ENGINEERING MATHEMATICS GATE Question Paper 2020

ELECTRONICS AND COMMUNICATION (EC) P1

Q. 1. If $V_1, V_2, \ldots, V_6$ are six vectors in $\mathbb{R}^4$, which one of the following statements is FALSE?

(a) It is not necessary that these vectors span $\mathbb{R}^4$.

(b) These vectors are not linearly independent.

(c) Any four of these vectors form a basis for $\mathbb{R}^4$.

(d) If $\{V_1, V_3, V_5, V_6\}$ spans, $\mathbb{R}^4$ then it forms a basic for $\mathbb{R}^4$.

Q. 2. For a vector field, $\vec{A}$ which one of the following is FALSE?

(a) $\vec{A}$ is solenoidal if $\nabla \cdot \vec{A} = 0$.

(b) $\nabla \times \vec{A}$ is another vector field.

(c) $\vec{A}$ is irrotational if $\nabla^2 \times \vec{A} = 0$.

(d) $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$.

Q. 3. The partial derivative of the function $f(x, y, z) = e^{1-x\cos y} + xze^{-1/(1+y^2)}$ with respect to x at point $(1, 0, e)$ is

(a) –1 **(b)** 0

(c) 1 **(d)** $\frac{1}{e}$

Q. 4. The general solution of $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$ is

(a) $y = C_1e^{3x} + C_2e^{-3x}$

(b) $y = (C_1 + C_2x)e^{-3x}$

(c) $y = (C_1 + C_2x)e^{3x}$

(d) $y = C_1e^{3x}$

Q. 5. The two sides of a *fair* coin are labeled as 0 and 1. The coin is tossed two times independently. Let M and N denote the labels corresponding to the outcomes of those tosses. For a random variable X, defined as X = min (M, N), the expected value E(X) (rounded off to two decimal places) is ______.

Q. 6. Consider the following system of linear equations.

$$x_1 + 2x_2 = b_1; \quad 2x_1 + 4x_2 = b_2;$$
$$3x_1 + 7x_2 = b_3; \quad 3x_1 + 9x_2 = b_4$$

Which one of the following conditions ensures that a solution exists for the above system?

(a) $b_2 = 2b_1$ and $6b_1 - 3b_3 + b_4 = 0$

(b) $b_3 = 2b_1$ and $6b_1 - 3b_3 + b_4 = 0$

(c) $b_2 = 2b_1$ and $3b_1 - 6b_3 + b_4 = 0$

(d) $b_3 = 2b_1$ and $3b_1 - 6b_3 + b_4 = 0$

Q. 7. Which one of the following options contains two solutions of the differential equation $\frac{dy}{dx} = (y-1)x$?

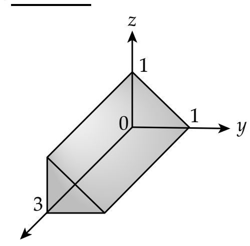
(a) $\ln|y-1| = 0.5x^2 + C$ and $y = 1$

(b) $\ln|y-1| = 2x^2 + C$ and $y = 1$

(c) $\ln|y-1| = 0.5x^2 + C$ and $y = -1$

(d) $\ln|y-1| = 2x^2 + C$ and $y = -1$

Q. 8. For the solid S shown below, the value of $\iiint_S x\,dx\,dy\,dz$ (rounded of to two decimal places) is______.



ELECTRICAL ENGINEERING (EE) P1

Q. 9. Which of the following is true for all possible non-zero choices of integers $m, n;\ m \neq n$, or all possible non-zero choices of real numbers. $p, q;\ p \neq q$, as applicable?

(a) $\frac{1}{\pi}\int_0^{\pi} \sin m\theta \sin n\theta \, d\theta = 0$

(b) $\frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \sin p\theta \sin q\theta \, d\theta = 0$

(c) $\frac{1}{2\pi}\int_{-\pi}^{\pi} \sin p\theta \cos q\theta \, d\theta = 0$

(d) $\lim_{\alpha \to \infty} \frac{1}{2\alpha}\int_{-\alpha}^{\alpha} \sin p\theta \sin q\theta \, d\theta = 0$

Q. 10. The value of following complex integral, with C representing the unit circle centred at origin in the counterclockwise sense, is:

$$\int_c \frac{z^2+1}{z^2-2z}dz$$

(a) $8\pi i$ **(b)** $-8\pi i$
(c) $-\pi i$ **(d)** πi

Q. 11. Consider the initial value problem below. The value of y at $x = \ln(2)$, (rounded off to 3 decimal places) is ______.

$$\frac{dy}{dx} = 2x - y, \qquad y(0) = 1$$

Q. 12. For real numbers, x and y, with $y = 3x^2 + 3x + 1$, the maximum and minimum value of y for $x \in [-2, 0]$ are respectively,

(a) 7 and $\frac{1}{4}$ **(b)** 7 and 1
(c) -2 and $\frac{-1}{2}$ **(d)** 1 and $\frac{1}{4}$

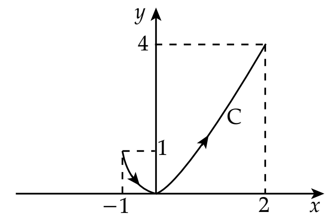
Q. 13. The vector function expressed by
$F = a_x(5y - k_1 z) + a_y(3z + k_2 x) + a_z(k_3 y - 4x)$
Represents a conservative field, where a_x, a_y, a_z are unit vectors along x, y and z directions, respectively. The values of constants k_1, k_2, k_3 are given by
(a) $k_1 = 3, k_2 = 3, k_3 = 7$
(b) $k_1 = 3, k_2 = 8, k_3 = 5$
(c) $k_1 = 4, k_2 = 5, k_3 = 3$
(d) $k_1 = 0, k_2 = 0, k_3 = 0$

Q. 14. The number of purely real elements in a lower triangular representation of the given 3 × 3 matrix, obtained through the given decomposition is______.

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}^T$$

(a) 5 **(b)** 6
(c) 8 **(d)** 9

Q. 15. Let a_x and a_y be unit vectors along x and y directions, respectively. A vector function is given by
$F = a_x y - a_y x$
The line integral of the above function
$\int_c F.dl$
along the curve C, which follows the parabola $y = x^2$ as shown below is ______ (rounded off to 2 decimal places).



MECHANICAL ENGINEERING (ME) P1

Q. 16. Multiplication of real valued square matrices of same dimension is
(a) associative.
(b) commutative.
(c) always positive definite.
(d) not always possible to compute.

Q. 17. The value of $\lim_{x\to 1}\left(\frac{1-e^{-c(1-x)}}{1-x\,e^{-c(1-x)}}\right)$ is

(a) c **(b)** $c + 1$
(c) $\frac{c}{c+1}$ **(d)** $\frac{c+1}{c}$

Q. 18. The Laplace transform of a function $f(t)$ is $L(f) = \frac{1}{(s^2+\omega^2)}$. Then $f(t)$ is

(a) $f(t) = \frac{1}{\omega^2}(1-\cos\omega t)$

(b) $f(t) = \frac{1}{\omega}\cos\omega t$

(c) $f(t) = \frac{1}{\omega}\sin\omega t$

(d) $f(t) = \frac{1}{\omega^2}(1-\sin\omega t)$

Q. 19. Which of the following function $f(z)$, of the complex variable z, is NOT analytic at all the points of the complex plane?
(a) $f(z) = z^z$ **(b)** $f(z) = e^z$
(c) $f(z) = \sin z$ **(d)** $f(z) = \log z$

Q. 20. The velocity field of an incompressible flow in a Cartesian system is represented by
$\vec{V} = 2(x^2 - y^2)\hat{i} + v\hat{j} + 3\hat{k}$
which one of the following expressions for v is valid?
(a) $-4xz + 6xy$ **(b)** $-4xy - 4xz$
(c) $4xz - 6xy$ **(d)** $4xy + 4xz$

Q. 21. For three vectors $\vec{A} = 2\hat{j} - 3\hat{k}, \vec{B} = -2\hat{i} + \hat{k}$ and $\vec{C} = 3\hat{i} - j$, where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along the axes of a right–handed rectangular/Cartesian coordinate system, the value of $(\vec{A}\cdot(\vec{B}\times\vec{C})+6)$ is ______.

Q. 22. A company is hiring to fill four managerial vacancies. The candidates are five men and three women. If every candidate is equally likely to be chosen then the probability that at least one women will be selected is______ (round off to 2 decimal places).

