables.

$$\text{Var}(Y) = \text{Var}(X_1) + \text{Var}(X_2)$$

$$\sigma^2 = \sigma_1^2 + \sigma_2^2$$

$$\Rightarrow \quad \sigma^2 = (1)^2 + (2)^2 = 5$$

27. Option (c) is correct.

Given, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\text{Div}(\vec{r}) = \frac{\partial(x)}{\partial x} + \frac{\partial(y)}{\partial y} + \frac{\partial(z)}{\partial y}$$

$$= 1 + 1 + 1 = 3$$

$\because$ Surface is closed with volume V so, by Gauss divergence theorem.

$$\iint_s \vec{r}.\vec{n}.ds = \int\iint_V \text{Div}(\vec{r})dV$$

$$\Rightarrow \quad \iint_s \vec{r}.\vec{n}.ds = \int\iint_V (3)dV$$

$$\Rightarrow \quad \iint_s \vec{r}.\vec{n}.ds = 3V$$

28. Correct answer is [2.59].

$$\frac{dy}{dt} = f(t, y) = y$$

Given, $y(0) = 1$

$\Rightarrow \quad t_0 = 0, y_0 = 1; h = 0.1$

By use of forward Euler method.

$$y_1 = y_0 + hf(t_0, y_0)$$

$$= y_0 + hy_0$$

$$= 1 + 0.1(1)$$

$$y_1 = 1.1$$

$$y_2 = y_1 + hf(t_1, y_1)$$

$$= y_1 + hy_1$$

$$\Rightarrow \quad y_2 = 1.1 + 0.1(1.1)$$

$$\Rightarrow \quad y_2 = 1.21$$

$$y_3 = y_2 + hf(t_2, y_2) = y_2 + hy_2$$

$$\Rightarrow \quad y_3 = 1.21 + 0.1(1.21)$$

$$\Rightarrow \quad y_3 = 1.331$$

$$y_4 = y_3 + hf(t_3, y_3)$$

$$= y_3 + hy_3$$

$$\Rightarrow \quad y_4 = 1.331 + 0.1(1.331)$$

$$\Rightarrow \quad y_4 = 1.4641$$

$$y_5 = y_4 + hf(t_4, y_4)$$

$$= y_4 + hy_4$$

$$\Rightarrow \quad y_5 = 1.4641 + 0.1(1.4641)$$

$$\Rightarrow \quad y_5 = 1.61051$$

$$y_6 = y_5 + hf(t_5, y_5)$$

$$= y_5 + hy_5$$

$$\Rightarrow \quad y_6 = 1.61051 + 0.1(1.61051)$$

$$\Rightarrow \quad y_6 = 1.771561$$

$$y_7 = y_6 + hf(t_6, y_6)$$

$$= y_6 + hy_6$$

$$\Rightarrow \quad y_7 = 1.771561 + 0.1(1.771561)$$

$$\Rightarrow \quad y_7 = 1.9487$$

$$y_8 = y_7 + hf(t_7, y_7)$$

$$= y_7 + hy_7$$

$$\Rightarrow \quad y_8 = 1.9487 + 0.1(1.9487)$$

$$\Rightarrow \quad y_8 = 2.14357$$

$$y_9 = y_8 + hf(t_8, y_8)$$

$$\Rightarrow \quad y_9 = y_8 + hy_8$$

$$\Rightarrow \quad y_9 = 2.14357 + 0.1(2.14357)$$

$$\Rightarrow \quad y_9 = 2.357927$$

$$y_{10} = y_9 + hf(t_9, y_9)$$

$$= y_9 + hy_9$$

$$\Rightarrow \quad y_{10} = 2.357927 + 0.1(2.357927)$$

$$\Rightarrow \quad y_{10} = 2.5937197$$

$$\Rightarrow \quad y_{10} = 2.5937$$

=2.59 up to decimal place

29. Correct answer is [0.50].

Given, $f(t) = 2t^2e^{-t}$

We know,

$$F(s) = L\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\Rightarrow \quad F(s) = L\{t^2\} = \frac{2!}{s^{2+1}} = \frac{2}{s^3}$$

$$\Rightarrow \quad F(s) = L\{2t^2\} = \frac{2.2}{s^3} = \frac{4}{s^3}$$

Now applying time shifting property of Laplace transform.

