NDA/NA

National Defence Academy / Naval Academy

MATHEMATICS

I

QUESTION PAPER 2017

Time : 2:30 Hour **Total Marks : 300**

Important Instructions :

1. *This test Booklet contains* **120** *items (questions). Each item is printed in* **English**. *Each item comprises four responses (answer's). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ONLY ONE response for each item.*
2. *You have to mark all your responses ONLY on the separate Answer Sheet provided.*
3. **All** *items carry equal marks.*
4. *Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions.*
5. *Penalty for wrong answers :*
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   - **(i)** *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, one·third of the marks assigned to that question will be deducted as penalty.*
   - **(ii)** *If a candidate gives more than one answer, it will be treated as a wrong answer even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   - **(iii)** *If a question is left blank, i.e., no answer is given by the candidate, there will be no penalty for that question.*

1. Let S be the set of all persons living in Delhi. We say that x, y in S are related if they were born in Delhi on the same day. Which one of the following is correct?

(a) The relation is an equivalent relation

(b) The relation is not reflexive but it is symmetric and transitive

(c) The relation is not symmetric but it is reflexive and transitive

(d) The relation is not transitive but it is reflexive and symmetric

2. Let A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Then the number of subsets of A containing two or three elements is

(a) 45 **(b)** 120

(c) 165 **(d)** 330

3. The value of $i^{2n} + i^{2n+1} + i^{2n+2} + i^{2n+3}$, where $i = \sqrt{-1}$, is

(a) 0 **(b)** 1

(c) i **(d)** $-i$

4. If the difference between the roots of the equation $x^2 + kx + 1 = 0$ is strictly less than $\sqrt{5}$, where $|k| \geq 2$, then k can be any element of the interval

(a) $(-3, -2] \cup [2, 3)$ **(b)** $(-3, 3)$

(c) $[-3, -2] \cup [2, 3]$ **(d)** None of the above

5. If the roots of the equation $x^2 + px + q = 0$ are in the same ratio as those of the equation $x^2 + lx + m = 0$, then which one of the following is correct?

(a) $p^2m = l^2q$ **(b)** $m^2p = l^2q$

(c) $m^2p = q^2l$ **(d)** $m^2p^2 = l^2q$

6. The value of $\left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n$

where n is not a multiple of 3 and $i = \sqrt{-1}$, is

(a) 1 **(b)** -1

(c) i **(d)** $-i$

7. Three-digit numbers are formed from the digits 1, 2 and 3 in such a way that the digits are not repeated. What is the sum of such three-digit numbers?

(a) 1233 **(b)** 1322

(c) 1323 **(d)** 1332

8. What is the sum of the series

0·3 + 0·33 + 0·333 + ... n terms?

(a) $\frac{1}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$ **(b)** $\frac{1}{3}\left[n - \frac{2}{9}\left(1 - \frac{1}{10^n}\right)\right]$

(c) $\frac{1}{3}\left[n - \frac{1}{3}\left(1 - \frac{1}{10^n}\right)\right]$ **(d)** $\frac{1}{3}\left[n - \frac{1}{9}\left(1 + \frac{1}{10^n}\right)\right]$

9. If 1, ω, ω^2 are the cube roots of unity, then $(1+\omega)(1+\omega^2)(1+\omega^3)(1+\omega+\omega^2)$ is equal to

(a) -2 **(b)** -1

(c) 0 **(d)** 2

10. If the sum of m terms of an AP is n and the sum of n terms is m, then the sum of $(m+n)$ terms is

(a) mn **(b)** $m+n$

(c) $2(m+n)$ **(d)** $-(m+n)$

11. The modulus and principal argument of the complex number

$$\frac{1+2i}{1-(1-i)^2}$$

are respectively

(a) 1, 0 **(b)** 1, 1

(c) 2, 0 **(d)** 2, 1

12. If the graph of a quadratic polynomial lies entirely above x-axis, then which one of the following is correct?

(a) Both the roots are real

(b) One root is real and the other is complex

(c) Both the roots are complex

(d) Cannot say

13. If $|z+4| \le 3$, then the maximum value of $|z+1|$ is

(a) 0 **(b)** 4

(c) 6 **(d)** 10

14. The number of roots of the equation $z^2 = 2\bar{z}$ is

(a) 2 **(b)** 3

(c) 4 **(d)** zero

15. If cot α and cot β are the roots of the equation $x^2 + bx + c = 0$ with $b \neq 0$, then the value of cot $(\alpha+\beta)$ is

(a) $\frac{c-1}{b}$ **(b)** $\frac{1-c}{b}$

(c) $\frac{b}{c-1}$ **(d)** $\frac{b}{1-c}$

16. The sum of the roots of the equation $x^2 + bx + c = 0$ (where b and c are non-zero) is equal to the sum of the reciprocals of their squares. Then $\frac{1}{c}$, b, $\frac{c}{b}$ are in

(a) A.P. **(b)** G.P.

(c) H.P. **(d)** None of the above

17. The sum of the roots of the equation $ax^2 + x + c = 0$ (where a and c are non-zero) is equal to the sum of the reciprocals of their squares. Then a, ca^2, c^2 are in

(a) A.P. **(b)** G.P.