Q. 23. The evaluation of the definite integral $\int_{-1}^{1.4} x|x|\,dx$ by using Simpson's 1/3rd (one-third) rule with step size $h = 0.6$ yields

(a) 0.914 **(b)** 1.248

(c) 0.581 **(d)** 0.592

Q. 24. A vector field of defined as

$$\vec{f}(x,y,z) = \frac{x}{\left[x^2+y^2+z^2\right]^{\frac{3}{2}}}\hat{i} + \frac{y}{\left[x^2+y^2+z^2\right]^{\frac{3}{2}}}\hat{j} + \frac{z}{\left[x^2+y^2+z^2\right]^{\frac{3}{2}}}\hat{k}$$

Where, $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the axes of a right–handed rectangular/Cartesian coordinate system. The surface integral $\iint \vec{f}.d\vec{S}$ (where $d\vec{S}$ is an elemental surface area vector) evaluated over the inner and outer surface of a spherical shell formed by two concentric spheres with origin as the center, and internal and external radii of 1 and 2, respectively is

(a) 0 **(b)** 2π

(c) 4π **(d)** 8π

Q. 25. Consider two exponentially distributed random variables X and Y, both having a mean of 0.50. Let Z = X + Y and r be the correlation coefficient between X and Y. If the variance of Z equals 0, then the value of r is _________(round off to 2 decimal places).

Q. 26. An analytic function of a complex variable $z = x + iy$ $(i = \sqrt{-1})$ is defined as $f(z) = x^2 - y^2 + i\,\Psi(x, y)$, where $\Psi(x, y)$, is a real function. The value of the imaginary part of $f(z)$ at $z = (i + i)$ is (round off to 2 decimal places).

MECHANICAL ENGINEERING (ME) P2

Q. 27. The sum of two normally distributed random variables X and Y is

(a) always normally distributed.

(b) normally distributed, only if X and Y are independent.

(c) normally distributed, only if X and Y have the same standard deviation.

(d) normally distributed, only if X and Y have the same mean.

Q. 28. A matrix P is decomposed into its symmetric part S and skew symmetric part V. If

$$S = \begin{pmatrix} -4 & 4 & 2 \\ 4 & 3 & \frac{7}{2} \\ 2 & \frac{7}{2} & 2 \end{pmatrix},\ V = \begin{pmatrix} 0 & -2 & 3 \\ 2 & 0 & \frac{7}{2} \\ -3 & \frac{-7}{2} & 0 \end{pmatrix},$$

then matrix P is

(a) $\begin{pmatrix} -4 & 6 & -1 \\ 2 & 3 & 0 \\ 5 & 7 & 2 \end{pmatrix}$ **(b)** $\begin{pmatrix} -4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 4 & -6 & 1 \\ -2 & -3 & 0 \\ -5 & -7 & -2 \end{pmatrix}$ **(d)** $\begin{pmatrix} -2 & \frac{9}{2} & -1 \\ -1 & \frac{81}{4} & 11 \\ -2 & \frac{45}{2} & \frac{73}{4} \end{pmatrix}$

Q. 29. Let $I = \int_{x=0}^{1}\int_{y=0}^{x^2} xy^2\,dy\,dx$. Then, I may also be expressed as

(a) $\int_{y=0}^{1}\int_{x=0}^{\sqrt{y}} xy^2\,dx\,dy$

(b) $\int_{y=0}^{1}\int_{x=\sqrt{y}}^{1} yx^2\,dx\,dy$

(c) $\int_{y=0}^{1}\int_{x=\sqrt{y}}^{1} xy^2\,dx\,dy$

(d) $\int_{y=0}^{1}\int_{x=0}^{\sqrt{y}} yx^2\,dx\,dy$

Q. 30. The solution of $\frac{d^2y}{dt^2} - y = 1$, which additionally satisfies $y|_{t=0} = \left.\frac{dy}{dt}\right|_{t=0} = 0$ in the Laplace – domain is

(a) $\frac{1}{s(s+1)(s-1)}$ **(b)** $\frac{1}{s(s+1)}$

(c) $\frac{1}{s(s-1)}$ **(d)** $\frac{1}{s-1}$

Q. 31. The directional derivative of $f(x, y, z) = xyz$ at point (–1, 1, 3) in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$ is

(a) $3\hat{i} - 3\hat{i} - \hat{k}$ **(b)** $-\frac{7}{3}$

(c) $\frac{7}{3}$ **(d)** 7

Q. 32. The function $f(z)$ of complex variable $z = x + iy$, where $i = \sqrt{-1}$, is given as $f(z) = (x^3 - 3xy^2) + i\,v(x, y)$. For this function to analytic, $v(x, y)$ should be

(a) $(3xy^2 - y^3)$ + constant

(b) $(3x^2y^2 - y^3)$ + constant

(c) $(x^3 - 3x^2y)$ + constant

(d) $(3x^2y - y^3)$ + constant

Q. 33. For the integral $\int_0^{\frac{\pi}{2}} (8 + 4\cos x)\,dx$, the absolute percentage error in numerical evaluation with the Trapezoidal rule, using only the end points, is ______ (round off to one decimal place).

Q. 34. A fair coin is tossed 20 times. The probability that 'head' will appear exactly 4 times in the first ten tosses, and 'tail' will appear exactly 4 times in the next ten tosses is (round off to 3 decimal places).

CIVIL ENGINEERING (CE) P1

Q. 35. In the following partial differential equation, θ is a function of t and z. and D and K are functions of θ $D(\theta)\frac{\partial^2\theta}{\partial z^2} + \frac{\partial K(\theta)}{\partial z} - \frac{\partial \theta}{\partial t} = 0$

The above equation is

(a) a second order linear equation.

(b) a second degree linear equation.

(c) a second order non-linear equation.

(d) a second degree non-linear equation.

Q. 36. The value of $\lim_{x\to\infty} \frac{x^2 - 5x + 4}{4x^2 + 2x}$ is

(a) 0 **(b)** $\frac{1}{4}$

(c) $\frac{1}{2}$ **(d)** 1

Q. 37. The true value of ln(2) is 0.69. If the value of ln(2) is obtained by linear interpolation between ln(1) and ln(6), the percentage of absolute error (round off to the nearest integer), is

(a) 35 **(b)** 48

(c) 69 **(d)** 84

Q. 38. The area of an ellipse represented by an equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(a) $\frac{\pi ab}{4}$ **(b)** $\frac{\pi ab}{2}$

(c) πab **(d)** $\frac{4\pi ab}{3}$

Q. 39. The probability that a 50 year flood may NOT occur at all during 25 years life of a project (round off to two decimal places), is______.

Q. 40. For the ordinary Differential Equation $\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + 6x = 0$ with initial conditions $x(0) = 0$ and $\frac{dx}{dt}(0) = 10$, the solution is

(a) $-5e^{2t} + 6e^{3t}$ **(b)** $5e^{2t} + 6e^{3t}$

(c) $-10e^{2t} + 10e^{3t}$ **(d)** $10e^{2t} + 10e^{3t}$

Q. 41. A continuous function $f(x)$ is defined. If the third derivative at Xi is to be computed by using the fourth order central finite–divided–difference scheme (with step length = h), the correct formula is

(a) $f'''(x_i) = \frac{-f(x_{i+3}) + 8f(x_{i+2}) - 13f(x_{i+1}) + 13f(x_{i-1}) - 8f(x_{i-2}) + f(x_{i-3})}{8h^3}$

(b) $f'''(x_i) = \frac{f(x_{i+3}) - 8f(x_{i+2}) - 13f(x_{i+1}) + 13f(x_{i-1}) + 8f(x_{i-2}) + f(x_{i-3})}{8h^3}$

(c) $f'''(x_i) = \frac{-f(x_{i+3}) - 8f(x_{i+2}) - 13f(x_{i+1}) + 13f(x_{i-1}) + 8f(x_{i-2}) - f(x_{i-3})}{8h^3}$

(d) $f'''(x_i) = \frac{f(x_{i+3}) - 8f(x_{i+2}) + 13f(x_{i+1}) + 13f(x_{i-1}) - 8f(x_{i-2}) - f(x_{i-3})}{8h^3}$

Q. 42. If C represents a line segment between (0, 0, 0) and (1, 1, 1) in Cartesian coordinate system, the value (*expressed as integer*) of the line integral

$$\int_c [(y+z)dx + (x+z)dy + (x+y)dz]$$

is______.