$$F(s) = L\{e^{-t}.2t^2\} = \frac{4}{(s+1)^3}$$

$$\Rightarrow \quad F(1) = \frac{4}{(1+1)^3} = \frac{4}{8} = 0.50$$

30. Option (c) is correct.

Given, Fourier series of even function.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx)$$

Expansion of Fourier series is –

$$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + ... \quad ...(i)$$

$$f(x) = \cos^2 x = \frac{1+\cos 2x}{2} = \frac{1}{2} + \frac{\cos 2x}{2} = \frac{1}{2} + \frac{1}{2}.(\cos 2x) \quad ...(ii)$$

Comparing the coefficients of (i) and (ii) we have.

$$a_0 = \frac{1}{2};$$

$$a_1 = 0;$$

$$a_2 = \frac{1}{2} = 0.5$$

31. Option (c) is correct.

Given, $\vec{u} = e^x(\cos y\hat{i} + \sin y\hat{j}) = u_1\hat{i} + u_2\hat{j}$

Divergence $= \nabla.\vec{u} = \frac{\partial}{\partial x}(u_1) + \frac{\partial}{\partial y}(u_2)$

$$\nabla.\vec{u} = \frac{\partial}{\partial x}(e^x \cos y) + \frac{\partial}{\partial y}(e^x \sin y)$$

$$= e^x \cos y + e^x \cos y$$

$$\Rightarrow \quad \nabla.\vec{u} = 2e^x \cos y$$

32. Option (b) is correct.

$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \rightarrow$ wave Equation

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \rightarrow$ heat equation

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \rightarrow$ Laplace equation

33. Option (c) is correct.

Given, DE is $y^3 \frac{dy}{dx} + x^3 = 0$

$$y^3 dy = -x^3 dx$$

$$\int y^3 dy = \int -x^3 dx$$

$$\frac{y^4}{4} = -\frac{x^4}{4} + c$$

$$\Rightarrow \quad \frac{y^4}{4} + \frac{x^4}{4} = c$$

$$\Rightarrow \quad \frac{y^4 + x^4}{4} = c \quad ...(i)$$

$$y(0) = 1$$

$$\Rightarrow \quad \frac{(1)^4 + (0)^4}{4} = c \Rightarrow c = \frac{1}{4}$$

Putting value of c in (i)

$$\Rightarrow \quad \frac{y^4 + x^4}{4} = \frac{1}{4}$$

$$\Rightarrow \quad y^4 + x^4 = 1$$

$$\Rightarrow \quad y^4 = 1 - x^4$$

at $x = -1$,

$$y^4 = 1 - (-1)^4 = 0$$

34. Correct answer is [0.25].

Given $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ is a triangular matrix, so determinant will be product of diagonal elements

$$\therefore \quad |A| = 4$$

$$\because \quad |A^{-1}| = \frac{1}{|A|}$$

$$\therefore \quad |A^{-1}| = \frac{1}{4} = 0.25$$

35. Option (a) is correct.

Given, $\oint_C \frac{1}{5z-4} dz = A\pi i; \quad C : |z| = 1$

Where pole is at $z = \frac{4}{5}$ lies inside the C.

$$\int \frac{1}{5z-4} dz = A\pi i$$

By Cauchy's integral formula

$$\oint_C \frac{f(z)dz}{z - z_0} = 2\pi i f(z_0)$$

$$\Rightarrow \int \frac{\frac{1}{5}}{\left(z - \frac{4}{5}\right)} dz = 2\pi i.f\left(\frac{4}{5}\right) = 2\pi i.\left(\frac{1}{5}\right)$$

$$\therefore \quad \frac{2}{5}\pi i = A\pi i$$

$$\Rightarrow \quad A = \frac{2}{5}$$

36. Option (a) is correct.

We know mean of exponential RV$=\frac{1}{\lambda}$

So, $E(X_1)=\frac{1}{\lambda_1}=0.5 \Rightarrow \lambda_1=2$

$$E(X_2)=\frac{1}{\lambda_2}=0.25 \Rightarrow \lambda_2=4$$

$$E(Y)=\frac{1}{\lambda}=\frac{1}{\lambda_1+\lambda_2}=\frac{1}{2+4}=\frac{1}{6}$$

Clearly, Y is exponentially distributed with mean $\frac{1}{6}$.

