PRODUCTION AND INDUSTRIAL ENGINEERING (PI)

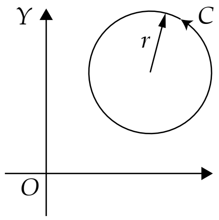
1. In the Taylor series expansion of sinz around $z=0$, the coefficient of the term z^3 is

(a) 0 **(b)** $\frac{1}{3}$ **(c)** $-\frac{1}{6}$ **(d)** $-\frac{1}{3}$

2. A vector field is given as $\mathbf{F}(x,y) = (100x + 100y)\, i + (-50x+200y)\, \boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the unit vectors along the x and y axes in the Cartesian frame, respectively. Then the value of

$$\oint_c \mathbf{F}(x, y)\ d\ell$$

where $d\ell = dx\,\boldsymbol{i}+dy\,\boldsymbol{j}$ is an elemental path taken over an anticlockwise circular contour C of radius $r = 2$ is



(a) -100π **(c)** -400π

(b) -600π **(d)** 400π

3. If work sampling is carried out using a large number of observations, then the required sample size is estimated using

(a) Poisson distribution

(b) Uniform distribution

(c) Normal distribution

(d) Exponential distribution

4. In a single server Markovian queuing system, if the customers arrive following the Poisson distribution, then the inter-arrival time follows

(a) Poisson distribution

(b) Uniform distribution

(c) Exponential distribution

(d) Binomial distribution

5. In a complex function

$$f(x,y) = u(x,y) + i\,v(x,y)$$

i is the imaginary unit, and $x,y,u(x,y)$ and $v(x,y)$ are real.

If $f(x,y)$ is analytic then which of the following equations is/are TRUE?

(a) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

(b) $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=0$

(c) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=0$

(d) $(\frac{\partial u}{\partial x})(\frac{\partial v}{\partial x})+(\frac{\partial u}{\partial y})(\frac{\partial v}{\partial y})=0$

6. If X is a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{K}{4}, & 0 \le x \le 10, \\ 0 & \text{otherwise} \end{cases}$$

then the value of K is ________ . (*Answer in integer*)

7. If $\lim_{x\to 1}\left(\frac{x^2-2ax+b}{x-1}\right) = 8$

then $(a-b)$ is _________ . (*Answer in integer*)

8. If $\mathbf{A} = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ is a matrix such that $\mathbf{A}^2 = \mathbf{I}$, where $\mathbf{I}$ is an identity matrix, then which of the following is TRUE?

(a) $1+ a^2 + bc = 0$

(b) $1- a^2 + bc = 0$

(c) $1- a^2 - bc = 0$

(d) $1+ a^2 - bc = 0$

9. Consider the following linear programming problem with two decision variables x_1 and x_2. There are three constraints involving resources R_1, R_2 and R_3 as indicated.

Maximize $Z = 6x_1 + 5x_2$

Subject to

$2x_1 + 5x_2 \le 40$ R_1

$2x_1 + x_2 \le 22$ R_2

$x_1 + x_2 \le 13$ R_3

$x_1 \ge 0,\quad x_2 \ge 0$

The optimal solution of the problem is: $x_1 = 9$ and $x_2 = 4$.

For which one of the following options, the shadow price of the resource(s) will have non-zero value(s)?

(a) R_1, R_2 and R_3

(b) R_1 and R_2

(c) R_2 and R_3

(d) R_1 only

10. The following differential equation governs the evolution of variable $x(t)$ with time t, $t \ge 0$.

$$\frac{d^2x}{dt^2} + 4x = e^{-t}$$

Given the initial conditions $x=0$ and $\frac{dx}{dt}=0$ at $t=0$, the value of x at $t = \pi/8$ is _________ . (*Rounded off to 3 decimal places*)

11. The values of function $y(x)$ at discrete values of x are given in the table. The value of $\int_0^4 y(x)dx$, using Trapezoidal rule is __________ . (*Rounded off to 1 decimal place*)

x	0	1	2	3	4
$y(x)$	1	3	6	9	12

MECHANICAL ENGINEERING (ME)

12. In order to numerically solve the ordinary differential equation $\frac{dy}{dt} = -y$ for $t > 0$, with an initial condition $y(0)=1$, the following scheme is employed

$$\frac{y_{n+1} - y_n}{\Delta t} = -\frac{1}{2}(y_{n+1} + y_n).$$

Here, Δt is the time step and $y_n = y(n\Delta t)$ for $n = 0, 1, 2, \ldots$. This numerical scheme will yield a solution with non-physical oscillations for $\Delta t >$ h. The value of h is

(a) $\frac{1}{2}$ **(b)** 1 **(c)** $\frac{3}{2}$ **(d)** 2

13. The value of the surface integral

$$\oiint_S z\,dx\,dy$$

where S is the external surface of the sphere $x^2+y^2+z^2=R^2$ is

(a) 0 **(b)** $4\pi R^3$ **(c)** $\frac{4\pi}{3}R^3$ **(d)** πR^3

14. Let $f(z)$ be an analytic function, where $z=x+iy$. If the real part of $f(z)$ is $\cosh x \cos y$, and the imaginary part of $f(z)$ is zero for $y=0$, then $f(z)$ is

(a) $\cosh x \exp(-iy)$ **(c)** $\cosh z \cos y$
(b) $\cosh z \exp z$ **(d)** $\cosh z$

15. Consider the system of linear equations

$$x + 2y + z = 5$$
$$2x + ay + 4z = 12$$
$$2x + 4y + 6z = b$$

The values of a and b such that there exists a non-trivial null space and the system admits infinite solutions are

(a) $a = 8, b = 14$ **(c)** $a = 8, b = 12$
(b) $a = 4, b = 12$ **(d)** $a = 4, b = 14$

16. Let $f(.)$ be a twice differentiable function from $\mathbb{R}^2 \to \mathbb{R}$. If $p, x_0 \in \mathbb{R}^2$ where $\|\boldsymbol{p}\|$ is sufficiently small (here $\|\cdot\|$ is the Euclidean norm or distance function), then $f(\boldsymbol{x_0}+\boldsymbol{p})=f(\boldsymbol{x_0})+\nabla f(\boldsymbol{x_0})^T\boldsymbol{p}+12\boldsymbol{p}^T\nabla^2 f(\psi)\boldsymbol{p}$ where $\psi \in \mathbb{R}^2$ is a point on the line segment joining $\boldsymbol{x_0}$ and $\boldsymbol{x_0}+\boldsymbol{p}$. If $\boldsymbol{x_0}$ is a strict local minimum of $f(\boldsymbol{x})$, then which one of the following statements is TRUE?

(a) $\nabla f(\boldsymbol{x_0})^T\boldsymbol{p} > 0$ and $\boldsymbol{p}^T\nabla^2 f(\psi)\boldsymbol{p} = 0$
(b) $\nabla f(\boldsymbol{x_0})^T\boldsymbol{p} = 0$ and $\boldsymbol{p}^T\nabla^2 f(\psi)\boldsymbol{p} > 0$
(c) $\nabla f(\boldsymbol{x_0})^T\boldsymbol{p} = 0$ and $\boldsymbol{p}^T\nabla^2 f(\psi)\boldsymbol{p} = 0$
(d) $\nabla f(\boldsymbol{x_0})^T\boldsymbol{p} = 0$ and $\boldsymbol{p}^T\nabla^2 f(\psi)\boldsymbol{p} < 0$

17. The velocity field of a two-dimensional, incompressible flow is given by

$$\vec{V} = 2\sinh x\,\hat{i} + v(x,y)\,\hat{j}$$

where $\hat{i}$ and $\hat{j}$ denote the unit vectors in x and y directions, respectively. If $v(x,0)=\cosh x$, then $v(0,-1)$ is

(a) 1 **(b)** 2 **(c)** 3 **(d)** 4

18. The matrix $\begin{bmatrix} 1 & a \\ 8 & 3 \end{bmatrix}$ (where $a > 0$) has a negative eigenvalue if a is greater than

(a) $\frac{3}{8}$ **(b)** $\frac{1}{8}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{1}{5}$

19. At the current basic feasible solution (bfs) $\boldsymbol{v_0}$ ($\boldsymbol{v_0} \in \mathbb{R}^5$), the simplex method yields the following form of a linear programming problem in standard form.

minimize $z = -x_1 - 2x_2$

s.t. $x_3 = 2 + 2x_1 - x_2$
$x_4 = 7 + x_1 - 2x_2$
$x_5 = 3 - x_1$
$x_1, x_2, x_3, x_4, x_5 \geq 0$

Here the objective function is written as a function of the non-basic variables. If the simplex method moves to the adjacent bfs $\boldsymbol{v_1}$ ($\boldsymbol{v_0} \in \mathbb{R}^5$) that best improves the objective function, which of the following represents the objective function at v_1, assuming that the objective function is written in the same manner as above?

(a) $z = -4 - 5x_1 + 2x_3$ **(c)** $z = -4 - 5x_1 + 2x_4$
(b) $z = -3 + x_5 - 2x_2$ **(d)** $z = -6 - 5x_1 + 2x_3$

20. If $x(t)$ satisfies the differential equation

$$t\frac{dy}{dt} + (t - x) = 0$$

subject to the condition $x(1) = 0$, then the value of $x(2)$ is ________ (*rounded off to 2 decimal places*).

21. Let X be a continuous random variable defined on [0, 1] such that its probability density function $f(x) = 1$ for $0 \leq x \leq 1$ and 0 otherwise. Let $Y = \log_e(X+1)$. Then the expected value of Y is_____. (rounded off to 2 decimal places)

CIVIL ENGINEERING (CE-1)

22. The smallest positive root of the equation

$$x^5 - 5x^4 - 10x^3 + 50x^2 + 9x - 45 = 0$$

lies in the range

(a) $0 < x \leq 2$ **(c)** $6 \leq x \leq 8$
(b) $2 < x \leq 4$ **(d)** $10 \leq x \leq 100$

23. The second-order differential equation in an unknown function $u : u(x, y)$ is defined as

$$\frac{\partial^2 u}{\partial x^2} = 2$$

Assuming $g : g(x)$, $f : f(y)$, and h : h (y), the general solution of the above differential equation is

(a) $u = x^2 + f(y) + g(x)$ **(c)** $u = x^2 + x\,f(y) + g(x)$
(b) $u = x^2 + x\,f(y) + h(y)$ **(d)** $u = x^2 + f(y) + y\,g(x)$

24. The probability that a student passes only in Mathematics is $\frac{1}{3}$. The probability that the student passes only in English is $\frac{4}{9}$. The probability that the student passes in both of these subjects is $\frac{1}{6}$. The probability that the student will pass in at least one of these two subjects is

(a) $\frac{17}{18}$ **(b)** $\frac{11}{18}$ **(c)** $\frac{14}{18}$ **(d)** $\frac{1}{18}$

25. For the following partial differential equation,

$$x\frac{\partial^2 f}{\partial x^2} + y\frac{\partial^2 f}{\partial y^2} = \frac{x^2+y^2}{2}$$

which of the following option(s) is/are CORRECT?

(a) elliptic for $x > 0$ and $y > 0$

(b) parabolic for $x > 0$ and $y > 0$

(c) elliptic for $x = 0$ and $y > 0$

(d) hyperbolic for $x < 0$ and $y > 0$

26. Consider the data of $f(x)$ given in the table.

i	0	1	2
x_i	1	2	3
$f(x_i)$	0	0.3010	0.4771

The value of $f(1.5)$ estimated using second-order Newton's interpolation formula is ______________ (rounded off to 2 decimal places).

27. What are the eigenvalues of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 2 \end{bmatrix}$?

(a) 1, 2, 5 **(c)** −5, 1, 2

(b) 1, 3, 4 **(d)** −5, − 1, 2

28. A vector field $\vec{p}$ and a scalar field r are given by

$\vec{p} = (2x^2-3xy+z^2)\,\hat{\imath} + (2y^2-3yz+x^2)\,\hat{\jmath} + (2z^2-3xz+x^2)\,\vec{k}$

$r = 6x^2+4y^2-z^2-9xyz-2xy+3xz-yz$

Consider the statements P and Q.