(c) H.P. **(d)** None of the above

18. The value of $[C(7,0)+C(7,1))+[C(7,1)+C(7,2)] + \ldots + [C(7,6)+C(7,7)]$ is

(a) 254 **(b)** 255

(c) 256 **(d)** 257

19. The number of different words (eight-letter words) ending and beginning with a consonant which can be made out of the letters of the word 'EQUATION' is

(a) 5200 **(b)** 4320

(c) 3000 **(d)** 2160

20. The fifth term of an AP of n terms, whose sum is $n^2 - 2n$, is

(a) 5 **(b)** 7

(c) 8 **(d)** 15

21. The sum of all the two-digit odd numbers is

(a) 2475 **(b)** 2530

(c) 4905 **(d)** 5049

22. The sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\ldots$ is equal to

(a) $2^n - n - 1$ **(b)** $1 - 2^{-n}$

(c) $2^{-n} + n - 1$ **(d)** $2^n - 1$

23. Consider the following in respect of sets A and B:

(1) $(A - B) \cup B = A$

(2) $(A - B) \cup A = A$

(3) $(A - B) \cap B = \phi$

(4) $A \subseteq B \Rightarrow A \cup B = B$

Which of the above are correct?

(a) 1, 2 and 3 **(b)** 2, 3 and 4

(c) 1, 3 and 4 **(d)** 1, 2 and 4

24. In the binary equation $(1p101)_2 + (10q1)_2 = (100r00)_2$ where p, q and r are binary digits, what are the possible values of p, q and r respectively?

(a) 0, 1 , 0 **(b)** 1, 1, 0

(c) 0, 0, 1 **(d)** 1, 0, 1

25. If $S = \{x : x^2 + 1 = 0, x \text{ is real}\}$, then S is

(a) $\{-1\}$ **(b)** $\{0\}$

(c) $\{1\}$ **(d)** an empty set

26. The expansion of $(x - y)^n$, $n \geq 5$ is done in the descending powers of x. If the sum of the fifth and sixth terms is zero, then $\frac{x}{y}$ is equal to

(a) $\frac{n-5}{6}$ **(b)** $\frac{n-4}{5}$

(c) $\frac{5}{n-4}$ **(d)** $\frac{6}{n-5}$

27. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $\det(A^3) = 125$, then α is equal to

(a) ± 1 **(b)** ± 2

(c) ± 3 **(d)** ± 5

28. If B is a non-singular matrix and A is a square matrix, then the value of $\det(B^{-1}AB)$ is equal to

(a) det (B) **(b)** det (A)

(c) $\det(B^{-1})$ **(d)** $\det(A^{-1})$

29. If $a \neq b \neq c$, then one value of x which satisfies the equation

$$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$$

is given by

(a) a **(b)** b

(c) c **(d)** 0

30. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

then what is AA^T equal to (where A^T is the transpose of A)?

(a) Null matrix **(b)** Identity matrix

(c) A **(d)** – A

31. The equations

$$x + 2y + 3z = 1$$
$$2x + y + 3z = 2$$
$$5x + 5y + 9z = 4$$

(a) have the unique solution

(b) have infinitely many solutions

(c) are inconsistent

(d) None of the above

32. $A = \begin{bmatrix} x+y & y \\ x & x-y \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$.

If AB = C, then what is A^2 equal to?

(a) $\begin{bmatrix} 4 & 8 \\ -4 & -16 \end{bmatrix}$ **(b)** $\begin{bmatrix} 4 & -4 \\ 8 & -16 \end{bmatrix}$

(c) $\begin{bmatrix} -4 & -8 \\ 4 & 12 \end{bmatrix}$ **(d)** $\begin{bmatrix} -4 & -8 \\ 8 & 12 \end{bmatrix}$

33. What is the value of the determinant

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+xyz & 1 \\ 1 & 1 & 1+xyz \end{vmatrix}?$$

(a) $1 + x + y + z$ **(b)** $2xyz$

(c) $x^2y^2z^2$ **(d)** $2x^2y^2z^2$

34. If $\begin{vmatrix} x & y & 0 \\ 0 & x & y \\ y & 0 & x \end{vmatrix} = 0$, then which one of the following is correct?

(a) $\frac{x}{y}$ is one of the cube roots of unity

(b) x is one of the cube roots of unity

(c) y is one of the cube roots of unity

(d) $\frac{x}{y}$ is one of the cube roots of – 1

35. Consider the set A of all matrices of order 3 × 3 with entries 0 or 1 only. Let B be the subset of A consisting of all matrices whose determinant is 1. Let C be the subset of A consisting of all matrices whose determinant is –1. Then which one of the following is correct?

(a) C is empty

(b) B has as many elements as C

(c) $A = B \cup C$

(d) B has thrice as many elements as C

36. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then what is A^3 equal to?

(a) $\begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$ **(b)** $\begin{bmatrix} \cos^3\theta & \sin^3\theta \\ -\sin^3\theta & \cos^3\theta \end{bmatrix}$

(c) $\begin{bmatrix} \cos 3\theta & -\sin 3\theta \\ \sin 3\theta & \cos 3\theta \end{bmatrix}$ **(d)** $\begin{bmatrix} \cos^3\theta & -\sin^3\theta \\ \sin^3\theta & \cos^3\theta \end{bmatrix}$

37. What is the order of

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}?$$

(a) 3 × 1 **(b)** 1 × 1

(c) 1 × 3 **(d)** 3 × 3

38. If $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, then the value of A^4 is

(a) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ **(b)** $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$

(c) $\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}$ **(d)** $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

39. If $\sin A=\frac{3}{5}$, where $450°<A<540°$, then $\cos\frac{A}{2}$ is equal to

(a) $\frac{1}{\sqrt{10}}$ **(b)** $-\sqrt{\frac{3}{10}}$

(c) $\frac{\sqrt{3}}{\sqrt{10}}$ **(d)** None of the above

40. What is $\frac{1}{\sin 10°}-\frac{\sqrt{3}}{\cos 10°}$ equal to?

(a) 0 **(b)** 1

(c) 2 **(d)** 4

41. From the top of a lighthouse, 100m high, the angle of depression of a boat is $\tan^{-1}\left(\frac{5}{12}\right)$ What is the distance between the boat and the lighthouse?