Q. 43. Consider the system of equations

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & -3 \\ 4 & 4 & -6 \\ 2 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

The value of x_3 (round off to the nearest integer), is______.

CIVIL ENGINEERING (CE) P2

Q. 44. The ordinary differential equation $\frac{d^2u}{dx^2} - 2x^2u + \sin x = 0$ is

(a) linear and homogeneous.
(b) linear and nonhomogeneous.
(c) nonlinear and homogeneous.
(d) nonlinear and nonhomogeneous.

Q. 45. The value of $\lim_{x \to \infty} \frac{\sqrt{9x^2 + 2020}}{x + 7}$ is

(a) 7/9 **(b)** 1
(c) 3 **(d)** indeterminable

Q. 46. The integral $\int_0^1 \left(5x^3 + 4x^2 + 3x + 2\right) dx$ is estimated numerically using three alternative methods namely the rectangular, trapezoidal and Simpson's rules with a common step size. In this context, which one of the following statements is TRUE?

(a) Simpson's rule as well as rectangular rule of estimation will give NON–zero error.
(b) Simpson's rule, rectangular rule as well as trapezoidal rule of estimation will give NON–zero error.
(c) Only the rectangular rule of estimation will give zero error.
(d) Only Simpson's rule of estimation will give zero error.

Q. 47. The following partial differential equation is defined for $u : u(x, y)$

$$\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial x^2};\ y \geq 0;\ x_1 \leq x \leq x_2$$

The set of auxiliary conditions necessary to solve the equation uniquely, is

(a) three initial conditions.
(b) three boundary conditions.
(c) two initial conditions and one boundary condition.
(d) one initial conditions and two boundary conditions.

Q. 48. A fair (unbiased) coin is tossed 15 times. The probability of getting exactly 8 Heads (round off to three decimal places), is ______.

Q. 49. An ordinary differential equation is given below

$$6\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$$

The general solution of the above equation (with constants C_1 and C_2), is

(a) $y(x) = C_1 e^{-\frac{x}{3}} + C_2 e^{\frac{x}{2}}$

(b) $y(x) = C_1 e^{\frac{x}{3}} + C_2 e^{-\frac{x}{2}}$

(c) $y(x) = C_1 x e^{-\frac{x}{3}} + C_2 e^{\frac{x}{2}}$

(d) $y(x) = C_1 e^{-\frac{x}{3}} + C_2 x e^{\frac{x}{2}}$

Q. 50. A 4 × 4 matrix [P] is given below

$$[P] = \begin{bmatrix} 0 & 1 & 3 & 0 \\ -2 & 3 & 0 & 4 \\ 0 & 0 & 6 & 1 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

The eigenvalues of [P] are

(a) 0, 3, 6, 6 **(b)** 1, 2, 3, 4
(c) 3, 4, 5, 7 **(d)** 1, 2, 5, 7

Q. 51. The Fourier series to represent $x - x^2$ for $-\pi \leq x \leq \pi$ is given by

$$x - x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

The value of a_0 (round off to two decimal places), is______.

COMPUTER SCIENCE (CS) P1

Q. 52. Consider the functions

I. e^{-x}
II. $x^2 - \sin x$
III. $\sqrt{x^3 + 1}$

Which of the above functions is/are increasing everywhere in [0, 1] ?

(a) III only **(b)** II only
(c) II and III only **(d)** I and III only

Q. 53. For parameters a and b, both of which are $\omega(1)$, $T(n) = T(n^{1/a}) + 1$ and $T(b) = 1$. Then $T(n)$ is

(a) $\Theta(\log_a \log_b n)$ **(b)** $\Theta(\log_{ab} n)$
(c) $\Theta(\log_b \log_a n)$ **(d)** $\Theta(\log_2 \log_2 n)$

Q. 54. Let R be the set of all binary relations on the set (1, 2, 3). Suppose a relation is chosen from R at random. The probability that the chosen relation is reflexive (round off to 3 decimal places) is______.

Q. 55. Let G be a group of 35 elements. Then the largest possible size of a subgroup of G other than G itself is______.

Q. 56. Let A and B be two $n \times n$ matrices over real numbers. Let rank (M) and det (M) denote the rank and determinant of matrix M, respectively. Consider the following statements.

I. rank(AB) = rank(A) rank (B)
II. det(AB) = det(A) det (B)
III. rank(A + B) ≤ rank(A) + rank (B)
IV. det(A + B) ≤ det(A) + det (B)
Which of the above statements are TRUE?
(a) I and II only **(b)** I and IV only
(c) II and III only **(d)** III and IV only

Q. 57. The number of permutations of the characters in LILAC so that no character appears in its original position, if the two L's are indistinguishable, is ______.

Q. 58. For $n > 2$, let $a \in \{0, 1\}^n$ be a non-zero vector. Suppose that x is chosen uniformly at random from $\{0, 1\}^n$. Then, the probability that $\sum_{i=1}^{n} a_i x_i$ is an odd number is______.

CHEMICAL ENGINEERING (CH) P1

Q. 59. Which one of the following methods requires specifying an initial interval containing the root (*i.e.*, bracketing) to obtain the solution of $f(x) = 0$, where $f(x)$ is a continuous non-linear algebraic function?
(a) Newton-Raphson method
(b) regula falsi method
(c) secant method
(d) fixed point iteration method

Q. 60. Consider the hyperbolic functions in Group–1 and their definitions in Group–2.

Group–1	**Group–2**
P. tan hx	**I.** $\dfrac{e^x + e^{-x}}{e^x - e^{-x}}$
Q. cot hx	**II.** $\dfrac{2}{e^x + e^{-x}}$
R. sec hx	**III.** $\dfrac{2}{e^x - e^{-x}}$
P. cosec hx	**IV.** $\dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

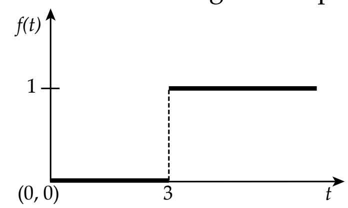
The correct combination is
(a) P – IV, Q – I, R – III, S – II
(b) P – II, Q – III, R – I, S – IV
(c) P – IV, Q – I, R – II, S – III
(d) P – I, Q – II, R – IV, S – III

Q. 61. Consider the following continuously differentiable function

$V(x, y, z) = 3x^2y\hat{i} + 8y^2z\hat{j} + 5xyz\hat{k}$

Where $\hat{i}$, $\hat{j}$, and $\hat{k}$ represent the respective unit vectors along the x, y, and z directions in the Cartesian coordinate system. The curl of this function is
(a) $-3x^2i - 8y^2j + 5z(x + y)k$
(b) $6xyi - 16yzj + 5xyk$
(c) $(5xz - 8y^2)i - 5yzj - 3x^2k$
(d) $y(11x + 16z)$

Q. 62. Consider the following unit step function.



The Laplace transform of this function is
(a) $\dfrac{e^{-3s}}{s}$ **(b)** $\dfrac{e^{-3s}}{s^2}$
(c) $\dfrac{e^{-3s}}{3s}$ **(d)** $\dfrac{e^{-6s}}{s}$

Q. 63. Sum of the eigenvalues of the matrix $\begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 9 \\ 12 & 1 & 7 \end{bmatrix}$ is _________ (round off to nearest integer).

Q. 64. In a box, there are 5 green balls and 10 blue balls. A person picks 6 balls randomly, The probability that the person has picked 4 green balls and 2 blue balls is
(a) $\dfrac{42}{1001}$ **(b)** $\dfrac{45}{1001}$
(c) $\dfrac{240}{1001}$ **(d)** $\dfrac{420}{1001}$

Q. 65. The maximum value of the function $f(x) = -\dfrac{5}{3}x^3 + 10x^2 - 15x + 16$ in the interval (0.5, 3.5) is
(a) 0 **(b)** 8
(c) 16 **(d)** 48

Q. 66. Given $\dfrac{dy}{dx} = y - 20$, and $y|_{x=0} = 40$, the value of y at $x = 2$ is_______ (round off to nearest integer).

Q. 67. Consider the following dataset.

x	1	3	5	15	25
$f(x)$	6	8	10	12	5

The value of the integral $\int_1^{25} f(x)\,dx$ using Simpson's $\left(\dfrac{1}{3}\right)^{\text{rd}}$ rule is ______ (round off to 1 decimal place).