P: Curl of the gradient of the scalar field r is a null vector.

Q: Divergence of curl of the vector field $\vec{p}$ is zero.

Which one of the following options is CORRECT?

(a) Both P and Q are FALSE

(b) P is TRUE and Q is FALSE

(c) P is FALSE and Q is TRUE

(d) Both P and Q are TRUE

CIVIL ENGINEERING (CE-2)

29. A partial differential equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

is defined for the two-dimensional field $T{:}T(x,y)$, inside a planar square domain of size 2m × 2m. Three boundary edges of the square domain are maintained at value $T=50$, whereas the fourth boundary edge is maintained at $T=100$.

The value of T at the center of the domain is

(a) 50.0 **(b)** 62.5 **(c)** 75.0 **(d)** 87.5

30. The statements P and Q are related to matrices A and B, which are conformable for both addition and multiplication.

P: $(\mathbf{A}+\mathbf{B})^{\mathbf{T}} = \mathbf{A}^{\mathbf{T}} + \mathbf{B}^{\mathbf{T}}$

Q: $(\mathbf{AB})^{\mathbf{T}} = \mathbf{A}^{\mathbf{T}}\mathbf{B}^{\mathbf{T}}$

Which one of the following options is CORRECT?

(a) P is TRUE and Q is FALSE

(b) Both P and Q are TRUE

(c) P is FALSE and Q is TRUE

(d) Both P and Q are FALSE

31. The second derivative of a function f is computed using the fourth-order Central Divided Difference method with a step length h.

The CORRECT expression for the second derivative is

(a) $\frac{1}{12h^2}[-f_{i+2}+16f_{i+1}-30f_i+16f_{i-1}-f_{i-2}]$

(b) $\frac{1}{12h^2}[f_{i+2}+16f_{i+1}-30f_i+16f_{i-1}-f_{i-2}]$

(c) $\frac{1}{12h^2}[-f_{i+2}+16f_{i+1}-30f_i+16f_{i-1}+f_{i-2}]$

(d) $\frac{1}{12h^2}[-f_{i+2}-16f_{i+1}+30f_i-16f_{i-1}-f_{i-2}]$

32. The function $f(x) = x^3 - 27x + 4,\ 1 \le x \le 6$ has

(a) Maxima point

(b) Minima point

(c) Saddle point

(d) Inflection point

33. Consider two Ordinary Differential Equations (ODEs):

P: $\frac{dy}{dx} = \frac{x^4 + 3x^2y^2 + 2y^4}{x^3y}$

Q: $\frac{dy}{dx} = \frac{-y^2}{x^2}$

Which one of the following options is CORRECT?

(a) P is a homogeneous ODE and Q is an exact ODE.

(b) P is a homogeneous ODE and Q is not an exact ODE.

(c) P is a nonhomogeneous ODE and Q is an exact ODE.

(d) P is a nonhomogeneous ODE and Q is not an exact ODE.

34. Three vectors $\vec{p}, \vec{q}$, and $\vec{r}$ are given as

$\vec{p} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\vec{q} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\vec{r} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$

Which of the following is/are CORRECT?

(a) $\vec{p} \times (\vec{q} \times \vec{r}) + \vec{q} \times (\vec{r} \times \vec{p}) + \vec{r} \times (\vec{p} \times \vec{q}) = \vec{0}$

(b) $\vec{p} \times (\vec{q} \times \vec{r}) = (\vec{p} \cdot \vec{r})\,\vec{q} - (\vec{p} \cdot \vec{q})\,\vec{r}$

(c) $\vec{p} \times (\vec{q} \times \vec{r}) = (\vec{p} \times \vec{q}) \times \vec{r}$

(d) $\vec{r} \cdot (\vec{p} \times \vec{q}) = (\vec{q} \times \vec{p}) \cdot \vec{r}$

35. Consider two matrices $\mathbf{A} = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 0 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -1 & 0 \\ 2 & 3 \\ 1 & 4 \end{bmatrix}$.

The determinant of the matrix **AB** is ________ (*in integer*).

COMPUTER SCIENCE & INFORMATION TECHNOLOGY (CS-1)

36. Let $f:\mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = \max\{x, x^3\}$, $x \in \mathbb{R}$, where $\mathbb{R}$ is the set of all real numbers. The set of all points where $f(x)$ is NOT differentiable is

(a) $\{-1, 1, 2\}$ **(c)** $\{0, 1\}$

(b) $\{-2, -1, 1\}$ **(d)** $\{-1, 0, 1\}$

37. The product of all eigenvalues of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is

(a) –1 **(b)** 0 **(c)** 1 **(d)** 2

38. Let A and B be two events in a probability space with $P(A) = 0.3$, $P(B) = 0.5$, and $P(A \cap B) = 0.1$. Which of the following statements is/are TRUE?

(a) The two events A and B are independent

(b) $P(A \cup B) = 0.7$

(c) $P(A \cap B^c) = 0.2$, where B^c is the complement of the event B

(d) $P(A^c \cap B^c) = 0.4$, where A^c and B^c are the complements of the events A and B, respectively

39. Let A and B be non-empty finite sets such that there exist one-to-one and onto functions (i) from A to B and (ii) from $A \times A$ to $A \cup B$. The number of possible values of $|A|$ is ________

40. Consider the following recurrence relation:

$$T(n) = \begin{cases} \sqrt{n}T(\sqrt{n}) + n & \text{for } n \geq 1, \\ 1 & \text{for } n = 1 \end{cases}$$

Which one of the following options is CORRECT?

(a) $T(n) = \theta(n \log \log n)$

(b) $T(n) = \theta(n \log n)$

(c) $T(n) = \theta(n^2 \log n)$

(d) $T(n) = \theta(n^2 \log \log n)$

41. Consider the operators ◊ and □ defined by $a \lozenge b = a + 2b$, $a \square b = ab$, for positive integers. Which of the following statements is/are TRUE?

(a) Operator ◊ obeys the associative law

(b) Operator □ obeys the associative law

(c) Operator ◊ over the operator □ obeys the distributive law

(d) Operator □ over the operator ◊ obeys the distributive law

COMPUTER SCIENCE & INFORMATION TECHNOLOGY (CS-2)

42. Let $T(n)$ be the recurrence relation defined as follows:

$T(0) = 1$,

$T(1) = 2$, and

$T(n) = 5T(n-1) - 6T(n-2) \quad \text{for } n \geq 2$

Which one of the following statements is TRUE?

(a) $T(n) = \theta(2^n)$

(b) $T(n) = \theta(n2^n)$

(c) $T(n) = \theta(3^n)$

(d) $T(n) = \theta(n3^n)$

43. Let $f(x)$ be a continuous function from $\mathbb{R}$ to $\mathbb{R}$ such that

$$f(x) = 1 - f(2-x)$$

Which one of the following options is the CORRECT value of $\int_0^2 f(x)dx$? ?

(a) 0 **(b)** 1 **(c)** 2 **(d)** –1

44. When six unbiased dice are rolled simultaneously, the probability of getting all distinct numbers (*i.e.*, 1, 2, 3, 4, 5, and 6) is

(a) $\frac{1}{324}$ **(b)** $\frac{5}{324}$ **(c)** $\frac{7}{324}$ **(d)** $\frac{11}{324}$

45. For a Boolean variable x, which of the following statements is/are FALSE?

(a) $x.1 = x$ **(c)** $x.x = 0$

(b) $x + 1 = x$ **(d)** $x + \bar{x} = 1$

46. Let P be the partial order defined on the set {1, 2, 3, 4} as follows

$$P = \{(x, x) \mid x \in \{1, 2, 3, 4\}\} \cup \{(1, 2), (3, 2), (3, 4)\}$$

The number of total orders on {1, 2, 3, 4} that contain P is ________

47. Let x and y be random variables, not necessarily independent, that take real values in the interval [0,1]. Let $z = xy$ and let the mean values of x,y,z be $\bar{x}, \bar{y}, \bar{z}$, respectively. Which one of the following statements is TRUE?

(a) $\bar{z} = \bar{x}\bar{y}$ **(c)** $\bar{z} \geq \bar{x}\bar{y}$

(b) $\bar{z} \leq \bar{x}\bar{y}$ **(d)** $\bar{z} \leq \bar{x}$

48. Let Z_n be the group of integers $\{0, 1, 2, \ldots, n-1\}$ with addition modulo n as the group operation. The number of elements in the group $Z_2 \times Z_3 \times Z_4$ that are their own inverses is ________

ELECTRONICS AND COMMUNICATION ENGINEERING (EC)

49. The general form of the complementary function of a differential equation is given by $y(t) = (At + B)e^{-2t}$, where A and B are real constants determined by the initial condition. The corresponding differential equation is _____.

(a) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = f(t)$

(b) $\frac{d^2y}{dt^2} + 4y = f(t)$

(c) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = f(t)$

(d) $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = f(t)$

50. In a number system of base r, the equation $x^2 - 12x + 37 = 0$ has $x = 8$ as one of its solutions. The value of r is _______.

51. Let $\mathbb{R}$ and $\mathbb{R}^3$ denote the set of real numbers and the three dimensional vector space over it, respectively. The value of α for which the set of vectors

$$\{[2\ \ -3\ \ \alpha],\ [3\ \ -1\ \ 3],\ [1\ \ -5\ \ 7]\}$$

does not form a basis of $\mathbb{R}^3$ is _______.

52. Let z be a complex variable. If $f(z) = \frac{\sin(\pi z)}{z^2(z-2)}$ and C is the circle in the complex plane with $|z| = 3$ then $\oint_C f(z)dz$ is _______.

(a) $\pi^2 j$ **(c)** $j\pi\left(\frac{1}{2} + \pi\right)$

(b) $j\pi\left(\frac{1}{2} - \pi\right)$ **(d)** $-\pi^2 j$

53. Consider a system S represented in state space as

$$\frac{dx}{dt} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r,\ y = [2\ \ -5]x.$$

Which of the state space representations given below has/have the same transfer function as that of S?

(a) $\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r,\ y = [1\ \ 2]x$

(b) $\frac{dx}{dt} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r,\ y = [0\ \ 2]x$

(c) $\frac{dx}{dt} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} -1 \\ 3 \end{bmatrix} r,\ y = [1\ \ 1]x$

(d) $\frac{dx}{dt} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} r,\ y = [1\ \ 2]x$

54. Let F_1, F_2, and F_3 be functions of (x, y, z). Suppose that for every given pair of points A and B in space, the line integral $\int_C (F_1 dx + F_2 dy + F_3 dz)$ evaluates to the same value along any path C that starts at A and ends at B . Then which of the following is/are true?

(a) For every closed path Γ, we have $\int_C (F_1 dx + F_2 dy + F_3 dz)$.

(b) There exists a differentiable scalar function $f(x, y, z)$ such that $F_1 = \frac{\partial f}{\partial x}, F_2 = \frac{\partial f}{\partial y}, F_3 = \frac{\partial f}{\partial z}$.

(c) $\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = 0$.

(d) $\frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$.

55. Consider the matrix $\begin{bmatrix} 1 & k \\ 2 & 1 \end{bmatrix}$, where k is a positive real number. Which of the following vectors is/are eigenvector(s) of this matrix?

(a) $\begin{bmatrix} 1 \\ -\sqrt{2/k} \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 \\ \sqrt{2/k} \end{bmatrix}$ **(c)** $\begin{bmatrix} \sqrt{2k} \\ 1 \end{bmatrix}$ **(d)** $\begin{bmatrix} \sqrt{2k} \\ -1 \end{bmatrix}$

ELECTRICAL ENGINEERING (EE)

56. Which one of the following matrices has an inverse?

(a) $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 0.5 & 2 & 4 \end{bmatrix}$ **(c)** $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 1 & 2 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 2 & 9 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 3 & 12 & 24 \end{bmatrix}$

57. Let X be a discrete random variable that is uniformly distributed over the set $\{-10, -9, \cdots, 0, \cdots, 9, 10\}$. Which of the following random variables is/are uniformly distributed?

