NDA/NA

National Defence Academy / Naval Academy

MATHEMATICS

I

QUESTION PAPER 2018

Time : 2:30 Hour **Total Marks : 300**

Important Instructions :

1. *This test Booklet contains* **120** *items (questions). Each item is printed in* **English**. *Each item comprises four responses (answer's). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ONLY ONE response for each item.*
2. *You have to mark all your responses ONLY on the separate Answer Sheet provided.*
3. **All** *items carry equal marks.*
4. *Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions.*
5. *Penalty for wrong answers :*
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   - **(i)** *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, one-third of the marks assigned to that question will be deducted as penalty.*
   - **(ii)** *If a candidate gives more than one answer, it will be treated as a wrong answer even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   - **(iii)** *If a question is left blank, i.e., no answer is given by the candidate, there will be no penalty for that question.*

1. If $n \in N$, then $121^n - 25^n + 1900^n - (-4)^n$ is divisible by which one of the following?
(a) 1904 **(b)** 2000
(c) 2002 **(d)** 2006

2. If $n = (2017)!$, then
$\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + ... + \frac{1}{\log_{2017} n}$ is equal to?
(a) 0 **(b)** 1
(c) $\frac{n}{2}$ **(d)** n

3. In the expansion of $(1 + x)^{43}$, if the coefficients of $(2r + 1)^{th}$ and $(r + 2)^{th}$ terms are equal, then what is the value of r $(r \neq 1)$?
(a) 5 **(b)** 14
(c) 21 **(d)** 22

4. What is the principal argument of $(-1 - i)$, where $i = \sqrt{-1}$?
(a) $\frac{\pi}{4}$ **(b)** $-\frac{\pi}{4}$
(c) $-\frac{3\pi}{4}$ **(d)** $\frac{3\pi}{4}$

5. Let α and β be real numbers and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct non-real roots with $Re(z) = 1$, then it is necessary that
(a) $\beta \in (-1, 0)$ **(b)** $|\beta| = 1$
(c) $\beta \in (1, \infty)$ **(d)** $\beta \in (0, 1)$

6. Let A and B be subsets of X and $C = (A \cap B') \cup (A' \cap B)$ where A′ and B′ are complements of A and B respectively in X. What is C equal to?
(a) $(A \cup B') - (A \cap B')$ **(b)** $(A' \cup B) - (A' \cap B)$
(c) $(A \cup B) - (A \cap B)$ **(d)** $(A' \cup B') - (A' \cap B')$

7. How many numbers between 100 and 1000 can be formed with the digits 5, 6, 7, 8, 9, if the repetition of digits is not allowed?
(a) 3^5 **(b)** 5^3
(c) 120 **(d)** 60

8. The number of non-zero integral solutions of the equation $|1 - 2i|^x = 5x$ is, where $i = \sqrt{-1}$
(a) Zero (No solution) **(b)** One
(c) Two **(d)** Three

9. If the ratio of AM to GM of two positive numbers a and b is 5 : 3, then $a : b$ is equal to
(a) 3 : 5 **(b)** 2 : 9
(c) 9 : 1 **(d)** 5 : 3

10. If the coefficients of a^m and a^n in the expansion of $(1 + a)^{m+n}$ are α and β, then which one of the following is correct?
(a) $\alpha = 2\beta$ **(b)** $\alpha = \beta$
(c) $2\alpha = \beta$ **(d)** $\alpha = (m + n)\beta$

11. If $x + \log_{15}(1 + 3^x) = x\log_{15} 5 + \log_{15} 12$, where x is an integer, then what is x equal to?

(a) -3 **(b)** 2

(c) 1 **(d)** 3

12. How many four-digit numbers divisible by 10 can be formed using 1, 5, 0, 6, 7 without repetition of digits?

(a) 24 **(b)** 36

(c) 44 **(d)** 64

Consider the information given below and answer the two items (02) that follow:

In a class, 54 students are good in Hindi only, 63 students are good in Mathematics only and 41 students are good in English only. There are 18 students who are good in both Hindi and Mathematics. 10 students are good in all three subjects.

13. What is the number of students who are good in either Hindi or Mathematics but not in English?

(a) 99 **(b)** 107

(c) 125 **(d)** 130

14. What is the number of students who are good in Hindi and Mathematics but not in English?

(a) 18 **(b)** 12

(c) 10 **(d)** 8

15. If α and β are different complex numbers with $|\alpha| = 1$, then what is $\left|\frac{\alpha - \beta}{1 - \alpha\bar{\beta}}\right|$ equal to?

(a) $|\beta|$ **(b)** 2

(c) 1 **(d)** 0

16. The equation $|1 - x| + x^2 = 5$ has

(a) a rational root and an irrational root

(b) two rational roots

(c) two irrational roots

(d) no real roots

17. The binary number expression of the decimal number 31 is

(a) 1111 **(b)** 10111

(c) 11011 **(d)** 11111

18. What is $i^{1000} + i^{1001} + i^{1002} + i^{1003}$ equal to (where $i = \sqrt{-1}$)?

(a) 0 **(b)** i

(c) $-i$ **(d)** 1

19. What is

$$\frac{1}{\log_2 N} + \frac{1}{\log_3 N} + \frac{1}{\log_4 N} + ... + \frac{1}{\log_{100} N}$$

equal to $(N \neq 1)$?