(a) 120 m **(b)** 180 m

(c) 240 m **(d)** 360 m

42. The maximum value of $\sin\left(x+\frac{\pi}{6}\right)+\cos\left(x+\frac{\pi}{6}\right)$ in the interval $\left(0,\frac{\pi}{2}\right)$ is attained at

(a) $\frac{\pi}{12}$ **(b)** $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ **(d)** $\frac{\pi}{2}$

43. If $K=\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$, than what is the value of K?

(a) $\frac{1}{2}$ **(b)** $\frac{1}{4}$

(c) $\frac{1}{8}$ **(d)** $\frac{1}{16}$

44. The expression $\frac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta}$ is equal to

(a) $\tan\left(\frac{\alpha+\beta}{2}\right)$ **(b)** $\cot\left(\frac{\alpha+\beta}{2}\right)$

(c) $\sin\left(\frac{\alpha+\beta}{2}\right)$ **(d)** $\cos\left(\frac{\alpha+\beta}{2}\right)$

45. If $\sin\theta=3\sin(\theta+2\alpha)$, then the value of $\tan(\theta+\alpha)+2\tan\alpha$ is equal to

(a) -1 **(b)** 0

(c) 1 **(d)** 2

46. What is the value of $\tan 18°$?

(a) $\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$ **(b)** $\frac{\sqrt{5}-1}{\sqrt{10+\sqrt{5}}}$

(c) $\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$ **(d)** $\frac{\sqrt{10+\sqrt{5}}}{\sqrt{5}-1}$

47. Let x, y, z be positive real numbers such that x, y, z are in G.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are in A.P. Then which one of the following is correct?

(a) $x=y=z$ **(b)** $xz=1$

(c) $x\neq y$ and $y=z$ **(d)** $x=y$ and $y\neq z$

48. If $\tan(\alpha+\beta)=2$ and $\tan(\alpha-\beta)=1$, then $\tan(2\alpha)$ is equal to

(a) -3 **(b)** -2

(c) $-\frac{1}{3}$ **(d)** 1

49. Consider the following for triangle ABC:

(1) $\sin\left(\frac{B+C}{2}\right)=\cos\left(\frac{A}{2}\right)$

(2) $\tan\left(\frac{B+C}{2}\right)=\cot\left(\frac{A}{2}\right)$

(3) $\sin(B+C)=\cos A$

(4) $\tan(B+C)=-\cot A$

Which of the above are correct?

(a) 1 and 3 **(b)** 1 and 2

(c) 1 and 4 **(d)** 2 and 3

50. If $\sec\theta-\text{cosec}\ \theta=\frac{4}{3}$, then what is $(\sin\theta-\cos\theta)$ equal to?

(a) -2 only

(b) $\frac{1}{2}$ only

(c) Both -2 and $\frac{1}{2}$

(d) Neither $\frac{1}{2}$ nor -2

51. If a vertex of a triangle is (1, 1) and the midpoints of two sides of the triangle through this vertex are (–1, 2) and (3, 2), then the centroid of the triangle is

(a) $\left(-\frac{1}{3}, \frac{7}{3}\right)$ **(b)** $\left(-1, \frac{7}{3}\right)$

(c) $\left(\frac{1}{3}, \frac{7}{3}\right)$ **(d)** $\left(1, \frac{7}{3}\right)$

52. The incentre of the triangle with vertices A $(1, \sqrt{3})$, B(0, 0) and C(2, 0) is

(a) $\left(1, \frac{\sqrt{3}}{2}\right)$ **(b)** $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$

(c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ **(d)** $\left(1, \frac{1}{\sqrt{3}}\right)$

53. If the three consecutive vertices of a parallelogram are (– 2, – 1), (1, 0) and (4, 3), then what are the coordinates of the fourth vertex?

(a) (1, 2) **(b)** (1, 0)

(c) (0, 0) **(d)** (1, – 1)

54. The two circles $x^2 + y^2 = r^2$ and $x^2 + y^2 - 10x + 16 = 0$ intersect at two distinct points. Then which one of the following is correct?

(a) $2 < r < 8$ **(b)** $r = 2$ or $r = 8$

(c) $r < 2$ **(d)** $r > 2$

55. What is the equation of the circle which passes through the points (3, – 2) and (– 2, 0) and having its centre on the line $2x - y - 3 = 0$?

(a) $x^2 + y^2 + 3x + 2 = 0$

(b) $x^2 + y^2 + 3x + 12y + 2 = 0$

(c) $x^2 + y^2 + 2x = 0$

(d) $x^2 + y^2 = 5$

56. What is the ratio in which the point $C\left(-\frac{2}{7}, -\frac{20}{7}\right)$ divides the line joining the points A(– 2, – 2) and B(2, – 4)?