(a) X^2 **(c)** $(X - 5)^2$

(b) X^3 **(d)** $(X + 10)^2$

58. Which of the following complex functions is/are analytic on the complex plane?

(a) $f(z) = j\text{Re}(z)$ **(c)** $f(z) = e^{|z|}$

(b) $f(z) = \text{Im}(z)$ **(d)** $f(z) = z^2 - z$

59. Consider the complex function $f(z) = \cos z + e^{z^2}$. The coefficient of z^5 in the Taylor series expansion of $f(z)$ about the origin is ______ (rounded off to 1 decimal place).

60. The sum of the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^2$ is ______ (rounded off to the nearest integer).

61. Let $X(\omega)$ be the Fourier transform of the signal

$$x(t) = e^{-t^4}\cos t, \quad -\infty < t < \infty.$$

The value of the derivative of $X(\omega)$ at $\omega = 0$ is _______ (rounded off to 1 decimal place).

62. Consider a vector $\bar{u} = 2\hat{x} + \hat{y} + 2\hat{z}$, where $\hat{x}, \hat{y}, \hat{z}$ represent unit vectors along the coordinate axes x, y, z respectively. The directional derivative of the function $f(x, y, z) = 2\ln(xy) + \ln(yz) + 3\ln(xz)$ at the point $(x, y, z) = (1, 1, 1)$ in the direction of $\bar{u}$ is

(a) 0 **(b)** $\frac{7}{5\sqrt{2}}$ **(c)** 7 **(d)** 21

63. If the Z-transform of a finite-duration discrete-time signal $x[n]$ is $X(z)$, then the Z-transform of the signal $y[n] = x[2n]$ is

(a) $Y(z) = X(z^2)$

(b) $Y(z) = \frac{1}{2}[X(z^{-1/2}) + X(-z^{-1/2})]$

(c) $Y(z) = \frac{1}{2}[X(z^{1/2}) + X(-z^{1/2})]$

(d) $Y(z) = \frac{1}{2}[X(z^2) + X(-z^2)]$

64. Let $f(t)$ be a real-valued function whose second derivative is positive for $-\infty < t < \infty$. Which of the following statements is/are always true?

(a) $f(t)$ has at least one local minimum.

(b) $f(t)$ cannot have two distinct local minima.

(c) $f(t)$ has at least one local maximum.

(d) The minimum value of $f(t)$ cannot be negative.

65. Consider the function $f(t) = (\max(0,t))^2$ for $-\infty < t < \infty$, where $\max(a,b)$ denotes the maximum of a and b. Which of the following statements is/are true?

(a) $f(t)$ is not differentiable.

(b) $f(t)$ is differentiable and its derivative is continuous.

(c) $f(t)$ is differentiable but its derivative is not continuous.

(d) $f(t)$ and its derivative are differentiable.

66. Which of the following differential equations is/are nonlinear?

(a) $tx(t) + \frac{dx(t)}{dt} = t^2e^t, \quad x(0) = 0$

(b) $\frac{1}{2}e^t + x(t)\frac{dx(t)}{dt} = 0, \quad x(0) = 0$

(c) $x(t)\cos t - \frac{dx(t)}{dt}\sin t = 1, \quad x(0) = 0$

(d) $x(t) + e^{\left(\frac{dx(t)}{dt}\right)} = 1, \quad x(0) = 0$

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	(c)	Taylor Series	Differential Calculus
2	(c)	Line Integral	Vector Calculus
3	(a)	Poisson Distribution	Probability Distribution
4	(c)	Queuing Theory	Operation Research
5	(a,b,d)	Function of a Complex Variable	Complex Analysis
6	4	Continuous Random Variable	Basic of Probabilty
7	4	Limits and Continuity	Differential Calculus
8	(c)	Matrix	Linear Algebra
9	(c)	Linear Programming Problem	Operation Research
10	0.064	Ordinary Differential Equations	Differential Equation
11	24.5	Numerical Integration	Numerical Analysis
12	(a)	Numerical Solution of Ode	Numerical Analysis
13	(c)	Surface Integral	Vector Integration
14	(d)	Analytic Function	Complex Analysis
15	(d)	Solution of Linear Equations	Linear Algebra
16	(b)	Maxima And Minima	Differential Calculus
17	(c)	Incompressible Fliud	Vector Differentiation
18	(a)	Eigen Values and Eigen Vectors	Linear Algebra
19	(a)	Simplex Method	Operation Research
20	–1.38	Boundary Value Problems	Differential Equations
21	0.39	Mathematical Expectation	Basics of Probabilty
22	(a)	Numerical Solution of Algebric Equation	Numerical Analysis
23	(b)	Partial Differential Equation	Differential Equation
24	(b)	Principal of Inclusion and Exclusion	Basic of Probabilty
25	(a)	Classification of PDE	Partial Differential Equation
26	0.17	Newton's Forward Interpolation Formula	Numerical Analysis
27	(a)	Eigen Values and Eigen Vectors	Linear Algebra
28	(d)	Vector Differentiation	Vector Calculus
29	(b)	Partial Differential Equation	Differential Equation
30	(a)	Matrix	Linear Algebra
31	(a)	Divided Differences	Numerical Analysis
32	(b)	Maxima and Minima	Differential Calculus
33	(b)	Ordinary Differential Equations	Differential Equations
34	(a,b)	Basic Properties of Vectors	Vector Calculus
35	10	Matrix	Linear Algebra
36	(d)	Function or Mapping	Set Theory
37	(b)	Eigen Values and Eigen Vectors	Matrix
38	(b,c)	Addition Law of Probability	Basic of Probabilty
39	2	Cardinality of a Set	Set Theory
40	(a)	Recurrence Relations	Discrete Mathematics
41	(b,c)	Basic Operation of Algebra	Linear Algebra
42	(a)	Recurrence Relations	Discrete Mathematics
43	(b)	Definite Integral	Integral Calculus
44	(b)	Basic of Probability	Basic of Probabilty
45	(b,c)	Basic Law of Boolean Algebra	Boolean Algebra
46	4	Poset and Hasses Diagram	Linear Algebra

47	(d)	Random Variables	Basic of Probability
48	4	Groups	Boolean Algebra
49	(a)	Differential Equation of Higher Order	Differential Equations
50	11	Quadratic Equations	Linear Algebra
51	5	L.I and L.D Vectors	Abstract Algebra
52	(d)	Cauchy's Integral Formula	Complex Analysis
53	(a,c)	Differential Equation	Differential Equation
54	(a,b,d)	Conservative Vector Field and Scalar Potential	Vector Calculus
55	(a,b)	Eigen Values and Eigen Vectors	Linear Algebra
56	(c)	Inverse of a Matrix	Linear Algebra
57	(b,d)	Random Variables	Basic of Probabilty
58	(d)	Analytic Function	Complex Analysis
59	0	Taylor Series	Complex Analysis
60	29	Eigen Values and Eigen Vectors	Linear Algebra
61	0	Fourier Transform	Fourier Transform
62	(c)	Directional Derivative	Vector Calculus
63	(c)	Z-Transform	Integral Transfrom
64	(b)	Maxima And Minima	Differential Calculus
65	(b)	Limits and Continuity, Differentiability	Differential Calculus
66	(b,d)	Linear and Non Linear Differential Equations	Differential Equation

Engineering Mathematics GATE Solved Papers 2024

PRODUCTION AND INDUSTRIAL ENGINEERING (PI)

1. Option (c) is correct.

Given $f(z) = \sin z$

We have the Taylor series Expansion

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + ---$$

$$f(z) = f(0) + (z-0)f'(0) + \frac{(z-0)^2}{2!}f''(0) + \frac{(z-0)^3}{3!}f'''(0) + ---$$

$$f(z) = \sin z,\ f'(z) = \cos z,\ f''(z) = -\sin z,\ f'''(z) = -\cos z$$

$$f(0) = \sin 0 = 0\ \ f'(0) = \cos 0 = 1\ \ f''(0) = -\sin 0 = 0\ \ f'''(0) = -\cos 0 = -1$$

$$f(z) = \sin z = 0 + z\times 1 + \frac{z^2}{2!}\times 0 + \frac{z^3}{3!}\times(-1) + ---$$

$$\sin z = z - \frac{z^3}{3!} + ---$$

So the coefficient of z^3 is $\frac{-1}{3!}$ i.e. $\frac{-1}{6}$

2. Option (c) is correct.

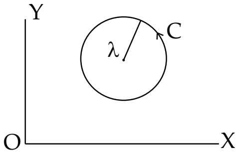
Given $f(x, y) = (100x + 100y)\hat{i} + (-50x + 200y)\hat{j}$

Then $\oint_C F(x, y)\, d\ell$ where $d\ell = dx\hat{i} + dy\hat{j}$

is an elemental parts taken over an anticlockwise circular contour C of Radius $r = 2$ is

$$\oint_C [(100x + 100y)\hat{i} + (-50x + 200y)\hat{j}]\,[dx\hat{i} + dy\hat{j}]$$

$$\Rightarrow \oint_C (100x + 100y)\, dx + (-50x + 200y)\, dy$$



$x = 2\cos\theta$

$y = 2\sin\theta$

$\theta = 0$ to $\theta = 2\pi$

$dx = -2\sin\theta\, d\theta$

$dy = 2\cos\theta\, d\theta$

$$\int_0^{2\pi} (100\times 2\cos\theta + 100\times 2\sin\theta)(-2\sin\theta)d\theta + (-50\times 2\cos\theta + 200\times 2\sin\theta)\, 2\cos\theta\, d\theta$$

$$\int_0^{2\pi} (-400(\cos\theta + \sin\theta)\sin\theta + (-100\cos\theta + 400\sin\theta).\, 2\cos\theta]d\theta$$

$$\int_0^{2\pi}\left[400\sin\theta\cos\theta - 200\sin^2\theta - 200\right]d\theta = \int_0^{2\pi} (200\sin 2\theta - 200\sin\theta - 200)d\theta$$

$$\Rightarrow 200\int_0^{2\pi}\left[\sin 2\theta - \sin^2\theta - 1\right]d\theta = 200\int_0^{2\pi}\left[\sin 2\theta - \frac{(1-\cos 2\theta)}{2} - 1\right]d\theta$$

$$= \frac{200}{2}\int_0^{2\pi}[2\sin 2\theta - 1 + \cos 2\theta - 2]d\theta$$

$$= 100\left[-\frac{\not{2}\cos 2\theta}{\not{2}} + \frac{\sin 2\theta}{2} - 3\theta\right]_0^{2\pi}$$

$$= 100[-\cos 4\pi + \cos 0 - 3\times 2\pi] = -100\times 6\pi = -600\pi$$

3. Option (a) is correct.

If work sampling is carried out using a large number of observations, then the required sample size is estimated using poisson distribution.

In a Poisson Distribution number of trials are very large and probability of success is very small. i.e. mean $m = nP$ is a finite quantity,

$$P(r) = \frac{e^{-\mu}\times\mu^r}{r!} \quad r = 0, 1, 2, 3\cdots\infty$$

4. Option (c) is correct.

The number of person in the given (Lq) is given by

$$L_q = \frac{\rho^2}{1-\rho},\quad \rho = \frac{\lambda}{\mu}$$

where λ = Arival rate

μ = Service rate

ρ = Utilization of the server.

As this is single server $m/m/1$

System (L_2) will be same for FCFS and LCFS.

In queueing theory, if the arrivals occur according to a poisson distribution, the interaxial time is exponential.

5. Options (a, b and d) are correct.

Given $f(x, y) = u(x, y) + iv(x, y)$ is Analytic fraction.

Where $u(x, y)$ and $v(x, y)$ are Harmonic functions, $u(x, y)$ and $v(x, y)$ satisfy Laplace equations,

i.e., $\nabla^2 u = 0$ $\qquad$ $\nabla^2 v = 0$

$$\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad \frac{\partial^2 \bar{v}}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

and $\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial x}\right) + \left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial v}{\partial y}\right) = 0$

$\because f(x, y)$ is an analytic function then its satisfy C-R equations

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y},\quad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\therefore -\frac{\partial v}{\partial y}\cdot\frac{\partial u}{\partial y} + \frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial y} = 0 \text{ satisfy}$$

6. Correct answer is [4].

Given probability density function

$$f(x) = \begin{cases} \frac{k}{4} & 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

We have by the condition of continuous random variable.