37. Option (c) is correct.

For $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

and $|\vec{r}|=\sqrt{x^2+y^2+z^2}$

gradient $\nabla\phi=\nabla \ln|\vec{r}|=\nabla\left\{\frac{1}{2}\ln(x^2+y^2+z^2)\right\}$

$$\nabla\phi=\frac{1}{2}\cdot\frac{2x}{x^2+y^2+z^2}\hat{i}+\frac{1}{2}\cdot\frac{2y}{x^2+y^2+z^2}\hat{j}+\frac{1}{2}\cdot\frac{2z}{x^2+y^2+z^2}\hat{k}$$

$$=\frac{x\hat{i}+y\hat{j}+z\hat{k}}{x^2+y^2+z^2}$$

$$\therefore \quad \nabla\phi=\frac{\vec{r}}{|\vec{r}|}=\frac{\vec{r}}{\vec{r}\cdot\vec{r}}$$

38. Correct answer is [1.46].

Given, DE is $\frac{d^2y}{dx^2}+\frac{dy}{dx}-6y=0$

$$y(0)=0;$$

$$\frac{dy}{dx}(0)=1$$

$$(D^2+D-6)y=0$$

AE: $(D+3)(D-2)=0$

$$\Rightarrow \quad D=2,-3$$

$$y=CF+PI=C_1e^{2x}+C_2e^{-3x} \quad \text{...(i)}$$

At $x=0,\ y=0$

$\Rightarrow \quad C_1+C_2=0$

$\Rightarrow \quad C_1=-C_2$

$$\frac{dy}{dx}=2C_1e^{2x}-3C_2e^{-3x} \quad \text{...(ii)}$$

Putting $\frac{dy}{dx}(0)=1$

$\Rightarrow \quad 2C_1-3C_2=1$

$\Rightarrow 2(-C_2)-3C_2=1$

$\Rightarrow \quad -5C_2=1$

$\Rightarrow \quad C_2=-\frac{1}{5}$ and $C_1=\frac{1}{5}$

Putting the value of C_1 and C_2 in (i)

$$y=\frac{1}{5}e^{2x}-\frac{1}{5}e^{-3x}$$

At $x=1$

$$y(1)=\frac{1}{5}e^{2(1)}-\frac{1}{5}e^{-3(1)}=1.4678$$

= 1.47 up to decimal

39. Option (c) is correct.

Matrix having determinant zero called singular matrix. So

$$A=\begin{bmatrix}2 & 4\\ 3 & 6\end{bmatrix}$$

$$|A|=12-12=0$$

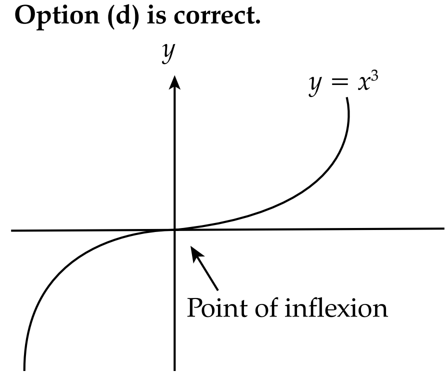
40. Option (c) is correct.

Given matrix $Q=\begin{bmatrix}\frac{3}{7} & \frac{2}{7} & \frac{6}{7}\\ \frac{-6}{7} & \frac{3}{7} & \frac{2}{7}\\ \frac{2}{7} & \frac{6}{7} & \frac{-3}{7}\end{bmatrix}$

For Orthogonal matrix $Q^{-1}=Q^T$

$$Q^{-1}=\begin{bmatrix}\frac{3}{7} & -\frac{6}{7} & \frac{2}{7}\\ \frac{2}{7} & \frac{3}{7} & \frac{6}{7}\\ \frac{6}{7} & \frac{2}{7} & \frac{-3}{7}\end{bmatrix}$$

41. Option (d) is correct.



Given $f(x)=x^3$

$$f'(x)=3x^2$$

solving $f'(x)=0$

$$3x^2=0$$

$\Rightarrow x=0$

so, at $x=0$ the function is neither minima nor maxima, it is the point of inflection.

