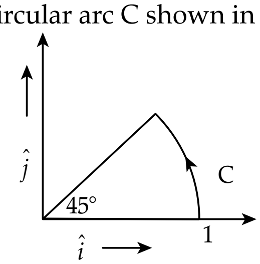
# ENGINEERING MATHEMATICS GATE Question Paper 2021

## ELECTRONICS AND COMMUNICATION (EC) P1

**Q. 1.** The vector function F $(r) = -x\hat{i} + y\hat{j}$ is defined over a circular arc C shown in the figure




The line integral of $\int_C F(r).dr$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{6}$ (d) $\frac{1}{3}$

**Q. 2.** Consider the differential equation given below.

$$\frac{dy}{dx} + \frac{x}{1-x^2}y = x\sqrt{y}$$

The integrating factor of the differential equation is

(a) $(1-x^2)^{-\frac{3}{2}}$ (b) $(1-x^2)^{-\frac{1}{4}}$

(c) $(1-x^2)^{-\frac{3}{2}}$ (d) $(1-x^2)^{-\frac{1}{2}}$

**Q. 3.** Two continuous random variables X and Y are related as Y = 2 X + 3

Let $\sigma_X^2$ and $\sigma_Y^2$ denote the variances of X and Y, respectively. The variances are related as

(a) $\sigma_Y^2 = 2\,\sigma_X^2$ (b) $\sigma_Y^2 = 4\,\sigma_X^2$

(c) $\sigma_Y^2 = 5\,\sigma_X^2$ (d) $\sigma_Y^2 = 25\,\sigma_X^2$

**Q. 4.** If the vectors (1. 0, −1. 0, 2. 0), (7. 0, 3. 0, $x$) and (2. 0, 3. 0, 1. 0) in $\mathbb{R}^3$ are linearly dependent, the value of $x$ is______.

**Q. 5.** Consider the vector field F = $a_x(4y - c_1 z) + a_y(4x + 2z) + a_z(2y + z)$ in a rectangular coordinate system $(x, y, z)$ with unit vectors $a_x$, $a_y$, and $a_z$. If the field F is irrational (conservative), then the constant $c_1$ (in integer) is______.

**Q. 6.** Consider the integral

$$\oint_C \frac{\sin(x)}{x^2(x^2+4)}dx$$

where C is a counter – clockwise oriented circle defined as $|x-i| = 2$ The value of the integral is

(a) $-\frac{\pi}{8}\sin(2i)$ (b) $\frac{\pi}{8}\sin(2i)$

(c) $-\frac{\pi}{4}\sin(2i)$ (d) $\frac{\pi}{4}\sin(2i)$

**Q. 7.** A box contains the following three coins.

**I.** A fair coin with head on one face and tail on the other face.

**II.** A coin with heads on both the faces.

**III.** A coin with tails on both the faces.

A coin is picked randomly from the box and tossed. Out of the two remaining coins in the box, one coin is then picked randomly and tossed. If the first toss results in a head, the probability of getting a head in the second toss is

(a) $\frac{2}{5}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

**Q. 8.** A real 2 × 2 non–singular matrix A with repeated eigenvalue is given as

$$A = \begin{bmatrix} x & -3.0 \\ 3.0 & 4.0 \end{bmatrix}$$

where $x$ is a real positive number. The value of $x$ (rounded off to one decimal place) is _______.

## ELECTRICAL ENGINEERING (EE) P1

**Q. 9.** Let $p$ and $q$ be real numbers such that $p^2 + q^2 = 1$.

The eigen values of the matrix $\begin{bmatrix} p & q \\ q & -p \end{bmatrix}$ are

(a) 1 and 1 (b) 1 and – 1

(c) $j$ and – $j$ (d) $pq$ and – $pq$

**Q. 10.** Let $p(z) = z^3 + (1+j)z^2 + (2+j)z + 3$, where $z$ is a complex number. Which one of the following is true?

**(a)** conjugate $\{p(z)\} = p$ (conjugate $\{z\}$) for all $z$

**(b)** The sum of the roots of $p(z) = 0$ is a real number

**(c)** The complex roots of the equation $p(z) = 0$ come in conjugate pairs

**(d)** All the roots cannot be real

**Q. 11.** Let $f(x)$ be a real – valued function such that $f'(x_0) = 0$ for some $x_0 \in (0, 1)$ and $f''(x) > 0$ for all $x \in (0, 1)$ Then $f(x)$ has

**(a)** no local minimum in (0, 1)

**(b)** one local maximum in (0, 1)

**(c)** exactly one local minimum in (0, 1)

**(d)** two distinct local minima in (0, 1)

**Q. 12.** Suppose the circles $x^2 + y^2 = 1$ and $(x-1)^2 + (y-1)^2 = r^2$ intersect each other orthogonally at the point $(u, v)$. Then $u + v =$ ________.

**Q. 13.** In the open interval (0, 1), the polynomial $p(x) = x^4 - 4x^3 + 2$ has

**(a)** two real roots **(b)** one real root

**(c)** three real roots **(d)** no real roots

**Q. 14.** Suppose the probability that a coin toss shows "head" is $p$, where $0 < p < 1$. The coin is tossed repeatedly until the first "head" appears.

The expected number of tosses required is

**(a)** $\frac{p}{(1-p)}$ **(b)** $\frac{(1-p)}{p}$

**(c)** $\frac{1}{p}$ **(d)** $\frac{1}{p^2}$

**Q. 15.** Let $(-1-j), (3-j), (3+j)$ and $(-1+j)$ be the vertices of a rectangle C in the complex plane. Assuming that C is traversed in counter–clockwise direction, the value of the contour integral $\oint_C \frac{dz}{z^2(z-4)}$ is

**(a)** $\frac{j\pi}{2}$ **(b)** 0

**(c)** $\frac{-j\pi}{8}$ **(d)** $\frac{j\pi}{16}$

**Q. 16.** Let A be a 10 × 10 matrix such that $A^5$ is a null matrix, and let I be the 10 × 10 identity matrix. The determinant of A + I is _______.

## MECHANICAL ENGINEERING (ME) P1

**Q. 17.** If $y(x)$ satisfies the differential equation $(\sin x)\frac{dy}{dx} + y\cos x = 1$, subject to the condition $y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$, then $y\left(\frac{\pi}{6}\right)$ is

**(a)** 0 **(b)** $\frac{\pi}{6}$

**(c)** $\frac{\pi}{3}$ **(d)** $\frac{\pi}{2}$

**Q. 18.** The value of $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$ is

**(a)** $\frac{1}{4}$ **(b)** $\frac{1}{3}$

**(c)** $\frac{1}{2}$ **(d)** 1

**Q. 19.** The Dirac–delta function $(\delta(t - t_0))$ for $t, t_0 \in \mathbb{R}$, has the following property

$$\int_a^b f(t)\delta(t-t_0)\,dt = \begin{cases} f(t_0) & a < t_0 < b \\ 0 & \text{otherwise} \end{cases}$$

The Laplace transform of the Dirac delta function $\delta(t - a)$ for $a > 0$; $\mathrm{L}(\delta(t-a)) = \mathrm{F}(s)$ is

**(a)** 0 **(b)** $\infty$

**(c)** $e^{sa}$ **(d)** $e^{-sa}$

**Q. 20.** The ordinary differential equation $\frac{dy}{dx} = -\pi y$ subject to an initial condition $y(0) = 1$ is solved numerically using the following scheme:

$$\frac{y(t_{n+1}) - y(t_n)}{h} = -\pi y(t_n)$$

where $h$ is the time step, $t_n = nh$, and $n = 0, 1, 2 \ldots$. This numerical scheme is stable for all values of $h$ in the interval_______.

**(a)** $0 < h < \frac{2}{\pi}$ **(b)** $0 < h < 1$

**(c)** $0 < h < \frac{\pi}{2}$ **(d)** for all $h > 0$

**Q. 21.** Consider a binomial random variable X. If $X_1, X_2, \ldots, X_n$ are independent and identically distributed samples from the distribution of X with sum $Y = \sum_{t=1}^{n} X_i$, then the distribution of Y as $n \to \infty$ can be approximated as

**(a)** Exponential **(b)** Bernoulli

**(c)** Binomial **(d)** Normal

**Q. 22.** Let C represent the unit circle centered at origin in the complex plane, and complex variable, $Z = x + iy$. The value of the contour integral $\oint_C \frac{\cosh 3z}{2z} dz$. (where integration is taken counter clockwise) is

**(a)** 0 **(b)** 2
**(c)** $\pi i$ **(d)** $2\pi i$

**Q. 23.** Customers arrive at a shop according to the Poisson distribution with a mean of 10 customers/hour. The manager notes that no customer arrives for the first 3 minutes after the shop opens. The probability that a customer arrives within the next 3 minutes is

**(a)** 0.39 **(b)** 0.86
**(c)** 0.50 **(d)** 0.61

**Q. 24.** Let $f(x) = x^2 - 2x + 2$ be a continuous function defined on $x \in [1, 3]$. The point $x$ at which the tangent of $f(x)$ becomes parallel to the straight line joining $f(1)$ and $f(3)$ is

**(a)** 0 **(b)** 1
**(c)** 2 **(d)** 3

## MECHANICAL ENGINEERING (ME) P2

**Q. 25.** Consider an $n \times n$ matrix A and a non-zero $n \times 1$ vector $p$. Their product $Ap = \alpha^2 p$, where $\alpha \in \mathbb{R}$ and $\alpha \notin \{-1, 0, 1\}$. Based on the given information, the eigen value of $A^2$ is:

**(a)** $\alpha$ **(b)** $\alpha^2$
**(c)** $\sqrt{\alpha}$ **(d)** $\alpha^4$

**Q. 26.** If the Laplace transform of a function $f(t)$ is given by $\frac{s+3}{(s+1)(s+2)}$, then $f(0)$ is

**(a)** 0 **(b)** $\frac{1}{2}$
**(c)** 1 **(d)** $\frac{3}{2}$

**Q. 27.** The mean and variance, respectively, of a binomial distribution for $n$ independent trials with the probability of success as $p$, are

**(a)** $\sqrt{np}, np(1-2p)$ **(b)** $\sqrt{np}, \sqrt{np(1-p)}$
**(c)** $np, np$ **(d)** $np, np(1-p)$

**Q. 28.** Value of $\int_4^{5.2} \ln x\, dx$ using Simpson's one-third rule with interval size 0.3 is

**(a)** 1.83 **(b)** 1.60
**(c)** 1.51 **(d)** 1.06

**Q. 29.** Value of $(1 + i)^8$, where $i = \sqrt{-1}$, is equal to

**(a)** 4 **(b)** 16
**(c)** $4i$ **(d)** $16i$

**Q. 30.** The value of $\int_0^{\frac{\pi}{2}} \int_0^{\cos\theta} r \sin\theta\, dr\, d\theta$ is

**(a)** 0 **(b)** $\frac{1}{6}$
**(c)** $\frac{4}{3}$ **(d)** $\pi$

**Q. 31.** Let the superscript T represent the transpose operation. Consider the function $f(x) = \frac{1}{2} x^T Q x - r^T x$, where $x$ and $r$ are $n \times 1$ vectors and Q is a symmetric $n \times n$ matrix. The stationary point of $f(x)$ is

**(a)** $Q^T r$ **(b)** $Q^{-1} r$
**(c)** $\frac{r}{r^T r}$ **(d)** $r$

**Q. 32.** Consider the following differential equation $(1+y)\frac{dy}{dx} = y$. The solution of the equation that satisfies the condition $y(1) = 1$ is

**(a)** $2ye^y = e^x + e$ **(b)** $y^2 e^y = e^x$
**(c)** $ye^y = e^x$ **(d)** $(1 + y)e^y = 2e^x$

**Q. 33.** Find the positive real root of $x^3 - x - 3 = 0$ using Newton-Raphson method.
If the starting guess $(x_0)$ is 2, the numerical value of the root after two iterations $(x_2)$ is _______. (round off to two decimal places).