Answer Key			
Q. No.	**Answer**	**Topic Name**	**Chapter Name**
1	(c)	Vectors in Plane and Space	Vector Calculus
2	(c)	Divergence and Curl	Vector Calculus
3	(b)	Partial Derivative	Calculus
4	(c)	Solution of 2nd Order Differential Equation	Differential Equation
5	0.25	Expected Value	Probability and Statistics
6	(a)	System of Linear Equations	Linear Algebra
7	(a)	First Order Differential Equation	Differential Equation
8	2.25	Volume Integrals	Calculus
9	(a, c, d)	Integrals	Calculus
10	(c)	Complex Integration	Complex Variable
11	0.8863	Solution of 1st Order Differential Equation	Differential Equation
12	(a)	Maximum and Minimum	Calculus
13	(c)	Curl	Vector Calculus
14	(a, b, c, d)	Lu Decomposition	Linear Algebra
15	–3	Line Integrals	Calculus
16	(a)	Matrices	Linear Algebra
17	(c)	Limit	Calculus
18	(c)	Inverse Laplace Transform	Transform Theory
19	(d)	Analytic Functions	Complex Variables
20	(b)	Divergence	Vector Calculus
21	6	Vector Operations	Vector Calculus
22	0.93	Conditional Probability	Probability and Statistics
23	(d)	Simpson’s 1/3rd Rule	Numerical Methods
24	(a)	Divergence Theorem	Vector Calculus
25	–1	Correlation	Probability and Statistics
26	2	Analytic Function	Complex Variables
27	(a, b, c, d)	Normal Distribution	Probability and Statistics
28	(b)	Matrices	Linear Algebra
29	(c)	Double Integral	Calculus
30	(a)	Applications of Laplace Transform	Differential Equation
31	(c)	Directional Derivative	Vector Calculus
32	(d)	Analytic Functions	Complex Variables
33	5.2	Trapezoidal Rule	Numerical Methods

Q. No.	Answer	Topic Name	Chapter Name
34	0.042	Conditional Probability	Probability and Statistics
35	(c)	Order of Differential	Differential Equation
36	(b)	Limit	Calculus
37	(b)	Interpolation	Numerical Analysis
38	(c)	Application of Integration	Calculus
39	0.603	Probability	Probability and Statistics
40	(c)	Solution of 2nd Order Differential Equation	Differential Equation
41	(a)	Finite Difference	Numerical Analysis
42	3	Line Integral	Calculus
43	3	System of Linear Equations	Linear Algebra
44	(b)	Linearity and Homogeneity	Differential Equation
45	(c)	Limit	Calculus
46	(d)	Simpson's Rule and Trapezoidal Rule	Numerical Analysis
47	(d)	Partial Differentiation	Differential Equations
48	0.196	Probability	Probability and Statistics
49	(b)	Solution of 2nd Order Differential Equation	Differential Equation
50	(d)	Eigen Values	Linear Algebra
51	–6.58	Fourier Series	Transform Theory
52	(a)	Functions	Calculus
53	(a)	Recurrences	Discrete Mathematics
54	0.125	Sets and Relation	Discrete Mathematics
55	7	Groups	Discrete Mathematics
56	(c)	Matrices	Linear Algebra
57	12	Probability	Probability and Statistics
58	0.5	Probability	Probability and Statistics
59	(b)	Non-Algebraic Solutions	Numerical Analysis
60	(c)	Trigonometry Hyperbolic Function	Calculus
61	(c)	Curl	Vector Calculus
62	(a)	Laplace Transform	Transform Theory
63	14	Eigen Values	Linear Algebra
64	(b)	Probability	Probability and Statistics
65	(c)	Maximum and Minimum	Calculus
66	168	First Order DE	Differential Equation
67	242	Simpson's Rule	Numerical Analysis

ANSWERS WITH EXPLANATIONS

1. Option (c) is correct.

Any four vectors may have linearly dependent vectors and therefore can't form a basis.

2. Option (c) is correct.

For Irrotational field curl of a vector should be zero *i.e.*, $\nabla \times \vec{A} = 0$

3. Option (b) is correct.

Given, $f(x, y, z) = e^{1-x\cos y} + xze^{-1/(1+y^2)}$

Differentiating w.r.t to x

$$\frac{\partial f}{\partial x} = e^{1-x\cos y}(0-\cos y) + ze^{-1/(1+y^2)}$$

$$\left(\frac{df}{dx}\right)_{(1,0,e)} = e^0(0-1) + e.e^{-1/(1+0)}$$

$$= -1+1 = 0$$

4. Option (c) is correct.

Given, differential equation is

$$\frac{\partial^2 y}{\partial x^2} - 6\frac{dy}{dx} + 9y = 0$$

or

$$\left(D^2 - 6D + 9\right)y = 0$$

$$\Rightarrow \quad (D-3)^2 y = 0$$

$\therefore$ AE $(m-3)^2 = 0$, $m = 3, 3$

So, solution of differential equation is

$$y = \left(C_1 + C_2 x\right)e^{3x}$$

5. Correct answer is [0.25].

For the possible values of x

$X = \text{Min}\{M, N\}$

$X = \text{Min}\{1, 1\} = 1$

$X = \text{Min}\{1, 0\} = 0$

$X = \text{Min}\{0, 1\} = 0$

$X = \text{Min}\{0, 0\} = 0$

$$P(X=1) = \frac{1}{4}$$

$$P(X=0) = \frac{3}{4}$$

Mean or expected value.

$$E(X) = \sum_i X_i P(x_i)$$

$$= 1\times\frac{1}{4} + 0\times\frac{3}{4} = 0.25$$

6. Option (a) is correct.

$$[A|B] = \begin{bmatrix} 1 & 2 & b_1 \\ 2 & 4 & b_2 \\ 3 & 7 & b_3 \\ 3 & 9 & b_4 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_3 - 3R_1$

$$[A|B] = \begin{bmatrix} 1 & 2 & b_1 \\ 0 & 0 & b_2 - 2b_1 \\ 0 & 1 & b_3 - 3b_1 \\ 0 & 3 & b_4 - 3b_1 \end{bmatrix}$$

$R_4 \to R_4 - 3R_3$

$$[A|B] = \begin{bmatrix} 1 & 2 & b_1 \\ 0 & 0 & b_2 - 2b_1 \\ 0 & 1 & b_3 - 3b_1 \\ 0 & 0 & b_4 - 3b_3 + 6b_1 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$[A|B] = \begin{bmatrix} 1 & 2 & b_1 \\ 0 & 1 & b_3 - 3b_1 \\ 0 & 0 & b_2 - 2b_1 \\ 0 & 0 & b_4 - 3b_3 + 6b_1 \end{bmatrix}$$

for solution to exist rank A = rank [A|B]

rank A = 2

$\therefore$ row 3, row 4 should be zero for [A|B]

$b_2 - 2b_1 = 0$ and $b_4 - 3b_3 + 6b_1 = 0$

$\Rightarrow b_2 - 2b_1 = 0$ and $6b_1 - 3b_3 + b_4 = 0$

7. Option (a) is correct.

Given, differential equation, $\frac{dy}{dx} = (y-1)x$

Using variable separable method

$$\int\frac{dy}{y-1} = \int x\,dx$$

$$\ln|y-1| = \frac{x^2}{2} + c;$$

(where $y \neq 1$) and second solution is for $y = 1$.

8. **Correct answer is [2.25].**

From figure the limits integration

$x : 0 \to 3;$

$y : 0 \to 1;$

$z : 0 \to 1-y$

$$= \int_{y=0}^{1}\int_{z=0}^{1-y}\int_{x=0}^{3} x\,dx\,dz\,dy = \int_{y=0}^{1}\int_{z=0}^{1-y}\left(\frac{x^2}{2}\right)_0^3 dy\,dz$$

$$= \int_{y=0}^{1}\frac{9}{2}(z)_0^{1-y}\,dy$$

$$= \frac{9}{2}\int_{y=0}^{1}(1-y)dy = \frac{9}{2}\left(y-\frac{y^2}{2}\right)_0^1$$

$$= \frac{9}{2}\left(1-\frac{1}{2}\right) = \frac{9}{4} = 2.25$$

9. **Option (a, c, d) is correct.**

For m and n being non-zero integers and $m \neq n$

$$\int_0^{\pi} \sin m\theta \cdot \sin n\theta \; d\theta = 0$$

For p and q being real numbers and $p \neq q$

$$\int_{-\pi}^{\pi} \sin p\theta \cdot \cos q\theta \; d\theta = 0$$

and $\lim\limits_{\alpha\to\infty} \frac{1}{2\alpha}\int_{-\alpha}^{\alpha} \sin p\theta \cdot \sin q\theta \; d\theta = 0$

but $\frac{1}{2\pi}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin p\theta \; \sin q\theta \; d\theta \neq 0$

for all p, q being real.