(a) 1 : 3 **(b)** 3 : 4

(c) 1 : 2 **(d)** 2 : 3

57. What is the equation of the ellipse having foci $(\pm 2, 0)$ and the eccentricity $\frac{1}{4}$?

(a) $\frac{x^2}{64} + \frac{y^2}{60} = 1$ **(b)** $\frac{x^2}{60} + \frac{y^2}{64} = 1$

(c) $\frac{x^2}{20} + \frac{y^2}{24} = 1$ **(d)** $\frac{x^2}{24} + \frac{y^2}{20} = 1$

58. What is the equation of the straight line parallel to $2x + 3y + 1 = 0$ and passes through the point (– 1, 2)?

(a) $2x + 3y - 4 = 0$ **(b)** $2x + 3y - 5 = 0$

(c) $x + y - 1 = 0$ **(d)** $3x - 2y + 7 = 0$

59. What is the acute angle between the pair of straight lines $\sqrt{2}x + \sqrt{3}y = 1$ and $\sqrt{3}x + \sqrt{2}y = 2$?

(a) $\tan^{-1}\left(\frac{1}{2\sqrt{6}}\right)$ **(b)** $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(c) $\tan^{-1}(3)$ **(d)** $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

60. If the centroid of a triangle formed by (7, x), (y, – 6) and (9, 10) is (6, 3), then the values of x and y are respectively

(a) 5, 2 **(b)** 2, 5

(c) 1, 0 **(d)** 0, 0

61. A straight line with direction cosines (0, 1, 0) is

(a) parallel to x-axis

(b) parallel to y-axis

(c) parallel to z-axis

(d) equally inclined to all the axes

62. (0, 0, 0), (a, 0, 0), (0, b, 0) and (0, 0, c) are four distinct points. What are the coordinates of the point which is equidistant from the four points?

(a) $\left(\frac{a+b+c}{3}, \frac{a+b+c}{3}, \frac{a+b+c}{3}\right)$

(b) (a, b, c)

(c) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

(d) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

63. The points P(3, 2, 4), Q(4, 5, 2), R(5, 8, 0) and S(2, – 1, 6) are

(a) vertices of a rhombus which is not a square

(b) non-coplanar

(c) collinear

(d) coplanar but not collinear

64. The line passing through the points (1, 2, – 1) and (3, – 1, 2) meets the yz-plane at which one of the following points?

(a) $\left(0, -\frac{7}{2}, \frac{5}{2}\right)$ **(b)** $\left(0, \frac{7}{2}, \frac{1}{2}\right)$

(c) $\left(0, -\frac{7}{2}, -\frac{5}{2}\right)$ **(d)** $\left(0, -\frac{7}{2}, \frac{5}{2}\right)$

65. Under which one of the following conditions are the lines $x = ay + b$; $z = cy + d$ and $x = ey + f$; $z = gy + h$ perpendicular?

(a) $ae + cg - 1 = 0$ **(b)** $ae + bf - 1 = 0$

(c) $ae + cg + 1 = 0$ **(d)** $ag + ce + 1 = 0$

66. If $\vec{a} = \hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{c} = \hat{i} + m\hat{j} + n\hat{k}$ are three coplanar vectors and $|\vec{c}| = \sqrt{6}$, then which one of the following is correct?

(a) $m = 2$ and $n = \pm 1$ **(b)** $m = \pm 2$ and $n = -1$

(c) $m = 2$ and $n = -1$ **(d)** $m = \pm 2$ and $n = 1$

67. Let ABCD be a parallelogram whose diagonals intersect at P and let O be the origin. What is $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$ equal to?

(a) $2\overrightarrow{OP}$ **(b)** $4\overrightarrow{OP}$

(c) $6\overrightarrow{OP}$ **(d)** $8\overrightarrow{OP}$

68. ABCD is a quadrilateral whose diagonals are AC and BD. Which one of the following is correct?

(a) $\overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{AC} + \overrightarrow{DB}$

(b) $\overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{BD} + \overrightarrow{CA}$

(c) $\overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{AC} + \overrightarrow{BD}$

(d) $\overrightarrow{BA} + \overrightarrow{CD} = \overrightarrow{BC} + \overrightarrow{AD}$

69. $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$ then which one of the following is correct?

(a) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs and $|\vec{a}| = |\vec{c}|$ and $|\vec{b}| = 1$

(b) $\vec{a}, \vec{b}, \vec{c}$ are non-orthogonal to each other

(c) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs but $|\vec{a}| \neq |\vec{c}|$

(d) $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs but $|\vec{b}| \neq 1$

70. If $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} + \lambda\hat{k}$ are perpendicular, then what is the value of λ?

