ENGINEERING MATHEMATICS GATE Question Paper 2019

ELECTRONICS AND COMMUNICATION (EC) P1

Q. 1. Which one of the following functions is analytic over the entire complex plane?

(a) $\ln(z)$ **(b)** $e^{\frac{1}{z}}$

(c) $\frac{1}{1-z}$ **(d)** $\cos(z)$

Q. 2. The families of curves represented by the solution of the equation

$$\frac{dy}{dx} = -\left(\frac{x}{y}\right)^n$$

for $n = -1$ and $n = +1$, respectively, are

(a) Parabolas and Circles

(b) Circles and Hyperbolas

(c) Hyperbolas and Circles

(d) Hyperbolas and Parabolas

Q. 3. The value of the contour integral

$$\frac{1}{2\pi j}\oint\left(z+\frac{1}{z}\right)^2 dz$$

evaluated over the circle $|z| = 1$ is______.

Q. 4. The number of distinct eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

is equal to________.

Q. 5. If X and Y are random variables such that E[2X + Y] = 0 and E[X + 2Y] = 33, then E[X] + E[Y] = ________.

Q. 6. The value of the integral $\int_0^{\pi}\int_y^{\pi}\frac{\sin x}{x}dx\,dy$, is equal to________.

Q. 7. Let Z be an exponential random variable with mean 1. That is, the cumulative distribution function of Z is given by

$$F_z(x) = \begin{cases} 1-e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then Pr($Z > 2 \mid Z > 1$), rounded off to two decimal places, is equal to _______.

Q. 8. Consider a differentiable function $f(x)$ on the set of real numbers such that $f(-1) = 0$ and $|f'(x)| \leq 2$. Given these conditions, which one of the following inequalities is necessarily true for all $x \in [-2, 2]$?

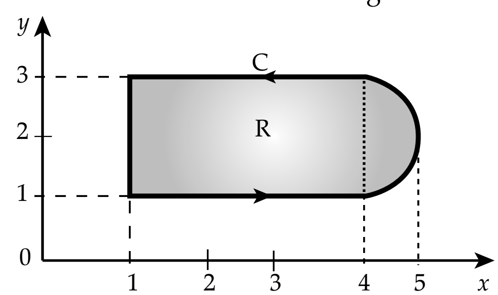
(a) $f(x) \leq \frac{1}{2}|x+1|$ **(b)** $f(x) \leq 2|x+1|$

(c) $f(x) \leq \frac{1}{2}|x|$ **(d)** $f(x) \leq 2|x|$

Q. 9. Consider the line integral

$$\int_c (x\,dy - y\,dx)$$

the integral being taken in a counter clockwise direction over the closed curve C that forms the boundary of the region R shown in the figure below. The region R is the area enclosed by the union of a 2 × 3 rectangle and a semi–circle of radius 1. The line integral evaluates to



(a) 6 + π/2 **(b)** 8 + π

(c) 12 + π **(d)** 16 + 2π

Q. 10. Consider the homogeneous ordinary differential equation

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0,$$

$x > 0$

with $y(x)$ as a general solution. Given that $y(1) = 1$ and $y(2) = 14$ the value of $y(1.5)$, rounded off to two decimal places, is ______.

ELECTRICAL ENGINEERING (EE) P1

Q. 11. The inverse Laplace transform of H(s) $= \frac{s+3}{s^2+2s+1}$ for $t \geq 0$ is

(a) $3te^{-t} + e^{-t}$ **(b)** $3e^{-t}$

(c) $2te^{-t} + e^{-t}$ **(d)** $4te^{-t} + e^{-t}$

Q. 12. M is a 2×2 matrix with eigenvalues 4 and 9. The eigenvalues of M^2 are

(a) 4 and 9 **(b)** 2 and 3
(c) –2 and –3 **(d)** 16 and 81

Q. 13. Which one of the following functions is analytic in the region $|z| \le 1$?

(a) $\frac{z^2-1}{z}$ **(b)** $\frac{z^2-1}{z+2}$
(c) $\frac{z^2-1}{z-0.5}$ **(d)** $\frac{z^2-1}{z+j0.5}$

Q. 14. If $f = 2x^3 + 3y^2 + 4z$, the value of line integral $\int_c \text{grad}\, f \cdot dr$ evaluated over contour C formed by the segments $(-3, -3, 2) \to (2, -3, 2) \to (2, 6, 2) \to (2, 6, -1)$ is ______.

Q. 15. The rank of the matrix, $M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ is______.

Q. 16. Consider a 2×2 matrix $M = [v_1\, v_2]$, where, v_1 and v_2 are the column vectors. Suppose

$$M^{-1} = \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix},$$

where u_1^T and u_2^T are the row vectors. Consider the following statements:

Statement I : $u_1^T v_1 = 1$ and $u_2^T v_2 = 1$

Statement II : $u_1^T v_2 = 0$ and $u_2^T v_1 = 0$

Which of the following options is correct?

(a) Statement I is true and statement II is false
(b) Statement II is true and statement I is false
(c) Both the statements are true
(d) Both the statements are false

Q. 17. The closed loop line integral $\oint_{|z|=5} \frac{z^3+z^2+8}{z+2}\, dz$ evaluated counter–clockwise, is

(a) $+8j\pi$ **(b)** $-8j\pi$
(c) $-4j\pi$ **(d)** $+4j\pi$

Q. 18. If $\mathbf{A} = 2xi + 3yj + 4zk$ and $u = x^2 + y^2 + z^2$, then div(uA) at (1, 1, 1) is ______.

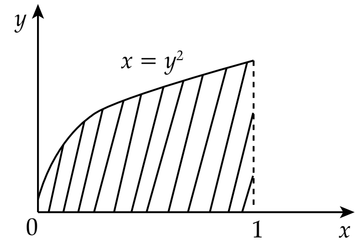
Q. 19. The probability of a resistor being defective is 0.02. There are 50 such resistors in a circuit. The probability of two or more defective resistors in the circuit (round off to two decimal places) is ______.

MECHANICAL ENGINEERING (ME) P1

Q. 20. Consider the matrix $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. The number of distinct eigenvalues of P is

(a) 0 **(b)** 1
(c) 2 **(d)** 3

Q. 21. A parabola $x = y^2$ with $0 \le x \le 1$ is shown in the figure. The volume of the solid of rotation obtained by rotating the shaded area by 360° around the x-axis is



(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{2}$
(c) π **(d)** 2π

Q. 22. For the equation $\frac{dy}{dx} + 7x^2 y = 0$, if $y(0) = \frac{3}{7}$, then the value of $y(1)$ is

(a) $\frac{7}{3}e^{-7/3}$ **(b)** $\frac{7}{3}e^{-3/7}$
(c) $\frac{3}{7}e^{-7/3}$ **(d)** $\frac{3}{7}e^{-3/7}$

Q. 23. The lengths of a large stock of titanium rods follow a normal distribution with a mean (μ) of 440 mm and a standard deviation (σ) of 1 mm. What is the percentage of rods whose lengths lie between 438 mm and 441 mm?

(a) 81.85% **(b)** 68.4%
(c) 99.75% **(d)** 86.64%

Q. 24. Evaluation of $\int_2^4 x^3 dx$ using a 2-equal-segment trapezoidal rule gives a value of ______.

Q. 25. The set of equations

$$x + y + z = 1$$
$$ax - ay + 3z = 5$$
$$5x - 3y + az = 6$$

has infinite solutions, if a =

(a) −3 **(b)** 3
(c) 4 **(d)** −4

Q. 26. A harmonic function is analytic if it satisfies the Laplace equation.

If $u(x, y) = 2x^2 - 2y^2 + 4xy$ is a harmonic function, then its conjugate harmonic function $v(x, y)$ is

(a) $4xy - 2x^2 + 2y^2 +$ constant

(b) $4y^2 - 4xy +$ constant

(c) $2x^2 - 2y^2 + xy +$ constant

(d) $-4xy + 2y^2 - 2x^2 +$ constant

Q. 27. The variable x takes a value between 0 and 10 with uniform probability distribution. The variable y takes a value between 0 and 20 with uniform probability distribution. The probability of the sum of variables $(x + y)$ being greater than 20 is

(a) 0 **(b)** 0.25

(c) 0.33 **(d)** 0.50

Q. 28. The value of the following definite integral $\int_1^e (x \ln x)\,dx$ is ______. (round off to three decimal places)

MECHANICAL ENGINEERING (ME) P2

Q. 29. In matrix equation [A] {X} = {R},

$$[A] = \begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix}, [X] = \begin{Bmatrix} 2 \\ 1 \\ 4 \end{Bmatrix}$$

and $[R] = \begin{Bmatrix} 32 \\ 16 \\ 64 \end{Bmatrix}$.