10. **Option (c) is correct.**

Given, $I = \int \frac{z^2+1}{z^2-2z}dz$; where C is the unit circle as origin is in the Counterclockwise direction.

By Cauchy's integral formula.

$$f(a) = \frac{1}{2\pi i}\int_c \frac{f(z)}{(z-a)}dz$$

Here we can apply Cauchy's integral theorem to it and $z = 2$ lies outside $|z| = 1$.

$$\therefore \quad I = \int \frac{\left(\frac{z^2+1}{(z-2)}\right)}{z}dz$$

$$= \int_c \frac{f(z)}{z}dz; \text{ where } f(z) = \int \frac{z^2+1}{(z-2)}dz$$

$$= 2\pi i\, f(0) \text{ (using integral theorem.)}$$

$$[\because z = z - a \text{ with } a = 0]$$

$$= 2\pi i \left(\frac{z^2+1}{(z-2)}\right)\Bigg|_{z=0}$$

$$= 2\pi i \times \frac{-1}{2}$$

$$= -\pi i$$

11. **Correct answer is [0.8863].**

Given, $\frac{dy}{dx} = 2x - y$

$\Rightarrow \quad \frac{dy}{dx} + y = 2x$

with $\quad$ P = 1, Q = $2x$

$$\text{IF} = e^{\int P dx} = e^{\int 1 dx} = e^x$$

As $\quad y(\text{IF}) = \int \text{Q(IF)}dx$

$\Rightarrow \quad ye^x = \int 2x.e^x dx = 2xe^x - 2e^x + c$

$\Rightarrow \quad y = 2x - 2 + ce^{-x}$ ...(i)

Given, $y(0) = 1$

$$1 = 0 - 2 + c \Rightarrow c = 3$$

Putting in (i)

$\therefore \quad y = 2x - 2 + 3e^{-x}$

For $\quad x = \ln 2$

$$y = 2(\ln 2) - 2 + 3e^{-\ln 2}$$

$$= 0.8863$$

12. **Option (a) is correct.**

Given, $\quad y = 3x^2 + 3x + 1$

$$\frac{dy}{dx} = 6x + 3$$

Put $\quad \frac{dy}{dx} = 0 \Rightarrow x = \frac{-1}{2}$

Checking function values at stationary points and end points.

$$f(-2) = 3(-2)^2 + 3(-2) + 1 = 7$$

$$f(0) = 1$$

$$f\left(-\frac{1}{2}\right) = 3\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) + 1 = \frac{1}{4}$$

Maximum value = 7 and minimum value = $\frac{1}{4}$

13. Option (c) is correct.

Given,

$\vec{F} = a_x(5y - k_1 z) + a_y(3z + k_2 x) + a_z(k_3 y - 4x)$

$\vec{F}$ is a conservative field. And for conservative field $\nabla \times \vec{F} = 0$.

$$\Rightarrow \begin{bmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (5y - k_1 z) & (3z + k_2 x) & (k_3 y - 4x) \end{bmatrix} = 0$$

$\Rightarrow \quad a_x(k_3 - 3) - a_y(-4 + k_1) + a_z(k_2 - 5) = 0$

$\therefore \quad k_3 - 3 = 0 \Rightarrow k_3 = 3$

$-4 + k_1 = 0 \Rightarrow k_1 = 4$

$k_2 - 5 = 0 \Rightarrow k_2 = 5$

14. Option (a, b, c and d) is correct.

Given,

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} . \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix} . \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ 0 & a_{22} & a_{32} \\ 0 & 0 & a_{33} \end{bmatrix}$$

Multiplying lower triangular matrix and upper triangular matrix and comparing with the elements of LHS matrix.

$$\Rightarrow \begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}^2 & a_{11}a_{21} & a_{11}a_{31} \\ a_{21}a_{11} & a_{21}^2 + a_{22}^2 & a_{21}a_{31} + a_{22}a_{32} \\ a_{31}a_{11} & a_{31}a_{21} + a_{32}a_{22} & a_{31}^2 + a_{32}^2 + a_{33}^2 \end{bmatrix}$$

$a_{11}^2 = 2 \Rightarrow a_{11} = \pm\sqrt{2}$

$a_{21}.\sqrt{2} = 3 \Rightarrow a_{21} = \pm\frac{3}{\sqrt{2}}$

$a_{31}.\sqrt{2} = 3 \Rightarrow a_{31} = \pm\frac{3}{\sqrt{2}}$

$\frac{3}{\sqrt{2}}.\frac{3}{\sqrt{2}} + a_{22}^2 = 2 \Rightarrow a_{22} = \pm\sqrt{\frac{-5}{2}} = \pm\frac{\sqrt{5}}{2}i$

imaginary element.

If a_{22} is imaginary, then a_{32} and a_{33} also becomes imaginary element with

$a_{32} = \pm\frac{7i}{\sqrt{10}}$, $a_{33} = \pm\sqrt{\frac{37}{5}}i$

So, purely real element in the matrix is 4.

15. Correct answer is [–3].

$\vec{F} = y\hat{a}_x - x\hat{a}_y$

$\vec{r} = x\hat{i} + y\hat{j}$

$d\vec{r} = dx\hat{i} + dy\hat{j}$

$\int_c F.d\vec{r} = \int_c ydx - xdy$

Where c is $y = x^2$

$dy = 2xdx$

Limit of x is –1 to 2.

$\int_c F.d\vec{l} = \int_{-1}^{2}\left[x^2 - 2x^2\right]dx = \int_{-1}^{2}(-x^2)dx$

$= -\left[\frac{x^3}{3}\right]_{-1}^{2} = -\left[\frac{8}{3} + \frac{1}{3}\right] = -3$

16. Option (a) is correct.

For three square matrices of same order A.(BC) = (AB).C; Multiplication of matrices are associative.

17. Option (c) is correct.

$\lim_{x\to 1}\left(\frac{1 - e^{-c(1-x)}}{1 - xe^{-c(1-x)}}\right) \to \frac{0}{0}$ form

Applying L' Hospital rule

$\lim_{x\to 1}\left(\frac{-ce^{-c(1-x)}}{-e^{-c(1-x)} - c\,xe^{-c(1-x)}}\right)$

$\frac{-c}{-1-c} = \frac{c}{1+c}$

18. Option (c) is correct.

$F(s) = \frac{1}{s^2 + \omega^2}$, $\left[\because \ L\{\sin \omega t\} = \frac{\omega}{s^2 + \omega^2}\right]$

$f(t) = L^{-1}\left[F(s)\right]\left(\frac{1}{s^2 + \omega^2}\right) = \frac{1}{\omega}\sin \omega t$

19. Option (d) is correct.

$f(z)$ = $\log z$ is differentiable and analytic everywhere except at origin.

Rest all three functions are analytic at all the points.

20. Option (b) is correct.

Given, $\vec{V} = 2(x^2 - y^2)i + vj + 3k$

For incompressible flow $\nabla.\vec{V} = 0$

$\Rightarrow \quad \nabla.\vec{V} = \frac{\partial}{\partial x}2(x^2 - y^2) + \frac{\partial}{\partial y}(v) + \frac{\partial}{\partial z}(3) = 0$

$\Rightarrow \quad 4x + \frac{\partial v}{\partial y} + 0 = 0$

$$\Rightarrow \quad \frac{\partial v}{\partial y} = -4x$$

$$\Rightarrow \quad v = \int -4x dy + f(x,z)$$

$$\Rightarrow \quad v = -4xy + f(x,z)$$

From given options considering $f(x, z) = -4xz$.