(a) 2 **(b)** 3

(c) 4 **(d)** 5

71. What is $\lim_{x \to 0} \frac{e^x - (1+x)}{x^2}$ equal to?

(a) 0 **(b)** $\frac{1}{2}$

(c) 1 **(d)** 2

72. What is $\int_0^{\pi/2} \frac{d\theta}{1 + \cos\theta}$ equal to?

(a) $\frac{1}{2}$ **(b)** 1

(c) $\sqrt{3}$ **(d)** None of the above

73. What is $\int \frac{dx}{x(x^7 + 1)}$ equal to?

(a) $\frac{1}{2}\ln\left|\frac{x^7 - 1}{x^7 + 1}\right| + c$ **(b)** $\frac{1}{7}\ln\left|\frac{x^7 + 1}{x^7}\right| + c$

(c) $\ln\left|\frac{x^7 - 1}{7x}\right| + c$ **(d)** $\frac{1}{7}\ln\left|\frac{x^7}{x^7 + 1}\right| + c$

74. The function $f : X \to Y$ defined by $f(x) = \cos x$, where $x \in X$, is one-one and onto if X and Y are respectively equal to

(a) $[0, \pi]$ and $[-1, 1]$

(b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[-1, 1]$

(c) $[0, \pi]$ and $(-1, 1)$

(d) $[0, \pi]$ and $[0, 1]$

75. If $f(x) = \frac{x}{x-1}$, then what is $\frac{f(a)}{f(a+1)}$ equal to?

(a) $f\left(-\frac{(a)}{(a+1)}\right)$ **(b)** $f(a^2)$

(c) $f\left(\frac{1}{a}\right)$ **(d)** $f(-a)$

76. What is $\int \frac{\left(x^{e-1} + e^{x-1}\right)dx}{x^e + e^x}$ equal to?

(a) $\frac{x^2}{2} + c$ **(b)** $\ln(x + e) + c$

(c) $\ln(x^e + e^x) + c$ **(d)** $\frac{1}{e}\ln\left(x^e + e^x\right) + c$

77. Let $f : [-6, 6] \to \mathbb{R}$ be defined by $f(x) = x^2 - 3$. Consider the following :

(1) $(fofof)(-1) = (fofof)(1)$

(2) $(fofof)(-1) - 4(fofof)(1) = (fof)(0)$

Which of the above is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

78. Let $f(x) = px + q$ and $g(x) = mx + n$. Then $f(g(x)) = g(f(x))$ is equivalent to

(a) $f(p) = g(m)$ **(b)** $f(q) = g(n)$

(c) $f(n) = g(q)$ **(d)** $f(m) = g(p)$

79. If $F(x)=\sqrt{9-x^2}$, then what is

$$\lim_{x\to 1}\frac{F(x)-F(1)}{x-1}$$

equal to?

(a) $-\frac{1}{4\sqrt{2}}$ **(b)** $\frac{1}{8}$

(c) $-\frac{1}{2\sqrt{2}}$ **(d)** $\frac{1}{2\sqrt{2}}$

80. What is $\frac{d^2x}{dy^2}$ equal to?

(a) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ **(b)** $\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-2}$

(c) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$ **(d)** $\left(\frac{d^2y}{dx^2}\right)^{-1}$

81. Let $f(x):\begin{cases}x, & x \text{ is rational}\\ 0, & x \text{ is irrational}\end{cases}$

and $g(x):\begin{cases}0, & x \text{ is rational}\\ x, & x \text{ is irrational}\end{cases}$

If $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$, then $(f-g)$ is

(a) one-one and into

(b) neither one-one nor onto

(c) many-one and onto

(d) one-one and onto

82. What is the length of the longest interval in which the function $f(x)=3\sin x-4\sin^3 x$ is increasing?

(a) $\frac{\pi}{3}$ **(b)** $\frac{\pi}{2}$

(c) $\frac{3\pi}{2}$ **(d)** π

83. If $xdy = y(dx + ydy)$; $y(1) = 1$ and $y(x) > 0$, then what is $y(-3)$ equal to?

(a) 3 **(b)** 2

(c) 1 **(d)** 0

84. What is the maximum value of the function $f(x)=4\sin^2 x+1$?

(a) 5 **(b)** 3

(c) 2 **(d)** 1

85. Let $f(x)$ be an indefinite integral of $\sin^2 x$. Consider the following statements:

Statement 1:

The function $f(x)$ satisfies $f(x+\pi)=f(x)$ for all real x.

Statement 2:

$\sin^2(x+\pi)=\sin^2 x$ for all real x.

Which one of the following is correct in respect of the above statements?

(a) Both the statements are true and Statement 2 is the correct explanation of Statement 1

(b) Both the statements are true but Statement 2 is not the correct explanation of Statement 1

(c) Statement 1 is true but Statement 2 is false

(d) Statement 1 is false but Statement 2 is true

86. What are the degree and order respectively of the differential equation

$$y=x\left(\frac{dy}{dx}\right)^2+\left(\frac{dx}{dy}\right)^2$$

(a) 1, 2 **(b)** 2, 1

(c) 1, 4 **(d)** 4, 1

87. What is the differential equation corresponding to $y^2-2ay+x^2=a^2$ by eliminating a?

(a) $(x^2-2y^2)p^2-4pxy-x^2=0$

(b) $(x^2-2y^2)p^2+4pxy-x^2=0$

(c) $(x^2+2y^2)p^2-4pxy-x^2=0$

(d) $(x^2+2y^2)p^2-4pxy+x^2=0$

where $p=\frac{dy}{dx}$.

88. What is the general solution of the differential equation

$$ydx-(x+2y^2)dy=0?$$

(a) $x=y^2+cy$ **(b)** $x=2cy^2$

(c) $x=2y^2+cy$ **(d)** None of the above

89. Let $f(x+y)=f(x)f(y)$ for all x and y. Then what is $f'(5)$ equal to [where $f(x)$ is the derivative of $f(x)$]?

(a) $f(5)f(0)$ **(b)** $f(5)-f(0)$

(c) $f(5)\,f(0)$ **(d)** $f(5)+f(0)$

90. If $f(x)$ and $g(x)$ are continuous functions satisfying $f(x)=f(a-x)$ and $g(x)+g(a-x)=2$, then what is $\int_0^a f(x)g(x)dx$ equal to?