Given $f(x)$ is a Pdf.

Then

(i) $f(x) \ge 0 \quad \forall \quad 0 \le x \le 1$ $\qquad$ (ii) $\int_{-\infty}^{\infty} f(x)dx = 1$

$\int_0^1 \frac{k}{4} dx = 1 \quad \therefore \int_0^1 k dx = 4$

$[kx]_0^1 = 4 \quad \therefore \quad K = 4$

7. Correct answer is [4].

If $\lim_{x\to 1}\left(\frac{x^2-2ax+b}{x-1}\right) = 8$

at $x \to 1$ $\quad x^2 - 2ax + b = 0$ for $\left[\frac{0}{0}\right]$ forms

$1 - 2a + b = 0 \quad \Rightarrow b - 2a = -1$ ...(1)

$\lim_{x\to 1}\left[\frac{x^2-2ax+b}{x-1}\right]$. By L-Hospital's Rule

$\lim_{x\to 1}\left[\frac{2x-2a}{1}\right] = 8 \Rightarrow 2 - 2a = 8$ ...(2)

$-2a = 8 - 2 \qquad \therefore\ a = \frac{6}{-2} = -3$

from 1 $b = -1 - 6 = -7 \qquad a = -3$

Hence $a = -3$ and $b = -7$

$\therefore\ a - b = -3 + 7 = 4$

8. Option (c) is correct.

If $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ such that $A^2 = I$

$A^2 = \begin{bmatrix} a & b \\ c & -a \end{bmatrix} \times \begin{bmatrix} a & b \\ c & -a \end{bmatrix} = \begin{bmatrix} a^2+bc & ab-ab \\ ac-ac & bc+a^2 \end{bmatrix} = I$

$\begin{bmatrix} a^2+bc & 0 \\ 0 & a^2+bc \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$a^2 + bc = 1$

$\therefore\ 1 - a^2 - bc = 0$

9. Option (c) is correct.

Given maximize $z = 6x_1 + 5x_2$

Subject to $\quad 2x_1 + 5x_2 \le 40 \quad R_1$

$2x_1 + x_2 \le 22 \quad R_2$

$x_1 + x_2 \le 13 \quad R_3$

$x_1 \ge 0, \quad x_2 \ge 0$

The optimal solution of the problem is $x_1 = 9$ and $x_2 = 4$

This solution only satisfy the Resources R_2 and R_3 but not R_1

$2 \times 9 + 4 = 22 \le 22 \quad R_2$

and $9 + 4 = 13 \le 13 \quad R_3$

but $2 \times 9 + 5 \times 4 = 18 + 20 = 38 \le 40 \quad R_1$

So that only the resources R_2 and R_3 will have non-zero values.

10. Correct answer is [0.064].

$\frac{d^2x}{dt^2} + 4x = e^{-t}$ ...(1)

For $\quad x(t), \quad t \ge 0$

Given $x = 0, \frac{dx}{dt} = 0$

at $t = 0$

$m^2 + 4 = 0 \quad \therefore\ m = 0 \pm 2i$ (Imaginary Root)

$C \cdot F = e^{\alpha x}\left[c_1 \cos\beta x + c_2 \sin\beta x\right]$

$= e^{0\times t}\left[c_1 \cos 2t + c_2 \sin 2t\right]$

$P \cdot I = \frac{Q(x)}{f(D)} = \frac{e^{-t}}{D^2+4} = \frac{e^{-t}}{1+4} = \frac{e^{-t}}{5}$

Complete Solution

$x = C \cdot F + P \cdot I \cdot = c_1 \cos 2t + c_2 \sin 2t + \frac{e^{-t}}{5}$

$x = c_1 \cos 2t + c_2 \sin 2t + \frac{e^{-t}}{5}$ ...(2)

Put $\quad t = 0 \quad x = 0 \quad$ Apply Boundary Condition

$0 = c_1 + c_2 \times 0 + \frac{1}{5} \quad \therefore c_1 = -\frac{1}{5}$

$x = -\frac{1}{5}\cos 2t + c_2 \sin 2t + \frac{e^{-t}}{5}$ ...(3)

Differential with respect to t

$\frac{dx}{dt} = \frac{2}{5}\sin 2t + c_2 \times 2\cos 2t - \frac{1}{5}e^{-t}$ ...(4)

Given $\frac{dx}{dt} = 0 \quad$ at $t = 0$

$0 = 0 + 2c_2 - \frac{1}{5} \quad \therefore\ 2c_2 = \frac{1}{5} \quad \therefore\ c_2 = \frac{1}{10}$

Then solution

$x = -\frac{1}{5}\cos 2t + \frac{1}{10}\sin 2t + \frac{1}{5}e^{-t}$

Put $t = \frac{\pi}{8}$

$x = -\frac{1}{5}\cos 2 \times \frac{\pi}{8} + \frac{1}{10}\sin 2 \times \frac{\pi}{8} + \frac{1}{5}e^{-\pi/8}$

$= -\frac{1}{5}\cos\frac{\pi}{4} + \frac{1}{10}\sin\frac{\pi}{4} + \frac{1}{5}e^{-\pi/8}$

$= -\frac{1}{5} \times \frac{1}{\sqrt{2}} + \frac{1}{10} \times \frac{1}{\sqrt{2}} + \frac{1}{5}e^{-\pi/8}$

$\frac{1}{\sqrt{2}}\left[\frac{-2+1}{10}\right] + \frac{1}{5}e^{\pi/8} \Rightarrow \frac{-1}{10\sqrt{2}} + \frac{1}{5}e^{-\pi/8}$

$e^{-\pi/8} = 0.005401$

$= -\frac{1}{10\sqrt{2}} + 0.001080$

$= -0.070711 + 0.001080$

$= 0.064$

11. Correct answer is [24.5].

x	0	1	2	3	4
$y(x)$	1	3	6	9	12
	y_0	y_1	y_2	y_3	y_4

We have by Trapezoidal Rule

$\int_0^4 y(x)dx = \frac{h}{2}\left[(y_0 + y_4) + 2(y_1 + y_2 + y_3)\right]$

divide two range [0, 4] into 4 equal parts

$h = \frac{b-a}{4} = \frac{4-0}{4} = 1$

$\int_0^4 y(x)dx = \frac{1}{2}[(1+12) + 2(3+6+9)]$

$= \frac{1}{2}[13+36] = \frac{1}{2}[49] = 24.5$

MECHANICAL ENGINEERING (ME)

12. Option (a) is correct.

Given $\frac{y_{n+1}-y_n}{\Delta t} = -\frac{1}{2}(y_{n+1}+y_n)$ ...(1)

$\Delta t > y_n$

$y_n = y(n\,\Delta\, t)$

For $n = 0, 1, 2$ ....

$$y_{n+1} - y_n = -\frac{\Delta t}{2}\,(y_{n+1} + y_n)$$

$$y_{n+1}\left(\frac{1+\Delta t}{2}\right) = \left(\frac{1-\Delta t}{2}\right) y_n$$

$$\Rightarrow y_{n+1}\left(\frac{2+\Delta t}{\not 2}\right) = \left(\frac{2-\Delta t}{\not 2}\right) y_n$$

$$y_{n+1} = \left[\frac{2-\Delta t}{2+\Delta t}\right] y_n \qquad ...(2)$$

For stability or non physical oscillations

$$\left|\frac{2-\Delta t}{2+\Delta t}\right| \le 1 \;\Rightarrow\; -1 \le \frac{2-\Delta t}{2+\Delta t} \le 1$$

Case-I $\frac{2-\Delta t}{2+\Delta t} \geqslant -1 \Rightarrow \frac{\Delta t - 2}{\Delta t+2} \leqslant 1 \Rightarrow \frac{\Delta t-2}{\Delta t+2} - 1 \leqslant 0$

$$\Rightarrow \frac{\not\Delta t - 2 - \not\Delta t - 2}{\Delta t + 2} \leqslant 0 \Rightarrow \frac{-4}{\Delta t+2} \leqslant 0 \qquad ...(3)$$

$$\Rightarrow \frac{4}{\Delta t+2} \geqslant 0 \Rightarrow \Delta t \geqslant -2$$

Case-II $\frac{2-\Delta t}{2+\Delta t} \leqslant 1 \Rightarrow \frac{\Delta t-2}{\Delta t+2} \geqslant -1$

$$\Rightarrow \frac{\Delta t-2}{\Delta t+2} + 1 \geqslant 0 \Rightarrow \frac{\Delta t - \not 2 + \Delta t + \not 2}{\Delta t + 2} \geqslant 0$$

$$\frac{2\Delta t}{\Delta t+2} \ge 0 \Rightarrow \Delta t \in (-\infty, -2] \cup [0, \infty) \qquad ...(4)$$

From (3) and (4) $\Delta t \in [0,\infty)$ or $h \in [0,\infty)$ all given option satisfy the given interval. The answer should be option (A), as it is a general convention that the factor relating y_{n+1} and y_n should be as least as possible and generally lie between 0 and 1.

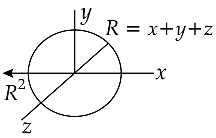
13. Option (c) is correct.

Evaluate $\oiint z\; dxdy$

Where S is the external surface of the sphere $x^2 + y^2 + z^2 = R^2$

$\oiint z\; dxdy$ = Volume of sphere

$= \frac{4}{3}\neq R^3 \quad x^2 + y^2 + z^2 = R^2$



14. Option (d) is correct.

Given $u(x, y) = \cos hx \cos y$ ...(1)

and $f(z) = u + iv$ is an analytic function

Its satisfy C-R equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Partial differential with respect to x and y of equation (1)

$$\frac{\partial u}{\partial x} = \sin hx \cos y, \qquad \frac{\partial u}{\partial y} = -\cos hx \sin y$$

Put $x = z$ and $y = 0$

$\phi, (z, 0) = \sinh z, \qquad \phi_2(z, 0) = 0$

We have by Milne Thomson Method

$$f(z) = \int \phi_1(z,0)dz - i\int \phi_2(z,0)dz + c$$

$$\Rightarrow f(z) = \int \sin hz\, dz + c$$

$$\Rightarrow f(z) = \cos hz + c$$

15. Option (d) is correct.

Given system of equations

$x + 2y + z = 5$ ...(1)

$2x + ay + 4z = 12$ ...(2)

$2x + 4y + 6z = b$ ...(3)

The values of a and b same that there exists a non-trivial null space and the system admits infinite solutions

Convert given equation into matrix form

$$\underset{A}{\begin{bmatrix} 1 & 2 & 1 \\ 2 & a & 4 \\ 2 & 4 & 6 \end{bmatrix}} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underset{B}{\begin{bmatrix} 5 \\ 12 \\ b \end{bmatrix}}$$

Augmented matrix

C = [A : B]

$$C = \begin{bmatrix} 1 & 2 & 1 & : & 5 \\ 2 & a & 4 & : & 12 \\ 2 & 4 & 6 & : & b \end{bmatrix}$$ Apply Row Transformation $R_2 \to R_2 - 2R_1, R_3 \to R_3 - 2R_1$

$$-\begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & a-4 & 2 & 2 \\ 0 & 0 & 4 & b-10 \end{bmatrix}$$ If $a - 4 = 0 \Rightarrow [a = 4]$ and $b - 10 = 4 \Rightarrow [b = 14]$

Then $\int(A) = \int(C) = 2 < 3$ then system is consistent and have infinite many solutions.