42. Option (b) is correct.

Let $\quad I=\int_0^{\pi} x\cos^2 x\,dx$

$$=\int_0^{\pi} x\left(\frac{1+\cos 2x}{2}\right)dx$$

$$=\int_0^{\pi}\left[\frac{x}{2}+\left(\frac{x\cos 2x}{2}\right)\right]dx$$

$$=\left(\frac{x^2}{4}\right)_0^{\pi}+\frac{1}{2}\left(\frac{x\sin 2x}{2}+\frac{\cos 2x}{4}\right)_0^{\pi}$$

$$=\frac{\pi^2}{4}+\frac{1}{2}\left\{\left(0+\frac{1}{4}\right)-\left(0+\frac{1}{4}\right)\right\}$$

$$I=\frac{\pi^2}{4}$$

43. Option (d) is correct.

$$\frac{c^2\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}$$

$$u(x,0)=f(x)$$

$$u_t(x,0)=g(x)$$

$$\frac{\partial^2 u}{\partial x^2}=25\frac{\partial^2 u}{\partial t^2}$$

$$f(x)=3x$$

$$g(x)=3$$

$$c^2=\frac{1}{25}$$

D'Alembert Formula

$$u(x,t)=\frac{1}{2}\left[f(x+ct)+f(x-ct)\right]+\frac{1}{2c}\int_{x-ct}^{x+ct} g(y)\,dy$$

$$=\frac{1}{2}\left[3\left(x+\frac{t}{5}\right)+3\left(x-\frac{t}{5}\right)\right]+\frac{1}{2\left(\frac{1}{5}\right)}\int_{x-\frac{t}{5}}^{x+\frac{t}{5}} 3\,dy$$

$$=\frac{1}{2}[6x]+\frac{3}{2}(5)\left[x+\frac{t}{5}-x+\frac{t}{5}\right]$$

$$u(x,t)=3x+3t$$

Now at $x=1$, $t=1$

$$u(x,t)=6$$

44. Correct answer is [0.36].

Given, $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+y=0;$

$$y(0)=1;\ \frac{dy}{dx}=-1$$

$$(D^2+2D+1)y=0$$

AE: $(D+1)^2=0 \Rightarrow \quad D=-1,-1$

$$CF=(C_1+C_2x)e^{-x}$$

$$y=C_1e^{-x}+C_2xe^{-x} \qquad \text{...(i)}$$

Putting $\quad x=0,\ y=1$ in (i)

$\Rightarrow \qquad 1=C_1$

Differentiating (i) w.r.t x

$$\frac{dy}{dx}=-C_1e^{-x}+C_2(-xe^{-x}+e^{-x}) \qquad \text{...(ii)}$$

Putting $\quad x=0, \dfrac{dy}{dx}=-1$

$\Rightarrow \qquad -1=-C_1+C_2$

$$C_2=0$$

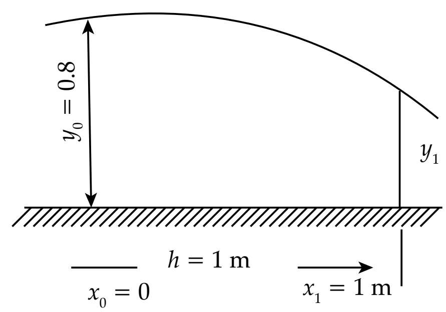
Substitute the value of C_1 and C_2 in (i)

$$y=e^{-x}$$

For $\quad x=1$

$$y=e^{-1}\approx 0.36$$

45. Correct answer is [0.79].



$$f(x,y)=\frac{dy}{dx}=\frac{1-e^{\frac{-10}{3}\ln(y)}}{250-45e^{-3\ln(y)}}$$

$y_1=y_0+hf(x_0,y_0)$, with $h=1$,

$x_0=0$, $y_0=1$

$$=0.8+1\left[\frac{1-e^{\frac{-10}{3}\ln(0.8)}}{250-45e^{-3\ln(0.8)}}\right]=0.8+1\left[\frac{-1.1039}{162.109}\right]$$