(a) $\int_0^a g(x)dx$ **(b)** $\int_0^a f(x)dx$

(c) $2\int_0^a f(x)dx$ **(d)** 0

91. What is the solution of the differential equation

$$\ln\left(\frac{dy}{dx}\right)-a=0?$$

(a) $y = xe^a + c$ **(b)** $x = ye^a + c$

(c) $y = \ln x + c$ **(d)** $x = \ln y + c$

92. Let $f(x)$ be defined as follows:

$$f(x) = \begin{cases} 2x+1, & -3 < x < -2 \\ x-1, & -2 \le x < 0 \\ x+2, & 0 \le x < 1 \end{cases}$$

Which one of the following statements is correct in respect of the above function?

(a) It is discontinuous at $x = -2$ but continuous at every other point.

(b) It is continuous only in the interval $(-3, -2)$.

(c) It is discontinuous at $x = 0$ but continuous at every other point.

(d) It is discontinuous at every point.

93. Consider the following statements:

(1) If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ both exist, then $\lim_{x \to a} \{f(x)\, g(x)\}$ exists.

(2) If $\lim_{x \to a} \{f(x)\, g(x)\}$ exists, then both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ must exist.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

94. Which one of the following functions is neither even nor odd?

(a) $x^2 - 1$ **(b)** $x + \frac{3}{x}$

(c) $|x|$ **(d)** $x^2 (x - 3)$

95. What is the derivative of $\log_{10} (5x^2 + 3)$ with respect to x?

(a) $\frac{x \log_{10} e}{5x^2 + 3}$ **(b)** $\frac{2x \log_{10} e}{5x^2 + 3}$

(c) $\frac{10x \log_{10} e}{5x^2 + 3}$ **(d)** $\frac{10x \log_e 10}{5x^2 + 3}$

96. Let $f(a) = \frac{a-1}{a+1}$.

Consider the following:

(1) $f(2a) = f(a) + 1$

(2) $f\left(\frac{1}{a}\right) = -f(a)$

Which of the above is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

97. What is the maximum area of a triangle that can be inscribed in a circle of radius a?

(a) $\frac{3a^2}{4}$ **(b)** $\frac{a^2}{2}$

(c) $\frac{3\sqrt{3}a^2}{4}$ **(d)** $\frac{\sqrt{3}a^2}{4}$

98. Let $f(x) = x + \frac{1}{x}$, where $x \in (0, 1)$. Then which one of the following is correct?

(a) $f(x)$ fluctuates in the interval

(b) $f(x)$ increases in the interval

(c) $f(x)$ decreases in the interval

(d) None of the above

99. Suppose the function $f(x) = x^n$, $n \neq 0$ is differentiable for all x. Then n can be any element of the interval

(a) $[1, \infty)$ **(b)** $(0, \infty)$

(c) $\left(\frac{1}{2}, \infty\right)$ **(d)** None of the above

100. What is $\int_{e^{-1}}^{e^2} \left|\frac{\ln x}{x}\right| dx$ equal to?

(a) $\frac{3}{2}$ **(b)** $\frac{5}{2}$

(c) 3 **(d)** 4

101. The variance of 20 observations is 5. If each observation is multiplied by 3, then what is the new variance of the resulting observations?

(a) 5 **(b)** 10

(c) 15 **(d)** 45

102. The mean of a group of 100 observations was found to be 20. Later it was found that four observations were incorrect, which were recorded as 21, 21, 18 and 20. What is the mean if the incorrect observations are omitted?

(a) 18 **(b)** 20

(c) 21 **(d)** 22

103. A committee of two persons is constituted from two men and two women. What is the probability that the committee will have only women?

(a) $\frac{1}{6}$ **(b)** $\frac{1}{3}$

(c) $\frac{1}{2}$ **(d)** $\frac{2}{3}$

104. A question is given to three students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the question will be solved?

(a) $\frac{1}{24}$ **(b)** $\frac{1}{4}$

(c) $\frac{3}{4}$ **(d)** $\frac{23}{24}$

105. The mean weight of 150 students in a certain class is 60 kg. The mean weight of boys in the class is 70 kg and that of girls is 55 kg. What is the number of boys in the class?

(a) 50 **(b)** 55

(c) 60 **(d)** 100

106. For two dependent events A and B, it is given that P(A) = 0.2 and P(B) = 0.5. If $A \subseteq B$, then the values of conditional probabilities P(A|B) and P(B|A) are respectively

(a) $\frac{2}{5}, \frac{3}{5}$

(b) $\frac{2}{5}, 1$

(c) $1, \frac{2}{5}$

(d) Information is insufficient

107. A point is chosen at random inside a circle. What is the probability that the point is closer to the centre of the circle than to its boundary?

(a) $\frac{1}{5}$ **(b)** $\frac{1}{4}$

(c) $\frac{1}{3}$ **(d)** $\frac{1}{2}$

108. If two regression lines between height (x) and weight (y) are $4y - 15x + 410 = 0$ and $30x - 2y - 825 = 0$, then what will be the correlation coefficient between height and weight?