16. Option (b) is correct.

Given $f(\cdot)$ be a twice differentiable function from $R^2 \to R$. If $P, x_0 \in R^2$ where $||P||$ is sufficient small (here $||\cdot||$ is the Euclidean norm or distance function)

Then $f(x_0 + P) = f(x_0) + \nabla f(x_0)^T P + \frac{1}{2} P^T \nabla^2 f(\psi) P$ where $\psi \in R^2$

It can be assumed that the function is parabolic nature i.e., of the form $ax^2 + bx + c$ for a strictly local minima to exist, The coefficient of x^2 ie, 'a' should be positive. Also, for a local minimum to exist, its first derivative should be zero $f'(x)$ while be second derivative should be positive $f''(x) = +ve$. Similar conditions can be observed in option b only.

17. Option (c) is correct.

Given $\vec{V} = 2\sin hx\hat{i} + v(x, y)\hat{j}$ velocity field incompressible flow.

and $v(x, o) = \cos hx$.

A vector field $\vec{V}$ is incompressible

$$\text{div}\,\bar{v} = 0 \Rightarrow \bar{\nabla} \cdot \bar{v} = 0$$

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot (2\sinh x\hat{i} + v(x \cdot y)\hat{j} + 0\hat{k}) = 0$$

$$\frac{\partial}{\partial x} 2\sin hx + \frac{\partial v}{\partial y} = 0$$

$$2\cos hx + \frac{\partial v}{\partial y} = 0 \Rightarrow \frac{\partial v}{\partial y} = -2\cos hx$$

$$\int \partial v = \int 2\cos hx\, \partial y$$

$v = -2\cos hx \cdot y + f(x)$ ... (1)

$v(x, 0) = \cos hx$, given $\cos hx = f(x)$

$\therefore$ $v = -2y \cos hx + \cos hx = (1-2y) \cos hx$

$v = (1-2y) \cos hx$ ... (2)

Then $v(0, -1) = (1 + 2) \cos h\, 0 = 3$

$v(0, -1) = 3$

18. Option (a) is correct.

Given matrix $\begin{bmatrix} 1 & a \\ 8 & 3 \end{bmatrix}$ (a > 0) has a negative eigen value

$\Rightarrow$ Product of eigen value < 0

$$\Rightarrow \begin{bmatrix} 1 & a \\ 8 & 3 \end{bmatrix} < 0 \Rightarrow 3 - 8a < 0$$

$$3 < 8a \therefore a > \frac{3}{8}$$

Hence, $a > \frac{3}{8}$

19. Option (a) is correct.

Minimize $z = -x_1 - 2x_2$

S.t $x_3 = 2 + 2x_1 - x_2$ ...(1)

$x_4 = 7 + x_1 - 2x_2$ ...(2)

$x_5 = 3 - x_1$ ...(3)

$x_1, x_2, x_3, x_4, x_5 \geq 0$ ...(4)

From (1)

$x_2 = 2 + 2x_1 - x_3$

Substituting this value in object function

$z = -x_1 - 2(2 + 2x_1 - x_3)$

$z = -x_1 - 4 - 4x_1 + 2x_3$

$z = -4 - 5x_1 + 2x_3$

From (3)

$x_1 = 3 - x_5$ Putting in objective function

$z = -(3 - x_5) - 2x_2$

$z = -3 - 2x_2 + x_5$

But as per the question asked, the correct option is [a].

20. Correct answer is [–1.38].

Given differential equation

$$t\frac{dx}{dt} + (t - x) = 0 \quad ...(1)$$

$$\frac{dx}{dt} + \left(\frac{t-x}{t}\right) = 0$$

$$\frac{dx}{dt} + 1 - \frac{1}{t}x = 0$$

$$\frac{dx}{dt} - \frac{1}{t}x = -1 \quad ...(2)$$

which is Linear deferential equation of first order

$$P = -\frac{1}{t}, \quad Q = -1$$

$$\text{I.F} = e^{\int pdt} = e^{\int -\frac{1}{t}dt} = e^{-\log t} = \frac{1}{t}$$

Solution for $x(t)$

$$x \cdot (I \cdot F) = \int (I \cdot F)\, Qdt + C$$

$$x \cdot \frac{1}{t} = \int \frac{1}{t}\, dt + c$$

$$\frac{x}{t} = -\log t + c \Rightarrow x(t) = -t\log t + ct \quad ...(3)$$

Put condition $x(1) = 0$

$0 = -1 \log 1 + c$ $\quad\therefore c = 0$

$x(t) = -t \log t$ ...(4)

Put $t = 2$

$x(2) = -2 \log 2 = -1.38$

21. Correct answer is [0.39].

Given x be a continuous random variable defined as [0, 1]

$$\text{Given pdf } f(x) = \begin{cases} 1 & 0 \leqslant x \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

let $y = \log_e (x + 1)$

Expected value of $y = E(y) = \int_{-\infty}^{\infty} y \cdot f(x) dx$

$$= \int_{-\infty}^{\infty} y \cdot f(x)dx = \int_0^1 \log_e (x+1) \cdot 1dx$$

$x + 1 = t$ $\quad\Rightarrow dx = dt$

$$\int_1^2 \log t dt = [t\log t - t]_1^2 = [2\log 2 - 2] - (1\log 1 - 1)$$

$= 2 \log 2 - 2 + 1 = 2 \log 2 - 1 = 1.386\text{-}1$

$= 0.386 \cong 0.39$

CIVIL ENGINEERING (CE-1)

22. Option (a) is correct.

Given equation

$$x^5 - 5x^4 - 10x^3 + 50x^2 + 9x - 45 = 0 \quad ...(1)$$

The smallest positive root

Taking option [a] $0 \leqslant x \leqslant 2$

$f(0) = 0 - 45 < 0$

$f(2) = 2^5 - 5 \times 2^4 - 10(2)^3 + 50(2)^2 + 9 \times 2 - 45$

$= 45 > 0$

$$\left.\begin{matrix} f(0) < 0 \\ f(2) > 0 \end{matrix}\right] \Rightarrow$$

Hence, there will be one root in this interval which lice will smallest root.

i.e., the smallest positive root of given equation lies between the interval [0, 2]

23. Option (b) is correct.

Given second order Partial Differential equation

$\frac{\partial^2 u}{\partial x^2} = 2$, $u : u\,(r, y)$

Assuming $g : g(x), f : f(y)$ and h: $h(y)$

Integrating with respect to x bats sides

$$\int \frac{\partial^2 u}{\partial x^2} = 2\int dx + f(y)$$

$$\frac{\partial u}{\partial x} = 2x + f(y)$$

Again Integrating basis sides

$$\int \partial u = \int 2x + f(y)dx + (y)$$

$$u = \frac{\cancel{2}x^2}{\cancel{2}} + xf(y) + h(y)$$

$$u = x^2 + xf(y) + h(y)$$

24. Option (b) is correct.

Given probability of a sudent passes only in Mathematics

i.e., $P(A) = \frac{1}{3}$

Passes only in English i.e., $P(B) = \frac{4}{9}$

Passes in between Maths and English

i.e., $P(A \cap B) = \frac{1}{6}$

We have Addition law of Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{3} + \frac{4}{9} - \frac{1}{6} = \frac{6+8-3}{18} = \frac{14-3}{18} = \frac{11}{18}$

So, the Probability that the student will pass in atleast one of there two subject is $\frac{11}{18}$

25. Option (a) is correct.

Given Partial Differential equation

$x\frac{\partial^2 f}{\partial x^2} + y\frac{\partial^2 f}{\partial y^2} = \frac{x^2+y^2}{2}$

$A = x,\ \ B = 0,\ \ C = y$

We check, $B^2 - 4AC$

$\Rightarrow -4xy$ if $x > 0,\ y > 0$

Then $B^2 - 4AC < 0$

So the nature of given Partial Differential equation is elliptic.

Condition

(i) $B^2 - 4AC > 0$ Hyperbolic

(ii) $B^2 - 4AC = 0$ Parabolic

(iii) $B^2 - 4AC < 0$ Elliptic in – Nature

26. Correct answer is [0.17].

i	0	1	2
x_i	1	2	3
$f(x_i)$	0	0.3010	0.4771

$h = 1$ at $x = 1.5$ $p = \frac{x - x_0}{h}$

$P = \frac{1.5-1}{1} = 0.5$

We have by Newtons forward Interpolation formula.

$$y = f(x) = f(x_0) + p\Delta f(x_0) + \frac{p(p-1)}{2!}\Delta^2 f(x_0) + \cdots$$

$$f(1.5) = f(1) + 0.5\Delta f(1) + 0.5\frac{(0.5-1)}{2!}\Delta^2 f(1) + \cdots$$

Difference Table

i	x_i	$f(x_i)$	$\Delta f(x_i)$	$\Delta^2 f(x_i)$
0	1	0	0.3010	– 0.1249
1	2	0.3010	0.1761	
2	3	0.4771		

$$f(1.5) = 0 + 0.5 \times 0.3010 + \frac{0.5 \times (0.5-1)}{2!} \times -0.1249$$

$= 0.1505 + 0.0156125 = 0.1661$

27. Option (a) is correct.

Given Matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Characteristic equation $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 4-\lambda & 1 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0$$

$\Rightarrow (2-\lambda)[(4-\lambda)(2-\lambda)-1] - 1[2-\lambda-1] + 1[1-(4-\lambda)] = 0$

$\Rightarrow (2-\lambda)\left[8 - 2\lambda - 4\lambda + \lambda^2 - 1\right] - [1-\lambda] + 1[-3+\lambda] = 0$

$\Rightarrow (2-\lambda)\left[\lambda^2 - 6\lambda + 7\right] - 1 + \lambda - 3 + \lambda = 0$

$\Rightarrow 2\lambda^2 - 12\lambda + 14 - \lambda^3 + 6\lambda^2 - 7\lambda - 4 + 2\lambda = 0$

$\Rightarrow -\lambda^3 + 8\lambda^2 - 17\lambda + 10 = 0$

$\Rightarrow -\lambda^3 - 8\lambda^2 + 7\lambda - 10 = 0$

which is characteristic equation.

Put $\lambda = 1$ is one root of given equation

$1 - 8 + 17 - 10 = 0$

Now, $\lambda^2 - 7\lambda + 10 = 0$

$\Rightarrow \lambda^2 - 2\lambda - 5\lambda + 10 = 0$

$\Rightarrow (\lambda - 2)(\lambda - 5) = 0$

$\Rightarrow \lambda = 2$ or 5

Hence, eigen values of given Matrix A one 1, 2, 5

28. Option (d) is correct.

Given vector function and scalar funcricus

$\bar{p} = \left(2x^2 - 3xy + z^2\right)\hat{i} + \left(2y^2 - 3yz + x^2\right)\hat{j}$

$+\left(2z^2 - 3xz + x^2\right)\hat{k}$

$r = 6x^2 + 4y^2 - z^2 - 9xyz - 2xy + 3xz - 4z$

Consider two statements P and Q

P : Curl of the gradient of the scalar field r is a Null vector

grad $r = \bar{\nabla} r = \frac{\partial r}{\partial x}\hat{i} + \frac{\partial r}{\partial y}\hat{j} + \frac{\partial r}{\partial z}\hat{k}$

$= (12x - 9yz - 2y + 3z)\hat{i} + (8y - 9xz - 2x - z)\hat{j}$

$+(-2z - 9xy + 3x - y)\hat{k}$

Curl (grad r) = $\bar{\nabla} \times$ (grad r)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$\hat{i}\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right) - \hat{j}\left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right) + \hat{k}\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)$$

$\hat{i}(-9x - 1 - (-9x - 1)) - \hat{j}(3 - 9x - (-9y + 3))\hat{j}$

$+(-9z - 2 - (-9z - 2))\hat{k}$

$\hat{i}(-\cancel{9x} - \cancel{y} + \cancel{9x} + \cancel{1}) - \hat{j}\left(-\cancel{9y} + \cancel{3} + \cancel{9y} - \cancel{3}\right) + \hat{k}$

$(-\cancel{9z} - \cancel{2} + \cancel{9z} + \cancel{2})$.