## CIVIL ENGINEERING (CE) P1

**Q. 34.** The rank of matrix $\begin{bmatrix} 1 & 2 & 2 & 3 \\ 3 & 4 & 2 & 5 \\ 5 & 6 & 2 & 7 \\ 7 & 8 & 2 & 9 \end{bmatrix}$ is

**(a)** 1 **(b)** 2
**(c)** 3 **(d)** 4

**Q. 35.** If $P = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $Q = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then $Q^T P^T$ is

**(a)** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$
**(c)** $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ **(d)** $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

**Q. 36.** The shape of the cumulative distribution function of Gaussian distribution is

**(a)** Horizontal line
**(b)** Straight line at 45 degree angle
**(c)** Bell – shaped
**(d)** S – shaped

**Q. 37.** Consider the limit:

$$\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$$

The limit (correct up to one decimal place) is ______.

**Q. 38.** The volume determined from $\iiint_V 8\,xyz\,dV$ for $V=[2,3]\times[1,2]\times[0,1]$ will be (in integer)____.

**Q. 39.** The solution of the second– order differential equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$ with boundary conditions $y(0)=1$ and $y(1)=3$ is

**(a)** $e^{-x}+(3e-1)xe^{-x}$
**(b)** $e^{-x}-(3e-1)xe^{-x}$
**(c)** $e^{-x}+\left[3e\sin\left(\frac{\pi x}{2}\right)-1\right]xe^{-x}$
**(d)** $e^{-x}-\left[3e\sin\left(\frac{\pi x}{2}-1\right)\right]xe^{-x}$

**Q. 40.** The value of $\int_0^1 e^x\,dx$ using the trapezoidal rule with four equal sub-intervals is

**(a)** 1.718 **(b)** 1.727
**(c)** 2.192 **(d)** 2.718

**Q. 41.** The values of abscissa ($x$) and ordinate ($y$) of a curve are as follows:

| $x$ | $y$ |
|---|---|
| 2.0 | 5.00 |
| 2.5 | 7.25 |
| 3.0 | 10.00 |
| 3.5 | 13.25 |
| 4.0 | 17.00 |

By Simpson's 1/3rd rule, the area under the curve (round off to two decimal places) is ______.

**Q. 42.** Vehicular arrival at an isolated intersection follows the Poisson distribution.

The mean vehicular arrival rate is 2 vehicle per minute. The probability (round off to two decimal places) that at least 2 vehicles will arrive in any given 1 – minute interval is ______.

## CIVIL ENGINEERING (CE) P2

**Q. 43.** The value of $\lim_{x\to\infty}\frac{x\ln(x)}{1+x^2}$ is

**(a)** 0 **(b)** 1.0
**(c)** 0.5 **(d)** $\infty$

**Q. 44.** The rank of the matrix $\begin{bmatrix}5 & 0 & -5 & 0\\ 0 & 2 & 0 & 1\\ -5 & 0 & 5 & 0\\ 0 & 1 & 0 & 2\end{bmatrix}$ is

**(a)** 1 **(b)** 2
**(c)** 3 **(d)** 4

**Q. 45.** The unit normal vector to the surface $X^2+Y^2+Z^2-48=0$ at the point (4, 4, 4) is

**(a)** $\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}$ **(b)** $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$
**(c)** $\frac{2}{\sqrt{2}},\frac{2}{\sqrt{2}},\frac{2}{\sqrt{2}}$ **(d)** $\frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}}$

**Q. 46.** If A is a square matrix then orthogonality property mandates

**(a)** $AA^T = I$ **(b)** $AA^T = 0$
**(c)** $AA^T = A^{-1}$ **(d)** $AA^T = A^2$

**Q. 47.** The value (round off to one decimal place) of $\int_{-1}^{1} x\,e^{|x|}\,dx$ is________.

**Q. 48.** If $k$ is a constant, the general solution of $\frac{dy}{dx}-\frac{y}{x}=1$ will be in the form of

**(a)** $y = x\ln(kx)$ **(b)** $y = k\ln(kx)$
**(c)** $y = x\ln(x)$ **(d)** $y = xk\ln(k)$

**Q. 49.** The smallest eigenvalue and the corresponding eigenvector of the matrix $\begin{bmatrix}2 & -2\\ -1 & 6\end{bmatrix}$, respectively, are

**(a)** 1.55 and $\begin{Bmatrix}2.00\\ 0.45\end{Bmatrix}$ **(b)** 2.00 and $\begin{Bmatrix}1.00\\ 1.00\end{Bmatrix}$
**(c)** 1.55 and $\begin{Bmatrix}-2.55\\ -0.45\end{Bmatrix}$ **(d)** 1.55 and $\begin{Bmatrix}2.00\\ -0.45\end{Bmatrix}$

**Q. 50.** A function is defined in Cartesian coordinate system as $f(x,y)=xe^y$. The value of the directional derivative of the function (in integer) at the point (2, 0) along the direction of the straight line segment from point (2, 0) to point $\left(\frac{1}{2},2\right)$ is ______.

**Q. 51.** Numerically integrate, $f(x) = 10x - 20x^2$ from lower limit $a = 0$ to upper limit $b = 0.5$. Use Trapezoidal rule with five equal subdivisions. The value (in units, round off to two decimal places) obtained is ______.

## COMPUTER SCIENCE (CS) P1

**Q. 52.** Let $p$ and $q$ be two propositions. Consider the following two formulae in propositional logic.

$S_1 : (\neg p \wedge (p \vee q)) \rightarrow q$

$S_2 : q \rightarrow (\neg p \wedge (p \vee q))$

Which one of the following choices is correct?

**(a)** Both $S_1$ and $S_2$ are tautologies.
**(b)** $S_1$ is a tautology but $S_2$ is not a tautology.
**(c)** $S_1$ is not a tautology but $S_2$ is a tautology.
**(d)** Neither $S_1$ nor $S_2$ is a tautology.

**Q. 53.** The lifetime of a component of a certain type is a random variable whose probability density function is exponentially distributed with parameter 2. For a randomly picked component of this type, the probability that its lifetime exceeds the expected lifetime (rounded to 2 decimal places ) is ______.

**Q. 54.** There are 6 jobs with distinct difficulty levels, and 3 computers with distinct processing speeds. Each job is assigned to a computer such that:

– The fastest computer gets the toughest job and slowest computer gets the easiest job.
– Every computer gets at least one job.

The number of ways in which this can be done is______.

**Q. 55.** Consider the following expression.

$$\lim_{x \to -3} \frac{\sqrt{2x+22}-4}{x+3}$$

The value of the above expression (rounded to 2 decimal places) is______.

**Q. 56.** Consider the following recurrence relation,

$$T(n) = \begin{cases} T\left(\frac{n}{2}\right) + T\left(\frac{2n}{5}\right) + 7n & \text{if } n > 0 \\ 1 & \text{if } n > 0 \end{cases}$$

Which one of the following options is correct ?

**(a)** $T(n) = \theta(n^{5/2})$ **(b)** $T(n) = \theta(n \log n)$
**(c)** $T(n) = \theta(n)$ **(d)** $T(n) = \theta[(\log n)^{5/2})]$

**Q. 57.** Consider the two statements.

$S_1$ : There exist random variable X and Y such that

$$\left(E\left[X - E(X)\left(Y - E(Y)\right)\right]\right)^2 > \mathrm{Var}[X]\mathrm{Var}[Y]$$

$S_2$ : For all random variables X and Y,

$$\mathrm{Cov}[X, Y] = E\left[\left|X - E[X]\right|\left|Y - E[Y]\right|\right]$$

Which one of the following choices is correct?

**(a)** Both $S_1$ and $S_2$ are true.
**(b)** $S_1$ is true, but $S_2$ is false.
**(c)** $S_1$ is false, but $S_2$ is true.
**(d)** Both $S_1$ and $S_2$ are false.

**Q. 58.** Consider the following matrix.

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

The largest eigenvalue of the above matrix is ________.

## COMPUTER SCIENCE (CS) P2

**Q. 59.** Choose the correct choice($s$) regarding the following propositional logic assertion

$S : ((P \wedge Q) \rightarrow R) \rightarrow ((P \wedge Q)) \rightarrow ((Q \rightarrow R))$

**(a)** S is neither a tautology nor a contradiction.
**(b)** S is a tautology.
**(c)** S is a contradiction.
**(d)** The antecedent of S is logically equivalent to the consequent of S.

**Q. 60.** For a given biased coin, the probability that the outcome of a toss is a head is 0.4.

This coin is tossed, 1,000 times. Let X denote the random variable whose value is the number of times that head appeared in these 1,000 tosses. The standard deviation of X (rounded to 2 decimal places) is_______.

**Q. 61.** Suppose that P is $a$ 4 × 5 matrix such that every solution of the equation $Px = 0$ is a scalar multiple of $[2\,5\,4\,3\,1]^T$. The rank of P is ______.