One of the eigenvalues of matrix [A] is

(a) 4 **(b)** 8

(c) 15 **(d)** 16

Q. 30. The directional derivative of the function $f(x, y) = x^2 + y^2$ along a line directed from (0, 0) to (1, 1), evaluated at the point $x = 1$, $y = 1$ is

(a) $\sqrt{2}$ **(b)** 2

(c) $2\sqrt{2}$ **(d)** $4\sqrt{2}$

Q. 31. The differential equation $\frac{dy}{dx} + 4y = 5$ is valid in the domain $0 \le x \le 1$ with $y(0) = 2.25$.

The solution of the differential equation is

(a) $y = e^{-4x} + 5$ **(b)** $y = e^{-4x} + 1.25$

(c) $y = e^{4x} + 5$ **(d)** $y = e^{4x} + 1.25$

Q. 32. An analytic function $f(z)$ of complex variable $z = x + iy$ may be written as $f(z) = u(x, y) + iv(x, y)$.

Then, $u(x, y)$ and $v(x, y)$ must satisfy

(a) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

(b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

(c) $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

(d) $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

Q. 33. If x is the mean of data 3, x, 2 and 4, then the mode is______.

Q. 34. Given a vector $\vec{u} = \frac{1}{3}\left(-y^3\hat{i} + x^3\hat{j} + z^3\hat{k}\right)$ and $\hat{n}$ as the unit normal vector to the surface of the hemisphere $\left(x^2 + y^2 + z^2 = 1; z \ge 0\right)$, the value of integral $\int(\nabla \times \vec{u}) \bullet \hat{n}\, dS$ evaluated on the curved surface of the hemisphere S is

(a) $-\frac{\pi}{2}$ **(b)** $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ **(d)** π

Q. 35. A differential equation is given as

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 4.$$

The solution of the differential equation in terms of arbitrary constants C_1 and C_2 is

(a) $y = C_1x^2 + C_2x + 2$ **(b)** $y = \frac{C_1}{x^2} + C_2x + 2$

(c) $y = C_1x^2 + C_2x + 4$ **(d)** $y = \frac{C_1}{x^2} + C_2x + 4$

Q. 36. The derivative of $f(x) = \cos(x)$ can be estimated using the approximation

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}.$$

The percentage error is calculated as

$$\left(\frac{\text{Exact value} - \text{Approximate value}}{\text{Exact value}}\right) \times 100.$$

The percentage error in the derivative of $f(x)$ at $x = \frac{\pi}{6}$ radian, choosing $h = 0.1$ radian, is

(a) < 0.1 %

(b) > 0.1 % and < 1 %

(c) > 1 % and < 5 %

(d) > 5 %

Q. 37. The probability that a part manufactured by a company will be defective is 0.05. If 15 such parts are selected randomly and inspected, then the probability that at least two parts will be defective is ______ (round off to two decimal places).

CIVIL ENGINEERING (CE) P1

Q. 38. Which one of the following is correct?

(a) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)=2$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1$

(b) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)=1$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1$

(c) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)=\infty$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1$

(d) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)=2$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=\infty$

Q. 39. For a small value of h, the Taylor series expansion for $f(x+h)$ is

(a) $f(x)+hf'(x)+\frac{h^2}{2!}f''(x)+\frac{h^3}{3!}f'''(x)+...\infty$

(b) $f(x)-hf'(x)+\frac{h^2}{2!}f''(x)-\frac{h^3}{3!}f'''(x)+...\infty$

(c) $f(x)+hf'(x)+\frac{h^2}{2}f''(x)+\frac{h^3}{3}f'''(x)+...\infty$

(d) $f(x)-hf'(x)+\frac{h^2}{2}f''(x)-\frac{h^3}{3}f'''(x)+...\infty$

Q. 40. The probability that the annual maximum flood discharge will exceed 25000 $\frac{m^3}{s}$, at least once in next 5 years is found to be 0.25. The return period of this flood event (in *years, round off to 1 decimal place*) is _______

Q. 41. Which one of the following is NOT a correct statement?

(a) The function $\sqrt[x]{x}$, ($x > 0$), has the global maxima at $x = e$

(b) The function $\sqrt[x]{x}$, ($x > 0$), has the global minima at $x = e$

(c) The function x^3 has neither global minima nor global maxima

(d) The function $|x|$ has the global minima at $x = 0$

Q. 42. A one–dimensional domain is discretized into N sub–domains of width Δx with node numbers i = 0, 1, 2, 3…., N. If the time scale is discretized in steps of Δt, the forward–time and centered space finite difference approximation at i^{th} node and n^{th} time step, for the partial differential $\frac{\partial v}{\partial t}=\beta\frac{\partial^2 v}{\partial x^2}$ equation is

(a) $\frac{v_i^{(n+1)}-v_i^{(n)}}{\Delta t}=\beta\left[\frac{v_{i+1}^{(n)}-2v_i^{(n)}+v_{i-1}^{(n)}}{(\Delta x)^2}\right]$

(b) $\frac{v_{i+1}^{(n+1)}-v_i^{(n)}}{\Delta t}=\beta\left[\frac{v_{i+1}^{(n)}-2v_i^{(n)}+v_{i-1}^{(n)}}{2\Delta x}\right]$

(c) $\frac{v_i^{(n)}-v_i^{(n-1)}}{\Delta t}=\beta\left[\frac{v_{i+1}^{(n)}-2v_i^{(n)}+v_{i-1}^{(n)}}{(\Delta x)^2}\right]$

(d) $\frac{v_i^{(n)}-v_i^{(n-1)}}{2\Delta t}=\beta\left[\frac{v_{i+1}^{(n)}-2v_i^{(n)}+v_{i-1}^{(n)}}{2\Delta x}\right]$

Q. 43. Consider two functions: $x=\psi\ln\phi$ and $y=\phi\ln\psi$. Which one of the following is the correct expression for $\frac{\partial\psi}{\partial x}$?

(a) $\frac{x\ln\psi}{\ln\phi\ln\psi-1}$ **(b)** $\frac{x\ln\phi}{\ln\phi\ln\psi-1}$

(c) $\frac{\ln\phi}{\ln\phi\ln\psi-1}$ **(d)** $\frac{\ln\psi}{\ln\phi\ln\psi-1}$

Q. 44. Consider the ordinary differential equation $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$. Given the values of $y(1) = 0$ and $y(2) = 2$, the value of $y(3)$ (round off to 1 decimal place), is ______.

CIVIL ENGINEERING (CE) P2

Q. 45. Euclidean norm (length) of the vector $[4\ -2\ -6]^T$ and is

(a) $\sqrt{12}$ **(b)** $\sqrt{24}$

(c) $\sqrt{48}$ **(d)** $\sqrt{56}$

Q. 46. The Laplace transform of sin h (a t) is

(a) $\frac{a}{s^2-a^2}$ **(b)** $\frac{a}{s^2+a^2}$

(c) $\frac{s}{s^2-a^2}$ **(d)** $\frac{s}{s^2+a^2}$

Q. 47. The following inequality is true for all x close to 0.

$$2-\frac{x^2}{3}<\frac{x\sin x}{1-\cos x}<2$$

What is the value of $\lim_{x\to 0}\frac{x\sin x}{1-\cos x}$?

(a) 0 **(b)** 1/2
(c) 1 **(d)** 2

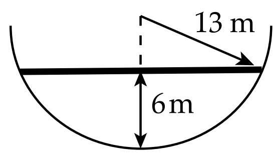
Q. 48. What is curl of the vector field $2x^2y\hat{i}+5z^2\hat{j}-4yz\hat{k}$?

(a) $6z\hat{i}+4x\hat{j}-2x^2\hat{k}$
(b) $6z\hat{i}-8xy\hat{j}+2x^2y\hat{k}$
(c) $-14z\hat{i}+6y\hat{j}+2x^2\hat{k}$
(d) $-14z\hat{i}-2x^2\hat{k}$

Q. 49. The probability density function of a continuous random variable distributed uniformly between x and y (for $y > x$) is

(a) $\frac{1}{x-y}$ **(b)** $\frac{1}{y-x}$
(c) $x-y$ **(d)** $y-x$

Q. 50. Consider the hemi–spherical tank of radius 13 m as shown in the figure (*not drawn to scale*). What is the volume of water (in m^3) when the depth of water at the centre of the tank is 6 m?



(a) 78π **(b)** 156π
(c) 396π **(d)** 468π

Q. 51. An ordinary differential equation is given below.

$$\left(\frac{dy}{dx}\right)(x\ln x)=y$$

The solution for the above equation is
(Note: K denotes a constant in the options)

(a) $y = \mathrm{K}x\ln x$ **(b)** $y = \mathrm{K}xe^{x}$
(c) $y = \mathrm{K}xe^{-x}$ **(d)** $y = \mathrm{K}\ln x$

Q. 52. The inverse of the matrix $\begin{bmatrix}2 & 3 & 4\\ 4 & 3 & 1\\ 1 & 2 & 4\end{bmatrix}$ is

(a) $\begin{bmatrix}10 & -4 & -9\\ -15 & 4 & 14\\ 5 & -1 & -6\end{bmatrix}$ **(b)** $\begin{bmatrix}-10 & 4 & 9\\ 15 & -4 & -14\\ -5 & 1 & 6\end{bmatrix}$

(c) $\begin{bmatrix}-2 & \frac{4}{5} & \frac{9}{5}\\ 3 & -\frac{4}{5} & -\frac{14}{5}\\ -1 & \frac{1}{5} & \frac{6}{5}\end{bmatrix}$ **(d)** $\begin{bmatrix}2 & -\frac{4}{5} & -\frac{9}{5}\\ -3 & \frac{4}{5} & \frac{14}{5}\\ 1 & -\frac{1}{5} & -\frac{6}{5}\end{bmatrix}$

Q. 53. A series of perpendicular offsets taken from a curved boundary wall to a straight survey line at an interval of 6 m are 1.22, 1.67, 2.04, 2.34, 2.14, 1.87, and 1.15 m. The area (in m^2, *round off to 2 decimal places*) bounded by the survey line, curved boundary wall, the first and the last offsets, determined using Simpson's rule, is ______.