(a) $\frac{1}{\log_{100!} N}$ **(b)** $\frac{1}{\log_{99!} N}$

(c) $\frac{99}{\log_{100!} N}$ **(d)** $\frac{99}{\log_{99!} N}$

20. The modulus-amplitude form of $\sqrt{3} + i$, where $i = \sqrt{-1}$ is

(a) $2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ **(b)** $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(c) $4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ **(d)** $4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

21. What is the number of non-zero terms in the expansion of $\left(1 + 2\sqrt{3}x\right)^{11} + \left(1 - 2\sqrt{3}x\right)^{11}$ (after simplification)?

(a) 4 **(b)** 5

(c) 6 **(d)** 11

22. What is the greatest integer among the following by which the number $5^5 + 7^5$ is divisible?

(a) 6 **(b)** 8

(c) 11 **(d)** 12

23. If $x = 1 - y + y^2 - y^3 + ...$ up to infinite terms, where $|y| < 1$, then which one of the following is correct?

(a) $x = \frac{1}{1+y}$ **(b)** $x = \frac{1}{1-y}$

(c) $x = \frac{y}{1+y}$ **(d)** $x = \frac{y}{1-y}$

24. What is the inverse of the matrix

$$A = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}?$$

(a) $\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **(b)** $\begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$ **(d)** $\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

25. If A is a 2×3 matrix and AB is a 2×5 matrix, then B must be a

(a) 3×5 matrix **(b)** 5×3 matrix

(c) 3×2 matrix **(d)** 5×2 matrix

26. If $A=\begin{pmatrix}1 & 2\\ 2 & 3\end{pmatrix}$ and $A^2 - kA - I_2 = O$, where I_2 is the 2×2 identity matrix, then what is the value of k?
(a) 4 **(b)** –4
(c) 8 **(d)** –8

27. What is the number of triangles that can be formed by choosing the vertices from a set of 12 points in a plane, seven of which lie on the same straight line?
(a) 185 **(b)** 175
(c) 115 **(d)** 105

28. What is $C(n, r) + 2C(n, r-1) + C(n, r-2)$ equal to?
(a) $C(n+1, r)$ **(b)** $C(n-1, r+1)$
(c) $C(n, r+1)$ **(d)** $C(n+2, r)$

29. Let $[x]$ denote the greatest integer function. What is the number of solutions of the equation $x^2 - 4x + [x] = 0$ in the interval $[0, 2]$?
(a) Zero (No solution) **(b)** One
(c) Two **(d)** Three

30. A survey of 850 students in a University yields that 680 students like music and 215 like dance. What is the least number of students who like both music and dance?
(a) 40 **(b)** 45
(c) 50 **(d)** 55

31. What is the sum of all two-digit numbers which when divided by 3 leave 2 as the remainder?
(a) 1565 **(b)** 1585
(c) 1635 **(d)** 1655

32. If $0 < a < 1$, the value of $\log_{10} a$ is negative. This is justified by
(a) Negative power of 10 is less than 1
(b) Negative power of 10 between 0 and 1
(c) Negative power of 10 is positive
(d) Negative power of 10 is negative

33. The third term of a GP is 3. What is the product of the first five terms?
(a) 216
(b) 226
(c) 243
(d) Cannot be determined due to insufficient data

34. If $x, \frac{3}{2}, z$ are in AP; $x, 3, z$ are in GP; then which one of the following will be in HP?
(a) $x, 6, z$ **(b)** $x, 4, z$
(c) $x, 2, z$ **(d)** $x, 1, z$

35. What is the value of the sum $\sum_{n=2}^{11}\left(i^n + i^{n+1}\right)$, where $i = \sqrt{-1}$?
(a) i **(b)** $2i$
(c) $-2i$ **(d)** $1 + i$

36. If $\sin x = \frac{1}{\sqrt{5}}$, $\sin y = \frac{1}{\sqrt{10}}$, where $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, then what is $(x + y)$ equal to?
(a) π **(b)** $\frac{\pi}{2}$
(c) $\frac{\pi}{4}$ **(d)** 0

37. What is $\frac{\sin 5x - \sin 3x}{\cos 5x + \cos 3x}$ equal to?
(a) $\sin x$ **(b)** $\cos x$
(c) $\tan x$ **(d)** $\cot x$

38. What is $\sin 105° + \cos 105°$ equal to?
(a) $\sin 50°$ **(b)** $\cos 50°$
(c) $\frac{1}{\sqrt{2}}$ **(d)** 0

39. In a triangle ABC if $a = 2$, $b = 3$ and $\sin A = \frac{2}{3}$, then what is angle B equal to?
(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ **(d)** $\frac{\pi}{6}$

40. What is the principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$?
(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ **(d)** $\frac{2\pi}{3}$

41. If x, $x - y$ and $x + y$ are the angles of a triangle (not an equilateral triangle) such that $\tan(x-y)$, $\tan x$ and $\tan(x + y)$ are in GP, then what is x equal to?
(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{3}$
(c) $\frac{\pi}{6}$ **(d)** $\frac{\pi}{2}$

42. ABC is a triangle inscribed in a circle with centre O. Let $\alpha = \angle BAC$, where $45° < \alpha < 90°$. Let

$\beta = \angle$ BOC. Which one of the following is correct?