**Q. 62.** Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function on the interval [–3, 3] and a differentiable function in the interval (–3, 3) such that for every $x$ in the interval, $f'(x) \leq 2$. If $f(-3) = 7$, then $f(3)$ is at most ______.

**Q. 63.** A bag has $r$ red balls and $b$ black balls. All balls are identical except for their colours. In a trial, a ball is randomly drawn from the bag, its colour is noted and the ball is placed back into the bag along with another ball of the same colour. Note that the number of balls in the bag will increase by one, after the trial. A sequence of four such trials is conducted. Which one of the following choices gives the probability of drawing a red ball in the fourth trial?

**(a)** $\dfrac{r}{r+b}$

**(b)** $\dfrac{r}{r+b+3}$

**(c)** $\dfrac{r+3}{r+b+3}$

**(d)** $\left(\dfrac{r}{r+b}\right)\left(\dfrac{r+1}{r+b+1}\right)\left(\dfrac{r+2}{r+b+2}\right)\left(\dfrac{r+3}{r+b+3}\right)$

**Q. 64.** For constants $a \geq 1$ and $b > 1$, consider the following recurrence defined on the non–negative integers:

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)$$

Which one of the following options is correct about the recurrence T($n$)?

**(a)** If $f(n)$ is $n \log_2 (n)$, then $T(n)$ is $\Theta(n \log_2(n))$.

**(b)** If $f(n)$ is $\dfrac{n}{\log_2(n)}$, then $T(n)$ is $\Theta(\log_2(n))$

**(c)** If $f(n)$ is $\Theta\left(n^{\log_b(a)-\epsilon}\right)$ for some $\epsilon > 0$, then $T(n)$ is $\Theta\left(n^{\log_b(a)}\right)$.

**(d)** If $f(n)$ is $\Theta\left(n^{\log_b(a)}\right)$, then $T(n)$ is $\Theta\left(n^{\log_b(a)}\right)$.

**Q. 65.** Let S be a set consisting of 10 elements. The number of tuples of the form (A, B) such that A and B are subsets of S, and $A \subseteq B$ is______.

## CHEMICAL ENGINEERING (CH) P1

**Q. 66.** An ordinary differential equation (ODE), $\dfrac{dy}{dx} = 2y$, with an initial condition $y(0) = 1$, has analytical solution $y = e^{2x}$.

Using Runge-Kutta second order method, numerically integrate the ODE to calculate $y$ at $x = 0.5$ using a step size of $h = 0.5$.

If the relative percentage error is defined as,

$$\varepsilon = \left|\frac{y_{\text{analytical}} - y_{\text{numerical}}}{y_{\text{analytical}}}\right|$$

Then the value of $\varepsilon$ at $x = 0.5$ is________.

**(a)** 0.06 **(b)** 0.8

**(c)** 4.0 **(d)** 8.0

**Q. 67.** The function cos ($x$) is approximated using Taylor series around $x = 0$ as $\cos(x) \approx 1 + ax + bx^2 + cx^3 + dx^4$. The values of $a$, $b$, $c$ and $d$ are

**(a)** $a = 1, b = -0.5, c = -1, d = -0.25$

**(b)** $a = 0, b = -0.5, c = 0, d = 0.042$

**(c)** $a = 0, b = 0.5, c = 0, d = 0.042$

**(d)** $a = -0.5, b = 0, c = 0.042, d = 0$

**Q. 68.** A principal amount is charged a nominal annual interest rate of 10%. If the interest rate is compounded continuously, the final amount at the end of one year would be

**(a)** higher than the amount obtained when the interest rate is compounded monthly

**(b)** lower than the amount obtained when the interest rate is compounded annually

**(c)** equal to 1.365 times the principal amount

**(d)** equal to the amount obtained when using an effective interest rate of 27.18%

**Q. 69.** For the function $f(x) = \begin{cases} -x, & x < 0 \\ x^2, & x \geq 0 \end{cases}$ the CORRECT statement(s) is/are

**(a)** $f(x)$ is continuous at $x = 1$

**(b)** $f(x)$ is differentiable at $x = 1$

**(c)** $f(x)$ is continuous at $x = 0$

**(d)** $f(x)$ is differentiable at $x = 0$

**Q. 70.** A, B, C and D are vectors of length 4.

$A = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}$

$B = \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix}$

$C = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \end{bmatrix}$

$D = \begin{bmatrix} d_1 & d_2 & d_3 & d_4 \end{bmatrix}$

It is known that B is not a scalar multiple of A also, C is linearly independent of A and B. Further, D = 3A + 2B + C. The rank of the matrix $\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix}$ is ______.

**Q. 71.** Let A be a square matrix of size $n \times n$ ($n > 1$). The elements of $A = \{a_{ij}\}$ are given by

$$a_{ij} = \begin{cases} i \times j & \text{if } i \geq j \\ 0, & \text{if } i < j \end{cases}$$

The determinant of A is

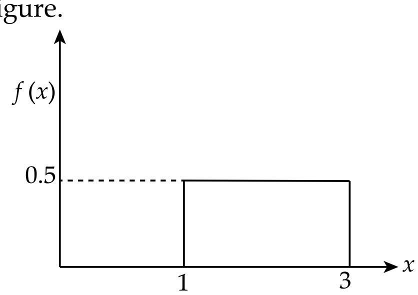
**(a)** 0 **(b)** 1

**(c)** $n!$ **(d)** $(n!)^2$

**Q. 72.** To solve an algebraic equation $f(x) = 0$, an iterative scheme of the type $x_{n+1} = g(x_n)$ is proposed, where $g(x) = x - \frac{f(x)}{f'(x)}$.

At the solution $x = s$, $g'(s) = 0$ and $g''(s) \neq 0$. The order of convergence for this iterative scheme near the solution is ____________.

**Q. 73.** The probability distribution function of a random variable X is shown in the following figure.




From this distribution, random samples with sample size $n = 68$ are taken.

If $\vec{X}$ is the sample mean, the standard deviation of the probability distribution of $\vec{X}$, *i.e.* $\sigma_{\bar{X}}$ is ______.

(round off to 3 decimal places).

**Q. 74.** For the ordinary differential equation

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y = 1$$

with initial conditions

$y(0) = y'(0) = y''(0) = y'''(0) = 0$, the value of $\lim_{t \to \infty} y(t) =$ ______.

(round off to 3 decimal places).

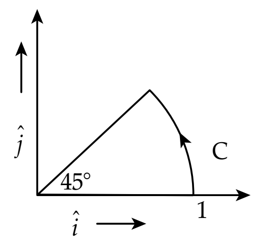
| **Answer Key** | | | |
|---|---|---|---|
| **Q. No.** | **Answer** | **Topic Name** | **Chapter Name** |
| 1 | (a) | Line Integral | Vector Calculus |
| 2 | (b) | 1st Order DE | Differential Equation |
| 3 | (b) | Mean and Variance | Probability and Statistics |
| 4 | 8 | Linearly Dependent | Vector Calculus |
| 5 | 0 | Curl | Vector Calculus |
| 6 | (a) | Residue Theorem | Complex Variable |
| 7 | (b) | Probability | Probability and Statistics |
| 8 | 10 | Eigen Values | Linear Algebra |
| 9 | (b) | Eigen Values | Linear Algebra |
| 10 | (d) | Complex roots | Functions |
| 11 | (c) | Maxima-Minima | Calculus |
| 12 | 1 | Application of Derivative | Calculus |
| 13 | (b) | Function | Calculus |
| 14 | (c) | Expected Value | Probability and Statistics |
| 15 | (c) | Cauchy's Integral Theorem | Complex Variable |
| 16 | 1 | Eigen Value | Linear Algebra |
| 17 | (c) | 1st Order DE | Differential Equation |
| 18 | (c) | Limit | Calculus |
| 19 | (d) | Laplace Transform | Transform Theory |
| 20 | (a) | Euler's Method | Numerical Methods |

| Q. No. | Answer | Topic Name | Chapter Name |
|---|---|---|---|
| 21 | (d) | Statistics | Probability and Statistics |
| 22 | (c) | Cauchy's Integral Theorem | Complex Variable |
| 23 | (d) | Poisson Distribution | Probability and Statistics |
| 24 | (c) | Mean Value Theorem | Calculus |
| 25 | (d) | Eigen Values | Linear Algebra |
| 26 | (c) | Laplace Transform | Transform Theory |
| 27 | (d) | Binomial Distribution | Probability and Statistics |
| 28 | (a) | Simpson's Rule | Numerical Method |
| 29 | (b) | Basics | Complex Variable |
| 30 | (b) | Double Integral | Calculus |
| 31 | (b) | Maxima-Minima | Calculus |
| 32 | (c) | 1st Order DE | Differential Equation |
| 33 | 1.67 | Newton Raphson | Numerical Method |
| 34 | (b) | Rank | Linear Algebra |
| 35 | (d) | Matrix | Linear Algebra |
| 36 | (d) | Normal Distribution | Probability and Statistics |
| 37 | 0.5 | Limit | Calculus |
| 38 | 15 | Volume Integral | Vector Calculus |
| 39 | (a) | 2nd Order DE | Differential Equation |
| 40 | (b) | Simpson's Rule | Numerical Method |
| 41 | 20.67 | Simpson's Rule | Numerical Method |
| 42 | 0.59 | Poisson Distribution | Probability and Statistics |
| 43 | (a) | Limit | Calculus |
| 44 | (c) | Rank | Linear Algebra |
| 45 | (b) | Gradient | Vector Calculus |
| 46 | (a) | Orthogonality of Matrix | Linear Algebra |
| 47 | 0 | Definite Integral | Calculus |
| 48 | (a) | 1st Order DE | Differential Equation |
| 49 | (a) | Eigen Value and Eigen Vector | Linear Algebra |
| 50 | 1 | Directional Derivative | Vector Calculus |
| 51 | 0.4 | Trapezoidal Rule | Numerical Method |
| 52 | (b) | Logic | Discrete Mathematics |
| 53 | 0.36 | Exponential Distribution | Probability and Statistics |
| 54 | 65 | Combinatory | Discrete Mathematics |
| 55 | 0.25 | Limit | Calculus |
| 56 | (c) | Recurrence Relation | Discrete Mathematics |
| 57 | (d) | Statistics | Probability and Statistics |

| Q. No. | Answer | Topic Name | Chapter Name |
|---|---|---|---|
| 58 | 3 | Eigen Value | Linear Algebra |
| 59 | (b),(d) | Logic | Discrete Mathematics |
| 60 | 15.49 | Statistics | Probability and Statistics |
| 61 | 4 | Rank | Linear Algebra |
| 62 | 19 | Mean Value Theorem | Calculus |
| 63 | (a) | Probability | Probability and Statistics |
| 64 | (c) | Recurrence Relation | Discrete Mathematics |
| 65 | 59049 | Sets | Discrete Mathematics |
| 66 | (d) | Runga-Kutta Method | Numerical Method |
| 67 | (b) | Taylor Series | Calculus |
| 68 | (a) | CI | Basic Maths |
| 69 | (a),(b),(c) | Continuity and Differentiability | Calculus |
| 70 | 3 | Rank | Linear Algebra |
| 71 | (d) | Determinant | Linear Algebra |
| 72 | 2 | Iterative Solution | Numerical Method |
| 73 | 0.07 | Probability and Statistics | Statistics |
| 74 | 0.167 | Differential Equation | Higher Order DE |

## ANSWERS WITH EXPLANATIONS

**1. Option (a) is correct.**




$$\int_C \vec{F}.d\vec{r} = \int\left(-x\hat{i} + y\hat{j}\right)\left(dx\hat{i} + dy\hat{j}\right)$$

$$= -x\,dx + y\,dy$$

Changing into polar form.