COMPUTER SCIENCE (CS) P1

Q. 54. Let $U=\{1,2,\ldots,n\}$. Let $A=\{(x,X)\,|\,x\in X, X\subseteq U\}$. Consider the following two statements on $|A|$.

I. $|A|=n2^{n-1}$

II. $|A|=\sum_{k=1}^{n}k\binom{n}{k}$

Which of the above statements is/are TRUE?

(a) Only I **(b)** Only II
(c) Both I and II **(d)** Neither I nor II

Q. 55. Let X be a square matrix. Consider the following two statements on X.

I. X is invertible.
II. Determinant of X is non–zero.

Which one of the following is TRUE?

(a) I implies II; II does not imply I.
(b) II implies I; I does not imply II.
(c) I does not imply II; II does not imply I.
(d) I and II are equivalent statements

Q. 56. Let G be an arbitrary group. Consider the following relations on G:

R_1**:** $\forall\, a, b \in G$, $a\,R_1\,b$ if and only if $\exists\, g \in G$ such that $a = g^{-1}bg$

R_2**:** $\forall\, a, b \in G$, $a\,R_2\,b$ if and only if $a = b^{-1}$

Which of the above is/are equivalence relation/relations?

(a) R_1 and R_2 **(b)** R_1 only
(c) R_2 only **(d)** Neither R_1 nor R_2

Q. 57. Compute $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$

(a) 1 **(b)** $\frac{53}{12}$
(c) $\frac{108}{7}$ **(d)** Limit does not exist

Q. 58. Two numbers are chosen independently and uniformly at random from the set {1, 2,..., 13}. The probability (rounded off to 3 decimal places) that their 4–bit (unsigned) binary representations have the same most significant bit is ___________.

Q. 59. Consider the first order predicate formula $\phi : \forall x\ [(\forall z\ z|x \Rightarrow((z = x) \vee (z = 1))) \Rightarrow \exists w\ (w > x) \wedge (\forall z\ z|w \Rightarrow ((w = z) \vee (z = 1)))]$

Here 'a|b' denotes that 'a divides b', where a and b are integers. Consider the following sets:

S_1. {1, 2, 3,…, 100}

S_2. Set of all positive integers

S_3. Set of all integers

Which of the above sets satisfy ϕ?

(a) S_1 and S_2 **(b)** S_1 and S_3

(c) S_2 and S_3 **(d)** S_1, S_2 and S_3

Q. 60. Consider the following matrix:

$$R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$$

The absolute value of the product of Eigen values of R is ________.

Q. 61. Suppose Y is distributed uniformly in the open interval (1, 6). The probability that the polynomial $3x^2 + 6x\text{Y} + 3\text{Y} + 6$ has only real roots is ______. (rounded off to 1 decimal place)

CHEMICAL ENGINEERING (CH) P1

Q. 62. A system of n homogeneous linear equations containing n unknowns will have non–trivial solutions if and only if the determinant of the coefficient matrix is

(a) 1 **(b)** –1

(c) 0 **(d)** ∞

Q. 63. The value of the expression $\lim_{x \to \frac{\pi}{2}} \left| \frac{\tan x}{x} \right|$ is

(a) ∞ **(b)** 0

(c) 1 **(d)** –1

Q. 64. The product of the eigenvalues of the matrix $\begin{pmatrix} 2 & 3 \\ 0 & 7 \end{pmatrix}$ is _______________ (rounded off to one decimal place).

Q. 65. The solution of the ordinary differential equation $\frac{dy}{dx} + 3y = 1$, subject to the initial condition $y = 1$ at $x = 0$, is

(a) $\frac{1}{3}\left(1+2e^{-x/3}\right)$ **(b)** $\frac{1}{3}\left(5-2e^{-x/3}\right)$

(c) $\frac{1}{3}\left(5-2e^{-3x}\right)$ **(d)** $\frac{1}{3}\left(1+2e^{-3x}\right)$

Q. 66. The value of the complex number $i^{-1/2}$ (where $i = \sqrt{-1}$) is

(a) $\frac{1}{\sqrt{2}}(1-i)$ **(b)** $-\frac{1}{\sqrt{2}}i$

(c) $\frac{1}{\sqrt{2}}i$ **(d)** $\frac{1}{\sqrt{2}}(1+i)$

Q. 67. If x, y and z are directions in a Cartesian coordinate system and, $\hat{i}$ $\hat{j}$ and $\hat{k}$ are the respective unit vectors, the directional derivative of the function $u(x, y, z) = x^2 - 3yz$ at the point (2, 0, –4) in the direction $\frac{\left(\hat{i} + \hat{j} - 2\hat{k}\right)}{\sqrt{6}}$ is ______. (rounded off to two decimal places).

Q. 68. Two unbiased dice are thrown. Each dice can show any number between 1 and 6. The probability that the sum of the outcomes of the two dice is divisible by 4 is ______. (rounded off to two decimal places).

Q. 69. The Newton–Raphson method is used to determine the root of the equation $f(x) = e^{-x} - x$. If the initial guess for the root is 0, the estimate of the root after two iterations is ______. (rounded off to three decimal places).

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	(d)	Analytic Functions	Complex Variables
2	(c)	Family of Curve	Differential Equation
3	0	Cauchy's Integral Theorem	Complex Variables
4	3	Eigen Values	Matrix and Determinant
5	11	Random Variable, Mean	Probability and Statistics
6	2	Double Integral	Calculus
7	0.37	Probability Distribution Function	Probability and Statistics
8	(b)	Mean Value Theorem	Calculus
9	(c)	Green's Theorem	Calculus
10	5.25	Cauchy Euler Equation	Differential Equation
11	(c)	Inverse Laplace Transform	Transform Theory
12	(d)	Eigen Values	Linear Algebra
13	(b)	Analytic Function	Complex Variable
14	139	Line Integral	Vector Calculus
15	3	Rank of Matrix	Linear Algebra
16	(c)	Matrix Operation	Linear Algebra
17	(a)	Residue Theorem	Complex Variable
18	45	Divergence	Vector Calculus
19	0.26	Probability	Probability and Statistics
20	(b)	Eigen Values	Linear Algebra
21	(b)	Volume of Solid of Revolution	Calculus
22	(c)	First Order DE	Differential Equation
23	(a)	Normal Distribution	Probability and Statistics
24	63	Trapazoidal Rule	Numerical Analysis
25	(c)	System of Linear Equations	Linear Algebra
26	(a)	Analytic Functions	Complex Variable
27	(b)	Random Variable	Uniform Distribution
28	2.096	Definite Integral	Calculus
29	(d)	Eigen Values	Linear Algebra
30	(c)	Direction Derivative	Calculus
31	(b)	First Order DE	Differential Equations
32	(b)	Analytic Function	Complex Analysis
33	3	Mode	Probability and Statistics
34	(c)	Surface Integral	Vector Calculus
35	(a)	Cauchy Euler Equations	Differential Equations

Q. No.	Answer	Topic Name	Chapter Name
36	(b)	Error Analysis	Numerical Method
37	0.17	Binomial Distributions	Probability and Statistics
38	(a)	Limit	Calculus
39	(a)	Taylor Series	Calculus
40	17.92	Probability	Probability and Statistics
41	(b)	Maxima and Minima	Calculus
42	(a)	Finite Difference Approximation	Numerical Methods
43	(d)	Partial Differentiation	Calculus
44	6	Cauchy Euler Equations	Differential Equations
45	(d)	Euclidean Norm	Calculus
46	(a)	Laplace Transform	Laplace Transform
47	(d)	Limit	Calculus
48	(d)	Curl	Calculus
49	(b)	Uniform Distribution	Probability and Statistics
50	(c)	Volume	Volume
51	(d)	ODE	Differential Equations
52	(c)	Inverse of Matrix	Linear Algebra
53	68.5	Simpsons Rule	Numerical Methods
54	(c)	Sets	Discrete Mathematics
55	(d)	Property of Matrix	Linear Algebra
56	(b)	Equivalence Relations	Discrete Mathematics
57	(c)	Limit	Calculus
58	0.461	Probability	Probability and Statistics
59	(c)	Sets	Discrete Mathematics
60	12	Eigen Values	Linear Algebra
61	0.8	Probability	Probability and Statistics
62	(c)	System of Linear Equations	Linear Algebra
63	(a)	Limit	Calculus
64	14	Eigen Values	Linear Algebra
65	(d)	ODE	Differential Equations
66	(a)	Complex Roots	Complex Variable
67	6.53	Directional Derivative	Calculus
68	0.25	Probability	Probability and Statistics
69	0.566	Newton Raphson Method	Numerical Methods

ENGINEERING MATHEMATICS | GATE | Solved Paper 2019

ANSWERS WITH EXPLANATIONS

1. Option (d) is correct.

$\ln(z)$ is not an analytic function at $z = 0$.

$e^{\frac{1}{z}}$ is not an analytic function at $z = 0$.

$\frac{1}{1-z}$ is also not an analytic function.

But $\cos(z)$ is analytic over the entire complex plane.

$\cos(z) = \cos(x+iy) = \cos x \cos(iy) - \sin x \sin(iy)$

$= \cos x \cos hy - i \sin x \sin hy$; $u + iv \rightarrow$ form.