(a) $\cos\beta = \dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}$ **(b)** $\cos\beta = \dfrac{1+\tan^2\alpha}{1-\tan^2\alpha}$

(c) $\cos\beta = \dfrac{2\tan\alpha}{1+\tan^2\alpha}$ **(d)** $\sin\beta = 2\sin^2\alpha$

43. If a flag-staff of 6 m height placed on the top of a tower throws a shadow of $2\sqrt{3}$ m along the ground, then what is the angle that the sun makes with the ground?

(a) 60° **(b)** 45°

(c) 30° **(d)** 15°

44. What is $\tan^{-1}\left(\dfrac{1}{4}\right)+\tan^{-1}\left(\dfrac{3}{5}\right)$ equal to?

(a) 0 **(b)** $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{3}$ **(d)** $\dfrac{\pi}{2}$

45. A spherical balloon of radius r subtends an angle α at the eye of an observer, while the angle of elevation of its centre is β. What is the height of the centre of the balloon (neglecting the height of the observer)?

(a) $\dfrac{r\sin\beta}{\sin\left(\dfrac{\alpha}{2}\right)}$ **(b)** $\dfrac{r\sin\beta}{\sin\left(\dfrac{\alpha}{4}\right)}$

(c) $\dfrac{r\sin\left(\dfrac{\beta}{2}\right)}{\sin\alpha}$ **(d)** $\dfrac{r\sin\alpha}{\sin\left(\dfrac{\beta}{2}\right)}$

46. If $\dfrac{\sin(x+y)}{\sin(x-y)}=\dfrac{a+b}{a-b}$, then what is $\dfrac{\tan x}{\tan y}$ equal to?

(a) $\dfrac{a}{b}$ **(b)** $\dfrac{b}{a}$

(c) $\dfrac{a+b}{a-b}$ **(d)** $\dfrac{a-b}{a+b}$

47. If $\sin\alpha + \sin\beta = 0 = \cos\alpha + \cos\beta$, where $0 < \beta < \alpha < 2\pi$, then which one of the following is correct?

(a) $\alpha = \pi - \beta$ **(b)** $\alpha = \pi + \beta$

(c) $\alpha = 2\pi - \beta$ **(d)** $2\alpha = \pi + 2\beta$

48. Suppose cos A is given. If only one value of $\cos\left(\dfrac{A}{2}\right)$ is possible, then A must be

(a) An odd multiple of 90°

(b) A multiple of 90°

(c) An odd multiple of 180°

(d) A multiple of 180°

49. If $\cos\alpha + \cos\beta + \cos\gamma = 0$, where $0 < \alpha \le \dfrac{\pi}{2}$, $0 < \beta \le \dfrac{\pi}{2}, 0 < \gamma \le \dfrac{\pi}{2}$, then what is the value of $\sin\alpha + \sin\beta + \sin\gamma$?

(a) 0 **(b)** 3

(c) $\dfrac{5\sqrt{2}}{2}$ **(d)** $\dfrac{3\sqrt{2}}{2}$

50. The maximum value of $\sin\left(x+\dfrac{\pi}{5}\right)+\cos\left(x+\dfrac{\pi}{5}\right)$, where $x \in \left(0,\dfrac{\pi}{2}\right)$, is attained at

(a) $\dfrac{\pi}{20}$ **(b)** $\dfrac{\pi}{15}$

(c) $\dfrac{\pi}{10}$ **(d)** $\dfrac{\pi}{2}$

51. What is the distance between the points which divide the line segment joining (4, 3) and (5, 7) internally and externally in the ratio 2 : 3 ?

(a) $\dfrac{12\sqrt{17}}{5}$ units **(b)** $\dfrac{13\sqrt{17}}{5}$ units

(c) $\dfrac{\sqrt{17}}{5}$ units **(d)** $\dfrac{6\sqrt{17}}{5}$ units

52. What is the angle between the straight lines $(m^2 - mn)\,y = (mn + n^2)\,x + n^3$ and $(mn + m^2)\,y = (mn - n^2)\,x + m^3$, where $m > n$?

(a) $\tan^{-1}\left(\dfrac{2mn}{m^2+n^2}\right)$ **(b)** $\tan^{-1}\left(\dfrac{4m^2n^2}{m^4-n^4}\right)$

(c) $\tan^{-1}\left(\dfrac{4m^2n^2}{m^4+n^4}\right)$ **(d)** 45°

53. What is the equation of the straight line cutting off an intercept 2 from the negative direction of y-axis and inclined at 30° with the positive direction of x-axis?

(a) $x-2\sqrt{3}y-3\sqrt{2}=0$ **(b)** $x+2\sqrt{3}y-3\sqrt{2}=0$

(c) $x+\sqrt{3}y-2\sqrt{3}=0$ **(d)** $x-\sqrt{3}y-2\sqrt{3}=0$

54. What is the equation of the line passing through the point of intersection of the lines $x + 2y - 3 = 0$ and $2x - y + 5 = 0$ and parallel to the line $y - x + 10 = 0$?

(a) $7x - 7y + 18 = 0$ **(b)** $5x - 7y + 18 = 0$

(c) $5x - 5y + 18 = 0$ **(d)** $x - y + 5 = 0$

55. Consider the following statements:

(1) The length p of the perpendicular from the origin to the line $ax + by = c$ satisfies the relation $p^2 = \frac{c^2}{a^2 + b^2}$.