$y_1=0.793$ m $=0.79$ m up to two decimal

46. Option (c) is correct.

Given, DE $x\dfrac{dy}{dx}+y=0$

$$\Rightarrow x\frac{dy}{dx}=-y \Rightarrow \frac{1}{y}dy=-\frac{1}{x}dx$$

$$\int\frac{1}{y}dy=-\int\frac{1}{x}dx$$

$$\Rightarrow \log y=-\log x+\log c$$

$$xy=c \Rightarrow y=\frac{c}{x}$$

At point (1,1) we have c=1,

$$\Rightarrow \quad y=\frac{1}{x}=x^{-1}$$

47. Option (a) is correct.

For $f(x)$ to be valid PDF

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\Rightarrow \int_0^1 f(x)dx + \int_1^2 f(x)dx + \int_2^3 f(x)dx = 1$$

By area under the curve

$$\Rightarrow \frac{1}{2}\cdot h + \frac{1}{2}\cdot 2h + \frac{1}{2}\cdot 3h = 1$$

$$\Rightarrow \quad 3h = 1 \Rightarrow h = \frac{1}{3}$$

48. Option (c) is correct.

For the probability distribution when right skew → Mode < Median < Mean

And given curve is Right skewed

49. Correct answer is [0.5].

Given, 5 red balls and 1 white ball.

Probability to choose 3 balls

$$P = \frac{5}{6}\times\frac{4}{5}\times\frac{3}{4} = 0.5$$

50. Correct answer is [1].

$$f(x) = 2x^2 - 3x + 3$$

$$\Rightarrow \quad f'(x) = 4x - 3$$

with $x_0 = 2$, first iteration gives

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)}$$

$$= 2 - \frac{5}{5} = 1$$

51. Correct answer is [18.2%].

Given, T=100 years; n=20 years

$$\text{Risk} = 1 - q^n$$

$$= 1 - \left(1 - \frac{1}{T}\right)^n$$

$$= 1 - \left(1 - \frac{1}{100}\right)^{20}$$

$$= 1 - (1 - 0.01)^{20} = 0.182 = 18.2\%$$

52. Option (d) is correct.

$$A = \begin{bmatrix} 2 & -4 \\ 4 & -2 \end{bmatrix}, |A - \lambda I| = 0$$

$$\Rightarrow \quad \begin{vmatrix} 2-\lambda & -4 \\ 4 & -2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad (2-\lambda)(-2-\lambda) + 16 = 0$$

$$\Rightarrow \quad \lambda^2 + 12 = 0$$

$$\Rightarrow \quad \lambda = \pm 2\sqrt{3}i \text{ are eigen values}$$

For $\lambda = 2\sqrt{3}i$, eigen vector

$$(A - \lambda I)\ X = 0$$

$$\Rightarrow \begin{bmatrix} 2-2\sqrt{3}i & -4 \\ 4 & -2-2\sqrt{3}i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore \quad (2-2\sqrt{3}i)\ x_1 - 4x_2 = 0$$

$$4x_1 + (-2-2\sqrt{3}i)x_2 = 0$$

gives eigen vector $\begin{bmatrix} 1+\sqrt{3}i \\ 2 \end{bmatrix}$

For $\lambda = -2\sqrt{3}i$, eigen vector

$$(A - \lambda I)X = 0$$

$$\Rightarrow \begin{bmatrix} 2+2\sqrt{3}i & -4 \\ 4 & -2+2\sqrt{3}i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore \quad (2+2\sqrt{3}i)\ x_1 - 4\ x_2 = 0$$

$$4x_1 + (-2+2\sqrt{3}i)x_2 = 0$$

gives eigen vector $\begin{bmatrix} 1-\sqrt{3}i \\ 2 \end{bmatrix}$

So, complex eigen values and complex eigen vector.

53. Option (b) is correct.

Given, $f(t) = e^{at}; t \geq 0$

Using Laplace transform property.

$$F(s) = \frac{1}{(s-a)}; (s-a) > 0$$

54. Option (b) is correct.

$$\text{Let} \quad A = \begin{bmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{bmatrix}$$

Reducing Matrix A into echelon form.