Where

$u(x, y) = \cos x \cos hy \quad v(x, y) = -\sin x \sin hy$

$u_x = -\sin x \cos hy \qquad v_x = -\cos x \sin hy$

$u_y = \cos x \sinh y \qquad v_y = -\sin x \cos hy$

$u_x = v_y$ and $u_y = -v_x$

2. Option (c) is correct.

Given, $\frac{dy}{dx} = -\left(\frac{x}{y}\right)^n$

For $n = -1$

$\frac{dy}{dx} = -\left(\frac{y}{x}\right)$

$\Rightarrow \quad \frac{dy}{y} = -\frac{dx}{x}$

On integration

$\Rightarrow \quad \ln y = -\ln x + \ln c$

$\Rightarrow \quad \ln yx = \ln c \Rightarrow xy = c$

This represents the rectangular hyperbola.

For $n = 1$

$\frac{dy}{dx} = -\frac{x}{y}$

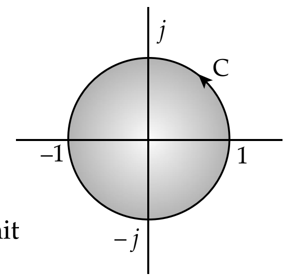
$\Rightarrow \quad ydy = -xdx$

On integration

$x^2 + y^2 = 2c$; this represents the family of circles.

$\Rightarrow \quad x^2 + y^2 = k$

3. Correct answer is [0].



$\frac{1}{2\pi j}\oint\left(z + \frac{1}{z}\right)^2 dz$

$I = \frac{1}{2\pi j}\oint\frac{\left(z^2+1\right)^2}{z^2}\, dz$;

$z = 0$ lies inside the unit circle with

$f(z_0) = (z_2 + 1)^2, z_0 = 0$

By CIT $I = f'(z_0)$

$\Rightarrow I = \left[\frac{d}{dz}\left(z^2+1\right)^2\right]_{z=0} \Rightarrow I = \left[2\left(z^2+1\right).2z\right]_{z=0} = 0$

4. Correct answer is [3].

For triangular matrix, diagonal elements are eigen value

So, eigen values are 2, 1, 3, 2

Distinct eigen values are 2, 1, 3

No. of distinct eigen values is 3

5. Correct answer is [11].

As E $[aX + bY] = a$E [X] + bE [Y]

Given, $E[2X + Y] = 0$ and $E[X + 2Y] = 33$

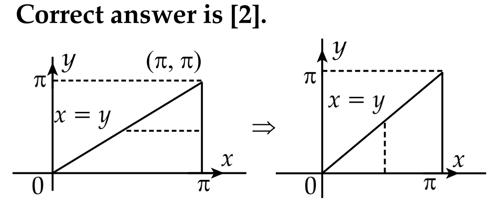
$2E[X] + E[Y] = 0$ and $E[X] + 2E[Y] = 33$

Adding both

$\Rightarrow \; 3E[X] + 3E[Y] = 0 + 33$

$E[X] + E[Y] = \frac{33}{3} = 11$

6. Correct answer is [2].



$I = \int_0^{\pi}\int_y^{\pi}\frac{\sin x}{x}dxdy$

Changing the order of integration

$I = \int_{x=0}^{\pi}\int_{y=0}^{x}\frac{\sin x}{x}dy\; dx = \int_{k=0}^{\pi}\left(\frac{\sin x}{x}\right).\left(y\right)\Big|_0^x dx$

$I = \int_0^{\pi}\frac{\sin x}{x}\cdot(x)dx = \int_0^{\pi}\sin x\, dx = \cos x\Big|_0^{\pi} = 2$

7. Correct answer is [0.367].

Given, CDF, $F_z(x) = \begin{cases} 1-e^{-x} & if x \geq 0 \\ 0 & if x < 0 \end{cases}$

$$f_z(x) = F'_z(x) = \begin{cases} e^{-x} & if x \geq 0 \\ 0 & if x < 0 \end{cases} \rightarrow \text{PDF}$$

Where Z be an exponential random variable with mean 1.

$$P_{req} = \frac{[P(z>2) \cap P(z>1)]}{P(z>1)} = \frac{P[z>2]}{P[z>1]}$$

$$P(z \leq 2) = 1 - e^{-2} \Rightarrow P(z>2) = e^{-2}$$

$$P(z \leq 1) = 1 - e^{-1} \Rightarrow P(z>1) = e^{-1}$$

Required probability $= \frac{P[z>2]}{P[z>1]}$

$$= \frac{e^{-2}}{e^{-1}} = e^{-1} = 0.367$$

8. Option (b) is correct.

For $f(-1) = 0$ condition option C and D are not preferable.

Now for $f'(x) \leq 2$

Option (A) $f(x) \leq \frac{1}{2}|x+1|$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{2}(x+1), & x \geq -1 \\ \frac{-1}{2}(x+1), & x < -1 \end{cases} \text{ then}$$

$$f'(x) = \begin{cases} \frac{1}{2} & x > -1 \\ \frac{-1}{2} & x < 1 \end{cases}$$

Option (B) $f(x) = \begin{cases} 2(x+1), & x \geq -1 \\ -2(x+1), & x < -1 \end{cases}$ then

$$f'(x) = \begin{cases} 2, & x > -1 \\ -2, & x < -1 \end{cases}$$

9. Option (c) is correct.

Here,

$$\int_c (xdy - ydx) = \int_c (-ydx + xdy)$$

By applying Green's theorem

$M = -y; \quad \frac{\partial M}{\partial y} = -1$ and $N = x; \quad \frac{\partial N}{\partial x} = 1$

$$\oint_c \left(\underset{M}{\underset{\downarrow}{-y}}\, dx + \underset{N}{\underset{\downarrow}{x}}\, dy \right) = \iint_R (1-(-1))dxdy$$

$= 2\iint_R dxdy = 2*$(area of region R.)

$=2$[area of rectangle + area of semi-circle]

$$= 2\left[3 \times 2 + \frac{1}{2}\pi(1)^2\right]$$

$$= 12 + \pi$$

10. Correct answer is [5.25].

$(x^2D^2 - 3xD + 3)y = 0 \qquad$ where $D = \frac{d}{dx}$

For $x = e^z \qquad xD = \theta;$

$\ln x = z \qquad x^2D^2 = \theta(\theta-1);$

where $\qquad \theta = \frac{d}{dz}$

$$(\theta(\theta-1) - 3\theta + 3)y = 0$$

$\Rightarrow \qquad (\theta^2 - 4\theta + 3)y = 0$

AE is $\qquad m^2 - 4m + 3 = 0$

$\Rightarrow \qquad m = 1, 3$

The solution is $y = C_1e^z + C_2e^{3z}$

$y = C_1x + C_2x^3 \qquad$ ...(i)

$\because \qquad [x = e^z]$

given $\qquad y(1) = 1$

$\Rightarrow \qquad 1 = C_1 + C_2 \qquad$ ...(ii)

also $\qquad y(2) = 14$

$\Rightarrow \qquad 14 = 2C_1 + 8C_2 \qquad$ ...(iii)

From (ii) and (iii) $C_1 = -1; C_2 = 2$

Putting the value of C_1 and C_2 in (i) $y = -x + 2x^3$

And $\qquad y(1.5) = -(1.5) + 2(1.5)^3$

$\Rightarrow \qquad y(1.5) = 5.25$

11. Option (c) is correct.

$$H(s) = \frac{s+3}{s^2+2s+1} = \frac{s+1+2}{(s+1)^2}$$

$$= \frac{s+1}{(s+1)^2} + \frac{2}{(s+1)^2}$$

$$\therefore H(s) = \frac{1}{s+1} + \frac{2}{(s+1)^2}$$

So, $h(t) = e^{-t} + 2\,t.e^{-t} \qquad$ (frequency shift)

12. Option (d) is correct.

Given, Eigen values of $M_{2\times2}$ are $\lambda = 4, 9$.

$\Rightarrow$ Eigen values of M^2 are λ^2 *i.e.*, $4^2, 9^2$.

So, Eigen values of M^2 are 16 and 81.

13. Option (b) is correct.

Given, region is $|z| \leq 1$.

$f(z) = \frac{z^2-1}{z+2}$; has singularity at $z = -2$ which lies outside $|z| = 1$.

So, $f(z) = \frac{z^2-1}{z+2}$ is analytic in region $|z| \leq 1$.