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$\theta \to 0 \text{ to } \frac{\pi}{4}$$

$$r = 1$$

$$= \int_{\theta=0}^{\frac{\pi}{4}} (-\cos\theta(-\sin\theta)d\theta + \sin\theta\cos\theta)d\theta$$

$$\int_{\theta=0}^{\frac{\pi}{4}} \sin 2\theta d\theta = \left.\frac{-\cos 2\theta}{2}\right|_0^{\frac{\pi}{4}} = \frac{1}{2}$$

**2. Option (b) is correct.**

Given, DE $\frac{dy}{dx} + \frac{x}{1-x^2}y = x\sqrt{y}$

Dividing both side by $\sqrt{y}$

$$\frac{1}{\sqrt{y}}\frac{dy}{dx} + \frac{x}{1-x^2}\frac{y}{\sqrt{y}} = x$$

$$\Rightarrow \quad \frac{1}{\sqrt{y}}\frac{dy}{dx} + \frac{x}{1-x^2}\sqrt{y} = x$$

Put $\sqrt{y} = t$

$$\frac{1}{2\sqrt{y}}\frac{dy}{dx} = \frac{dt}{dx}$$

$$2\frac{dt}{dx} + \frac{x}{1-x^2}t = x$$

$$\frac{dx}{dt} + \frac{1}{2}\frac{x}{1-x^2}t = \frac{x}{2}$$

Comparing with $\frac{dt}{dx} + P(x)t = Q$ standard form.

$$IF = e^{\int P(x)dx} = e^{\int (1/2)(x/1-x^2)dx}$$

$$= e^{(-1/4)\log(1-x^2)}$$

$$= e^{\log(1-x^2)^{-1/4}}$$

$$I.F = (1-x^2)^{-1/4}$$

**3. Option (b) is correct.**

Given, $Y = 2X + 3$

$$\text{var}[Y] = E\left[\left(Y - \overline{Y}\right)^2\right]$$

$$E[Y] = \overline{Y} = 2\overline{X} + 3$$

$$\text{var}[Y] = E\left[\left(2X + 3 - 2\overline{X} - 3\right)^2\right]$$

$$E = \left[4\left(X - \overline{X}\right)^2\right]$$

$$= 4E\left[\left(X - \overline{X}\right)^2\right]$$

$$\sigma_Y^2 = 4\sigma_X^2$$

**4. Correct answer is [8].**

$(1, -1, 2)$

$(7, 3, x)$

$(2, 3, 1)$ vectors are linearly dependent then

$$\begin{vmatrix} 1 & -1 & 2 \\ 7 & 3 & x \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad 1(3-3x) - 1(7-2x) + 2(21-6) = 0$$

$$\Rightarrow \quad (3-3x) - (7-2x) + 2(15) = 0$$

$$-5x = -40$$

$$x = 8$$

**5. Correct answer is [0].**

If the field is irrotational.

$$\nabla \times F = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4y - c_1 z & 4x + 2z & 2y + z \end{vmatrix} = 0$$

$$\hat{i}(2-2) - \hat{j}(0 + c_1) + \hat{k}(4-4) = 0$$

$$c_1 = 0$$

**6. Option (a) is correct.**

Poles are $x^2(x^2 - 4) = 0$

$$x^2 = 0$$

$\Rightarrow x = 0$ is a pole of order 2. And lie inside the circle $|z - i| = 2$.

$$x^2 - 4 = 0$$

$$\Rightarrow \quad x = \pm 2i$$

$x = 2i$ lie inside the circle.

$x = -2i$ lie outside the circle.

$$\operatorname{Re} s_0 = \frac{1}{(2-1)} \lim_{x \to 0} \frac{d}{dz}\left[(x-0)^2 \frac{\sin x}{x^2(x^2-4)}\right]$$

$$= \lim_{x \to 0} \frac{(x^2-4)\cos x - \sin x(2x)}{(x^2-4)^2} = \frac{1}{4}$$

$$\operatorname{Re} s_{2i} = \lim_{x \to 2i} (x - 2i) \frac{\sin x}{x^2 (x+2i)(x-2i)}$$

$$= \frac{\sin(2i)}{(-4)(4i)} = \frac{\sin(2i)}{-16i}$$

$$\oint_C f.dx = 2\pi i \left[\frac{1}{4} + \frac{\sin(2i)}{-16i}\right]$$

The value of integral $= -\frac{\pi}{8}\sin(2i)$

**7. Option (b) is correct.**

Given, (i) HT (ii) HH (iii) TT

Probability of getting head in the first toss

$$P(H_1) = \frac{1}{2}$$

$$P\left(\frac{H_2}{H_1}\right) = \frac{P(H_2 \cap H_1)}{P(H_1)}$$

For Head in 2nd toss with head in 1st toss

**Case I :** First coin – Fair coin

2nd coin be both head coin then

$$\left(\frac{1}{3} \times \frac{1}{2}\right) \times \left(\frac{1}{2} \times 1\right) = \frac{1}{12}$$

2nd coin be both tail coin then

$$\left(\frac{1}{3} \times \frac{1}{2}\right) \times \left(\frac{1}{2} \times 0\right) = \frac{1}{12}$$

**Case II :** First coin be Both head coin

2nd coin be fair coin then

$$\left(\frac{1}{3} \times 1\right) \times \left(\frac{1}{2} \times \frac{1}{2}\right) = \frac{1}{12}$$

2nd coin be both tail coin then

$$\left(\frac{1}{3} \times 1\right) \times \left(\frac{1}{2} \times 0\right) = 0$$

Since $P(H_2 \cap H_1) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$

$$\therefore \quad P\left(\frac{H_2}{H_1}\right) = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

**8. Correct answer is [10].**

For matrix A

The Characteristic equation is

$$|A - \lambda I| = 0 \;\Rightarrow\; \begin{vmatrix} x - \lambda & -3 \\ 3 & 4 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad \lambda^2 - (4+x)\lambda + (4x+9) = 0$$

For repeated roots

$$b^2 - 4ac = 0$$

$$(4+x)^2 - 4(4x+9) = 0$$

$$\Rightarrow \quad 16 + x^2 + 8x - 16x - 36 = 0$$

$$\Rightarrow \quad x^2 - 8x - 20 = 0$$

$$\Rightarrow \quad (x - 10)(x + 2) = 0$$

$$\Rightarrow \quad x = 10, -2$$

$\because$ $x$ is a positive real number.

$$x = 10$$

**9. Option (b) is correct.**

$$A = \begin{bmatrix} p & q \\ q & -p \end{bmatrix}$$

Characteristic equation

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} p - \lambda & q \\ q & -p - \lambda \end{vmatrix} = 0$$

$$\lambda^2 - \lambda(-p+p) + (-p^2 - q^2) = 0$$

$$\lambda^2 - p^2 - q^2 = 0$$

$$\lambda^2 - 1 = 0 \quad [\because p^2 + q^2 = 1]$$

$$\lambda = \pm 1$$

**10. Option (d) is correct.**

Given, $p(z) = z^3 + (1+j)z^2 + (2+j)z + 3$

Here polynomial has complex coefficient for $z^2$, which will be sum of roots. So, all roots are not real.

**11. Option (c) is correct.**

$f'(x_0) = 0$ and $f''(x_0) > 0$ for all $x \in (0, 1)$

$\therefore$ $f(x)$ has exactly one local minima in (0, 1)

**12. Correct answer is [1].**

$x^2 + y^2 = 1$

Differentiating w.r.t. $x$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2x = -2y\frac{dy}{dx}$$

$$x = -y\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$m_1 = \frac{-x}{y}$$

and $(x-1)^2 + (y-1)^2 = r^2$

Differentiating w.r.t. $x$

$$2(x-1) + 2(y-1)\frac{dy}{dx} = 0$$

$$2(x-1) = -2(y-1)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-(x-1)}{(y-1)} = m_2$$

When circles are in orthogonality their slope

$$m_1 m_2 = -1 \text{ at } (u, v)$$

$$\left(\frac{-x}{y}\right)\left(\frac{-(x-1)}{(y-1)}\right) = -1$$

$$\Rightarrow \quad \left(\frac{x}{y}\right)\left(\frac{x-1}{y-1}\right) = -1$$

$$\Rightarrow \quad \frac{x^2 - x}{y^2 - y} = -1$$

$$\Rightarrow \quad x^2 - x = -y^2 + y$$

$$\left(x^2 + y^2 = x + y\right)_{(u,v)} = (u^2 + v^2 = u + v)$$

$$u + v = 1 \qquad [\because u^2 + v^2 = 1]$$

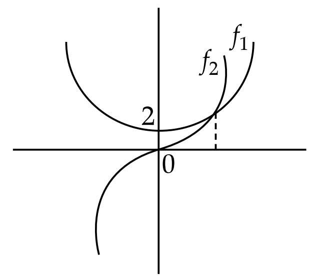
**13. Option (b) is correct.**

Given, polynomial $p(x) = x^4 - 4x^3 + 2$

$$x^4 + 2 = 4x^3$$

$$f_1(x) = x^4 + 2$$

$$f_2(x) = 4x^3$$




The point of intersection of function of graph

Root of $p(x) = 0$

$p(0)$ and $p(1)$ have opposite signs.

$\therefore$ Root of $p(x) = 0$ in (0, 1)

And from graph there is one point of intersection.