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 4R_1$

$$= \begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & -3 & 3 & 9 \end{bmatrix}$$

$R_3 \rightarrow R_3 - \frac{3}{2}R_2$

$$= \begin{bmatrix} 1 & 1 & 0 & -2 \\ 0 & -2 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

No. of non-zero rows are 2. So, Rank = 2

55. Correct answer is [0.67].

Given, $\vec{F}(\vec{r}) = x^2\hat{i} + y^2\vec{j}$

$$\int_C \vec{F}(\vec{r})d\vec{r} = \int_C x^2dx + y^2dy$$

Equation of straight line joining the points (0, 0) to (1, 1).

$$y = x$$

$\Rightarrow$ $dy = dx$

$\therefore$ $\int_C x^2dx + y^2dy = \int_0^1 x^2dx + x^2dx = \int_0^1 2x^2dx$

$$= 2\left(\frac{x^3}{3}\right)_0^1 = \frac{2}{3} = 0.67$$

56. Option (d) is correct.

Let G be a generating function for the given sequence $\{a_n\}$.

where $a_n = 2n+3$

$$G(x) = \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty}(2n+3)x^n$$

$$= \sum_{n=0}^{\infty}(2n)x^n + \sum_{n=0}^{\infty}(3)x^n$$

$$= 2\sum_{n=0}^{\infty} nx^n + 3\sum_{n=0}^{\infty} x^n = 2P + 3Q$$

Here $P = \sum_{n=0}^{\infty} nx^n = 0 + x + 2x^2 + 3x^3 + \ldots$ is a AGP series with 1st term is 0, common difference is 1 and ratio is x.

Sum of infinite AGP series

$$= \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$

So, the sum of the AGP

$$= \frac{0}{1-x} + \frac{x}{(1-x)^2} = \frac{x}{(1-x)^2}$$

And $Q = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots = \frac{1}{1-x}$

$\Rightarrow$ $G(x) = 2P + 3Q$

$$= 2.\frac{x}{(1-x)^2} + 3.\frac{1}{1-x} = \frac{2x+3-3x}{(1-x)^2} = \frac{3-x}{(1-x)^2}$$

57. Correct answer is [0.023].

Probability of tie $= \frac{6}{36} = \frac{1}{6}$

Probability not to tie $= 1 - \frac{1}{6} = \frac{5}{6}$

Probability of wining in the third tie= (1st tie) × (2nd tie) × (3rd)

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{216} = 0.023$$

58. Correct answer is [0.28].

$$I = \int_0^{\frac{\pi}{4}} x\cos(x^2)dx$$

Putting $x^2 = t \Rightarrow 2xdx = dt$

$$x = 0 \Rightarrow t = 0$$

$$x = \frac{\pi}{4} \Rightarrow t = \frac{\pi^2}{16}$$

$$\therefore I = \int_0^{\frac{\pi^2}{16}} \frac{\cos t}{2}\, dt = \left(\frac{\sin t}{2}\right)_0^{\frac{\pi^2}{16}} = \frac{1}{2}\left\{\sin\left(\frac{\pi^2}{16}\right) - \sin 0\right\}$$

$= 0.289$

≈ 0.29

59. Correct answer is [3].

Given, $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix};\ v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$A = uv^T$$

$$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

For eigen values of 2 × 2 matrix, characteristic equation form is given by

$\lambda^2 - (\text{trace of A})\lambda + |A| = 0$

$\Rightarrow$ $\lambda^2 - 3\lambda + 0 = 0$

$\Rightarrow$ $\lambda(\lambda - 3) = 0$

$\Rightarrow$ $\lambda = 0, 3$

Largest eigen value is 3.

60. Correct answer is [42].

Since G is a finite group. According to Lagrange theorem for any finite Group G, the order of subgroup H of group G divides the order of G. The order of the group represents the number of elements. Largest possible divisor of 84 is 42. So, largest possible subgroup of G is 42.

61. Option (d) is correct.

Given, for a matrix P only eigen vector multiples of $\begin{bmatrix} 1 \\ 4 \end{bmatrix} \Rightarrow$ eigen values is repeated.

$\therefore$ If eigen values are distinct then we have one more independent eigen vector. So, statement II is true here that P has repeated eigen values.