21. Correct answer is [6].

Given,

$\vec{A} = 2\hat{j} - 3\hat{k}$, $\vec{B} = -2\hat{i} + \hat{k}$ and $\vec{C} = 3\hat{i} - \hat{j}$

$$\vec{B} \times \vec{C} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$$

$$= \hat{i}(1) - \hat{j}(-3) + \hat{k}(2)$$

$$\vec{B} \times \vec{C} = \hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{A}.(\vec{B} \times \vec{C}) = (2\hat{j} - 3\hat{k}).(\hat{i} + 3\hat{j} + 2\hat{k})$$

$$= 6 - 6 = 0$$

$$\therefore \vec{A}.(\vec{B} \times \vec{C}) + 6 = 0 + 6 = 6$$

22. Correct answer is [0.93].

No. of men = 5

No. of women = 3

Candidates to be hired = 4

The required probability that at least one woman will be selected.

$$P = \frac{{}^5C_3 - {}^3C_1}{{}^8C_4} + \frac{{}^5C_2 - {}^3C_2}{{}^8C_4} + \frac{{}^5C_1 - {}^3C_3}{{}^8C_4}$$

$$= \frac{3}{7} + \frac{3}{7} + \frac{1}{14} = \frac{13}{14} = 0.928 \approx 0.93$$

23. Option (d) is correct.

$$\int_{-1}^{1.4} x|x|\, dx$$

x	–1	–0.4	0.2	0.8	1.4
$f(x)$	–1	–0.16	0.04	0.64	1.96
	y_0	y_1	y_2	y_3	y_4

By Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule

$$\int_{-1}^{1.4} x|x|dx = \frac{h}{3}[(y_0 + y_4) + 2y_2 + 4(y_1 + y_3)],$$

with $h = 0.6$

$$= \frac{0.6}{3}[(-1 + 1.96) + 2(0.04) + (-0.16 + 1.96)]$$

$= 0.592$

24. Option (a) is correct.

Given,

$$\vec{F}(x,y,z) = \frac{x}{\left[x^2 + y^2 + z^2\right]^{\frac{3}{2}}}\hat{i} + \frac{y}{\left[x^2 + y^2 + z^2\right]^{\frac{3}{2}}}\hat{j} + \frac{z}{\left[x^2 + y^2 + z^2\right]^{\frac{3}{2}}}\hat{k}$$

By Gauss Divergence theorem

$$\int\int_s \vec{F}\hat{n}ds = \iiint_V \nabla.\vec{F}dV \quad \text{...(i)}$$

$$\nabla.\vec{F} = \frac{\partial}{\partial x}\left(\frac{x}{[x^2 + y^2 + z^2]^{\frac{3}{2}}}\right) + \frac{\partial}{\partial y}\left(\frac{y}{[x^2 + y^2 + z^2]^{\frac{3}{2}}}\right) + \frac{\partial}{\partial z}\left(\frac{z}{[x^2 + y^2 + z^2]^{\frac{3}{2}}}\right)$$

$$= \frac{1}{\left[x^2 + y^2 + z^2\right]^{\frac{3}{2}}} - \frac{\frac{3}{2}\cdot x \cdot 2x}{\left[x^2 + y^2 + z^2\right]^{\frac{5}{2}}} + \frac{1}{\left[x^2 + y^2 + z^2\right]^{\frac{3}{2}}} - \frac{\frac{3}{2}\cdot y \cdot 2y}{\left[x^2 + y^2 + z^2\right]^{\frac{5}{2}}} + \frac{1}{\left[x^2 + y^2 + z^2\right]^{\frac{3}{2}}} - \frac{\frac{3}{2}\cdot z \cdot 2z}{\left[x^2 + y^2 + z^2\right]^{\frac{5}{2}}}$$

$$= \frac{x^2 + y^2 + z^2 - 3x^2 + x^2 + y^2 + z^2 - 3y^2 + x^2 + y^2 + z^2 - 3z^2}{\left[x^2 + y^2 + z^2\right]^{\frac{5}{2}}} = 0 \quad \text{...(ii)}$$

So, by substituting the value of $\nabla.\vec{F} = 0$ from (ii) in equation (i) we get result of integration is zero.

25. Correct answer is [–1].

$$X \sim E(\lambda_1);\ \text{mean} = \frac{1}{\lambda_1} = 0.5$$

$$\text{Variance; } x = \frac{1}{\lambda^2_1} = 0.25$$

$$Y \sim E(\lambda_2);\ \text{mean} = \frac{1}{\lambda_2} = 0.5$$

$$\text{Variance; } y = \frac{1}{\lambda^2_2} = 0.25$$

Given, $\text{Var}(z) = 0$

$$\text{Var}(z) = \text{Var}(x) + \text{Var}(y) + 2\text{COV}(x,y)$$

$$\Rightarrow \quad 0 = 0.25 + 0.25 + 2\text{COV}(x,y)$$

$\Rightarrow$ COV$(x, y) = -0.25$

Correlation coefficient,

$$\rho = \frac{COV(x,y)}{\sigma_x \sigma_y} = \frac{-0.25}{\sqrt{0.25}\sqrt{0.25}} = -1$$

26. Correct answer is [2].

$$w = f(z) = u + iv$$

Given $u = x^2 + y^2$

$$v = \psi(x, y) \quad ...(i)$$

As $f(z)$ is analytic function, it satisfies CR equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 2x$$

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} = 2y$$

So, $dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$ (Total Derivative)

$$\Rightarrow \quad dv = -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$$

$$\Rightarrow \quad dv = 2ydx + 2xdy$$

$$\Rightarrow \int dv = 2\int d(xy)$$

$$\Rightarrow \quad v = 2xy \quad ...(ii)$$

For $z = 1 + i$

Putting $x = 1;\ y = 1$ in equation (ii)

$$v = \psi(x, y)_{(1,1)} = 2$$

27. Option (a, b, c and d) is correct.

$$X_1 \sim N(\mu_1, \sigma_1);\ X_2 \sim N(\mu_2, \sigma_2)$$

$$X_1 + X_2 \sim N\left(\mu_1 + \mu_2, \sqrt{\sigma^2{}_1 + \sigma^2{}_2}\right)$$

If X and Y are independent normal random variables

sum of two normally distributed random variable is always normally distributed.

28. Option (b) is correct.

For matrix P decomposed

$$\text{Symmetric part, S} = \begin{bmatrix} -4 & 4 & 2 \\ 4 & 3 & \frac{7}{2} \\ 2 & \frac{7}{2} & 2 \end{bmatrix}$$

$$\text{Skew Symmetric part, V} = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & \frac{7}{2} \\ -3 & -\frac{7}{2} & 0 \end{bmatrix}$$

Symmetric part $S = \frac{P + P^T}{2}$

Skew-symmetric part $V = \frac{P - P^T}{2}$

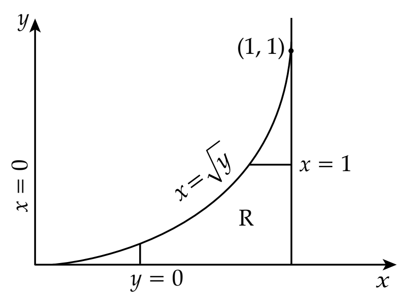
$$P = S + V = \begin{bmatrix} -4 & 4 & 2 \\ 4 & 3 & \frac{7}{2} \\ 2 & \frac{7}{2} & 2 \end{bmatrix} + \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & \frac{7}{2} \\ -3 & -\frac{7}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 2 & 5 \\ 6 & 3 & 7 \\ -1 & 0 & 2 \end{bmatrix}$$

29. Option (c) is correct.

From curve we can observe

$$I = \int_{x=0}^{x=1} \int_{y=0}^{y=x^2} xy^2 dy dx$$



Changing order of integration

$$I = \int_{y=0}^{1} \int_{x=\sqrt{y}}^{x=1} xy^2 dx dy$$

30. Option (a) is correct.

Given,

$$y'' - y = 1$$

$$y(0) = 0$$

$$y'(0) = 0$$

Applying Laplace transform

$$s^2Y(s) - sy(0) - y'(0) - Y(s) = \frac{1}{s} \Rightarrow Y(s)(s^2 - 1) = \frac{1}{s}$$

$$\Rightarrow \quad Y(s) = \frac{1}{s(s^2 - 1)}$$

$$= \frac{1}{s(s-1)(s+1)}$$

31. Option (c) is correct.

Given, $f = xyz$

$$\nabla f = \frac{df}{dx}\hat{i} + \frac{df}{dy}\hat{j} + \frac{df}{dz}\hat{k}$$

$$= yz\hat{i} + xz\hat{j} + xy\hat{k}$$

$(\nabla f)_{(-1,1,3)} = 3\hat{i} - 3\hat{j} - \hat{k}$

For $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

Directional derivative in the direction of $\vec{a}$

$$\nabla f \cdot \frac{\vec{a}}{|a|} = \frac{3(1)+(-3)(-2)+(-1)(2)}{\sqrt{1^2+(-2)^2+(2)^2}}$$

$$= \frac{7}{3}$$

32. Option (d) is correct.

$f(z) = u + iv$

$u = x^3 + 3xy^2$

$v = v(x, y)$

For $f(z)$ to be analytical it should satisfy CR equations

$u_x = 3x^2 - 3y^2 = v_y,$

$u_y = -bxy = -v_x,$

$v_x = 6xy$, integrating w.r.t x,

$v = 3x^2 y + \phi(y)$ ...(i)