$\therefore$ One real root exists in (0, 1).

**14. Option (c) is correct.**

Let $x$ is number of tosses required.

| $x$ | 1 | 2 | 3 | ... |
|---|---|---|---|---|
| $p(x)$ | $p$ | $qp$ | $q^2p$ | ... |

Where $q = 1 - p$

$$E(x) = \sum x_i \; p(x_i)$$

$$= 1.p + 2.qp + 3.q^2p + ....$$

$$= p\left(1 + 2q + 3q^2 + ....\right)$$

$$= p(1-q)^{-2} \quad \left[\because (1-x)^2 = 1 + 2x + 3x^2 + 4x^3 + ...\right]$$

$$= p.p^{-2} = p^{-1} = \frac{1}{p}$$

**15. Option (c) is correct.**

$$\oint_C \frac{dz}{z^2(z-4)}$$

Singularities are $z^2(z-4) = 0$

$\therefore$ roots are $z = 0, 0, 4$

$z = 0$ and lies inside the circle C.

$z = 4$ and lies outside the circle C.

$$\text{Res} = \frac{1}{(2-1)}\lim_{z \to 0}\frac{d}{dz}\left[(z-0)^2 \frac{1}{z^2(z-4)}\right]$$

$$= \frac{-1}{(0-4)^2} = \frac{-1}{16}$$

Using Cauchy's Residue theorem

$$\oint_C \frac{dz}{z^2(z-4)} = 2\pi j \,\text{Re}\, s_0 = 2\pi j\left[\frac{-1}{16}\right] = \frac{-\pi j}{8}$$

**16. Correct answer is [1].**

Given, $A^5 = 0$

$\therefore$ A is a nilpotent matrix and $\lambda^5 = 0$

So, For $\lambda = 0$, eigen value of A + I is given by $\lambda + 1 \Rightarrow 0 + 1 = 1$

$\det(A + \lambda I)$ = Product of eigen values.

$1\times1\times1\times1\times1\times1\times1\times1\times1\times1 = 1$

**17. Option (c) is correct.**

$$(\sin x)\frac{dy}{dx} + y\cos x = 1$$

$$(y\cos x - 1)dx + \sin x dy = 0 \qquad \text{...(i)}$$

$$[\because Mdx + Ndy = 0]$$

$$\frac{\partial M}{\partial y} = \cos x = \frac{\partial N}{\partial x} = \cos x$$

$\therefore$ equation (i) is an exact differential equation Conceptual/numerical errors differential equation.

General solution $\int(y\cos x - 1)dx + \int(0)dy = c$

$$\Rightarrow \quad y\sin x - x = c \qquad \text{...(ii)}$$

given $\quad y\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \qquad \text{...(iii)}$

From equation (ii) and (iii)

$$\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right) - \frac{\pi}{2} = c$$

$$\Rightarrow \quad c = 0$$

$\therefore$ solution $\quad y = \dfrac{x}{\sin x}$

$$y\left(\frac{\pi}{6}\right) = \frac{\frac{\pi}{6}}{\sin\frac{\pi}{6}} = \frac{\pi}{6}\left(\frac{1}{\frac{1}{2}}\right) = \frac{\pi}{3}$$

**18. Option (c) is correct.**

$$\lim_{x\to0}\left(\frac{1-\cos x}{x^2}\right)\left(\frac{0}{0}\text{form}\right)$$

Applying L' Hospital Rule.

$$\lim_{x\to0}\left(\frac{\sin x}{2x}\right)\left(\frac{0}{0}\text{form}\right)$$

Again Applying L' Hospital Rule

$$\lim_{x\to0}\left(\frac{\cos x}{2}\right) = \frac{1}{2}$$

**19. Option (d) is correct.**

$$\delta(t-a) = \lim_{\in\to0}\begin{cases}\dfrac{1}{\in}, & a \le t \le a+\in \\ 0, & \text{otherwise}\end{cases}$$

$$\int^{\infty} f(t)\delta(t-t_0)\,dt = f(t_0)$$

$$L\{\delta(t-a)\} = \int_0^{\infty} e^{-st}.\delta(t-a)dt$$

$$= e^{-as}$$

$$\therefore \quad L\{\delta(t-a)\} = e^{-as}$$

**20. Option (a) is correct.**

Given, $\quad \dfrac{dy}{dt} = -\pi y \qquad \text{...(i)}$

$$\left(\because \frac{dy}{dx} = f(x, y)\right)$$

Also $y(0) = 1 \qquad \text{...(ii)}$

$$\frac{y(t_{n+1}) - y(t_n)}{h} = -\pi y(t_n)$$

$$y(t_{n+1}) - y(t_n) = h(-\pi y(t_n))$$

$$y(t_{n+1}) = y(t_n) + hf(t_n, y_n)$$

This is Euler method.

$$y_{n+1} = y_n + h(-\pi y_n)$$

$$y_{n+1} = (1 - h\pi)y_n \qquad \text{...(iii)}$$

This is in the form $y_{n+1} = Ey_n + hK$

For Euler method to be stable, $|E| < 1$

$$|1 - h\pi| < 1$$

$$\Rightarrow \quad -1 < 1 - h\pi < 1$$

$$\Rightarrow \quad -1 - 1 < -1 + 1 - h\pi < -1 + 1$$

$$\Rightarrow \quad -2 < -h\pi < 0$$

$$\Rightarrow \quad \frac{2}{\pi} > h > \frac{0}{\pi}$$

$$\therefore \quad 0 < h < \frac{2}{\pi} \text{ or } h \in \left(0, \frac{2}{\pi}\right)$$

**21. Option (d) is correct.**

"Normal"

**22. Option (c) is correct.**

Let $\quad I = \oint_C \dfrac{\cos h(3z)}{2z}dz$ where $|z| = 1$

$$I = \oint_C \frac{\frac{\cos h(3z)}{2}}{[z-0]}dz, \text{is of the form} \oint_C \frac{f(z)}{z-z_0}dz$$

$\therefore \quad I = 2\pi i f(z_0)$

$$\Rightarrow \quad I = 2\pi i\left(\frac{\cos h(3z)}{2}\right)_{z=0}$$

$$\Rightarrow \quad I = \pi i(\cos h(0)) \quad \left[\because \cos h(0) = \frac{e^0 + e^0}{2} = 1\right]$$

$$\therefore \quad I = \pi i$$

**23. Option (d) is correct.**

Probability of no customer in three minutes $\lambda T = 0.5$.

$$P_1 = (0,3) = \frac{e^{-0.5} \times (0.5)^0}{0!} = 0.606$$

Probability of one customer in first six minutes $\lambda T = 1$.

$$P_3 = (0,6) = \frac{e^{-1} \times (1)^1}{1!} = 0.3678$$

$\Rightarrow \quad P_1 \times P_2 = P_3$

$\Rightarrow \quad 0.606 \times P_2 = 0.3678$

$\Rightarrow \quad P_2 = 0.6069 \approx 0.61$

**24. Option (c) is correct.**

$$f(x) = x^2 - 2x + 2 \text{ and } [a, b] = (1, 3)$$

$$f'(x) = 2x - 2$$

By Lagrangian mean value theorem

$\Rightarrow \quad f'(c) = \dfrac{f(3) - f(1)}{3 - 1}$

$\Rightarrow \quad 2c - 2 = \dfrac{5 - 1}{2} \qquad [\because f(3) = 5,\ f(1) = 1]$

$\Rightarrow \quad c - 1 = 1 \Rightarrow c = 2$ or $x = 2$

**25. Option (d) is correct.**

$$Ap = \alpha^2 p$$

Comparing with $\quad AX = \lambda X$

$\Rightarrow \quad \lambda = \alpha^2$

$\therefore$ Eigen value of A is $\alpha^2$, so eigen value of $A^2$ is $\alpha^4$.

**26. Option (c) is correct.**

$$L[f(t)] = \frac{(s+3)}{(s+1)(s+2)}$$

$\Rightarrow \quad \dfrac{(s+3)}{(s+1)(s+2)} = \dfrac{A}{(s+1)} + \dfrac{B}{(s+2)}$

$\Rightarrow \quad (s+3) = A(s+2) + B(s+1)$

Comparing the coefficients on both sides.

$\Rightarrow \quad A + B = 1$ and $2A + B = 3$

$\Rightarrow \quad A = 2$ and $B = -1$

$\Rightarrow \quad \dfrac{(s+3)}{(s+1)(s+2)} = \dfrac{2}{(s+1)} - \dfrac{1}{(s+2)}$

$$f(t) = L^{-1}\left(\frac{2}{s+1}\right) - L^{-1}\left(\frac{1}{s+2}\right)$$

$$f(t) = 2e^{-t} - e^{-2t}$$

$$f(0) = 2e^{-0} - e^{-2(0)} = 1$$

**27. Option (d) is correct.**

For binomial distribution

Mean = $np$

$p$ is the probability of occurrence.

Variance = $npq = np(1 - p) \qquad [\because q = 1 - p]$

**28. Option (a) is correct.**

| $x$ | 4 | 4.3 | 4.6 | 4.9 | 5.2 |
|---|---|---|---|---|---|
| $y = \ln x$ | ln 4<br>$y_0$ | ln 4.3<br>$y_1$ | ln 4.6<br>$y_2$ | ln 4.9<br>$y_3$ | ln 5.2<br>$y_4$ |

According to Simpson's 1/3rd rule.

$$I = \frac{h}{3}\left[(y_0 + y_4) + 4(y_1 + y_3) + 2y_2\right]$$

$$= \frac{0.3}{3}\left[(\ln 4 + \ln 5.2) + 4(\ln 4.3 + \ln 4.9) + 2(\ln 4.6)\right]$$

$I = 1.83 \qquad$ [where $h = 0.3$]

**29. Option (b) is correct.**

Given, $(1+i)^8$

$$z = 1 + i$$

$$r = |z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$$

$$(1+i)^8 = \left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^8 = \sqrt{2}^8 \cdot e^{\left(i\frac{\pi}{4}\right)^8}$$

$$= 16 \times e^{i2\pi}$$

$$= 16(\cos 2\pi + i \sin 2\pi)$$

$$= 16 \times 1 \quad [\sin 2\pi = 0] = 16$$

**30. Option (c) is correct.**

Given, $\displaystyle\int_{\theta=0}^{\frac{\pi}{2}} \int_{r=0}^{\cos\theta} r \sin\theta \, dr \, d\theta$

$$= \int_{\theta=0}^{\frac{\pi}{2}} \left[\frac{r^2}{2}\right]_{r=0}^{\cos\theta} \sin\theta d\theta$$

$$= \frac{1}{2}\int_{\theta=0}^{\frac{\pi}{2}} \cos^2\theta \sin\theta d\theta$$

Put $\cos\theta = t$

$-\sin\theta d\theta = dt$

at $\theta = \dfrac{\pi}{2}$; $t = 0$ and $\theta = 0$; $t = 1$

$$= \int_1^0 \frac{-t^2}{2} dt = -\frac{1}{2}\left[\frac{t^3}{3}\right]_1^0 = -\frac{1}{2}\left(-\frac{1}{3}\right) = \frac{1}{6}$$