(2) The length p of the perpendicular from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$ satisfies the relation $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

(3) The length p of the perpendicular from the origin to the line $y = mx + c$ satisfies the relation $\frac{1}{p^2} = \frac{1 + m^2 + c^2}{c^2}$.

Which of the above is/are correct?

(a) 1, 2 and 3 **(b)** 1 only
(c) 1 and 2 only **(d)** 2 only

56. What is the equation of the ellipse whose vertices are $(\pm 5, 0)$ and foci are at $(\pm 4, 0)$?

(a) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ **(b)** $\frac{x^2}{16} + \frac{y^2}{9} = 1$
(c) $\frac{x^2}{25} + \frac{y^2}{16} = 1$ **(d)** $\frac{x^2}{9} + \frac{y^2}{25} = 1$

57. What is the equation of the straight line passing through the point (2, 3) and making an intercept on the positive y-axis equal to twice its intercept on the positive x-axis?

(a) $2x + y = 5$ **(b)** $2x + y = 7$
(c) $x + 2y = 7$ **(d)** $2x - y = 1$

58. Let the coordinates of the points A, B, C be (1, 8, 4), (0, – 11, 4) and (2, – 3, 1) respectively. What are the coordinates of the point D which is the foot of the perpendicular from A on BC?

(a) (3, 4, – 2) **(b)** (4, – 2, 5)
(c) (4, 5, – 2) **(d)** (2, 4, 5)

59. What is the equation of the plane passing through the points (– 2, 6, – 6), (– 3, 10, – 9) and (– 5, 0, – 6)?

(a) $2x - y - 2z = 2$ **(b)** $2x + y + 3z = 3$
(c) $x + y + z = 6$ **(d)** $x - y - z = 3$

60. A sphere of constant radius r through the origin intersects the coordinate axes in A, B and C. What is the locus of the centroid of the triangle ABC?

(a) $x^2 + y^2 + z^2 = r^2$ **(b)** $x^2 + y^2 + z^2 = 4r^2$
(c) $9(x^2 + y^2 + z^2) = 4r^2$ **(d)** $3(x^2 + y^2 + z^2) = 2r^2$

61. The coordinates of the vertices P, Q and R of a triangle PQR are (1, –1, 1), (3, –2, 2) and (0, 2, 6) respectively. If $\angle RQP = \theta$, then what is $\angle PRQ$ equal to?

(a) $30° + \theta$ **(b)** $45° - \theta$
(c) $60° - \theta$ **(d)** $90° - \theta$

62. The perpendiculars that fall from any point of the straight line $2x + 11y = 5$ upon the two straight lines $24x + 7y = 20$ and $4x - 3y = 2$ are

(a) 12 and 4 respectively
(b) 11 and 5 respectively
(c) Equal to each other
(d) Not equal to each other

63. The equation of the line, when the position of it intercepted between the axes is divided by the point (2, 3) in the ratio of 3 : 2 is

(a) Either $x + y = 4$ or $9x + y = 12$
(b) Either $x + y = 5$ or $4x + 9y = 30$
(c) Either $x + y = 4$ or $x + 9y = 12$
(d) Either $x + y = 5$ or $9x + 4y = 30$

64. What is the distance between the straight lines $3x + 4y = 9$ and $6x + 8y = 15$?

(a) $\frac{3}{2}$ units **(b)** $\frac{3}{10}$ unit
(c) 6 unit **(d)** 5 units

65. What is the equation to the sphere whose centre is at (–2, 3, 4) and radius is 6 units?

(a) $x^2 + y^2 + z^2 + 4x - 6y - 8z = 7$
(b) $x^2 + y^2 + z^2 + 6x - 4y - 8z = 7$
(c) $x^2 + y^2 + z^2 + 4x - 6y - 8z = 4$
(d) $x^2 + y^2 + z^2 + 4x + 6y + 8z = 4$

66. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = 2, |\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, then what is the acute angle between $\vec{a}$ and $\vec{b}$?

(a) 30° **(b)** 45°
(c) 60° **(d)** 90°

67. Let $\vec{p}$ and $\vec{q}$ be the position vectors of the points P and Q respectively with respect to origin O. The points R and S divide PQ internally and externally respectively in the ratio 2 : 3. If $\vec{OR}$ and $\vec{OS}$ are perpendicular, then which one of the following is correct?

(a) $9p^2 = 4q^2$ **(b)** $4p^2 = 9q^2$
(c) $9p = 4q$ **(d)** $4p = 9q$

68. What is the moment about the point $\hat{i}+2\hat{j}-\hat{k}$ of a force represented by $3\hat{i}+\hat{k}$ acting through the point $2\hat{i}-\hat{j}+3\hat{k}$?

(a) $-3\hat{i}+11\hat{j}+9\hat{k}$ **(b)** $3\hat{i}+2\hat{j}+9\hat{k}$

(c) $3\hat{i}+4\hat{j}+9\hat{k}$ **(d)** $\hat{i}+\hat{j}+\hat{k}$

69. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ and $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\lambda\left(\vec{b}\times\vec{c}\right)$, then what is the value of λ?