14. Correct answer is [139].

$$f = 2x^3 + 3y^2 + 4z$$

$$\text{grad } f = i\frac{\partial f}{\partial x} + j\frac{\partial f}{\partial y} + k\frac{\partial f}{\partial z}$$

$$= 6x^2\ i + 6yj + 4k$$

$$\therefore \int_c \text{grad } f.dr = \int_c (6x^2\ i + 6yj + 4k)(dxi + dyj + dzk)$$

$$= \int_c (6x^2 dx + 6ydy + 4dz)$$

$$= \int_{(-3,-3,2)}^{(2,6,-1)} (6x^2 dx + 6ydy + 4dz)$$

Equation of segment formed by $(-3,-3,2)$ and $(2,6,-1)$

$$x = 5t - 3;$$
$$\Rightarrow \quad dx = 5dt$$
$$y = 9t - 3$$
$$\Rightarrow \quad dy = 9dt$$
$$z = -3t + 2$$
$$\Rightarrow \quad dz = -3dt$$

and $t = 0$ to 1

$$= \int_0^1 (6(5t-3)^2 \cdot 5dt + 6(9t-3)9dt + 4(-3dt))$$

$$= \int_0^1 (30(25t^2 - 30t + 9)dt + 54(9t-3)dt - 12dt$$

$$= \int_0^1 (750t^2 - 900t + 270)dt + (486t - 162)dt - 12dt$$

$$\int_0^1 (750t^2 - 414t + 96)dt = 139$$

15. Correct answer is [3].

For Matrix $M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Interchanging Row R_1 and R_3; $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$$R_2 \to R_2 - R_1 \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$R_3 \to R_3 + R_2 \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

With no. of non-zero row in Echelon form = 3

Rank of M = 3

16. Option (c) is correct.

Let $\quad M = [v_1 \quad v_2] = \begin{bmatrix} p & r \\ q & s \end{bmatrix}$

and $\quad M^{-1} = \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix} = \begin{bmatrix} u & v \\ w & x \end{bmatrix}$

$$\because \quad M^{-1}M = I$$

$$\Rightarrow \begin{bmatrix} u & v \\ w & x \end{bmatrix} \cdot \begin{bmatrix} p & r \\ q & s \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} up + vq & ur + vs \\ wp + xq & wr + xs \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$up + vq = 1$$
$$\Rightarrow \quad u_1^T v_1 = 1$$
$$ur + vs = 0$$
$$\Rightarrow \quad u_1^T v_2 = 0$$
$$wp + xq = 0$$
$$\Rightarrow \quad u_2^T v_1 = 0$$
$$wr + xs = 1$$
$$\Rightarrow \quad u_2^T v_2 = 1$$

$\therefore$ both statements are correct.

17. Option (a) is correct.

For $\quad I = \oint_{|z|=5} \frac{z^3 + z^2 + 8}{z+2} dz$

$$f(z) = \frac{z^3 + z^2 + 8}{z+2}$$

$f(z)$ has singularity at $z = -2$ which lies inside the curve $|z| = 5$.

So, $[\text{Res } f(z)]_{z=-2} = \lim_{z \to -2} \left((z+2)\frac{z^3 + z^2 + 8}{z+2} \right) = 4$

Using Cauchy's residue theorem.

$$\oint_c \frac{z^3 + z^2 + 8}{z+2} dz = 2\pi j \times 4 = 8\pi j$$

18. Correct answer is [45].

Given $\vec{A} = 2xi + 3yj + 4zk$ and

$$u = x^2 + y^2 + z^2$$

$$\nabla.(u\vec{A}) = (\nabla u).\vec{A} + u(\nabla.\vec{A}) \quad \text{...(i)}$$

$$\nabla u = \frac{\partial u}{\partial x}i + \frac{\partial u}{\partial y}j + \frac{\partial u}{\partial z}k$$

$$\Rightarrow \quad \nabla u = 2xi + 2yj + 2zk$$

$$(\nabla u).\vec{A} = (2xi + 2yj + 2zk).(2xi + 3yj + 4zk) \quad \text{...(ii)}$$

$$\Rightarrow \quad (\nabla u).\vec{A} = 4x^2 + 6y^2 + 8z^2$$

$$\Rightarrow \quad \nabla.\vec{A} = \frac{\partial A_x}{\partial x}i + \frac{\partial A_y}{\partial y}j + \frac{\partial A_z}{\partial z}k$$

$$\Rightarrow \quad \nabla.\vec{A} = 2 + 3 + 4 = 9$$

$\Rightarrow \quad u(\nabla.\vec{A}) = (x^2 + y^2 + z^2)9$ ...(iii)

Substitute the values of (ii) and (iii) in (i)

$$\text{Div}(u.\vec{A}) = 4x^2 + 6y^2 + 8z^2 + (x^2 + y^2 + z^2)9$$

$$= (13x^2 + 15y^2 + 17z^2)\Big|_{(1,1,1,)} = 45$$

19. Correct answer is [0.26].

$P_{defective} = 0.02$

$n = 50 n$ being large Poisson Distribution

$\therefore \quad \lambda = np = 1$

Let X be the number of defective registers.

$$P[X \geq 2] = 1 - P[X < 2]$$

$$= 1 - \{P(X = 0) + P(X = 1)\}$$

$$= 1 - \left[\frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!}\right] \quad \because f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= 1 - e^{-1}(1+1)$$

$$= 1 - 2e^{-1}$$

$$= 0.26$$

20. Option (b) is correct.

Clearly, this is an upper triangular matrix. Eigen values are same as the diagonal elements of the matrix. *i.e.*, 1, 1, 1. Distinct eigen values is 1. So, number of distinct eigen values is 1.

21. Option (b) is correct.

Given, $y^2 = x;\ 0 \leq x \leq 1$

As Rotation is about x axis.

$$V = \int_0^1 \pi y^2 dx = \pi\int_0^1 x dx = \pi\left[\frac{x^2}{2}\right]_0^1 = \frac{\pi}{2}$$

22. Option (c) is correct.

Given, $\frac{dy}{dx} = -7x^2 y = 0; y(0) = \frac{3}{7}$

$$\int \frac{dy}{y} = -\int 7x^2\ dx$$

$\Rightarrow \quad \ln y = -7\frac{x^3}{3} + c$ ...(i)

At $x=0$,

$y = \frac{3}{7}$ putting in (i)

$\Rightarrow \quad c = \ln\left(\frac{3}{7}\right)$ ...(ii)

Substituting the value of c from (ii) in (i)

$$\Rightarrow \quad \ln y = -7\frac{x^3}{3} + \ln\left(\frac{3}{7}\right)$$

$$\Rightarrow \ln\left(\frac{7y}{3}\right) = -7\frac{x^3}{3} \quad \Rightarrow \frac{7}{3}y = e^{\frac{-7}{3}x^3}$$

$$\Rightarrow \quad y = \frac{3}{7}e^{-\frac{7}{3}x^3}$$

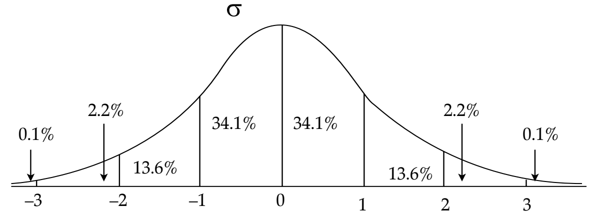
$$y(1) = \frac{3}{7}e^{-\frac{7}{3}}$$

23. Option (a) is correct

Given, $\mu = 440$ mm

$\sigma = 1$ mm

$$Z = \frac{x - \mu}{\sigma}$$



$$Z(x = 438) = \frac{438 - 440}{1} = -2$$

$$P(Z = -2) = 2.28\%$$

$$Z(x = 441) = \frac{441 - 440}{1} = 1$$

$$P(Z = 1) = 84.13\%$$

So, the required percentage of rods lying between 438 mm and 441 mm.

$$P(Z = 1) - P(Z = -2) = 84.13\% - 2.28\% = 81.85\%$$

24. Correct answer is [63].

Let $y = f(x) = x^3$; with $n = 2 \Rightarrow h = \frac{4-2}{2} = 1$

x	2	3	4
$y = f(x)$	8 y_0	27 y_1	64 y_2

According to trapezoidal rule

$$\int_2^4 x^3 dx = \frac{h}{2}\{(y_0 + y_n) + 2(y_1 + y_2 + ...y_{n-1})\}$$

$$= \frac{1}{2}\{y_1 + y_2 - 2y_1\} = \frac{1}{2}\{(8 + 64) + 2(27)\} = 63$$

25. Option (c) is correct.

Given, $\begin{bmatrix} 1 & 1 & 1 \\ a & -a & 3 \\ 5 & -3 & a \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$

This form is $AX = B$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ a & -a & 3 \\ 5 & -3 & a \end{bmatrix}$;

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$$

So, augmented matrix $[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ a & -a & 3 & 5 \\ 5 & -3 & a & 6 \end{array}\right]$

$R_2 \to R_2 - aR_1$
$R_3 \to R_3 - 5R_1$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2a & 3-a & 5-a \\ 0 & -8 & a-5 & 1 \end{array}\right]$$

$R_3 \to aR_3 - 4R_2$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2a & 3-a & 5-a \\ 0 & 0 & a^2-a-12 & 5a-20 \end{array}\right]$$

For infinite solutions

$\rho(A|B) = \rho(A) <$ No. of unknowns

$a^2 - a - 12 = 0$ and $5a - 20 = 0$

$\Rightarrow \quad a = 4, -3$ and $a = 4$.