**31. Option (b) is correct.**

$$f(x)=\frac{1}{2}x^{T}Qx-r^{T}x$$

Let Q be (2 × 2) matrix

$$Q_{2\times2}=\begin{bmatrix}a & c\\ c & b\end{bmatrix},$$

$$x_{2\times1}=\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$R_{2\times1}=\begin{bmatrix}r_1\\ r_2\end{bmatrix}$$

$$f(x)=\frac{1}{2}\left[\begin{bmatrix}x_1 & x_2\end{bmatrix}\begin{bmatrix}a & c\\ c & b\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}\right]-\begin{bmatrix}r_1 & r_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$=\frac{1}{2}\left[ax^2{}_1+bx^2{}_2+2cx_1\cdot x_2\right]-\left[r_1x_1+r_2x_2\right]$$

*i.e.* for $f(x)=U(x_1,x_2)$

$$=\frac{1}{2}ax^2{}_1+\frac{1}{2}bx^2{}_2+cx_1.x_2-r_1x_1-r_2x_2$$

For critical point $\frac{\partial U}{\partial x_1}=0$ and $\frac{\partial U}{\partial x_2}=0$

$\Rightarrow \quad a_1x_1+cx_2-r_1=0$ and $bx_2+cx_1-r_2=0$

In matrix form

$$\begin{bmatrix}a & c\\ c & b\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}r_1\\ r_2\end{bmatrix}$$

So, $\qquad Qx=R \qquad$ ...(i)

Multiplying above equation both side by $Q^{-1}$

$$x=Q^{-1}R$$

**32. Option (c) is correct.**

Here, $(1+y)\frac{dy}{dx}=y$

$$\frac{(1+y)}{y}dy=dx$$

$$\left(\frac{1}{y}+1\right)dy=dx$$

Integrating both sides

$$\log y+y=x+c$$

at $x=1, y=1$

$$\log 1+1=1+c\Rightarrow c=0$$

$\therefore \quad \log y+y=x$

$\Rightarrow \quad \log y=x-y$

$\Rightarrow \quad y=e^{x-y}=\frac{e^x}{e^y}$

$\Rightarrow \quad ye^y=e^x$

**33. Correct answer is [1.67].**

$$f(x)=x^3-x-3$$

$$f'(x)=3x^2-1$$

$$x_0=2$$

$$f(x_0)=x^3-x-3=3$$

$$f'(x_0)=3x^2-1=11$$

Using Newton Raphson method

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=2-\frac{3}{11}=1.727$$

$$x_2=1.727-\frac{\left[(1.727)^3-1.727-3\right]}{\left[3(1.727)^2-1\right]}=1.67$$

**34. Option (b) is correct.**

$$A=\begin{bmatrix}1 & 2 & 2 & 3\\ 3 & 4 & 2 & 5\\ 5 & 6 & 2 & 7\\ 7 & 8 & 2 & 9\end{bmatrix}$$

$R_2\to R_2-3R_1$

$R_3\to R_3-5R_1$

$R_4\to R_4-7R_1$

$$=\begin{bmatrix}1 & 2 & 2 & 3\\ 0 & -2 & -4 & -4\\ 0 & -4 & -8 & -8\\ 0 & -6 & -12 & -12\end{bmatrix}$$

$R_3\to R_3-2R_2$

$R_4\to R_4-3R_2$

$$=\begin{bmatrix}1 & 2 & 2 & 3\\ 0 & -2 & -4 & -4\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

No. of non-zero rows are two.

$\Rightarrow \quad \rho(A)=2$

**35. Option (d) is correct.**

$$PQ=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}2 & 1\\ 4 & 3\end{bmatrix}$$

$$(PQ)^T=\begin{bmatrix}2 & 4\\ 1 & 3\end{bmatrix}$$

According to reversal law

$$Q^TP^T=(PQ)^T=\begin{bmatrix}2 & 4\\ 1 & 3\end{bmatrix}$$

**36. Option (d) is correct.**

'S' Shaped

**37. Correct answer is [0.5].**

For $\lim_{x\to1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)(\infty-\infty \text{ form})$

$$\lim_{x\to1}\left(\frac{x-1-\ln x}{\ln x(x-1)}\right)\left(\frac{0}{0}\text{ form}\right)$$

Applying L' Hospital Rule

$$\lim_{x\to1}\left(\frac{1-\frac{1}{x}}{\frac{1}{x}(x-1)+\ln x}\right)\frac{0}{0}\text{ form}$$

Applying L' Hospital Rule again

$$\lim_{x\to1}\left(\frac{\frac{1}{x^2}}{\frac{1}{x^2}+\frac{1}{x}}\right)=\frac{1}{2}=0.5$$

as $\frac{1}{x}(x-1)=1-\frac{1}{x}$

**38. Correct answer is [15].**

Volume $=\iiint_V 8xy\,zd\,V$

$$=8\int_{z=0}^{1}\int_{y=1}^{2}\int_{x=2}^{3} xyz\ dx\ dy\ dz$$

$$=8\int_0^1 zdz\int_1^2 ydy\int_2^3 xdx$$

$$=8\left(\frac{z^2}{2}\right)_0^1\left(\frac{y^2}{2}\right)_1^2\left(\frac{x^2}{2}\right)_2^3$$

$$=8\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}(1-0)(4-1)(9-4)$$

$$=1\cdot3\cdot5=15$$

**39. Option (a) is correct.**

$$\left(D^2+2D+1\right)y=0$$

$$AE\to m^2+2m+1=0$$

$\Rightarrow$ $(m+1)^2=0$

$\Rightarrow$ $m=-1,-1$

$$CF=\left(C_1+C_2x\right)e^{-x}$$

$$PI=0$$

$$y=CF+PI$$

So, $y=\left(C_1+C_2x\right)e^{-x}$

At $x=0, y=1$

$$C_1=1$$

At $x=1, y=3$

$$C_2=3e-1$$

$\therefore$ $y=\left(1+(3e-1)x\right)e^{-x}$

$\Rightarrow$ $y=e^{-x}+(3e-1)xe^{-x}$

**40. Option (b) is correct.**

With 4 Sub intervals

Step size

$$h=\frac{1-0}{4}=\frac{1}{4}$$

| $x$ | 0 | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | 1 |
|---|---|---|---|---|---|
| $f(x)=e^x$ | 1 | $e^{\frac{1}{4}}$ | $e^{\frac{1}{2}}$ | $e^{\frac{3}{4}}$ | $e$ |

According to Trapezoidal rule

$$I=\frac{h}{2}\left[(y_0+y_4)+2(y_1+y_2+y_3)\right]$$

$$=\frac{0.25}{2}\left[(1+e)+2\left(e^{\frac{1}{4}}+e^{\frac{1}{2}}+e^{\frac{3}{4}}\right)\right]$$

$$=\frac{0.25}{2}(13.8177)=1.72721\approx1.727$$

**41. Correct answer is [20.67].**

$$\text{Area}=\int_1^4 f(x)$$

According to Simpson's 1/3rd Rule

$$I=\frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2y_2\right]$$

$$=\frac{1}{6}\left[(5+17)+4(7.25+13.25)+2(10)\right]=20.67$$

| | $x$ | $y$ | |
|---|---|---|---|
| $x_0$ | 2.0 | 5.00 | $y_0$ |
| $x_1$ | 2.5 | 7.25 | $y_1$ |
| $x_2$ | 3.0 | 10.00 | $y_2$ |
| $x_3$ | 3.5 | 13.25 | $y_3$ |
| $x_4$ | 4.0 | 17.00 | $y_4$ |

**42. Correct answer is [0.59].**

Given, (Mean) $\lambda=2$ vehicles/minute.

Probability at least two vehicles arrive in 1 minute, $P(x\geq2)=1-\left[P(x=0)+P(x=1)\right]$

$$\because\ P(x=0)+P(x=1)=\frac{e^{-2}(2)^0}{0!}+\frac{e^{-2}(2)^1}{1!}$$

$$=e^{-2}(1+2)$$

$$=0.406$$

So, the Required Probability $=1-0.406$

$$=0.593\simeq0.59$$

**43. Option (a) is correct.**

$$\lim_{x\to\infty}\left(\frac{x\ln x}{1+x^2}\right)\left(\frac{\infty}{\infty}\right)\text{form}$$

Applying L' Hospital Rule

$$= \lim_{x\to\infty}\left(\frac{x\frac{1}{x}+\ln x}{2x}\right)\to\left(\frac{\infty}{\infty}\right)\text{form}$$

Applying L' Hospital again

$$= \lim_{x\to\infty}\frac{\left(0+\frac{1}{x}\right)}{2}$$

$$= \frac{0+\frac{1}{\infty}}{2} = \frac{0}{2} = 0$$

**44. Option (c) is correct.**

$$A = \begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 2 & 0 & 1 \\ -5 & 0 & 5 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$R_3 \to R_3 + R_1$

$$= \begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$R_4 \to R_4 - \frac{R_2}{2}$

$$= \begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.5 \end{bmatrix}$$

The no. of non-zero row is 3.