(a) 2 **(b)** 3

(c) 4 **(d)** 6

70. If the vectors $\vec{k}$ and $\vec{A}$ are parallel to each other, then what is $k\vec{k}\times\vec{A}$ equal to?

(a) $k^2\vec{A}$ **(b)** $\vec{0}$

(c) $-k^2\vec{A}$ **(d)** $\vec{A}$

71. Which one of the following is correct in respect of the function $f:\mathbb{R}\to\mathbb{R}^+$ defined as $f(x)=|x+1|$?

(a) $f(x^2)=[f(x)]^2$ **(b)** $f(|x|)=|f(x)|$

(c) $f(x+y)=f(x)+f(y)$ **(d)** None of these

72. Suppose $f:\mathbb{R}\to\mathbb{R}^+$ is defined by $f(x)=\dfrac{x^2}{1+x^2}$. What is the range of the function?

(a) [0, 1) **(b)** [0, 1]

(c) (0, 1] **(d)** (0, 1)

73. If $f(x)=|x|+|x-1|$, then which one of the following is correct?

(a) $f(x)$ is continuous at $x=0$ and $x=1$

(b) $f(x)$ is continuous at $x=0$ but not at $x=1$

(c) $f(x)$ is continuous at $x=1$ but not at $x=0$

(d) $f(x)$ is neither continuous at $x=0$ nor at $x=1$

74. Consider the function $f(x)=\begin{cases}x^2\ln|x|, & x\neq 0\\ 0, & x=0\end{cases}$

What is $f'(0)$ equal to?

(a) 0 **(b)** 1

(c) -1 **(d)** It does not exist

75. What is the area of the region bounded by the parabolas $y^2=6(x-1)$ and $y^2=3x$?

(a) $\dfrac{\sqrt{6}}{3}$ sq. units **(b)** $\dfrac{2\sqrt{6}}{3}$ sq. units

(c) $\dfrac{4\sqrt{6}}{3}$ sq. units **(d)** $\dfrac{5\sqrt{6}}{3}$ sq. units

Consider the following information for the next three (03) items that follow:

Three sides of a trapezium are each equal to 6 cm. Let $\alpha\in\left(0,\dfrac{\pi}{2}\right)$ be the angle between a pair of adjacent sides.

76. If the area of the trapezium is the maximum possible, then what is 'α' equal to?

(a) $\dfrac{\pi}{6}$ **(b)** $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{3}$ **(d)** $\dfrac{2\pi}{5}$

77. If the area of the trapezium is maximum, what is the length of the fourth side?

(a) 8 cm **(b)** 9 cm

(c) 10 cm **(d)** 12 cm

78. What is the maximum area of the trapezium?

(a) $36\sqrt{3}$ cm^2 **(b)** $30\sqrt{3}$ cm^2

(c) $27\sqrt{3}$ cm^2 **(d)** $24\sqrt{3}$ cm^2

79. What is $\int_0^{\pi} e^x\sin x\,dx$ equal to?

(a) $\dfrac{e^{\pi}+1}{2}$ **(b)** $\dfrac{e^{\pi}-1}{2}$

(c) $e^{\pi}+1$ **(d)** $\dfrac{e^{\pi}+1}{4}$

80. If $f(x)=\dfrac{x^2-9}{x^2-2x-3}$, $x\neq 3$ is continuous at $x=3$, then which one of the following is correct?

(a) $f(3)=0$ **(b)** $f(3)=1.5$

(c) $f(3)=3$ **(d)** $f(3)=-1.5$

81. What is $\int_1^e x\ln x\,dx$ equal to?

(a) $\dfrac{e+1}{4}$ **(b)** $\dfrac{e^2+1}{4}$

(c) $\dfrac{e-1}{4}$ **(d)** $\dfrac{e^2-1}{4}$

82. What is $\int_0^{\sqrt{2}}\left[x^2\right]dx$ equal to (where [.] is the greatest integer function)?

(a) $\sqrt{2}-1$ **(b)** $1-\sqrt{2}$

(c) $2\left(\sqrt{2}-1\right)$ **(d)** $\sqrt{3}-1$

83. What is the maximum value of $16\sin\theta - 12\sin^2\theta$?

(a) $\frac{3}{4}$ **(b)** $\frac{4}{3}$

(c) $\frac{16}{3}$ **(d)** 4

84. If $f : \mathbb{R} \to S$ defined by $f(x) = 4\sin x - 3\cos x + 1$ is onto, then what is S equal to?

(a) $[-5, 5]$ **(b)** $(-5, 5)$

(c) $(-4, 6)$ **(d)** $[-4, 6]$

85. For f to be a function, what is the domain of f, if $f(x) = \frac{1}{\sqrt{|x| - x}}$?

(a) $(-\infty, 0)$ **(b)** $(0, \infty)$

(c) $(-\infty, \infty)$ **(d)** $(-\infty, 0]$

86. What is the solution of the differential equation $xdy - ydx = 0$?

(a) $xy = c$ **(b)** $y = cx$

(c) $x + y = c$ **(d)** $x - y = c$

87. What is the derivative of the function

$f(x) = e^{\tan x} + \ln(\sec x) - e^{\ln x}$, at $x = \frac{\pi}{4}$?