$0\hat{i} + 0\hat{j} + 0\hat{k}$ = Null vector

Q: Divergence of curl of the vector field $\vec{p}$ is zero

$$\text{Curl } \vec{p} = \bar{\nabla} \times \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$\hat{i}(0-(-3y)) - \hat{j}(-3z+2x-(2z)) + \hat{k}(2x+3x)$$

$$3y\hat{i} - (2x-5z)\hat{j} + \hat{k}(5x)$$

$$\text{div curl } \vec{p} = \nabla \cdot (\bar{\nabla} \times \vec{p})$$

$$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right) \cdot (3y\hat{i} - (2x-5z)\hat{j} + 5x\hat{k}$$

$$\frac{\partial}{\partial x}(3y) + \frac{\partial}{\partial y}[-(2x-5z)] + \frac{\partial}{\partial z}(5x)$$

$0 + 0 + 0 = 0$

Hence, both P and Q are true.

CIVIL ENGINEERING (CE-2)

29. Option (b) is correct.

Given Partial Differential equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad \text{...(1)}$$

T: T (x, y)

Temperature at centre = Average of Temperature at sides

	y	
	100	
50	C°	50
	50	x

$$\frac{50+50+50+100}{4} = 62.5$$

30. Option (a) is correct.

Statement

P: $(A + B)^T = A^T + B^T$ it is True because Transpose is distributed over addition i.e.,

$(A + B)^T = A^T + B^T$

Q: $(AB)^T = A^T \cdot B^T$ it is false because in Matrix Multiplication Transpose should follow Reversal law i.e., $(AB)^T = B^T \cdot A^T$ so that statement P is true and Q is false.

31. Option (a) is correct.

The standard formula for finding 2^{nd} derivative using 4^{th} order divided difference formula is as fallow:

$$f''(x) = \frac{1}{12h^2}[-f_{i+2} + 16f_{i+1} - 30f_i + 16f_{i-1} - f_{i-2}]$$

32. Option (b) is correct.

$f(x) = x^3 - 27x + 4, \quad 1 \le x \le 6$

Differentiation with respect to x

$$\frac{dy}{dx} = f'(x) = 3x^2 - 27 = 0 \qquad \therefore x^2 = 9$$

$$x = \pm 3$$

$$\frac{d^2y}{dx^2} = f''(x) = 6x$$

$x = 3$ lie in this range $1 \le x \le 6$

$f''(3) = 6 \times 3 = 18 + ve$

Has Point of Minima

33. Option (b) is correct.

Consider two Ordinary Differential equations

$$\text{P} : \frac{dy}{dx} = \frac{x^4 + 3x^2y^2 + 2y^4}{x^3y}$$

Given ODE is Homogeneous Linear Differential equation of first order

because $f_1(x, y) = x^4 + 3x^2y^2 + 2y^4$

is of same 4, degree Term

Same as $f_2(x, y) = x^3y$ having 4 degree term.

$$\text{Q}: \frac{dy}{dx} = \frac{-y^2}{x^2}$$

$x^2dy = -y^2dx \Rightarrow y^2dx + x^2dy = 0$

$Mdx + Ndy = 0$

$M = y^2, \qquad N = x^2$

We check the condition exactness

$$\frac{\partial M}{\partial y} = 2y, \quad \frac{\partial N}{\partial x} = 2x$$

Hence $\dfrac{\partial M}{\partial \dot{y}} \ne \dfrac{\partial N}{\partial x}$ so given differential equation is not exact differential equation.

Statement: P is Homogeneous ODE and Q is not an Exact ODE.

34. Options (a, b) are correct.

Three vectors $\vec{p} + \vec{q} - \vec{r}$ are given as

$$\vec{p} = \hat{i} + \hat{j} + \hat{k} \qquad \vec{q} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{r} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

by Observation $\vec{p} + \vec{q} - \vec{r} = 0$

$\Rightarrow \vec{r} = \vec{p} + \vec{q}$ i.e., $\vec{p} + \vec{q}$ and $\vec{r}$ are linearly dependent vectors

$\Rightarrow$ $\vec{p} + \vec{q} - \vec{r}$ lies in a same plane i.e., $\vec{p} + \vec{q} - \vec{r}$ are coplanar

$\Rightarrow$ S.T.P. = 0

Option [b] is also correct as it is the standard result.

Now checking [a]

$$= \vec{p} \times (\vec{q} \times \vec{r}) + \vec{q} \times (\vec{r} \times \vec{p}) + \vec{r} \times (\vec{p} \times \vec{q})$$

$$= \vec{p} \times [\vec{q} \times (\vec{p} + \vec{q})] + \vec{q} \times [(\vec{p} + \vec{q}) \times \vec{p}] + \vec{r} \times (\vec{p} \times \vec{q})$$

$$= \vec{p} \times (\vec{q} \times \vec{p}) + 0 + 0 + \vec{q} \times (\vec{q} + \vec{p}) - \vec{r} \times (\vec{q} \times \vec{p})$$

$$= (\vec{q} \times \vec{p})[\vec{p} + \vec{q} - \vec{r}] = (\vec{q} \times \vec{p})(\bar{0}) = \bar{0}$$

35. Correct answer is [10].

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 0 & 3 \end{bmatrix}_{2\times 3} \qquad B = \begin{bmatrix} -1 & 0 \\ 2 & 3 \\ 1 & 4 \end{bmatrix}_{3\times 2}$$

$$AB = \begin{bmatrix} 2 & 1 & 4 \\ 1 & 0 & 3 \end{bmatrix} \times \begin{bmatrix} -1 & 0 \\ 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 19 \\ 2 & 12 \end{bmatrix}_{2\times 2}$$

So determinant of A B

$$|AB| = \begin{vmatrix} 4 & 19 \\ 2 & 12 \end{vmatrix} = 48 - 38 = 10 \ne 0$$

COMPUTER SCIENCE & INFORMATION TECHNOLOGY (CS-1)

36. Option (d) is correct.

Let $f : R \rightarrow R$ be a function such that $f(x)$ = Max $\{x, x^3\}, x \in R$.

when R is set of all Real Numbers.

Let

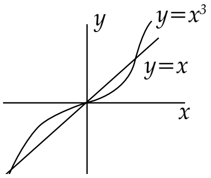
$x^3 = x \Rightarrow \quad x^3 - x = 0$

$\Rightarrow x(x^2 - 1) = 0$

$x = 0, 1, -1$

Sharp edges at $h = -1, 0, 1$

$\therefore f(n)$ is not diff. at $x = -1, 0, 1$



Graph of $f(x)$

37. Option (b) is correct.

Given Matrix A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

We have

The product of all eigen value of a Matrix A

= The Determinant of A i.e., |A|

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$= 1(45 - 48) - 2(36 - 42) + 3\,(32 - 35)$

$-3 + 2 \times 6 - 9 = -3 + 12 - 9 = 0$

Hence the product of all eigen values of A = 0

38. Options (b, c) are correct.

Given

P(A) = 0.3

P(B) = 0.5 and P(A ∩ B) = 0.1

We have by the addition low of Probability

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

= 0.3 + 0.5 – 0.1

= 0.8 – 0.1 = 0.7

Two events A and B are Independent

If P(A ∩ B) = P(A) · P(B)

$0.1 = 0.3 \times 0.5$

$0.1 \neq 0.15$

Hence A and B are not independent-events

So option [a] is false.

We have $P(A \cap B^c) = P(A) - P(A \cap B)$

= 0.3 – 0.1

= 0.2 where B^c is the complement of B.

39. Correct answer is (2).

$\because f : A \rightarrow B$ is one-one and onto Mapping

$\Rightarrow |A| = |B| = n$

We check, $f : (A \times A) \rightarrow (A \cup B)$

$$m^2 = \begin{cases} m & \text{if } A = B \\ 2n & \text{if } A \neq B \quad A \cap B = \phi \end{cases}$$

$\because f$ is one-one

$|A \times B| = |A \cup B|$

So either $n^2 = n$ or $n^2 = 2n$

$n = 0, 1 \qquad n = 0, 2$

$\therefore n = 1, 2$

$\therefore$ Number of possible values of Cardinality of A = 2

40. Option (a) is correct.

Given Recurrence

Relation $T(n) = \begin{cases} \sqrt{n}\, T(\sqrt{n}) + n & \text{for } n \geqslant 1 \\ 1 & \text{for } n = 1 \end{cases}$

$$T(n) = \sqrt{n}\, T\left(n^{1/2}\right) + n$$

$$= n^{1/2}\left[\left(n^2\right)^{1/2} T\left(n^{2^{1/2}}\right) + n^{1/2}\right] + n$$

$$= n^{\frac{3}{2^2}} \cdot T\left(n^{2^{1/2}}\right) + n + n$$

$$= n^{2^{\frac{3}{2}}}\left[n^{2^{1/3}} + T\left(n^{2^{1/3}}\right) + n^{2^{1/2}}\right] + 2n$$

$$= n^{\frac{7}{2^3}} \cdot T\left(n^{2^{\frac{1}{3}}}\right) + 3n$$

K times

$$= \frac{2^k - 1}{n^{2k}} \cdot T\left(\frac{1}{n^{2k}}\right) + km \left\{\therefore \frac{1}{n^{2k}} = 2\right.$$

$$2^K = \log_2 n$$

$$= \frac{n}{n^{\frac{1}{2k}}} \cdot T(2) + n \cdot \log_2 \log_2 n \qquad k = \log_2 \log_2 n$$

$$\therefore T(n) = \frac{n}{2} + n \log_2 \log_2 n = \theta\left(n \log_2 \log_2 n\right)$$

41. Options (b, c) are correct.

Correct answer is checked by trial option (a) operator ◊ obeys the associative law

$a \lozenge (b \lozenge c) = a \lozenge (b + 2c)$

$= a + 2b + 4c$

Consider

$(a \lozenge b) \lozenge c = (a + 2b) \lozenge c = a + 2b + 2c$

$\therefore \quad a \lozenge (b \lozenge c) \neq (a \lozenge b) \lozenge c$

Option [a] is false.

Operator □ over the operator ◊ obeys the distributive law

i.e., $a \lozenge (b \,\square\, c) = a \lozenge (abc) = a + 2bc$

$(a \lozenge b) \,\square\, (a \lozenge c) = (a + 2b) \,\square\, (a + 2c)$

$= (a + 2b) \cdot (a + 2c)$

Hence $a \lozenge (b \,\square\, c) \neq (a \lozenge b) \,\square\, (a \lozenge c)$ option [d] is false.

option [c] operator ◊ over the operator □ obeys the distributive law

$a \,\square\, (b \lozenge c) = (a \,\square\, b) \lozenge (a \,\square\, c)$

$a \,\square\, (b + 2c) = a(b + 2c)$ = L.H.S

R.H.S

$(a \,\square\, b) \lozenge (a \,\square\, c) = (ab \lozenge ac) = (ab + 2ac) = ab + 2ac$

L.H.S = R.H.S

Option [b] operator □ obeys the associative law

$a \,\square\, (b \,\square\, c) = (a \,\square\, b) \,\square\, c$

$a \,\square\, (bc)$

$abc = ab \,\square\, c = abc$

COMPUTER SCIENCE & INFORMATION TECHNOLOGY (CS-2)

42. Option (a) is correct.

Let T(n) be the recurrence relation defined as

T (0) = 1, T (1) = 2, and

T (n) = 5 T (n – 1) – 6T (n – 2) for $n \geq 2$

Put n = 2

T(2) = 5T (1) – 6T (0) = 5 × 2 – 6 × 1 = 10 – 6 = 4

Put $n = 3$

$T(3) = 5T(2) - 6T(1)$

$= 5 \times 4 - 6 \times 2 \quad = 20 - 12 = 8$

$T(0) = 2^0, \quad T(1) = 2^1, \quad T(2) = 2^2, \quad T(3) = 2^3$

So that from the above sequence

$T(n) = \theta(2^n) \qquad n = 0, 1, 2, 3$...........

43. Option (b) is correct.

Let $f(x)$ be a continuous function from R to R such that $f(x) = 1 - f(2 - x)$

Then $\int_0^2 f(x)dx \quad \therefore f(x) + f(2 - x) = 1 \qquad ... (1)$

Let $I = \int_0^2 f(x)dx = \int_0^2 [1 - f(2 - x)]dx$ = by property of definite integration

$$I = \int_0^2 \left[1 - f\left(2 - (2 - x)\right)\right]dx$$

$$I = \int_0^2 1dx - \int_0^2 f(x)dx \Rightarrow I = [x]_0^2 - I$$

$\Rightarrow 2I = 2$

$\therefore \quad I = 1$

44. Option (b) is correct.

Total case of six dice $= 6^6$

Number of favorable outcomes (Cases) = 6!