$\therefore \quad \rho(A) = 3 = \text{Rank}.$

**45. Option (b) is correct.**

Surface $\phi = x^2 + y^2 + z^2 - 48 = 0$

grad $\phi = \nabla.\phi = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$

$$\nabla.\phi_{(4,4,4)} = 2\times 4\hat{i} + 2\times 4\hat{j} + 2\times 4\hat{k}$$

$$= 8\hat{i} + 8\hat{j} + 8\hat{k}$$

$$\hat{n} = \frac{\nabla.\phi}{|\nabla.\phi|} = \frac{8\hat{i}+8\hat{j}+8\hat{k}}{\sqrt{8^2+8^2+8^2}} = \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$$

**46. Option (a) is correct.**

If the product of a matrix and its transpose is equal to identity matrix then square matrix is called orthogonal matrix that is $AA^T = I$

**47. Correct answer is [0].**

$$\int_{-1}^{1} xe^{|x|}dx = 0$$

[$\because$ The given function is odd function, and the integration of odd function is zero]

**48. Option (a) is correct.**

Given, $\quad \dfrac{dy}{dx} - \dfrac{y}{x} = 1$

$$\Rightarrow \quad \frac{dy}{dx} + \underbrace{\left(-\frac{1}{x}\right)}_{P} y = \underset{Q}{1}$$

$$\text{IF} = e^{\int P dx} = e^{\int\left(-\frac{1}{x}\right)dx} = e^{-\ln x} = x^{-1} = \frac{1}{x}$$

Solution of DE $y.\left(\dfrac{1}{x}\right) = \int 1.\left(\dfrac{1}{x}\right)dx + C$

$$\frac{y}{x} = \ln x + \ln k$$

$$\frac{y}{x} = \ln(kx)$$

$$y = x\ln(kx)$$

**49. Option (a) is correct.**

Let $A = \begin{bmatrix} 2 & -2 \\ -1 & 6 \end{bmatrix}$

Characteristics equation is

$$|A - \lambda I| = 0$$

$$\Rightarrow \quad \begin{vmatrix} 2-\lambda & -2 \\ -1 & 6-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad (2-\lambda)(6-\lambda) - 2 = 0$$

$$\Rightarrow \quad 12 - 8\lambda + \lambda^2 - 2 = 0$$

$$\Rightarrow \quad \lambda^2 - 8\lambda + 10 = 0$$

$$\therefore \quad \lambda = \left(4+\sqrt{6}\right) \text{ and } \left(4-\sqrt{6}\right)$$

Smallest eigen value $\left(4-\sqrt{6}\right)$

As $\quad AX = \lambda X$

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} 2-\left(4-\sqrt{6}\right) & -2 \\ -1 & 6-\left(4-\sqrt{6}\right) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + \sqrt{6}x_1 - 2x_2 = 0$$

$$x_1 = \left(\frac{2}{-2+\sqrt{6}}\right)x_2$$

let $x_2 = k$

$$x_1 = \left(\frac{2}{-2+\sqrt{6}}\right)k$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{2}{-2+\sqrt{6}}k \\ k \end{bmatrix} = \begin{bmatrix} \frac{2}{-2+\sqrt{6}} \end{bmatrix} = \begin{bmatrix} 2.00 \\ 0.45 \end{bmatrix}$$

for $\lambda = 4 - \sqrt{6} \approx 1.55$

**50. Correct answer is [1].**

$$f(x, y) = xe^y$$

$\mathrm{A}(2,0)$ and $\mathrm{B}\left(\frac{1}{2}, 2\right)$

$$\text{grad } f = (e^y)\hat{i} + (xe^y)\hat{j} + (0)\hat{k}$$

$$(\text{grad } f)_{\mathrm{P}} = \hat{i} + 2\hat{j}$$

$$\overrightarrow{\mathrm{AB}} = \left(\frac{1}{2} - 2\right)\hat{i} + (2-0)\hat{j} = \frac{-3}{2}\hat{i} + 2\hat{j}$$

Directional derivative

$$(\text{grad } f)_{\mathrm{P}} . \overrightarrow{\mathrm{AB}} = (\hat{i} + 2\hat{j}) . \frac{\left(\frac{-3}{2}\hat{i} + 2\hat{j}\right)}{\sqrt{\left(\frac{3}{2}\right)^2 + 2^2}}$$

$$= \frac{\frac{-3}{2} + 4}{\sqrt{\frac{25}{4}}} = \frac{\frac{-3+8}{2}}{\frac{5}{2}} = 1$$

**51. Correct answer is [0.40]**

$$f(x) = 10x - 2x^2$$

$a = 0, b = 0.5$ and $n = 5$

$$h = \frac{b-a}{n} = \frac{0.5-0}{5} = 0.1$$

| $x$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|---|
| $y = f(x)$ | 0<br>$y_0$ | 0.8<br>$y_1$ | 1.2<br>$y_2$ | 1.2<br>$y_3$ | 0.8<br>$y_4$ | 0<br>$y_5$ |

According to trapezoidal rule

$$\mathrm{I} = \frac{h}{2}\left[(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)\right]$$

$$= \frac{0.1}{2}\left[(0+0) + 2(0.8 + 1.2 + 1.2 + 0.8)\right]$$

$$= \frac{0.1 \times 8}{2} = 0.40$$

**52. Option (b) is correct.**

$$\mathrm{S}_1 : (\neg p \wedge (p \vee q)) \to q$$

$$\mathrm{S}_2 : q \to (\neg p \wedge (p \vee q))$$

$$\mathrm{S}_1 : [p'(p+q)] \to q$$

$\equiv [p'p + p'q] \to q$ [Distributive Law]

$\equiv p'q \to q$ $[p'\ p = c]$

$\equiv (p'q)' + q$ [Demorgan Law]

$\equiv p + q' + q$ $[q + q' = t]$

$\equiv p + 1$

$\equiv 1$ [tautology]

$$\mathrm{S}_2 : q \to (p'(p+q))$$

$\equiv q \to (p'p + p'q)$ [Distributive Law]

$\equiv q \to p'q$

$\equiv q' + p'q$

$\equiv (q' + p')$ (contingency)

Not a tautology.

**53. Correct answer is [0.36].**

For $t$ = {lifetime of component} and $\mu = 2$

Expected lifetime $\mathrm{E}(x) = \frac{1}{\mu}$

$$f(t) = \begin{cases} \mu e^{-\mu t} & t > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\mathrm{P}\left(t > \frac{1}{\mu}\right)$$

$$= \int_{\frac{1}{2}}^{\infty} \mu e^{-\mu t} dt$$

$$= -\left[e^{-\mu t}\right]_{\frac{1}{2}}^{\infty}$$

$= -[0 - e^{-1}]$ [for $\mu = 2$]

$$= e^{-1} = 0.367$$

**54. Correct answer is [65].**

Given the fastest computer gets the toughest job and the slowest computer gets the easiest job.

$\therefore$ Remaining 4 jobs to assign 3 computers in such a way that every computer gets at least one job.

Now only computer with medium speed has no job yet, so we need to remove combination of with no job in computer with medium speed.

$$= 3^4 - 1 \times (3-1)^4$$

$$= 81 - 16 = 65$$

**55. Correct answer is [0.25].**

$$\lim_{x\to -3}\left(\frac{\sqrt{2x+22}-4}{x+3}\right)\left(\frac{0}{0}\text{form}\right)$$

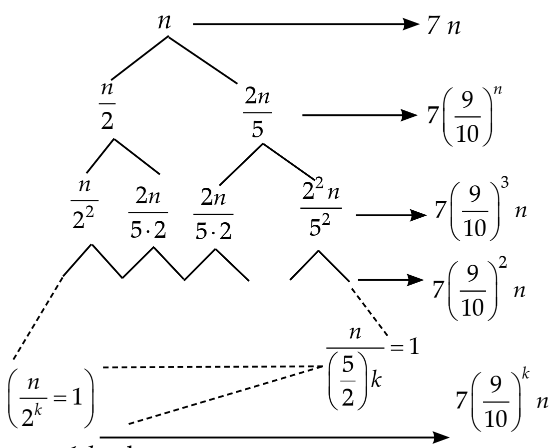
Applying L – H Rule

$$=\lim_{x\to -3}\left(\frac{\frac{2}{2\sqrt{2x+22}}}{1}\right)$$

$$=\lim_{x\to -3}\left(\frac{1}{\sqrt{2x+22}}\right)$$

$$=\frac{1}{\sqrt{16}}=0.25$$

**56. Option (c) is correct.**




$= 1\ k = \log_2 n$

$$T(n)=7n\left[1+\frac{9}{10}+\left(\frac{9}{10}\right)^2+\left(\frac{9}{10}\right)^3+\ldots+\left(\frac{9}{10}\right)^k\right]$$

$$=\frac{7n\left[1-\left(\frac{9}{10}\right)^{k+1}\right]}{1-\frac{9}{10}}$$

$$=\frac{7n}{\frac{1}{10}}\left[1-\left(\frac{9}{10}\right)^{\log_2 n+1}\right] \quad \text{decreasing term.}$$

$$T(n)=\theta(n)$$

**57. Option (d) is correct.**

$S_2 : \text{Cov}(x,y)=E\{|x-\bar{x}||y-\bar{y}|\}$ is false.

Expression is not always true. So, we can make conclusion $S_2$ is false.

$S_1$: is false.

$$\because\ E\left[\{|x-\bar{x}||y-\bar{y}|\}\right]^2 \le \text{var}(x).\text{var}(y)$$

So, both $S_1$ and $S_2$ are false.

**58. Correct answer is [3].**

$$A=\begin{bmatrix}0&1&1&1\\1&0&1&1\\1&1&0&1\\1&1&1&0\end{bmatrix}$$

For $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix}-\lambda&1&1&1\\1&-\lambda&1&1\\1&1&-\lambda&1\\1&1&1&-\lambda\end{vmatrix}=0$$

$$\Rightarrow \begin{vmatrix}-\lambda+3&1&1&1\\-\lambda+3&-\lambda&1&1\\-\lambda+3&1&-\lambda&1\\-\lambda+3&1&1&-\lambda\end{vmatrix}=0\ C_1\to C_1+C_2+C_3+C_4$$

taking $(-\lambda + 3)$ common from $C_1$

$$\Rightarrow \quad (-\lambda+3)\begin{vmatrix}1&1&1&1\\1&-\lambda&1&1\\1&1&-\lambda&1\\1&1&1&-\lambda\end{vmatrix}=0$$

Applying $R_2\to R_2-R_1$, $R_3\to R_3-R_1$,
$R_4\to R_4-R_1$

$$\Rightarrow \quad (-\lambda+3)\begin{vmatrix}1&1&1&1\\0&-\lambda-1&0&0\\0&0&-\lambda-1&0\\0&0&0&-\lambda-1\end{vmatrix}=0$$

$$\Rightarrow \quad (-\lambda+3)(-\lambda-1)^3=0$$

$\Rightarrow \quad \lambda=3,-1,-1,-1$ are the roots

Clearly, maximum eigen value is 3.