(a) $\frac{1}{3}$ **(b)** $\frac{1}{2}$

(c) $\frac{2}{3}$ **(d)** $\frac{3}{4}$

109. In an examination, 40% of candidates got second class. When the data are represented by a pie chart, what is the angle corresponding to second class?

(a) 40° **(b)** 90°

(c) 144° **(d)** 320°

110. Consider the following statements:

Statement 1:
Range is not a good measure of dispersion.

Statement 2:
Range is highly affected by the existence of extreme values.

Which one of the following is correct in respect of the above statements?

(a) Both Statement 1 and Statement 2 are correct and Statement 2 is the correct explanation of Statement 1

(b) Both Statement 1 and Statement 2 are correct but Statement 2 is not the correct explanation of Statement 1

(c) Statement 1 is correct but Statement 2 is not correct

(d) Statement 2 is correct but Statement 1 is not correct

111. A card is drawn from a well-shuffled ordinary deck of 52 cards. What is the probability that it is an ace?

(a) $\frac{1}{13}$ **(b)** $\frac{2}{13}$

(c) $\frac{3}{13}$ **(d)** $\frac{1}{52}$

112. If the data are moderately non symmetrical, then which one of the following empirical relationships is correct?

(a) 2 × Standard deviation = 5 × Mean deviation

(b) 5 × Standard deviation = 2 × Mean deviation

(c) 4 × Standard deviation = 5 × Mean deviation

(d) 5 × Standard deviation = 4 × Mean deviation

113. Data can be represented in which of the following forms?

(1) Textual form

(2) Tabular form

(3) Graphical form

Select the correct answer using the code given below.

(a) 1 and 2 only **(b)** 2 and 3 only

(c) 1 and 3 only **(d)** 1, 2 and 3

114. For given statistical data, the graphs for less than ogive and more than ogive are drawn. If the point at which the two curves intersect is P, then abscissa of point P gives the value of which one of the following measures of central tendency?

(a) Median **(b)** Mean

(c) Mode **(d)** Geometric mean

115. Consider the following statements:

(1) Two events are mutually exclusive if the occurrence of one event prevents the occurrence of the other.

(2) The probability of the union of two mutually exclusive events is the sum of their individual probabilities.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

116. If the regression coefficient of x on y and y on x are $-\frac{1}{2}$ and $-\frac{1}{8}$ respectively, then what is the correlation coefficient between x and y?

(a) $-\frac{1}{4}$ **(b)** $-\frac{1}{16}$

(c) $\frac{1}{16}$ **(d)** $\frac{1}{4}$

117. A sample of 5 observations has mean 32 and median 33. Later it is found that an observation was recorded incorrectly as 40 instead of 35. If we correct the data, then which one of the following is correct?

(a) The mean and median remain the same

(b) The median remains the same but the mean will decrease

(c) The mean and median both will decrease

(d) The mean remains the same but median will decrease

118. If two fair dice are thrown, then what is the probability that the sum is neither 8 nor 9?

(a) $\frac{1}{6}$ **(b)** $\frac{1}{4}$

(c) $\frac{3}{4}$ **(d)** $\frac{5}{6}$

119. Let A and B are two mutually exclusive events with $P(A)=\frac{1}{3}$ and $P(B)=\frac{1}{4}$. What is the value of $P(\overline{A}\cap\overline{B})$?

(a) $\frac{1}{6}$ **(b)** $\frac{1}{4}$

(c) $\frac{1}{3}$ **(d)** $\frac{5}{12}$

120. The mean and standard deviation of a binomial distribution are 12 and 2 respectively. What is the number of trials?

(a) 2 **(b)** 12

(c) 18 **(d)** 24

Answers

Q. No.	Answer Key	Topic Name	Chapter Name
1	(a)	Types of Relation	Relation and Function
2	(c)	Number of Subset	Permutation and Combination
3	(a)	Properties of Complex Number	Complex Number
4	(a)	Inequality	Quadratic Equation
5	(a)	Properties of Roots	Quadratic Equation
6	(b)	Cube Roots of Unity	Complex Number
7	(d)	Permutation	Permutation and Combination
8	(a)	Sum of n Terms of G.P.	Sequence and Series
9	(c)	Cube Roots of Unity	Complex Number
10	(d)	Sum of n Terms of A.P.	Sequence and Series
11	(a)	Modulus and Argument	Complex Number
12	(c)	Graph of $y = ax^2 + bx + c$	Quadratic Equation
13	(c)	Inequality	Complex Number
14	(c)	Conjugate and its Properties	Complex Number
15	(b)	Properties of Roots	Quadratic Equation
16	(c)	Properties of Roots	Quadratic Equation
17	(a)	Properties of Roots	Quadratic Equation
18	(a)	Properties of Combination	Binomial Theorem
19	(b)	Permutation	Permutation and Combination
20	(b)	Properties of A.P.	Sequence and Series
21	(a)	Sum of A.P.	Sequence and Series
22	(c)	Sum of G.P.	Sequence and Series
23	(b)	Operations on Sets	Sets
24	(a)	Binary Number	Binary Number
25	(d)	Classification/Types of Set	Sets
26	(b)	General Term	Binomial Theorem
27	(c)	Properties of Determinant	Matrices and Determinants
28	(b)	Properties of Determinant and Matrix	Matrices and Determinants
29	(d)	Value of Determinant	Matrices and Determinants
30	(b)	Transpose of Matrix	Matrices and Determinants
31	(a)	System of Equations	Matrices and Determinants
32	(d)	System of Equations	Matrices and Determinants
33	(c)	Properties of Determinant	Matrices and Determinants
34	(d)	Properties of Determinant	Matrices and Determinants
35	(b)	Properties of Determinant	Matrices and Determinants
36	(a)	Multiplication of Matrix	Matrices and Determinants
37	(b)	Multiplication of Matrix	Matrices and Determinants