So, $\quad a = 4$

26. Option (a) is correct.

Let $w = f(z) = u(x, y) + i\,v(x, y)$ be analytic function

For real part; $u = 2x^2 - 2y^2 + 4xy$

Using Milne Thomson Method.

$$\frac{\partial u}{\partial x} = 4x + 4y = \varphi_1(x, y)$$

$$\varphi_1(z, 0) = 4z \quad \text{...(i)}$$

and $\dfrac{\partial u}{\partial y} = -4y + 4x = \varphi_2(x, y)$

$$\varphi_2(z,\ 0) = 4z \quad \text{...(ii)}$$

$$f(z) = \int \left[\varphi_1(z,\ 0) - i\varphi_2(z,\ 0)\right] + c$$

From (i) and (ii)

$$= \int [4z - i4z]dz + c = 4(1-i)\int z\,dz + c$$

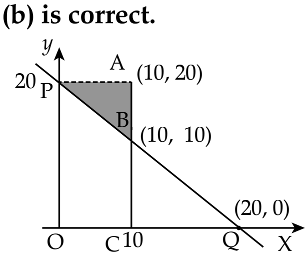
$$= 4(1-i)\frac{z^2}{2} + c$$

$$= 2(1-i)\left[x^2 - y^2 + 2ixy\right] + c$$

$$f(z) = \left(2x^2 - 2y^2 + 4xy\right) + i(4xy - 2x^2 + 2y^2 + c)$$

So, $\quad v = 4xy - 2x^2 + 2y^2 + c$

27. Option (b) is correct.



Variable x values $\Rightarrow 0 \le x \le 10$

Variable y values $\Rightarrow 0 \le y \le 20$

Variable x and y together constitute a rectangle OPAC.

With PQ: $x + y = 20$

$$P_{req} = P\left[(x+y) > 20\right] = \frac{\text{shaded area}}{\text{total area}} = \frac{50}{200} = 0.25$$

28. Correct answer is [2.096].

By applying ILATE Rule

$$I = \left(\ln x . \int x\,dx\right)_1^e - \int_1^e \frac{1}{x} . \left(\int x\,dx\right) dx$$

$$= \left(\ln e . \frac{e^2}{2} - \ln(1) . \frac{1^2}{2}\right) - \int_1^e \left(\frac{x}{2}\right) dx$$

$$= \frac{e^2}{2} - \left[\frac{x^2}{4}\right]_1^e = \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} = 2.097$$

29. Option (d) is correct.

Given, $\quad [A][X] = [R]$

$$\Rightarrow \begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 32 \\ 16 \\ 64 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = 16\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$\because \quad [A][X] = \lambda[X]$

So, one of the eigen value is

$$\lambda = 16$$

30. Option (c) is correct.

Given, $f(x, y) = x^2 + y^2$

Direction vector; $\vec{a} = (1,1) - (0,0) = \vec{i} + \vec{j}$

Let the point Q $(x,\ y) = (1,1)$

The directional derivative of $f(x, y)$ at point (1, 1)

$$\Rightarrow (\nabla f)_{(1,1)} \frac{\vec{a}}{|\vec{a}|} = \left(2x\vec{i} + 2y\vec{j}\right)_{(1,1)} \frac{\vec{i} + \vec{j}}{\sqrt{1^2 + 1^2}}$$

$$= (2i + 2j)\frac{(i+j)}{\sqrt{2}} = \frac{2+2}{\sqrt{2}} = 2\sqrt{2}$$

31. Option (b) is correct.

Given, $\dfrac{dy}{dx} + 4y = 5; 0 \le x \le 1$

Equation is in the form of $\dfrac{dy}{dx} + P(x)y = Q(x)$

Here P = 4 and Q = 5

$$IF = e^{\int 4dx} = e^{4x}$$

General solution $y.e^{4x} = \int 5.e^{4x} dx + c$

$$\Rightarrow \quad y.e^{4x} = \frac{5}{4}.e^{4x} + c \quad ...(i)$$

At $x = 0$; $y = 2.25$ put in (i)

$$2.25 = \frac{5}{4} + c \Rightarrow c = 1$$

Using the value of c in (i)

$$ye^{4x} = \frac{5}{4}e^{4x} + 1 \quad \Rightarrow y = 1.25 + e^{-4x}$$

32. Option (b) is correct.

Given, $f(z) = u(x, y) + iv(x, y)$ is an analytic function.

Then $u(x, y)$, $v(x, y)$ satisfies the Cauchy-Riemann equation as

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

33. Correct answer is [3].

Given, mean = x

$$x = \frac{3 + x + 2 + 4}{4}$$

$$\Rightarrow 4x = 9 + x \Rightarrow \quad 3x = 9$$

$$\Rightarrow \quad x = 3$$

Therefore, the data is 3, 3, 2,4. And as mode is the value which occur in the data maximum number of times. So, mode is 3.

34. Option (c) is correct.

Given, $\vec{u} = \frac{1}{3}\left(-y^3\hat{i} + x^3\hat{j} + z^3\hat{k}\right)$

$$\Rightarrow \nabla \times \vec{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{-y^3}{3} & \frac{x^3}{3} & \frac{z^3}{3} \end{vmatrix} = 0i + 0j + \left(x^2 + y^2\right)\hat{k}$$

Surface, $\phi = x^2 + y^2 + z^2 - 1 = 0$

$$\hat{n} = \frac{\nabla\phi}{|\nabla\phi|_{z\geq 0}} = \frac{2x\hat{i} + 2y\hat{j} + 2z\hat{k}}{\sqrt{(2x)^2 + (2y)^2 + (2z)^2}} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$[\because x^2 + y^2 + z^2 = 1]$$

$$\Rightarrow \left(\nabla \times \vec{u}\right).\hat{n} = (x^2 + y^2)z$$

$$\int((\nabla \times \vec{u}).\hat{n})ds = \iint z.\left(x^2 + y^2\right)ds$$

Let P be the projection of surface on $x - y$ plane.

$$\int_s (\nabla \times \vec{u})d\vec{s} = \iint_P z.\left(x^2 + y^2\right)\frac{dxdy}{\left|\hat{n}.\hat{k}\right|}$$

$$= \iint_P \left(x^2 + y^2\right)dx\,dy \text{ with} \left|\hat{n}.\hat{k}\right| = z \quad ...(i)$$

Where $P = x^2 + y^2 = 1$

Using polar coordinates in (i);

$$x = r\cos\theta;$$
$$y = r\sin\theta;$$
$$dxdy = rdrd\theta$$
$$\therefore \quad x^2 + y^2 = 2$$

$$= \iint_P r^2 rdrd\theta = \int_{r=0}^{1}\int_{\theta=0}^{2\pi} r^3 drd\theta = \left(\frac{r^4}{4}\right)_0^1 (\theta)_0^{2\pi}$$

$$= \left(\frac{1}{4}\right)2\pi = \frac{\pi}{2}$$

35. Option (a) is correct.

Given, $(x^2D^2 - 2Dx + 2)y = 4,$ ...(i)

where; $\frac{d}{dx} = D$

This is Euler Cauchy's DE.

Let $x = e^z$

$$\Rightarrow \quad \log x = z;$$
$$xD = \theta$$
$$x^2D^2 = \theta(\theta - 1) \quad ...(ii)$$

where $\frac{d}{dz} = \theta$

Substitute the values from (ii) in (i)

$[\theta(\theta - 1) - 2\theta + 2]y = 4$

$\Rightarrow \quad [\theta^2 - 3\theta + 2]y = 4$

$$\Rightarrow \quad f(\theta)y = Q(z);$$
$$Q(z) = 4$$
$$AE = m^2 - 3m + 2 = 0$$
$$\Rightarrow \quad m = 1, 2$$
$$y_{CF} = C_1e^{2z} + C_2e^z$$
$$= C_1x^2 + C_2x \quad [\because x = e^z]$$

as $y_{PI} = \frac{Q(z)}{f(\theta)_{\theta=a}}$

$$\therefore \quad y_{PI} = \frac{1}{\theta^2 - 3\theta + 2}4.e^{0.z}$$

Put $\theta = 0$ then, $[\because a = 0]$

So, $y_{PI} = \frac{4}{2}$

$$\Rightarrow \quad y_{PI} = 2$$
$$\therefore \quad y = y_{CF} + y_{PI}$$
$$\Rightarrow \quad C_1x^2 + C_2x + 2$$

36. Option (b) is correct.

Given, $f(x) = \cos x$

Exact value

$$f'(x) = -\sin x$$

$f'(x)$ at $x = \frac{\pi}{6}$

$$f'\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

Approximate value

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

$$= \frac{\cos(x+0.1) - \cos(x-0.1)}{2(0.1)}$$

$$\frac{(\cos x \cos(0.1) - \sin x \sin(0.1)) - (\cos(0.1)\cos x + \sin x \sin(0.1))}{2(0.1)}$$

$$= \frac{-\sin x \sin(0.1) - \sin x \sin(0.1)}{0.2}$$

$$= \frac{-2\sin x \sin(0.1)}{2(0.1)}$$

$$f'\left(\frac{\pi}{6}\right) = -0.49916$$

Error = exact value – Approximate value

= –0.5 – (–0.49916)

= –0.000 84

$$\% \text{Error} = \frac{-0.00084}{-0.5} \times 100\% = 0.168\%$$

$0.1\% < 0.168\% < 1\%$

37. Correct answer is [0.17].

Given, $n = 15, p = 0.05, q = 1 - p = 0.95$

$P(x \geq 2) = 1 - P(x < 2)$

$= 1 - \{P(x = 0) + P(x = 1)\}$

$= 1 - \{{}^{15}C_0 p^0 q^{15} + {}^{15}C_1 p^1 q^{14}\}$

(using binomial distribution)

$= 1 - \{q^{15} + 15pq^{14}\}$

$= 1 - \{0.95^{15} + 15(0.05)(0.95)^{14}\}$

$= 1 - \{0.46 + 0.37\}$

$= 1 - 0.83 = 0.17$

38. Option (a) is correct.

Given, $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)$

$$= \lim_{x\to 0}\left(\frac{4x\frac{\sin 4x}{4x}}{\frac{\sin 2x}{2x}.2x}\right) = \frac{4}{2} = 2$$

$$\lim_{x\to 0}\frac{\tan x}{x} = 1$$

39. Option (a) is correct.

Taylor series expansion for small h of $f(x+h)$

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) \ldots + \ldots$$

40. Correct answer is [17.92].

Uncertainty $= 1 - q^n$

$\Rightarrow \quad 0.25 = 1 - q^5$

$\Rightarrow \quad q^5 = 0.75$

$\Rightarrow \quad 5\log q = \log(0.75)$

$\Rightarrow \quad \log q = \frac{-0.1249}{5}$

$\Rightarrow \quad q = 0.9442$

$\Rightarrow \quad p = 1 - q$

$\Rightarrow \quad 0.0558$

$\Rightarrow \quad p = \frac{1}{T} \Rightarrow T = 17.92$ years

41. Option (b) is correct.

Let $\quad y = x^{\frac{1}{x}}, x > 0$

Taking log on both sides.