(a) $\frac{e}{2}$ **(b)** e

(c) $2e$ **(d)** $4e$

88. Which one of the following differential equation has a periodic solution?

(a) $\frac{d^2x}{dt^2} + \mu x = 0$ **(b)** $\frac{d^2x}{dt^2} - \mu x = 0$

(c) $x\frac{dx}{dt} + \mu t = 0$ **(d)** $\frac{dx}{dt} + \mu xt = 0$

where $\mu > 0$.

89. What is the period of the function $f(x) = \sin x$?

(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{2}$

(c) π **(d)** 2π

90. What is $\int \frac{dx}{2^x - 1}$ equal to?

(a) $\log(2^x - 1) + c$ **(b)** $\frac{\ln(1 - 2^{-x})}{\ln 2} + c$

(c) $\frac{\ln(2^{-x} - 1)}{2\ln 2} + c$ **(d)** $\frac{\ln(1 + 2^{-x})}{\ln 2} + c$

91. The order and degree of the differential equation $y^2 = 4a(x - a)$, where 'a' is an arbitrary constant, are respectively

(a) 1, 2 **(b)** 2, 1

(c) 2, 2 **(d)** 1, 1

92. What is the value of $\int_{-\pi/4}^{\pi/4} (\sin x - \tan x)\, dx$?

(a) $-\frac{1}{\sqrt{2}} + \ln\left(\frac{1}{\sqrt{2}}\right)$ **(b)** $\frac{1}{\sqrt{2}}$

(c) 0 **(d)** $\sqrt{2}$

93. If $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$, then what are the values of a and b respectively?

(a) $-1, 1$ **(b)** 1, 1

(c) 0, 0 **(d)** 2, -2

94. What is $\int_0^1 x(1-x)^9\, dx$ equal to?

(a) $\frac{1}{110}$ **(b)** $\frac{1}{132}$

(c) $\frac{1}{148}$ **(d)** $\frac{1}{240}$

95. What is $\lim_{x \to 0} \frac{\tan x}{\sin 2x}$ equal to?

(a) $\frac{1}{2}$ **(b)** 1

(c) 2 **(d)** Limit does not exist

96. What is $\lim_{h \to 0} \frac{\sqrt{2x + 3h} - \sqrt{2x}}{2h}$ equal to?

(a) $\frac{1}{2\sqrt{2x}}$ **(b)** $\frac{3}{\sqrt{2x}}$

(c) $\frac{3}{2\sqrt{2x}}$ **(d)** $\frac{3}{4\sqrt{2x}}$

97. If $f(x)$ is an even function, whare $f(x) \neq 0$, then which one of the following is correct?

(a) $f'(x)$ is an even function.

(b) $f'(x)$ is an odd function.

(c) $f'(x)$ may be an even or odd function depending on the type of function.

(d) $f'(x)$ is a constant function.

98. If $y = e^{x^2}\sin 2x$, then what is $\frac{dy}{dx}$ at $x = \pi$ equal to?

(a) $(1+\pi)e^{\pi^2}$ **(b)** $2\pi e^{\pi^2}$

(c) $2e^{\pi^2}$ **(d)** e^{π^2}

99. What is the solution of $(1 + 2x)dy - (1 - 2y)dx = 0$?

(a) $x - y - 2xy = c$ **(b)** $y - x - 2xy = c$

(c) $y + x - 2xy = c$ **(d)** $x + y + 2xy = c$

100. What are the order and degree, respectively, of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)^2 = y^4 + \left(\frac{dy}{dx}\right)^5 ?$$

(a) 4, 5 **(b)** 2, 3

(c) 3, 2 **(d)** 5, 4

101. In a binomial distribution, the mean is three times its variance. What is the probability of exactly 3 successes out of 5 trials?

(a) $\frac{80}{243}$ **(b)** $\frac{40}{243}$

(c) $\frac{20}{243}$ **(d)** $\frac{10}{243}$

102. Consider the following statements:

(1) $P(\overline{A} \cup B) = P(\overline{A}) + P(B) - P(\overline{A} \cap B)$

(2) $P(A \cap \overline{B}) = P(B) - P(A \cap B)$

(3) $P(A \cap B) = P(B)P(A \mid B)$

Which of the above statements are correct?

(a) 1 and 2 only **(b)** 1 and 3 only

(c) 2 and 3 only **(d)** 1, 2 and 3

103. If the correlation coefficient between x and y is 0.6, covariance is 27 and variance of y is 25, then what is the variance of x?

(a) $\frac{9}{5}$ **(b)** $\frac{81}{25}$

(c) 9 **(d)** 81

104. The probabilities that a student will solve Question A and Question B are 0.4 and 0.5 respectively. What is the probability that he solves at least one of the two questions?

(a) 0.6 **(b)** 0.7

(c) 0.8 **(d)** 0.9

105. Let $\bar{x}$ be the mean of $x_1, x_2, x_3, \ldots, x_n$. If $x_i = a + cy_i$ for some constants a and c, then what will be the mean of $y_1, y_2, y_3, \ldots, y_n$?

(a) $a + c\bar{x}$ **(b)** $a - \frac{1}{c}\bar{x}$

(c) $\frac{1}{c}\bar{x} - a$ **(d)** $\frac{\bar{x} - a}{c}$

106. Consider the following statements:

(1) If the correlation coefficient $r_{xy} = 0$, then the two lines of regression are parallel to each other.