Q. No.	Answer Key	Topic Name	Chapter Name
38	**(a)**	Multiplication of Matrix	Matrix and Determinant
39	**(d)**	Multiple and Sub-Multiple Angle	Trigonometric ratios and Identities
40	**(d)**	Sum and Difference Formula	Trigonometric ratios and Identities
41	**(c)**	Height and Distance	Height and Distance
42	**(a)**	Maximum and Minimum Value	Trigonometric Ratios and Identities
43	**(c)**	Identities	Trigonometric Ratios and Identities
44	**(a)**	Sum and Difference Formula	Trigonometric Ratios and Identities
45	**(b)**	Sum and Difference Formula	Trigonometric Ratios and Identities
46	**(a)**	Values	Trigonometric Ratios and Identities
47	**(a)**	Properties of A.P. and G.P.	Sequence and Series
48	**(a)**	Identities	Trigonometric Ratios and Identities
49	**(b)**	Allied Angles	Trigonometric Ratios and Identities
50	**(c)**	Multiple and Sub-Multiple Angle	Trigonometric Ratios and Identities
51	**(d)**	Centroid	Point and Straight Line
52	**(d)**	Centroid	Point and Straight Line
53	**(a)**	Mid Point	Point and Straight Line
54	**(a)**	Interaction of Circles	Circle
55	**(b)**	Equation of Circle	Circle
56	**(b)**	Section Formula	Point and Straight Line
57	**(a)**	Equation of Ellipse	Ellipse
58	**(a)**	Equation of Line	Point and Straight Line
59	**(a)**	Angle between Two Lines	Point and Straight Line
60	**(a)**	Centroid	Point and Straight Line
61	**(b)**	Direction Cosine	3D
62	**(c)**	Distance Formula	3D
63	**(c)**	Distance Formula	3D
64	**(d)**	Equation of Line	3D
65	**(c)**	Equation of Line	3D
66	**(d)**	Coplanarity	Vector
67	**(b)**	Mid Point	Vector
68	**(b)**	Position Vector	Vector
69	**(a)**	Cross Product of Vectors	Vector
70	**(b)**	Dot Product of Vectors	Vector
71	**(b)**	L′ Hospital Rule	Limits
72	**(b)**	Standard Formula of Integration	Definite Integration
73	**(d)**	Substitution Method	Indefinite Integration
74	**(a)**	One-One and onto Function	Function
75	**(b)**	Value of Function	Function
76	**(d)**	Substitution Method	Indefinite Integration
77	**(c)**	Composite Function	Function
78	**(c)**	Composite Function	Function
79	**(c)**	L′ Hospital Rule	Limits

Q. No.	Answer Key	Topic Name	Chapter Name
80	**(c)**	Differential Coefficient	Differentiation
81	**(d)**	One-One and onto Function	Function
82	**(a)**	Increasing Function	Application of Derivative
83	**(a)**	Variable Separable Method	Differential Equation
84	**(a)**	Maximum and Minimum Value	Trigonometric Ratios and Identities
85	**(b)**	Integration of Trigonometric Function	Indefinite Integration
86	**(c)**	Order and Degree	Differential Equation
87	**(a)**	Formation of Differential Equation	Differential Equation
88	**(c)**	Linear Differential Equation	Differential Equation
89	**(a)**	First Principle of Differentiability	Differentiability
90	**(b)**	Properties of Definite Integration	Definite Integration
91	**(a)**	Variable Separable Method	Differential Equation
92	**(c)**	Continuity in Interval	Continuity
93	**(a)**	Algebra of Limit	Limits
94	**(d)**	Even and Odd Function	Function
95	**(c)**	Differential Coefficient	Differentiation
96	**(b)**	Value of Function	Function
97	**(c)**	Maxima and Minima	Application of Derivative
98	**(c)**	Increasing and Decreasing	Application of Derivative
99	**(a)**	Differential Coefficient	Differentiation
100	**(b)**	Properties of Definite Integration	Definite Integration
101	**(d)**	Variance	Statistics
102	**(b)**	Mean	Statistics
103	**(a)**	Classical Definition of Probability	Probability
104	**(c)**	Independent Events	Probability
105	**(a)**	Mean	Statistics
106	**(b)**	Conditional Probability	Probability
107	**(b)**	Geometrical Probability	Probability
108	**(b)**	Correlation	Correlation and Regression
109	**(c)**	Pie Chart	Statistics
110	**(a)**	Dispersion	Statistics
111	**(a)**	Classical Definition of Probability	Probability
112	**(c)**	Standard Deviation	Statistics
113	**(b)**	Data Representation	Statistics
114	**(a)**	Median, Ogive	Statistics
115	**(c)**	Type of Events	Probability
116	**(a)**	Correlation	Correlation and Regression
117	**(b)**	Median	Statistics
118	**(c)**	Complement of an Event	Probability
119	**(d)**	Mutually Exclusive Events, Addition Theorem	Probability
120	**(c)**	Binomial Distribution	Probability