Therefore, the Probability of getting all distinct

Numbers (i.e., 1, 2, 3, 4, 5 and 6) is = $\frac{6!}{6^6}$

$$= \frac{6!}{6^6} = \frac{\cancel{6} \times 5!}{6^{\cancel{6}5}} = \frac{5!}{6^5} = \frac{5 \times 4 \times \cancel{6}}{6^{\cancel{5}4}}$$

$$= \frac{20}{6^4} = \frac{20}{6^3 \times 6} = \frac{\cancel{20}\,5}{6^2 \times \cancel{36}\,9} = \frac{5}{36 \times 9} = \frac{5}{324}$$

45. Options (b, c) are correct.

Option [a] $x \cdot 1 = x$ [By Identity law]

True. in Boolean $a \cdot 1 = a$

Option [b] $x + 1 = x$ is false because its not follow any law of the Boolean Algebra.

Option [c] $x \cdot x = 0$ is also false because by Idempotent law $(a \cdot a = a)$

Option [d] is correct because its fallow the Inverse law i.e., $\begin{Bmatrix} x + \bar{x} = 1 \\ a + \bar{a} = 1 \\ a \cdot \bar{a} = 0 \end{Bmatrix}$

46. Correct answer is [4].

Given A = {1, 2, 3, 4}

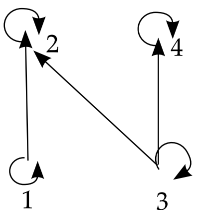
and P is a Partial order set our A.

$P = \{\{(x, x) : x \in A\} \cup \{(1,2)(3,2)(3,4)\}$

$\because$ P is a Partial order Set so it is Reflexive

$xRx \;\; \forall x \in Ap$

So that Hasses diagram of given Relation



Hence, Topological Order

$$\left.\begin{matrix} 1 & 3 & 2 & 4 \\ 1 & 3 & 4 & 2 \\ 3 & 1 & 2 & 4 \\ 3 & 1 & 4 & 2 \end{matrix}\right\}$$

So the total orders are A that contains P [4]

47. Option (d) is correct.

For options (a) and (c),

Assume x is a random variable uniformly distributed in the interval [0, 1] and $y = 1 - x$

So, $E(x) = \frac{1}{2}$ and $E(y) = E(1 - x) = E(1) - E(x)$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

Given, $z = xy$

$$E(z) = E(xy) = \int_0^1 x(1 - x)dx = \frac{1}{6}$$

Therefore, $E(z) \le E(x)E(y)$ (Here covariance is negative.)

This is not true.

For option (b),

Assume x is a random variable uniformly distributed in the interval [0, 1] and $y = x$.

So, $E(x) = \frac{1}{2}, E(y) = \frac{1}{2}$

Given $z = xy$

$$E(z) = E(xy) = \int_0^1 x \times x\,dx = \frac{1}{3}$$

Therefore, $E(z) \geqslant E(x)E(y)$

[Here covariance is negative.]

This is not true.

For option (d),

Given, x and y takes values in the range [0, 1].

So, $0 \leqslant y \leqslant 1 \Rightarrow 0 \leqslant xy \leqslant x$

$\Rightarrow E(0) \leqslant E(xy) \leqslant E(x)$

$\Rightarrow 0 \leqslant E(z) \leqslant E(x)$

$\Rightarrow 0 \leqslant z \leqslant x$

Hence, $\bar{z} \le \bar{x}$ is true.

48. Correct answer is [4].

$Z_2 \times Z_3 \times Z_4 = \{(p, q, r) \mid p\{0, 1\}, q\{0, 1, 2\}, r \in \{0,1,2,3\}\}$

So, $Z_2 \times Z_3 \times Z_4$ is a set of triplets in which first element comes from the set {0, 1}, second element from {0, 1, 2} and third element from the set {0, 1, 2, 3}.

It forms a group say G under the operation □.

Now, $(p_1, q_1, r_1) \oplus (p_2, q_2, r_2) = (p_1 \oplus_2 p_2, q_1 \oplus_3 q_2, r_1 \oplus_4 r_2)$

Here $x \,\square_m\, y = (x + y) \bmod m$

Now, $x = x^{-1}$ means $x^2 = e$, where x and identity element e belongs G.

To find the number of elements x of G such that $x \,\square\, x = e$.

For G, the unique identity element = (0, 0, 0)

Because (p, q, r) □ (0, 0, 0)

$= (p \oplus_2 0, q \oplus_3 0, r \oplus_4 0)$

$= (p, q, r)$

To find x such that x □ $x = e$

(p_1, q_1, r_1)□$(p_1, q_1, r_1) = (p_1 \oplus_2 p_1, q_1 \oplus_3 q_1 r_1 \oplus_4 r_1) = (0,0,0)$

Point (1): for x_1, we have 2 options that is 0 and 1 i.e. 0 $□_2$ 0 = 0 and 1 $□_2$ 1 = 0

Point (2): for x_2 we have 1 option that is 0.

i.e. 0 $□_3$ 0 = 0 and for $x_2 = 1, 2$

1 $□_3$ 1 ≠ 0 and 2 $□_3$ 2 ≠ 0.

Point (3): for x_3 we have 2 options that is 0 and 2.

i.e. 0 $□_4$ 0 = 0, 2 $□_4$ 2 = 0 and for x = 1, 3 i.e. 1 $□_4$ 1 ≠ 0 and 3 $□_4$ 3 ≠ 0

Therefore, for x_1, we have 2 options, for x_2 we have 1 option and for x_3 we have 2 options

So, total options for $x^2 = e$ is 2 × 1 × 2 i.e., 4

ELECTRONICS AND COMMUNICATION ENGINEERING (EC)

49. Option (a) is correct.

Given complementary function $(C \cdot F) = (At + B)e^{-2t}$

$\cong (c_1 + c_2 t)e^{-2t}$ i.e., the roots of Auxiliary equation are $m = -2, -2$ (Real and equal)

So $A \cdot E$ becomes

$(m + 2)^2 = 0$

$m^2 + 4m + 4 = 0$

$M \to D$ Replace

$(D^2 + 4D + 4)y = 0 \qquad D = \dfrac{d}{dt}$

∴ Required Differential equation will be

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = f(t)$$

50. Correct answer is [11].

Given equation $x^2 - 12x + 37 = 0$

whose one root $x = 8$,

let the base is $r \to$

$r^0 x^2 - (1 \times r + 2 \times r^0)x + 3 \times r^1 + 7 \times r^0 = 0$

$x^2 - (r + 2)x + 3r + 7 = 0$

Put $x = 8$ (is one solution)

$64 - (r + 2) \times 8 + 3r + 7 = 0$

$64 - 8r - 16 + 3r + 7 = 0$

$71 - 16 - 5r = 0$

$55 - 5r = 0 \ \Rightarrow 5r = 55$

∴ $(r = 11)$

Hence, base value is 11.

51. Correct answer is [5].

Let R and R^3 denote the set of real numbers and the three dimensional vector space over it.

$\{[2 - 3\ \alpha], [3 - 1, 3], [1 - 5, 7]\}$

Given vectors form basis so there must be Linearly Independent (L·I) and its condition is

$$|A| \neq 0$$

Model Matrix $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$\begin{bmatrix} 2 & -3 & \alpha \\ 3 & -1 & 3 \\ 1 & -5 & 7 \end{bmatrix}$$

$$|A| \neq 0 \quad \begin{vmatrix} 2 & -3 & \alpha \\ +3 & -1 & 3 \\ 1 & -5 & 7 \end{vmatrix} \neq 0$$

$2(-7 + 15) + 3(21 - 3) + \alpha(-15 + 1) \neq 0$

$16 + 54 - 14\alpha \neq 0$

$70 \neq 14\alpha \ : \ \alpha \neq 5$

52. Option (d) is correct.

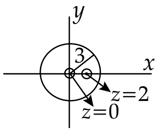
$\int_C \dfrac{\sin \pi z}{z^2(z-2)} dz$ where c is the circle in the complex plane $|z| = 3$

Pole: equating Denominator of $f(z)$ to zero

$z^2(z - 2) = 0 \quad \therefore z = 0$ is a pole of order 2 and $z = 2$ is a simple pole.

Both pole lies in the Region R so by Cauchy Integral formula

$\dfrac{1}{2\pi i}\int_c \dfrac{f(z)}{z-a} dz = f(a) = [f(z)]_{z=a}$



$\int_c \dfrac{\sin \pi z}{z^2} dz + \int_c \dfrac{\frac{\sin \pi z}{z^2}}{z-2} dz$

$= 2\pi i \left[\dfrac{\sin \pi z}{z^2}\right]_{z=2} + i\pi i \times \dfrac{1}{\lfloor 1}\left[\dfrac{d}{dz}\dfrac{\sin \pi z}{z - z}\right]_{z=0}$

$= 2\pi i \left[\dfrac{\sin 2\pi}{4} + \left\{\dfrac{\pi(z-2)\cos \pi z - \sin \pi z}{(z-2)^2}\right\}_{z=0}\right]$

$= 2\pi i \left[0 + \dfrac{(-2\pi)}{(-2)^2}\right] = -\dfrac{4\pi^2 i}{4} = -\pi^2 i$

53. Options (a, c) are correct.

Given state space

$\dfrac{dx}{dt} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r, y = [2, -5]x$

from the given Model

$TF = C(SI - A^{-1}B) + D \qquad \ldots (1)$

$= \dfrac{2s + 1}{s^2 + 3s + 2}$

Forms this controllable form can be written as

$x_0 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = [1, 2]x + [0]u$

Another possibility

$T \cdot f = \dfrac{25 + 1}{5^2 + 35 + 2} = \dfrac{-1}{5 + 1} + \dfrac{3}{5 + 2}$

By Partial fraction

$x^0 = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} -1 \\ 3 \end{bmatrix} v$

$y = [1, 1]x + [0]u$

Diagonal form of state space.

54. Options (a, b, d) are correct.

$\int_c F_1 dx + f_2 dy + f_3 dz = \int_C F \cdot d\bar{r}$ is Independent of Contour C

Then $\bar{F}$ is conservative vector field

$\therefore \bar{F} = \bar{\nabla}\phi =$ grad ϕ where ϕ is called Scalar potential

$$F_1\hat{i} + F_3\hat{j} + F_3\hat{k} = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$\Rightarrow F_1 = \frac{\partial\phi}{\partial x} \quad F_2 = \frac{\partial\phi}{\partial y}, \quad F_3 = \frac{\partial\phi}{\partial z}$$

$\because \bar{F}$ is Conservative vector field then curl $\bar{F} = 0$

i.e., $\bar{\nabla} \times \bar{F} = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = 0$$

$$\hat{i}\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right) - \hat{j}\left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right) + \hat{k}\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) = 0$$
$$= 0\hat{i} + 0\hat{j} + 0\hat{k}$$

Equating both sides

$$\frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z}, \quad \frac{\partial F_3}{\partial x} = \frac{\partial f_1}{\partial z}, \quad \frac{\partial f_2}{\partial x} = \frac{\partial f_1}{\partial y}$$

If $\bar{F}$ is rotational then

$$\int_C F_1 dx + F_2 dy + f_3 d_2 = 0$$

But

$$\bar{\nabla} \cdot \bar{F} = \bar{\nabla} \cdot (\bar{\nabla}\phi) \neq 0$$

55. Options (a, b) are correct.

Given Matrix $\begin{bmatrix} 1 & k \\ 2 & 1 \end{bmatrix}$ where k is a positive Real Number.