**59. Option (b, d) are correct.**

Antecedent of S: $(P\wedge Q)\to R$

$\equiv \sim(P\wedge Q)\vee R$

$\equiv \sim P\vee \sim Q\vee R$ [Demorgan's Law]

Consequent of S: $(P\wedge Q)\to(Q\to R)$

$\equiv (P\wedge Q)\to(\sim Q\vee R)$

$\equiv \sim(P\wedge Q)\vee(\sim Q\vee R)$

$\equiv \sim P\vee \sim Q\vee(\sim Q\vee R)$ [Demorgan's Law]

$\equiv \sim P\vee \sim Q\vee R$

Antecedent of S is equivalent to Consequent of S. Hence option (d) is correct

$A\to A$ is always tautology. Option (b) is correct.

**60. Correct answer is [15.49].**

Given, $n = 1000$, $p = 0.4$, $q = 0.6$.

For binomial distribution standard deviation is given by

$$SD = \sqrt{npq} = \sqrt{1000 \times 0.4 \times 0.6} = 15.49$$

**61. Correct answer is [4].**

$P_{4\times5} \rightarrow$ No. of unknowns $(n) = 5$ in $PX = 0$.

Also, Nullity $PX = 0$ is one that is $N(P) = 1$.

As number of independent solution is one.

Nullity = No. of unknowns – rank

$1 = 5 - \rho(P)$

$\rho(P) = 4$

**62. Correct answer is [19].**

Given, $f'(x) \le 2$ and $f(-3) = 7$

From MUT $f'(x) = \dfrac{f(3) - f(-3)}{3-(-3)}$

$$\Rightarrow \quad f'(x) = \frac{f(3)-7}{3-(-3)}$$

$$\Rightarrow \quad f'(x) = \frac{f(3)-7}{6}$$

$$\Rightarrow \quad 6f'(x) = f(3) - 7$$

$$\Rightarrow \quad f(3) = 6f'(x) + 7$$

$$\Rightarrow \quad f(3) = 6(2) + 7 = 19$$

$[\because$ max value of $f'(x) = 2]$

**63. Option (a) is correct.**

Let after $i^{th}$ iteration we have $r$ red balls and $b$ black balls.

So, the probability of picking red ball in $(i + 1)^{th}$ iteration will be–

$$= \frac{r}{r+b}$$

So, the probability of picking red ball in $(i + 2)^{th}$ iteration will be–

Considering in $(i+1)^{th}$ iteration. We could choose either red or black ball.

$$= \frac{r}{r+b} \times \frac{r+1}{r+b+1} + \frac{b}{r+b} \times \frac{r}{r+b+1}$$

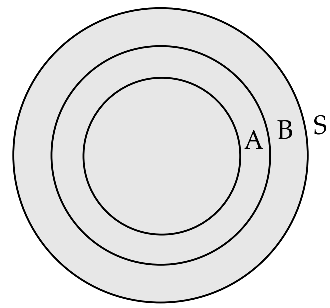
$$= \frac{r}{r+b}$$

From the initial condition we have $r$ red balls and $b$ black balls. So, in every iteration, the probability of picking a red ball will be $= \dfrac{r}{r+b}$.

**64. Option (c) is correct.**

From Standard Master theorem only Option (c) is correct.

**65. Correct answer is [59049].**




The Venn diagram for given problem is –

Every element $x$ in Set S has only three options–

I. $x \in A$ or

II. $x \in B - A$ or

III. $x \in S - B$

$\therefore$ No. of ways to choose A and B such that $A \subseteq B \subseteq S$ is $3^{10} = 59049$

**66. Option (d) is correct.**

Given, $f(x, y) = 2y = \dfrac{dy}{dx}$

$x_0 = 0,\ y_0 = 1,\ h = 0.5$

at $x = 0.5 \Rightarrow y = e^{2\times0.5} = e \approx 2.72$ is exact solution (analytical solution)

Using $2^{nd}$ order R-K Method.

$$y_1 = y_0 + \frac{1}{2}[k_1 + k_2]$$

$$k_1 = hf(x_0, y_0) = 0.5f(0,1) = 0.5 \times 2 \times 1 = 1$$

$$k_2 = hf(x_0 + h, y_0 + k_1) = 0.5f(0.5, 2)$$
$$= 0.5 \times 2 \times 2 = 2$$

$$\therefore \quad y_1 = 1 + \frac{1}{2}[1+2] = 2.5$$

is numerical (approximate value)

$$\varepsilon = \left|\frac{2.72 - 2.5}{2.72}\right| \times 100 = 8.08$$

**67. Option (b) is correct.**

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - .......$$

(Standard Maclaurin's Series) ...(i)

Now, Comparing equation (i)

$$\cos(x) = a + bx + cx^2 + dx^3......$$

We have $a = 0$, $b = -0.5$, $c = 0$, $d = \dfrac{1}{24} = 0.042$

**68. Option (a) is correct.**

Given, I = 10% = 0.1

Time = $n$ = 1 year

$\therefore$ Final amount at the end of one year is calculated as.

$$S = Pe^{rn} = Pe^{0.1} = 1.105\ P$$

$$i_{eff} = e^r - 1 = e^{0.1} - 1 = 10.5\%$$

For monthly compounding

$$S = \left(1 + \frac{r}{n}\right)^n P$$

$$= \left(1 + \frac{0.10}{12}\right)^{12} P$$

$$= 1.10\,47\ P$$

$$i_{eff} = S - 1$$

$$= 1.1047 - 1$$

$$= 10.47\%$$

$S = 1.105\,P$ means the final amount at the end of one year is greater than the amount obtained if the interest rate is compounded monthly.

**69. Option (a, b, c) is correct.**

At $x = 0$

LHL: $\lim_{x\to 0^-} f(x) = \lim_{x\to 0}(-x) = 0$ and $f(0) = 0$

RHL: $\lim_{x\to 0^+} f(x) = \lim_{x\to 0}(x^2) = 0$

$\therefore \lim_{x\to 0} f(x) = f(0) = f(x)$ is continuous at $x = 0$.

And $f(x) = x^2, x \geq 0$ (is a polynomial function and is continuous at $x = 1$.)

At $x = 0$

$$\text{LHD}: \lim_{x\to 0^-}\left(\frac{f(x)-f(0)}{x-0}\right) = \lim_{x\to 0}\left(\frac{-x-0}{x-0}\right) = -1$$

$$\text{RHD}: \lim_{x\to 0^+}\left(\frac{f(x)-f(0)}{x-0}\right) = \lim_{x\to 0}\left(\frac{x^2-0}{x-0}\right) = 0$$

LHD $\neq$ RHD $\Rightarrow f(x)$ is not differentiable at $x = 0$.

And $f(x) = x^2, x \geq 0$ (is a polynomial function)

$\Rightarrow f'(x) = 2x \Rightarrow f(x)$ is differentiable at $x = 1$.

**70. Correct answer is [3].**

Given, Matrix B is not scalar multiple of A.

$\Rightarrow$ B is Linearly independent on A. ...(i)

Also,

C is Linearly independent on A and B. ...(ii)

$D = 3A + 2B + C$

[from equation (i) and (ii) A, B, C are linearly independent.]

*i.e.*, D can be expressed as the Linear combination of A, B, C. D is Linear dependent on A, B, C.

$\therefore$ Number of linearly independent rows are 3.

$\therefore$ Rank of matrix is 3.

**71. Option (d) is correct.**

Let $n = 2$

$$\Rightarrow \quad A_{2\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$$

$$\Rightarrow \quad |A|_{1\times 4} = (1\times 2)^2 = (2!)^2$$

$$A_{3\times 3} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 6 & 9 \end{pmatrix}$$

$$\Rightarrow \quad |A| = 1\times 4\times 9 = (1\times 2\times 3)^2 = (3!)^2$$

Similarly, $|A_{n\times n}| = (n!)^2$

**72. Correct answer is [2].**

Given, $x_{n+1} = g(x_n)$

where $g(x) = x - \dfrac{f(x)}{f'(x)}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

This is Newton Raphson iterative formula.

Therefore, the order of convergence is 2.

**73. Correct answer is [0.070].**

Pdf of X $\rightarrow f(x) = \begin{cases} 0.5 & 1 < x < 3 \\ 0 & \text{otherwise} \end{cases}$

$$n = 68,\ E(X) = \bar{x} = \int xf(x)dx = \frac{1}{2}\int_1^3 xdx$$

$$= \frac{1}{4}\left[x^2\right]_1^3 = 2$$

$$E(X^2) = \int x^2 f(x)dx = \frac{1}{2}\int_1^3 x^2 dx = \frac{1}{6}\left[x^3\right]_1^3$$

$$= \frac{26}{6} = \frac{13}{3}$$

$$\therefore \quad \text{var}(X) = E(X^2) - [E(X)]^2$$

$$= \frac{13}{3} - 4 = \frac{1}{3}$$

$$\therefore \quad SD = \sigma = \frac{1}{\sqrt{3}}$$

Therefore, the standard deviation of Pdf

$$\bar{x} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{3\times 68}} = \frac{1}{\sqrt{204}} = 0.070$$

**74. Correct answer is [0.167].**

Given,

DE is $y'''(t)+6y''(t)+11y'(t)+6y(t)=1$

Taking Laplace Transform on both side of DE.

$$L\{y'''(t)\}+6L\{y''(t)\}+11L\{y'(t)\}+6L\{y(t)\} = L\{1\}$$

$$\Rightarrow \quad s^3Y(s)-s^2y(0)-sy'(0)-y''(0)+6\{s^2Y(s)-sy(0)-y'(0)\}+11\{sY(s)-y(0)\}+6Y(s)=\frac{1}{s}$$

$$\Rightarrow (s^3+6s^2+11s+6)Y(s)=\frac{1}{s}$$

$$\left[\because s^3+6s^2+11s+6=(s+1)(s+2)(s+3)\right]$$

$$\Rightarrow \quad Y(s)=\frac{1}{s(s+1)(s+2)(s+3)}$$

$$=\frac{A}{s}+\frac{B}{(s+1)}+\frac{C}{(s+2)}+\frac{D}{(s+3)}$$

(using Partial Fraction)

$$Y(s)=\frac{1}{6s}-\frac{1}{2}\frac{1}{(s+1)}+\frac{1}{2}\frac{1}{(s+2)}-\frac{1}{6}\frac{1}{(s+3)}$$

Taking Inverse Laplace transform

$$y(t)=\frac{1}{6}-\frac{1}{2}e^{-t}+\frac{1}{2}e^{-2t}-\frac{1}{6}e^{-3t}$$

$$\therefore \quad \lim_{t\to\infty} y(t)=\frac{1}{6}=0.167$$