Statement I need not to be true since P has repeated eigen values and it will be non zero.

For statement III as is given that P has only one independent eigen vector. So, it cannot be diagonalised.

62. Option (d) is correct.

P: set of rational numbers are countable. Rational numbers are of the form of $\frac{p}{q}$; where both p and q are integers. By enumeration procedure take $p+q$ and we can write all possible values.

Q : set of functions from (0, 1) to N and possible functions are N^2 So, countable.

R : set of functions from N to {0,1} and possible functions are 2^N. from theorem ⇒ If a set S is countable the P(S) mean 2^S is uncountable.

⇒ this statement is uncountable.

S : Set of finite subsets of N. They are countable. According to theorem – Every subset of a countable set is either countable or finite.

63. Option (a) is correct.

Let us say ϕ has model of size 7 – $\{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ and let us say $s = a_2$, $t = a_5$, $u = a_1$ and this works for all values of v, w, x, y then we can reduce this model size to $3\{a_1, a_2, a_5\}$. This can further reduce to model of size less than 3 if $s = t$ or $t = u$.

64. Option (c) is correct.

Given, DE are

$$\frac{dx}{dt}+x+y=0 \quad \text{...(i)}$$

$$\frac{dy}{dt}-x=0 \quad \text{...(ii)}$$

Differentiating (ii) we have

$$\frac{d^2y}{dt^2}-\frac{dx}{dt}=0 \Rightarrow \frac{dx}{dt}=\frac{d^2y}{dt^2} \quad \text{...(iii)}$$

From equation (ii) ⇒ $x=\frac{dy}{dt}$ putting in (i) and

From (iii) putting the value of $\frac{dx}{dt}$ in (i) we have

$$\Rightarrow \frac{d^2y}{dt^2}+\frac{dy}{dt}+y=0$$

65. Option (d) is correct.

Given, $\frac{dy}{dx}=f(x,y)$

$$y(x+h)=y(x)+\frac{1}{6}(k_1+2k_2+2k_3+k_4) \quad \text{...(i)}$$

$$k_1=hf(x,y)$$

$$k_2=hf\left(x+\frac{h}{2}, y+\frac{k_1}{2}\right)$$

$$k_3=hf\left(x+\frac{h}{2}, y+\frac{k_2}{2}\right)$$

$$k_4=hf(x+h, y+k_3)$$

According to question the function f depends solely on x,

⇒ $\frac{dy}{dx}=f(x)$ and y values of k_1, k_2, k_3, k_4 will be eliminated, new k_1, k_2, k_3, k_4 becomes

$$k_1=hf(x)$$

$$k_2=hf\left(x+\frac{h}{2}\right)$$

$$k_3=hf\left(x+\frac{h}{2}\right)$$

$$k_4=hf(x+h)$$

Put all the new values of k_1, k_2, k_3, k_4 in (i)

$$y(x+h)=y(x)+\frac{1}{6}\left[hf(x)+2hf\left(x+\frac{h}{2}\right)+2hf\left(x+\frac{h}{2}\right)+hf(x+h)\right]$$

$$y(x+h)=y(x)+\frac{1}{6}\left[hf(x)+4hf\left(x+\frac{h}{2}\right)+hf(x+h)\right]$$

On simplification we get Simpson's $\left(\frac{1}{3}\right)^{\text{rd}}$ rule.

$$y(x+h)=y(x)+\frac{\left(\frac{h}{2}\right)}{3}\left[f(x)+4f\left(x+\frac{h}{2}\right)+f(x+h)\right]$$

66. Option (c) is correct.

Let the two electronic circuits (EC's) be EC_1 and EC_2 respectively.

P_r (EC_1, failure) = 0.1 and P_r (EC_2, failure) = 0.1

P_r (EC_1 functioning without failure) = 1 – 0.1 = 0.9

P_r (EC_2 functioning without failure) = 0.9

∴ P_r (watch functioning without failure)

= $P_r\left(\overline{EC_1}\cap\overline{EC_2}\right)$ → both EC's are required for functioning of watch.

$= P_r\left(\overline{EC_1}\right)\times P_r\left(\overline{EC_2}\right)$

(∵ EC_1 and EC_2 are independent.)