Characteristic equation of A

$$|A - \lambda I| = 0 \qquad \text{i.e.,} \quad \begin{vmatrix} 1-\lambda & k \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)^2 - 2k = 0 \quad \Rightarrow \quad (1-\lambda)^2 = 2k$$

$$\therefore (1-\lambda) = \pm\sqrt{2k}$$

$\lambda = 1 + \sqrt{2k} = 1 + \sqrt{2k}, 1 - \sqrt{2k}$, are called Eigen values.

Let $X_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{2\times 1}$ We the Eigen vector corresponding to Eigen value $\lambda = 1 + \sqrt{2k}$.

Matrix equation will be

$$[A - \lambda F]x_1 = 0$$

$$\begin{bmatrix} \cancel{1} - \cancel{1} - \sqrt{2k} & k \\ 2 & \cancel{1} - \cancel{1} - \sqrt{2k} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\sqrt{2k} & k \\ 2 & -\sqrt{2k} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-\sqrt{2k}x_1 + kx_2 = 0 \Rightarrow -\sqrt{2k}x_1 = -kx_2$$

$$x_1 = \frac{k}{\sqrt{2k}}x_2 = \sqrt{\frac{k}{2}}x_2 \quad \text{let } x_2 = 1$$

$$x_1 = \begin{bmatrix} \sqrt{\frac{k}{2}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ \sqrt{\frac{2}{k}} \end{bmatrix}$$

Similarly, Eigen vector at $\lambda = 1 - \sqrt{2k}$

$$\begin{bmatrix} \sqrt{2k} & k \\ 2 & \sqrt{2k} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\sqrt{2k}x_1 + kx_2 = 0 \Rightarrow x_2 = -\frac{k}{\sqrt{2k}}x_2$$

$$x_1 = -\sqrt{\frac{k}{2}}x_2 \quad x_2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\sqrt{\frac{k}{2}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -\sqrt{\frac{2}{k}} \end{bmatrix}$$

Hence eigen vectors are $\begin{bmatrix} 1 \\ \sqrt{\frac{2}{k}} \end{bmatrix}, \begin{bmatrix} 1 \\ -\sqrt{\frac{2}{k}} \end{bmatrix}$

at $\lambda = 1 \pm \sqrt{2k}$

ELECTRICAL ENGINEERING (EE)

56. Option (c) is correct.

Only the matrix

$$\begin{bmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 1 & 2 & 4 \end{bmatrix} \text{ has an Inverse}$$

Because its determinant not equal to zero.

$$|A| = \begin{vmatrix} 1 & 4 & 8 \\ 0 & 4 & 2 \\ 1 & 2 & 4 \end{vmatrix} \quad \begin{aligned} &= 1(16-4) - 4(0-2) + 8(0-4) \\ &= 12 + 8 - 32 \\ &= 20 - 32 = -12 \neq 0 \end{aligned}$$

$$\because A^{-1} = \frac{\text{Adj}\,A}{|A|}$$

and rest of the Matrix has zero determinant.

57. Options (b, d) are correct.

$X = \{-10, -9 \ldots\ldots 0 \ldots\ldots 9, 10\}$

X be any discrete random variable

$X^2 = \{100, 81, 64, \ldots\ldots 4, 1, 0, 1, 4, 9 \ldots\ldots 81, 100\}$

which consist Total 21 Terms,

$P(0) = \frac{1}{21}$ and $P(1) = \frac{2}{21}$

so X^2 is not uniformly distributed.

$(X + 10)^2 = \{0, 1, 4, 9, 16 \ldots\ldots 81, 100\ 121, -400\}$

$\therefore$ P(choosing any No.) $= \frac{1}{21}$ so it is uniformly distributed.

Now $X^3 = \{-1000, -729, -512 \ldots. -8, -1, 0, 1, 8, \ldots 729, 1000\}$

Again P(choosing any No.) $= \frac{1}{21}$ = constant so it is also uniformly distributed.

58. Option (d) is correct.

Which of the following complex function is/an analytic an the complex plane.

A function $w = f(z) = u + iv$ is called Analytic function if it satisfy C-R equation

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}$$

Given function $w = f(z) = z^2 - z$ when $z = x + iy$

$w = f(z) = (x + iy)^2 - (x + iy)$

$= x^2 + i^2y^2 + 2xyi - x - iy$

$=(x^2 - y^2 - x) + (2xy - y)\hat{i}$

Hence

$u(x, y) = x^2 - y^2 - x \qquad v(x, y) = 2xy - y$

$\frac{\partial y}{\partial x}=2x-1$ ...(1) $\qquad \frac{\partial u}{\partial y}=-2y$ ...(2)

$\frac{\partial v}{\partial x}=2y$ ....(3) $\qquad \frac{\partial v}{\partial y}=2x-1$ ...(4)

from (1) and (4) we have

$\left[\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}\right]$ and from (2) and (3) we have $\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$

59. Correct answer is [0].

Given

$f(z) = \cos z + e^{z^2}$ is a function of complex variable z.

we have $\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}\cdots\cdots$

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\cdots$$

Then the coefficient of z^5 in Taylor Series Expansion of $f(z)$ about $z = 0$ (origin)

$$f(z)=\left(1-\frac{z^2}{2!}+\frac{z^4}{4!}-\cdots\right)+\left(1+z^2+\frac{z^4}{2!}+\frac{z^6}{3!}+\cdots\right)$$

$$f(z)=2+\left(1-\frac{1}{2!}\right)z^2+\left(\frac{1}{2!}+\frac{1}{4!}\right)z^4+\cdots$$

all the terms in the expansion of Taylor series are even Paurer Term.

So that the coefficient of z^5 is zero.

60. Correct answer is [29].

We have sum of the eigen values of a matrix is equal to the Trace of A.

Given matrix $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}^2=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\times\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}=\begin{bmatrix}7 & 10\\ 15 & 22\end{bmatrix}$

Let λ_1 and λ_2 be the eigen values of A

Then $\lambda_1 + \lambda_2$ = Trace of A

$\lambda_1 + \lambda_2 = 7 + 22 = 29$

61. Correct answer is [0].

We have by

Fourier Transform

$$F\{f(x)\}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{isx}f(x)dx=f(s)$$

$x(t)=e^{-t^4}\cos t \qquad -\infty < t < \infty$

$F\{x(t)\}=x(\omega)$

We have $tx(t) \rightleftarrows j\frac{dx(\omega)}{d\omega}$

$f(t)=\frac{tx(t)}{j}=\frac{dx}{d\omega}=F(\omega)$

$\left[\frac{dx(\omega)}{d\omega}\right]_{at\omega=0}=[F(\omega)]_{\omega=0}$ = Area of $f(t)$

$=\int_{-\infty}^{\infty}\frac{tx(t)}{j}dt =$ A react odd – function

$= 0$

$\because x(t) =$ even function

$\therefore t\frac{x(t)}{v} =$ odd-function

62. Option (c) is correct.

Given $f(x, y, z) = 2\log(xy) + \log(yz) + 3\log(xz)$

Direction vector $\bar{u}=2\hat{i}+\hat{j}+2\hat{k}$

At the point (1, 1, 1). We have the directional derivative of $f(x, y, z)$ in the direction of the vector is at (1, 1, 1)

$$=\text{grad} f\cdot\hat{a}=\bar{\nabla}f\cdot\frac{\bar{a}}{|\bar{a}|}=\bar{\nabla}\cdot f\cdot\frac{\bar{u}}{|\bar{u}|}$$

$$\text{grad } f=\bar{\nabla}f=\frac{\partial f}{\partial x}\hat{i}+\frac{\partial f}{\partial y}\hat{j}+\frac{\partial f}{\partial z}\hat{k}$$

$$=\left(\frac{2}{xy}\cdot y+\frac{3}{xz}\cdot z\right)\hat{i}+\left(\frac{2}{xy}\cdot x+\frac{1}{4z}\cdot z\right)\hat{j}+\left(\frac{1}{yz}\cdot y+\frac{3}{xz}\cdot x\right)\hat{k}$$

$$=\left(\frac{2}{x}+\frac{3}{x}\right)\hat{i}+\left(\frac{2}{y}+\frac{1}{y}\right)\hat{j}+\left(\frac{1}{z}+\frac{3}{z}\right)\hat{k}$$

$5\hat{i}+3\hat{j}+4\hat{k}$ at (1, 1, 1)

D.D of f = grad $f\cdot\hat{u}=(5\hat{i}+3\hat{j}+4\hat{k})\cdot\frac{2\hat{i}+\hat{j}+2\hat{k}}{\sqrt{4+1+y}}$

$$=\frac{10+3+8}{3}=\frac{21}{3}=7$$

63. Option (c) is correct.

We have by the z-trans form of any Sequence be

$$Z\{m\}=\sum_{n=-\infty}^{\infty}mz^{-n}=\bar{u}(Z) \qquad ...(1)$$

given sequence $Y[n] = X[2n]$

$$Z\{Y(n)\}=\sum_{n=-\infty}^{\infty}Y[n]z^{-n}=\sum_{n=-\infty}^{\infty}x[2n]z^{-n}$$

Let $2n = K \therefore n = k/2$

$$=\sum_{k=\text{ even Integer}}X(k)z^{-k/2}=\sum_{k=-\infty}^{\infty}\left[\frac{X(k)+(-1)^kX(k)}{2}\right]z^{-k/2}$$

$$=\frac{1}{2}\left[\sum_{k=-\infty}^{\infty}X(k)z^{-k/2}+\sum_{k=-\infty}^{\infty}(-1)^kX(k)z^{-k/2}\right]$$

$$=\frac{1}{2}\left[\sum_{k=-\infty}^{\infty}X(k)\left(z^{+1/2}\right)^{-k}+\sum_{k=-\infty}^{\infty}X(k)\left(-z^{1/2}\right)^{-k}\right]$$

$$=\frac{1}{2}\left[X\left(z^{1/2}\right)+X\left(-z^{1/2}\right)\right]=Y(z)$$

64. Option (b) is correct.

Given $f(t)$ be a real valued function whose second derivative is positive for $-\infty < t < \infty$

If $\frac{d^2f}{dt^2}=f''(t)=+ve$ from $-\infty < t < \infty$ then function is Minima

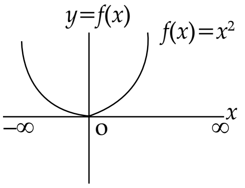
$= -ve$ from $-\infty < t < \infty$ the function is Mixima

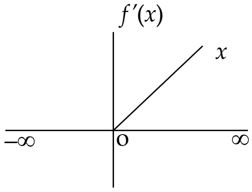
Then $f(t)$ can not have two distinct local maxima.

65. Option (b) is correct.

Given $f(t) = (\text{Max}\ (0, t)^2)$ for $-\infty < t < \infty$

when Max (a, b)





$\therefore$ $f(x)$ is differentiable and $f'(x)$ is continuous but $f''(x)$ is not continuous.

$f(t)$ is differentiable and its derivative is continuous.

66. Options (b, d) are correct.

Which of the following Differential equation is/are non linear?

Option (a) $\frac{dx(t)}{dt} + tx(t) = t^2e^t \quad x(0) = 0$ is a linear differential equation.

Option (b) $\frac{1}{2}e^t + x(t)\frac{d(x(t))}{dt} = 0, \qquad x(0) = 0$

$x(t)\frac{dx(t)}{dt} + \frac{1}{2}e^t = 0, \qquad x(0) = 0$ is non linear differential equation

Because $x\frac{dx}{dt}$ i.e., product of dependent variable and derivative is appear.

Option (c) $x(t)\cos t - \frac{dx}{dt}\sin t = 1 \quad x(0) = 0$

$-\sin t\frac{dx}{dt} + x\cos t = 1$

$\frac{dx}{dt} - x\cot t = -\text{cosec}\ t$ so it is also linear differential equation of first order.

Option (d) $x(t) + e^{\frac{dx(t)}{dt}} = 1, \quad x(0) = 0$

$e^{\frac{dx}{dt}} = 1 - x$ taking log both sides.

$\frac{dx}{dv} = \log(1 - x)$ is non linear differential equation because dependent variables is in log or the derivative $\frac{dx}{dt}$ is in the from $e^{\frac{dx}{dt}}$