(2) If the correlation coefficient $r_{xy} = 1$, then the two lines of regression are perpendicular to each other.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

107. If $4x - 5y + 33 = 0$ and $20x - 9y = 107$ are two lines of regression, then what are the values of $\bar{x}$ and $\bar{y}$ respectively?

(a) 12 and 18 **(b)** 18 and 12

(c) 13 and 17 **(d)** 17 and 13

108. Consider the following statements:

(1) Mean is independent of change in scale and change in origin.

(2) Variance is independent of change in scale but not in origin.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

109. Consider the following statements:

(1) The sum of deviations from mean is always zero.

(2) The sum of absolute deviations is minimum when taken around median.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

110. What is the median of the numbers

4.6, 0, 9.3, – 4.8, 7.6 , 2.3, 12.7, 3.5, 8.2, 6.1, 3.9, 5.2?

(a) 3.8 **(b)** 4.9

(c) 5.7 **(d)** 6.0

111. In a test in Mathematics, 20% of the students obtained "first class". If the data are represented by a Pie-Chart, what is the central angle corresponding to "first class"?

(a) 20° **(b)** 36°

(c) 72° **(d)** 144°

112. The mean and standard deviation of a set of values are 5 and 2 respectively. If 5 is added to each value, then what is the coefficient of variation for the new set of values?

(a) 10 **(b)** 20

(c) 40 **(d)** 70

113. A train covers the first 5 km of its journey at a speed of 30 km/hr and the next 15 km at a speed of 45 km/hr. What is the average speed of the train?

(a) 35 km/hr **(b)** 37.5 km/hr

(c) 39.5 km/hr **(d)** 40 km/hr

114. Two fair dice are rolled. What is the probability of getting a sum of 7?

(a) $\frac{1}{36}$ **(b)** $\frac{1}{6}$

(c) $\frac{7}{12}$ **(d)** $\frac{5}{12}$

115. If A and B are two events such that $2P(A) = 3P(B)$, whre $0 < P(A) < P(B) < 1$, then which one of the following is correct?

(a) $P(A|B) < P(B|A) < P(A \cap B)$

(b) $P(A \cap B) < P(B|A) < P(A|B)$

(c) $P(B|A) < P(A|B) < P(A \cap B)$

(d) $P(A \cap B) < P(A|B) < P(B|A)$

116. A box has ten chits numbered 0, 1, 2, 3, ..., 9. First, one chit is drawn at random and kept aside. From the remaining, a second chit is drawn at random. What is the probability that the second chit drawn is "9"?

(a) $\frac{1}{10}$ **(b)** $\frac{1}{9}$

(c) $\frac{1}{90}$ **(d)** None of these

117. One bag contains 3 white and 2 black balls, another bag contains 5 white and 3 black balls. If a bag is chosen at random and a ball is drawn from it, what is the chance that it is white?

(a) $\frac{3}{8}$ **(b)** $\frac{49}{80}$

(c) $\frac{8}{13}$ **(d)** $\frac{1}{2}$

118. Consider the following in respect of two events A and B.

(1) P(A occurs but not B) = P(A) – P(B), if $B \subset A$

(2) P(A alone or B alone occurs) $= P(A) + P(B) - P(A \cap B)$

(3) $P(A \cup B) = P(A) + P(B)$, if A and B are mutually exclusive

Which of the above is/are correct?

(a) 1 only **(b)** 1 and 3 only

(c) 2 and 3 only **(d)** 1 and 2 only

119. A committee of three has to be chosen from a group of 4 men and 5 women. If the selection is made at random, what is the probability that exactly two members are men?

(a) $\frac{5}{14}$ **(b)** $\frac{1}{21}$

(c) $\frac{3}{14}$ **(d)** $\frac{8}{21}$

120. The standard deviation σ of the first N natural numbers can be obtained using which one of the following formulae?

(a) $\sigma = \frac{N^2 - 1}{12}$ **(b)** $\sigma = \sqrt{\frac{N^2 - 1}{12}}$

(c) $\sigma = \sqrt{\frac{N - 1}{12}}$ **(d)** $\sigma = \sqrt{\frac{N^2 - 1}{6N}}$

Answers

Q. No.	Answer Key	Topic Name	Chapter Name
1	(b)	Mathematical Induction	Mathematical Induction
2	(b)	Logarithm	Logarithm
3	(b)	General Term	Binomial Theorem
4	(c)	Argument	Complex Number
5	(c)	Properties of Complex Number	Complex Number
6	(c)	Properties of Sets	Sets
7	(d)	Combination	Permutation and Combination
8	(a)	Modulus	Complex Number
9	(c)	Properties of A.P. and G.P.	Sequence and Series
10	(b)	General Term	Binomial Theorem
11	(c)	Properties of Log	Logarithm
12	(a)	Combination	Permutation and Combination
13	(c)	Properties of Sets	Sets
14	(d)	Properties of Sets	Sets
15	(c)	Properties of Complex Number	Complex Number
16	(a)	Properties of Quadratic Equation	Quadratic Equation
17	(d)	Binary Number	Binary Number
18	(a)	Properties of Complex Number	Complex Number
19	(a)	Properties of Log	Logarithm
20	(b)	Argument	Complex Number
21	(c)	Number of Terms	Binomial Theorem
22	(d)	Divisibility	Binomial Theorem
23	(a)	Sum of Infinite G.P.	Sequence and Series
24	(a)	Properties of Matrix	Matrix
25	(a)	Order of Matrix	Matrix
26	(a)	Properties of Matrix	Matrix
27	(a)	Number of Triangle	Combination
28	(d)	Properties of Combination	Combination
29	(b)	Properties of Quadratic Equation	Quadratic Equation
30	(b)	Properties of Sets	Sets
31	(c)	Sum of A.P.	Sequence and Series
32	(b)	Properties of Log	Logarithm
33	(c)	n^{th} Term of G.P.	Sequence and Series
34	(a)	Properties of A.P. and G.P.	Sequence and Series
35	(c)	Properties of Complex Number	Complex Number
36	(c)	Properties of Trigonometry	Trigonometry
37	(c)	Properties of Trigonometry	Trigonometry