$\Rightarrow \quad \log y = \frac{\log x}{x}$

$$f(x) = \frac{\log x}{x}$$

$$f'(x) = \frac{x\left(\frac{1}{x}\right) - \log x}{x^2} = \frac{1 - \log x}{x^2}$$

$$f'(x) = 0 \Rightarrow \frac{1 - \log x}{x^2} = 0$$

$\Rightarrow \quad 1 - \log x = 0$

$\Rightarrow \quad \log x = 1$

$\Rightarrow \quad x = e$

$$f''(x) = \frac{x^2\left(\frac{-1}{x}\right) - (1 - \log x)(2x)}{x^4} = \frac{-3x + 2x\log x}{x^4}$$

$$f''(e) = \frac{-3e + 2e\log .e}{e^4} = \frac{-e}{e^4} < 0$$

Therefore, $y = \sqrt[x]{x}$ is maximum at $x = e$ as with $f(x)$ maximum y will be maximum

42. Option (a) is correct.

Given, $\frac{\partial v}{\partial t} = \beta \frac{\partial^2 v}{\partial x^2}$

Putting time derivative with a forward difference and a spatial derivative with a central difference we obtain

$$= \frac{(v_i^{(n+1)} - v_i^{(n)})}{\Delta t} = \left[\frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{(\Delta x)^2}\right]\beta$$

43. Option (d) is correct.

Taking partial derivative of both equations w.r.t. x.

$$1=\frac{\partial\psi}{\partial x}\ln\phi+\frac{\psi}{\phi}\frac{\partial\phi}{\partial x}\Rightarrow 1-\frac{\partial\psi}{\partial x}\ln\phi=\frac{\psi}{\phi}\frac{\partial\phi}{\partial x} \quad \text{...(i)}$$

$$0=\frac{\phi}{\psi}\frac{\partial\psi}{\partial x}+\ln\psi\frac{\partial\phi}{\partial x}\Rightarrow -\frac{1}{\ln\psi}\frac{\partial\psi}{\partial x}=\frac{\psi}{\phi}\frac{\partial\phi}{\partial x} \quad \text{...(ii)}$$

Comparing the value of $\frac{\psi}{\phi}\frac{\partial\phi}{\partial x}$

$$1-\frac{\partial\psi}{\partial x}\ln\phi=-\frac{1}{\ln\psi}\frac{\partial\psi}{\partial x}$$

$$\Rightarrow \quad 1=-\frac{1}{\ln\psi}\frac{\partial\psi}{\partial x}+\frac{\partial\psi}{\partial x}\ln\phi$$

$$\Rightarrow \quad 1=\left(-\frac{1}{\ln\psi}+\ln\phi\right)\frac{\partial\psi}{\partial x}$$

$$\Rightarrow \quad 1=\left(\frac{-1+\ln\phi\ln\psi}{\ln\psi}\right)\frac{\partial\psi}{\partial x}$$

$$\frac{\ln\psi}{\ln\phi\ln\psi-1}=\frac{\partial\psi}{\partial x}$$

44. Correct answer is [6].

Given, differential equation is

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0;$$

$$y(1)=0,$$

$$y(2)=2$$

Let $\quad x=e^z$

$\Rightarrow \quad \log x=z$

$$\frac{d}{dx}=\text{D},\frac{d}{dz}=\theta$$

$x\text{D}=\theta;\ x^2\text{D}^2=\theta(\theta-1)$

then DE $[\theta(\theta-1)-2\theta+2]y=0$

$\Rightarrow \quad [\theta^2-3\theta+2]y=0$

$\text{AE}\rightarrow m^2-3m+2=0\Rightarrow m=1,2$

Solution $\quad \rightarrow y=\text{C}_1e^z+\text{C}_2e^{2z}$

$$y=\text{C}_1x+\text{C}_2x^2$$

Given $\quad y(1)=0$

$\Rightarrow \quad 0=\text{C}_1+\text{C}_2 \quad$ ...(i)

$$y(2)=2$$

$\Rightarrow \quad 2=2\text{C}_1+4\text{C}_2 \quad$ ...(ii)

From (i) and (ii)

$$\text{C}_1=-1,\text{C}_2=1$$

$$y=-x+x^2$$

$$y(3)=-3+(3)^2=6$$

45. Option (d) is correct.

$$\text{A}=\begin{bmatrix}4\\-2\\-6\end{bmatrix}$$

$$\text{Norm A}=\|\text{A}\|=\sqrt{(4)^2+(-2)^2+(-6)^2}=\sqrt{56}$$

46. Option (a) is correct.

$$\text{L}\{\sin h(\text{at})\}=\frac{a}{s^2-a^2}$$

47. Option (d) is correct.

Here

$$\lim_{x\to0}\left(2-\frac{x^2}{3}\right)=2$$

$$\lim_{x\to0}(2)=2$$

So, by Squeeze theorem

$$\lim_{x\to0}\left(\frac{x\sin x}{1-\cos x}\right)=2$$

48. Option (d) is correct.

Given, $\vec{\text{A}}=2x^2y\hat{i}+5z^2\hat{j}-4yz\hat{k}$

$$\text{curl}\vec{\text{A}}=\nabla\times\vec{\text{A}}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\\frac{\partial}{\partial x}&\frac{\partial}{\partial y}&\frac{\partial}{\partial z}\\2x^2y&5z^2&-4yz\end{vmatrix}$$

$$=(-4z-10z)\hat{i}-(0-0)\hat{j}+(0-2x^2)\hat{k}$$

$$\nabla\times\vec{\text{A}}=-14z\hat{i}-2x^2\hat{k}$$

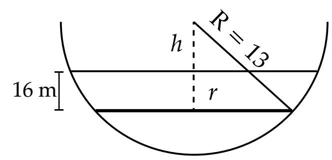
49. Option (b) is correct.

PDF of uniform distribution in $[x, y]$ is

$$f(t)=\begin{cases}\dfrac{1}{y-x} & x\le t\le y\\ 0 & \text{otherwise}\end{cases}$$

50. Option (c) is correct.

$$\text{V}=\int_7^{13}\pi r^2dh$$



By Pythagoras theorem

$$r^2=\text{R}^2-h^2$$

$$\text{So, V}=\pi\int_7^{13}(\text{R}^2-h^2)dh=\pi\left[\text{R}^2h-\frac{h^3}{3}\right]_7^{13}$$

$$=\pi\left\{13^2(13-7)-\frac{1}{3}\left(13^3-7^3\right)\right\}$$

$$=396\pi$$

51. Option (d) is correct.

Given, $\frac{dy}{dx}(x \ln x) = y$

$$\frac{dy}{y} = \frac{dx}{(x \ln x)}$$

$$\int \frac{dy}{y} = \int \frac{dx}{(x \ln x)}$$

Putting $\ln x = t \Rightarrow \frac{1}{x} dx = dt$

$\Rightarrow \ln y = \int \frac{1}{t} dt + \ln k$

$\Rightarrow \ln y = \ln t + \ln k = \ln kt$

Taking antilog $y = kt$

$$y = k \ln x$$

52. Option (c) is correct.

Given $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

$$|A| = 2(12-2) - 3(16-1) + 4(8-3) = -5$$

$$[M_{ij}]_{i \times j} = \text{Minors of } A = \begin{bmatrix} 10 & 15 & 5 \\ 4 & 4 & 1 \\ -9 & -14 & -6 \end{bmatrix}$$

Cofactors of $A = \begin{bmatrix} 10 & -15 & 5 \\ -4 & 4 & -1 \\ -9 & 14 & -6 \end{bmatrix}$

$[\because C_{ij} = (-1)_{i+j} M_{ij}]$

Adjoint of A = $(\text{Cofactors of } A)^T = \begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$

$$A^{-1} = \frac{\text{Adj}(A)}{|A|} \Rightarrow A^{-1} = \begin{bmatrix} -2 & \frac{4}{5} & \frac{9}{5} \\ 3 & \frac{-4}{5} & \frac{-14}{5} \\ -1 & \frac{1}{5} & \frac{6}{5} \end{bmatrix}$$

53. Correct answer is [68.5].

$$\text{Area} = A = \frac{d}{3}\{(y_0 + y_n) + 4\underbrace{(y_1 + y_3 + y_5 ....)}_{\text{odd terms}} + 2\underbrace{(y_2 + y_4 + ...)}_{\text{even terms}}\}$$

$$= \frac{6}{3}\{(1.12 + 1.15) + 2(2.04 + 2.14) + 4(1.67 + 2.34 + 1.87)\}$$

$$= 2\{34.25\} = 68.5 \text{ m}^2$$

54. Option (c) is correct.

Given, U= {1, 2, ……. n}

$$A = \{(x, X) | x \in X \text{ and } X \subset U\}$$

No. of non–empty subset of

$X = C(n,1) + C(n,2) + ... + C(n,n)$

No. of elements in $A = \sum_{k=1}^{n} kC(n,k) = \sum_{k=1}^{n} k \begin{pmatrix} n \\ k \end{pmatrix}$

By combinational Identity

$$\sum_{k=1}^{n} k\, C(n,k) = n.2^{n-1}$$

$\therefore$ both statements are correct.