$v_y = 3x^2 - 3y^2$, integrating w.r.t y,

$v = 3x^2 y - y^3 + \psi(x)$ ...(ii)

From (i) and (ii)

$\phi(y) = -y^3$ and $\psi(x) = 0$

$v = 3x^2 y - y^3 + C$

$v = 3x^2 y - y^3 +$ constant

33. Correct answer is [5.2].

Exact value

$$\int_0^{\frac{\pi}{2}} (8 + 4\cos x)\, dx = [8x + 4\sin x]_0^{\frac{\pi}{2}}$$

Approximate value $= 4\pi + 4 = 16.566$

Using trapezoidal rule

x	0	$\frac{\pi}{2}$
$f(x)$	12 y_0	8 y_1

Step size

$$h = \frac{\pi}{2}$$

$$I = \frac{h}{2}\{y_0 + y_1\}$$

$$I = \frac{\pi}{4}[12 + 8] = 5\pi = 15.708$$

Absolute error

$= |\text{Exact value–Approximate value}|$

$\Rightarrow |16.566 - 15.708| = 0.858$

Absolute percentage error

$$= \frac{0.858}{16.566} \times 100 = 5.179\% \approx 5.2\%$$

34. Correct answer is [0.042].

For coin is tossed 20 times.

Required probability, P = P [4 heads in first 10 tosses]. P [4 tails in next 10 tosses]

$$= {}^{10}C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^6 . {}^{10}C_4 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^4$$

$$\Rightarrow \frac{{}^{10}C_4\, {}^{10}C_4}{2^{10} \cdot 2^{10}} = 0.042$$

35. Option (c) is correct.

In equation first term contains product of dependent variable, with its derivative, this shows it is non-linear also derivative order is two, so it is second order.

36. Option (b) is correct.

Given, $\lim\limits_{x\to\infty}\left(\frac{x^2 - 5x + 4}{4x^2 + 2x}\right)$

We can see here $\left(\frac{\infty}{\infty}\right)$ form, so by applying L' Hospital rule.

$$= \lim_{x\to\infty}\left(\frac{2x-5}{8x+2}\right) \to \left(\frac{\infty}{\infty}\right)$$

Applying L' Hospital Rule again

$$= \lim_{x\to\infty} \frac{2}{8} = \frac{1}{4}$$

37. Option (b) is correct.

Given,

true value of ln 2 = 0.69

x	$y = \ln x$
$x_0 = 1$	0
$x_1 = 6$	1.79

Divided differentiation

$$\frac{1.79 - 0}{6 - 1} = 0.358 = f[x_0, x_1]$$

Approximate value $\ln 2 = f[x_0] + (x - x_0) f[x_0, x_1]$

$= 0 + (2 - 1)0.358$

$= 0.358$

$$\% \text{ error} = \frac{\text{true value} - \text{approx. value}}{\text{true value}} \times 100$$

$$= \frac{0.69 - 0.358}{0.69} \times 100 = 48.11\%$$

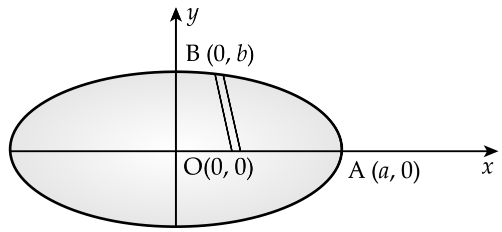
38. Option (c) is correct.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\Rightarrow \quad \frac{y^2}{b^2}=1-\frac{x^2}{a^2}$$

$$\Rightarrow \quad y^2=b^2\left(\frac{a^2-x^2}{a^2}\right)$$

$$\Rightarrow \quad y=\frac{b}{a}\sqrt{a^2-x^2}$$



Consider a positive coordinate OAB. And taking strip parallel to y-axis,

Limit for x varies from 0 to a

$$A=4\int_0^b y\,dx=4\int_0^b \frac{b}{a}\sqrt{a^2-x^2}\,dx$$

$$=4\frac{b}{a}\int_0^b \sqrt{a^2-x^2}\,dx=4\frac{b}{a}\int_0^a \sqrt{a^2-x^2}\,dx$$

$$=4\frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$=4\frac{b}{a}\left[0+\frac{a^2}{2}\sin^{-1}\left(\frac{a}{a}\right)\right]=4\frac{b}{a}.\frac{a^2}{2}\sin^{-1}(1)$$

$$=4\frac{b}{a}.\frac{a^2}{2}.\frac{\pi}{2}=\pi ab \qquad \left[\because \sin^{-1}(1)=\frac{\pi}{2}\right]$$

39. Correct answer is [0.603].

$$P=\frac{1}{50}=0.02$$

Probability of flood not happening

$q=1-P=0.98$

∴ probability of flood not happening in 25 years $=q^n=(0.98)^{25}=0.603$

40. Option (c) is correct.

Given,

$$\frac{d^2x}{dt^2}-5\frac{dx}{dt}+6x=0$$

A.E. is $\quad m^2-5m+6=0$

$\Rightarrow \quad m=2,3$ so, CF$=C_1e^{2t}+C_2e^{3t}$

So, $\quad x=C_1e^{2t}+C_2e^{3t}$ ...(i)

$$\frac{dx}{dt}=2C_1e^{2t}+3C_2e^{3t} \qquad \text{...(ii)}$$

For $x(0)=0$ in (i)

$0=C_1+C_2$

For $x'(0)=10$ in (ii)

$10=2C_1+3C_2$

$C_1=-10,\ C_2=10$

$x=-10e^{2t}+10e^{3t}$

41. Option (a) is correct.

Central finite difference fourth order scheme for third derivatives is given by.

$$\left.\frac{d^3u}{dx^3}\right|_{x_i}=\frac{\begin{array}{c}u_{i+3}+8u_{i+2}-13u_{i+1}\\+13u_{i-1}-8u_{i-2}+u_{i-3}\end{array}}{8\Delta x^3}+\frac{7\Delta x^4}{120}\cdot\left.\frac{\partial^7u}{\partial x^7}\right|_{x_i}+\ldots$$

$$\therefore\ f'''(x_i)=\frac{\begin{array}{c}-f(x_{i+3})+8f(x_{i+2})-13f(x_{i+1})\\+13f(x_{i-1})-8f(x_{i-2})+f(x_{i-3})\end{array}}{8h^3}$$

42. Correct answer is [3].

$$I=\int_c[(y+z)dx+(x+z)dy+(x+y)dz]$$

$$=\int_c[d(yx)+d(xz)+d(zy)]=[yx+xz+zy]_{0,0,0}^{1,1,1}$$

$(1+1+1)-(0+0+0)=3$

43. Correct answer is [3].

$$[AB]=\begin{bmatrix}1&3&2&1\\2&2&-3&1\\4&4&-6&2\\2&5&2&1\end{bmatrix}$$

In echelon from it is written as

$$=\begin{bmatrix}1&3&2&1\\0&-1&-2&-1\\0&0&1&3\\0&0&0&0\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}1&3&2\\0&-1&-2\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\1\\3\\1\end{bmatrix}$$

From third row

$x_3=3$

44. Option (b) is correct.

In this equation dependent variable is u, all the derivatives are raised to power 1 and coefficients are of independent variable x. Hence this is linear and non-homogenous.

45. Option (c) is correct.

Given,

$$\lim_{x\to\infty}\left(\frac{\sqrt{9x^2+2020}}{x+7}\right)$$

$$=\lim_{x\to\infty}\left(\frac{3x\sqrt{1+\frac{2020}{9x^2}}}{x\left(1+\frac{7}{x}\right)}\right)=\frac{3\sqrt{1+\frac{2020}{\infty}}}{\left(1+\frac{7}{\infty}\right)}$$

$$=3$$

46. Option (d) is correct.

Here, integrand is a polynomial of 3rd degree. So, Simpson's rule will give zero error.