= 0.9 × 0.9 = 0.81

67. Option (b) is correct.

We know $y = \lim_{x\to 0} \frac{\sin x}{x} = 1$

$$\frac{\sin x}{x} = +ve \text{ always till 3.14 value.}$$

Only option (b) is showing the graph.

68. Correct answer is [3.85].

Given, $\frac{dL}{dt} = -k\sqrt{t}$ ...(i)

Where $k = 0.6$

When $t = 0 \Rightarrow L = 4$

When $t = 0.5 \Rightarrow L = ?$

From (i) $dL = -k\sqrt{t}dt$ this is variable separable form.

Integrating both sides.

$$\int dL = -k\int \sqrt{t}\, dt$$

$$\Rightarrow \quad L = -k\frac{t^{3/2}}{3/2} + c$$

$$L = -\frac{2}{3}kt^{3/2} + c \quad \text{...(ii)}$$

Using given conditions at $t = 0 \Rightarrow L = 4$ we have

$\Rightarrow \quad 4 = 0 + c \Rightarrow c = 4$

Put value of c in (ii)

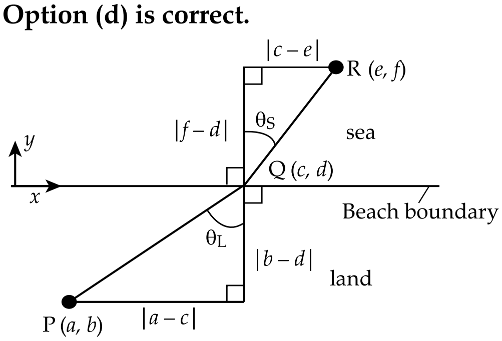
$$L = \frac{-2}{3}kt^{3/2} + 4$$

Now when $t = 0.5$

$$L = \frac{-2}{3}(0.6)(0.5)^{3/2} + 4$$

$\Rightarrow \quad L = -0.14142 + 4 = 3.8585\text{m}$

69. Option (d) is correct.



Distance $PQ = \sqrt{(c-a)^2 + (d-b)^2}$

Distance $QR = \sqrt{(e-c)^2 + (f-d)^2}$

Now time taken travelling from P to Q and Q to R.

$$T = \frac{\sqrt{(c-a)^2 + (d-b)^2}}{V_L} + \frac{\sqrt{(e-c)^2 + (f-d)^2}}{V_S}$$

Taking partial differentiation w.r.t. c

$$\frac{dT}{dc} = \frac{1}{2}\frac{2(c-a)}{V_L\sqrt{(c-a)^2 + (d-b)^2}} + \frac{1}{2}\frac{2(-1)(e-c)}{V_S\sqrt{(e-c)^2 + (f-d)^2}}$$

For optimum time $\frac{dT}{dc} = 0$

$$\Rightarrow \frac{(c-a)}{V_L\sqrt{(c-a)^2 + (d-b)^2}} - \frac{(e-c)}{V_S\sqrt{(e-c)^2 + (f-d)^2}} = 0$$

$$\Rightarrow \frac{(c-a)}{V_L\sqrt{(c-a)^2 + (d-b)^2}} = \frac{(e-c)}{V_S\sqrt{(e-c)^2 + (f-d)^2}}$$

$$\Rightarrow \quad \frac{\sin\theta_L}{V_L} = \frac{\sin\theta_S}{V_S}$$

$$\Rightarrow \quad \frac{\sin\theta_L}{\sin\theta_S} = \frac{V_L}{V_S}$$

70. Correct answer is [0].

Given, $y = e^{-x^2}$

$$\frac{dy}{dx} = -2xe^{-x^2}$$

$$\lim_{x\to\infty}\frac{1}{x}\frac{dy}{dx} = \lim_{x\to\infty}\frac{1}{x}\left(-2xe^{-x^2}\right) = -2e^{-\infty} = 0$$

71. Correct answer is [1].

Given, $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$$|AA^T| = |A|\cdot|A^T|$$

$$= \{|A|\}^2 \qquad (\because\ |A| = |A^T|)$$

$$= \left\{\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}\right\}^2$$

$$= \{\cos^2\theta + \sin^2\theta\}^2$$

$$|A\,A^T| = 1$$