55. Option (d) is correct.

If matrix X is invertible.

This means $\det(X) \neq 0$.

And this is true otherway around.

So, I and II equivalent statements

56. Option (b) is correct.

Given, G be an arbitrary group. Consider two relations R_1 and R_2.

$R_1 : \forall\, a, b \in G, aR_1 b$ if and only if $\exists g \in G$ such that $a = g^{-1}bg$.

This statement is reflexive as $a = g^{-1}ag$ can be satisfied by $g = e$, identity e always exists in a group.

This is also symmetric as $aRb \Rightarrow a = g^{-1}bg$

for some g $\Rightarrow b = gag^{-1} = (g^{-1})^{-1} ag^{-1}$

g^{-1} will always exists for every $g \in G$.

Transitive : aRb and $bRc \Rightarrow a = g_1^{-1}bg_1$ and

$b = g_2^{-1}cg_2$ for some $g_1 g_2 \in G$

$a = g_1^{-1}g_2^{-1}cg_2g_1 = (g_2g_1)^{-1} cg_2g_1$

$g_1 \in G, g_2 \in G \Rightarrow g_2\, g_1 \in G$

as group is closed so aRb and $bRc \Rightarrow aRc$

$\Rightarrow$ So, it is transitive.

i.e. R_1 is equivalence relation.

R_2 is not equivalence it need not be reflexive,

$\because aR_2a \Rightarrow a = a^{-1} \forall a$ which is not true in a group.

57. Option (c) is correct.

Given, $\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3} \to \left(\frac{0}{0}\right)$ form

Applying LH Rule $\lim_{x \to 3} \frac{4x^3}{4x - 5} = \frac{108}{7}$

58. Correct answer is [0.461].

To represent 4 – digit binary number, we need to represent 1 to 13. In which 7 numbers have MSB '0' and 6 have MSB '1'.

Required Probability

$$= \text{Both MSB1} + \text{Both MSB0}$$

$$= \frac{6}{13}\cdot\frac{6}{13}+\frac{7}{13}\cdot\frac{7}{13}$$

$$= 0.5029 \approx 0.503$$

59. Option (c) is correct.

"For each prime 'x', there exists prime 'w' where $w>x$".

S_1 : {1, 2, 3…} being a finite set fail to satisfy the ϕ because when x = 97 there is no w exists.

S_2 and S_3 are infinite sets, so for every x we can find w.

$\therefore S_2$ and S_3 satisfy the ϕ.

60. Correct answer is [12].

We know that product of eigen values = determinant of matrix.

Given matrix R can be written as

$$\begin{bmatrix} 1 & 2 & 2^2 & 2^3 \\ 1 & 3 & 3^2 & 3^3 \\ 1 & 4 & 4^2 & 4^3 \\ 1 & 5 & 5^2 & 5^3 \end{bmatrix}$$

$R_2 - R_1$; $R_3 - R_1$; $R_4 - R_1$ we have

$$= \begin{bmatrix} 1 & 2 & 2^2 & 2^3 \\ 0 & 3-2 & 3^2-2^2 & 3^3-2^3 \\ 0 & 4-2 & 4^2-2^2 & 4^3-2^3 \\ 0 & 5-2 & 5^2-2^2 & 5^3-2^3 \end{bmatrix}$$

$$= (3-2)\cdot(4-2)\cdot(5-2)\begin{bmatrix} 1 & 5 & 19 \\ 1 & 6 & 28 \\ 1 & 7 & 39 \end{bmatrix}$$

$R_3 \to R_3 - R_1$, $R_2 \to R_2 - R_1$

$$= 1\cdot 2\cdot 3\begin{bmatrix} 1 & 5 & 19 \\ 0 & 1 & 9 \\ 0 & 2 & 20 \end{bmatrix} = 1\cdot 2\cdot 3\cdot 1\begin{vmatrix} 1 & 9 \\ 2 & 20 \end{vmatrix}$$

$$= 1 \cdot 2 \cdot 3 \cdot 2 = 12$$

61. Correct answer is [0.8].

$$\text{PDF of } Y = f(y) = \begin{cases} \dfrac{1}{6-1} & 1 \le y \le 6 \\ 0 & \text{otherwise} \end{cases}$$

Given, $3x^2 + 6xY + 3Y + 6$ polynomial in x has only real roots.

For real roots $D \ge 0$

$$b^2 - 4ac \ge 0$$

$$(6Y)^2 - 4.3.(3Y+6) \ge 0$$

$$\Rightarrow 36Y^2 - 36Y - 72 \ge 0$$

$$\Rightarrow \quad Y^2 - Y - 2 \ge 0$$

$$\Rightarrow (Y+1)(Y-2) \ge 0$$

$$Y \in (-\infty, -1] \cap [2, \infty)$$

$\Rightarrow Y \in [2, 6)$, as for $(-\infty, -1]$ $f(y) = 0$

$\because$ y is uniformly distributed function.

$$f(y) = \frac{1}{5},\ 1 < y < 6$$

$$P_{Req} = \int_2^6 \frac{1}{5}dy = \frac{1}{5}y\Big|_2^6 = \frac{4}{5} = 0.8$$

62. Option (c) is correct.

A system of n homogeneous linear equations containing n unknown will have non trival solution if and only if $\Leftrightarrow |A| = 0$.

63. Option (a) is correct.

$$\lim_{x\to\frac{\pi}{2}}\left|\frac{\tan x}{x}\right| = \left|\frac{\tan\frac{\pi}{2}}{\frac{\pi}{2}}\right| = \infty$$

64. Correct answer is [14].

Product of eigen values of matrix $= |A|$

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 7 \end{bmatrix}$$

$$|A| = 14 - 0 = 14$$

65. Option (d) is correct.

Given, $\dfrac{dy}{dx} + 3y = 1$ ...(i)

From (i) P = 3, Q = 1

For $\dfrac{dy}{dx} + Py = \theta$

$$IF = e^{\int 3dx} = e^{3x}$$

Solution of DE $y.e^{3x} = \int 1.e^{3x}$

$$\Rightarrow \quad y.e^{3x} = \frac{e^{3x}}{3} + c$$

$$\Rightarrow \quad y = \frac{1}{3} + c.e^{-3x} \quad \text{...(ii)}$$

At $x = 0, y = 1$

$$1 = \frac{1}{3} + c \Rightarrow c = \frac{2}{3}$$

Substitute the value of c in (ii)

$$y = \frac{1}{3} + \frac{2}{3}.e^{-3x} \Rightarrow y = \frac{1}{3}\left(1 + 2e^{-3x}\right)$$

66. Option (a) is correct.

$$i^{\frac{-1}{2}} = \left[e^{i\left(\frac{\pi}{2}\right)}\right]^{\frac{-1}{2}} = e^{i\left(\frac{-\pi}{4}\right)} = \cos\left(\frac{-\pi}{4}\right) + i\sin\left(\frac{-\pi}{4}\right)$$

$$= \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}(1-i)$$

67. Correct answer is [6.53].

Given, $u(x, y, z) = x^2 - 3yz$

$$\nabla u = 2x\hat{i} - 3z\hat{j} - 3y\hat{k}$$

$$\nabla u_{(2,0,-4)} = \left(4\hat{i} + 12\hat{j}\right)$$

$$\text{D.D.}(u) = \nabla u_{(2,0,4)} \cdot \frac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$$

$$= \left(4\hat{i} + 12\hat{j}\right) \cdot \frac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$$

$$= \frac{4+12}{\sqrt{6}} = \frac{16}{\sqrt{6}} = 6.53$$

68. Correct answer is [0.25].

Given, Two unbiased dice are thrown.

Total number of outcomes are 36.

Events where sum of the outcomes of the two dice is divisible by 4 = [(1, 3) (3, 1), (2, 2), (2, 6) (6, 2) (3, 5) (5, 3) (4, 4) (6, 6)] = Favourable outcomes = 9

Probability

$$= \frac{\text{No. of favourable outcomes}}{\text{total outcomes}} = \frac{9}{36} = \frac{1}{4} = 0.25$$

69. Correct answer is [0.566].

Given,

$f(x) = e^{-x} - x$ and $x_0 = 0$

$f'(x) = -e^{-x} - 1$

Newton Raphson method, the iteration formula.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

First iteration

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$\Rightarrow \quad x_1 = 0 - \frac{1}{-1-1} = -\frac{1}{-2} = 0.5$$

Second iteration

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\Rightarrow x_2 = 0.5 - \frac{f(0.5)}{f'(0.5)}$$

$$= 0.5 - \frac{0.107}{-1.607} = 0.566$$