47. Option (d) is correct.

$\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial x^2}$; $y\geq 0$, $x_1\leq x\leq x_2$ is in the form of parabolic partial differential equation, which requires 2 boundary and 1 initial condition to solve.

48. Correct answer is [0.196].

Given,

Fair coin is tossed 15 times *i. e.*, $n = 15$

$$p=\frac{1}{2}$$

$$q=\frac{1}{2}$$

$$r=8$$

Using Binomial Distribution

$$P(X=8)={}^{n}C_r p^r q^{n-r}$$

$$={}^{15}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^7$$

$$=0.196$$

49. Option (b) is correct.

A.E. is $D^2 + D - 1 = 0 \Rightarrow 6D^2 + 3D - 2D - 1 = 0$

$\Rightarrow (3D - 1)(2D + 1) = 0 \quad \Rightarrow \quad D=-\frac{1}{2},\ \frac{1}{3}$

So, the general solution

$$y=C_1e^{-\frac{1}{2}x}+C_2e^{\frac{1}{3}x}$$

50. Option (d) is correct.

$\lambda_1+\lambda_2+\lambda_3+\lambda_4 = \text{Trace P}$

$0 + 3 + 6 + 6 = 15$

$\lambda_1\cdot\lambda_2\cdot\lambda_3\cdot\lambda_4=|P|$

$$|P|=70$$

So, this is satisfied by option (d) with □ = 1, 2, 5, 7

51. Correct answer is [–6.58].

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx=\frac{1}{\pi}\int_{-\pi}^{\pi}(x-x^2)dx$$

$$=-\frac{2\pi^2}{3}=-6.5797\approx -6.58$$

52. Option (a) is correct.

For a function to be increasing in the given interval, its first derivative should be greater than zero.

$$f(x)=\sqrt{x^3+1}$$

$$f'(x)=\frac{1}{2\sqrt{x^3+1}}\times 3x^2>0 \text{ in } [0,1]$$

for $g(x) = e^{-x}$

$g'(x) = -e^{-x} < 0$

for $h(x) = x^2 - \sin x$

$h'(x) = 2x - \cos x$

which is negative in the given interval

So, $f(x)$ is increasing function everywhere in [0, 1].

53. Option (a) is correct.

$$T(n)=T\left(n^{\frac{1}{a}}\right)+1$$

By use of substitution method

$$T(n)=T\left(n^{\frac{1}{a^2}}\right)+2$$

$$T(n)=T\left(n^{\frac{1}{a^3}}\right)+3$$

.

.

.

$$T(n)=T\left(n^{\frac{1}{a^k}}\right)+k \qquad \text{...(i)}$$

When $n^{\frac{1}{a^k}}=b$, $\Rightarrow a^k=\log_b n$

$k=\log_a.\log_b n$

$T(n)=T(b)+\log_a.\log_b n$ in (i)

$\Rightarrow \quad T(n)=1+\log_a.\log_b n \qquad$ [given $T(b)=1$]

54. Correct answer is [0.125].

$S = \{1, 2, 3\}$

$n=|S|=3$

Number of relations on

$$S=2^{n^2}=2^{3^2}=2^9$$

Number of reflexive relations on

$$S=2^{n^2-n}=2^{3^2-3}=2^6$$

P (reflexive relation)

$$=\frac{2^6}{2^9}=\frac{1}{2^3}=\frac{1}{8}=0.125$$

55. Correct answer is [7].

Given,

35 elements in a group G.

Order of subgroup divides the order of group.

Possible order of subgroup is– 1, 5, 7, 35 itself.

So, the largest subgroup other than G is 7.

56. Option (c) is correct.

For two square matrices of order n,

Determinant property

$|AB| = |A|.|B|$

And rank of matrix properties,

$\text{Rank}(A+B) \le \text{Rank}(A) + \text{Rank}(B)$

57. Correct answer is [12].

Given, the two L's are indistinguishable.

L's can be arranged in ${}^3C_2$ways.

For each case remaining 3 letters can be positioned in 2 □ 2! Ways.

So total ways are = ${}^3C_2$ 2 □ 2! = 12 ways.

58. Correct answer is [0.5].

$$a = \begin{bmatrix} a_1 \\ a_2 \\ : \\ : \\ a_n \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ : \\ : \\ x_n \end{bmatrix}$$

It is given that elements of a are non–zero.

$a_i = 0$ or 1

$x_i = 0$ or 1

$\sum_{i=1}^{n} a_i x_i$ values lie between 0 to n.

$a_i x_i$	0	1	2	……….	n
Favourable outcome	${}^nC_0$	${}^nC_1$	${}^nC_2$	……….	${}^nC_n$

$$P\left[\sum_{i=1}^{n} a_i x_i \text{ is odd}\right] = \frac{{}^nC_1 + {}^nC_3 + {}^nC_5 ......}{2^n}$$

$$= \frac{2^{n-1}}{2^n} = \frac{1}{2} = 0.5$$

$\left[\because \text{total no. of cases} = 2^n\right]$

59. Option (b) is correct.

To obtain the solution of continuous nonlinear algebraic function, $f(x) = 0$; Regula Falsi method requires an initial interval containing root *i.e.* (bracketing).

Newton Raphson method needs only an initial guess. And in secant method the direction of next root is always same.

60. Option (c) is correct.

By hyperbolic function formulas.

61. Option (c) is correct.

$$\nabla \times \vec{V} = \begin{bmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2y & 8y^2z & 5xyz \end{bmatrix}$$

$$= (5xz - 8y^2)i - (5yz - 0)j + (0 - 3x^2)k$$

$$= (5xz - 8y^2)i - 5yzj - 3x^2k$$

62. Option (a) is correct.

The waveform is step function. And changing at time 3 units.

$\therefore \quad f(t) = u(t-3)$

$$L\{f(t)\} = \frac{e^{-3s}}{s}$$

63. Correct answer is [14].

Sum of eigen values = sum of diagonal elements = 2 + 5 + 7 = 14

64. Option (b) is correct.

Given, green balls = 5; blue balls = 10

No. of balls selected = 6

$$P = \frac{{}^5C_4 \times {}^{10}C_2}{{}^{15}C_6} = \frac{45}{1001}$$

65. Option (c) is correct.

Given,

$$f(x) = -\frac{5}{3}x^3 + 10x^2 - 15x + 16$$

$$f'(x) = -5x^2 + 20x - 15$$

For stationary points

$$f'(x) = 0$$

$$\Rightarrow \quad -5x^2 + 20x - 15 = 0$$

$$\Rightarrow \quad x^2 - 4x + 3 = 0$$

$$\Rightarrow \quad x = 3, 1$$

Function values at end points and stationary points

$$f(3) = 16$$

$$f(1) = \frac{28}{3}$$

$$f(0.5) = 10.79$$

$$f(3.5) = 14.54$$

So, maximum value is 16 in the given interval.

66. Correct answer is [168].

Given, $\frac{dy}{dx} = y - 20$

□ $\int \frac{dy}{y-20} = \int dx$

$\Rightarrow \quad \ln|y-20| = x + c$ ...(i)

$x = 0,\ y = 40$ putting in (i)

$c = \ln 20$

Now putting value of c in (i)

$\ln|y-20| = x + \ln 20$ ...(ii)

$x = 2,\ y = ?$

Putting $x = 2$ in (ii) to get the value of y.

$\ln|y-20| = 2 + \ln 20$

$\Rightarrow \quad y - 20 = e^{2+ln20}$

$\Rightarrow \quad y = 168$

67. Correct answer is [242].

$$I = \int_1^{25} f(x)dx$$

$$= \int_1^5 f(x)dx + \int_5^{25} f(x)dx$$

$$I = I_1 + I_2$$

For I_1 ; $h = 2$

For I_2 ; $h = 10$

X	1	3	5
Y	6	8	10
	y_0	y_1	y_2

X	5	10	15
Y	10	12	15
	y_0	y_1	y_2

$$\Rightarrow \quad I_1 = \int_1^5 f(x)dx = \frac{h}{3}\{(y_0 + y_2) + 4(y_1)\}$$

$$= \frac{2}{3}\{(6+10) + 4(8)\} = 32$$

$$I_2 = \int_5^{25} f(x)dx = \frac{h}{3}\{(y_0 + y_2) + 4(y_1)\}$$

$$= \frac{10}{3}\{(10+5) + 4(12)\}$$

$$= 210$$

$$I = I_1 + I_2 = 242$$

■■■■■