Q. No.	Answer Key	Topic Name	Chapter Name
38	**(c)**	Properties of Trigonometry	Trigonometry
39	**(b)**	Properties of Triangle	Trigonometry
40	**(c)**	Inverse Trigonometry Function	Trigonometry
41	**(b)**	Properties of Triangle	Trigonometry
42	**(a)**	Formulas	Trigonometry
43	**(a)**	Height and Distance	Trigonometry
44	**(b)**	Inverse Trigonometry	Trigonometry
45	**(a)**	Height and Distance	Trigonometry
46	**(a)**	Properties of Trigonometry	Trigonometry
47	**(b)**	Properties of Trigonometry	Trigonometry
48	**(c)**	Half Angle Formulas	Trigonometry
49	**(b)**	Properties of Trigonometry	Trigonometry
50	**(a)**	Maximum and Minimum Value	Application of Derivative
51	**(a)**	Section Formula	2D
52	**(b)**	Angle between Two Lines	2D
53	**(d)**	Straight Line	2D
54	**(c)**	Equation of Line	2D
55	**(c)**	Properties of Straight Line	2D
56	**(a)**	Equation of Ellipse	2D
57	**(b)**	Equation of Line	2D
58	**(c)**	Foot of Perpendicular	3D
59	**(a)**	Equation of Plane	3D
60	**(c)**	Locus	3D
61	**(d)**	Properties of Triangle	3D
62	**(c)**	Perpendicular Distance	2D
63	**(d)**	Equation of Line	2D
64	**(b)**	Distance between Two Lines	2D
65	**(a)**	Equation of Sphere	3D
66	**(a)**	Properties of Vector	Vector
67	**(a)**	Properties of Vector	Vector
68	**(a)**	Momentum	Vector
69	**(d)**	Properties of Vector	Vector
70	**(b)**	Properties of Vector	Vector
71	**(d)**	Properties of Function	Function
72	**(a)**	Range	Function
73	**(a)**	Continuous Function	Continuity
74	**(a)**	Limits	Limits
75	**(c)**	Area Under Curves	Application of Integration
76	**(c)**	Maximum and Minimum Value	Application of Derivative

Q. No.	Answer Key	Topic Name	Chapter Name
77	**(d)**	Maximum and Minimum Value	Application of Derivative
78	**(c)**	Maximum and Minimum Value	Application of Derivative
79	**(a)**	Definite Integration	Integration
80	**(b)**	Value of Function	Function
81	**(b)**	Definite Integration	Integration
82	**(a)**	Definite Integration	Integration
83	**(c)**	Maximum and Minimum Value	Trigonometry
84	**(d)**	Maximum and Minimum Value	Trigonometry
85	**(a)**	Domain	Function
86	**(b)**	Differential Equation	Equation
87	**(c)**	Differential Coefficient	Equation
88	**(a)**	Differential Equation	Differential
89	**(d)**	Period of Function	Function
90	**(b)**	Indefinite Integration	Integration
91	**(a)**	Order and Degree	Differential Equation
92	**(c)**	Definite Integration	Integration
93	**(a)**	Definite Integration	Integration
94	**(a)**	Definite Integration	Integration
95	**(a)**	Limits	Limits
96	**(d)**	Limits	Limits
97	**(b)**	Differential Coefficient	Differential
98	**(c)**	Differential Coefficient	Differential
99	**(a)**	Differential Equation	Equation Differential
100	**(c)**	Order and Degree	Differential Equation
101	**(a)**	Binomial Distribution	Probability
102	**(b)**	Conditional Probability	Probability
103	**(d)**	Co-Variance	Statistics
104	**(b)**	Probability	Probability
105	**(d)**	Mean	Statistics
106	**(d)**	Correlation Coefficient	Statistics
107	**(c)**	Regression Line	Statistics
108	**(d)**	Properties of Mean and Variance	Statistics
109	**(c)**	Properties of Mean and Variance	Statistics
110	**(b)**	Median	Statistics
111	**(c)**	Pie Chart	Statistics
112	**(b)**	Coefficient of Variance	Statistics
113	**(d)**	Average Speed	Speed and Time
114	**(b)**	Probability	Probability
115	**(b)**	Conditional Probability	Probability

Q. No.	Answer Key	Topic Name	Chapter Name
116	**(c)**	Probability	Probability
117	**(b)**	Total Probability	Probability
118	**(b)**	Probability	Probability
119	**(a)**	Probability	Probability
120	**(b)**	Standard Deviation	Statistics