NDA/NA

National Defence Academy / Naval Academy

MATHEMATICS

II

QUESTION PAPER 2019

Time : 2:30 Hour **Total Marks : 300**

Important Instructions :

1. *This test Booklet contains* **120** *items (questions). Each item is printed in* **English**. *Each item comprises four responses (answer's). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ONLY ONE response for each item.*
2. *You have to mark all your responses ONLY on the separate Answer Sheet provided.*
3. **All** *items carry equal marks.*
4. *Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions.*
5. *Penalty for wrong answers :*
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   - **(i)** *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, one-third of the marks assigned to that question will be deducted as penalty.*
   - **(ii)** *If a candidate gives more than one answer, it will be treated as a wrong answer even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   - **(iii)** *If a question is left blank, i.e., no answer is given by the candidate, there will be no penalty for that question.*

1. If both p and q belong to the set (1, 2, 3, 4), then how many equations of the form $px^2 + qx + 1 = 0$ will have real roots?
(a) 12 **(b)** 10
(c) 7 **(d)** 6

2. What is the value of
$1 - 2 + 3 - 4 + 5 - ..... + 101$?
(a) 51 **(b)** 55
(c) 110 **(d)** 111

3. If A, B, and C are subsets of a given set, then which one of the following relations is **not** correct?
(a) $A \cup (A \cap B) = A \cup B$
(b) $A \cap (A \cup B) = A$
(c) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
(d) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

4. If the sum of first n terms of a series is $(n + 12)$, then what is its third term?
(a) 1 **(b)** 2
(c) 3 **(d)** 4

5. What is the value of k for which the sum of the squares of the roots of $2x^2 - 2(k-2)x - (k+1) = 0$ is minimum?
(a) -1 **(b)** 1
(c) $\frac{3}{2}$ **(d)** 2

6. If the roots of the equation
$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal, then which one of the following is correct?
(a) a, b and c are in AP
(b) a, b and c are in GP
(c) a, b and c are in HP
(d) a, b and c do not follow any regular pattern

7. $|x^2 - 3x + 2| > x^2 - 3x + 2$, then which one of the following is correct?
(a) $x \leq 1$ or $x \geq 2$
(b) $1 \leq x \leq 2$
(c) $1 < x < 2$
(d) x is any real value except 3 and 4

8. A geometric progression (GP) consists of 200 terms. If the sum of odd terms of the GP is m, and the sum of even terms of the GP is n, then what is its common ratio?
(a) $\frac{m}{n}$ **(b)** $\frac{n}{m}$
(c) $m + \left(\frac{n}{m}\right)$ **(d)** $n + \left(\frac{m}{n}\right)$

9. If a set A contains 3 elements and another set B contains 6 elements, then what is the minimum number of elements that (A ∪ B) can have?

(a) 3 **(b)** 6

(c) 8 **(d)** 9

10. What is the number of diagonals of an octagon?

(a) 48 **(b)** 40

(c) 28 **(d)** 20

11. What is the value of the determinant?

$$\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix} ?$$

(a) 0 **(b)** 12

(c) 24 **(d)** 36

12. What are the values of x that satisfy the equation

$$\begin{vmatrix} x & 0 & 2 \\ 2x & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 3x & 0 & 2 \\ x^2 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 0 ?$$

(a) $-2 \pm \sqrt{3}$ **(b)** $-1 \pm \sqrt{3}$

(c) $-1 \pm \sqrt{6}$ **(d)** $-2 \pm \sqrt{6}$

13. If $x + a + b + c = 0$, then what is the value of

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} ?$$

(a) 0 **(b)** $(a + b + c)^2$

(c) $a^2 + b^2 + c^2$ **(d)** $a + b + c - 2$

14. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then the expression $A^3 - 2A^2$ is

(a) a null matrix **(b)** an identity matrix

(c) equal to A **(d)** equal to –A

15. Let m and n $(m < n)$ be the roots of the equation $x^2 - 16x + 39 = 0$. If four terms p, q, r and s are interested between m and n to form an AP, then what is the value of $p + q + r + s$?

(a) 29 **(b)** 30

(c) 32 **(d)** 35

16. Under which one of the following conditions will be quadratic equation $x^2 + mx + 2 = 0$ always have real roots?

(a) $2\sqrt{3} \le m^2 < 8$ **(b)** $\sqrt{3} \le m^2 < 4$

(c) $m^2 \ge 8$ **(d)** $m^2 \le \sqrt{3}$

17. What is the value of

$$\left[\frac{i+\sqrt{3}}{2}\right]^{2019} + \left[\frac{i-\sqrt{3}}{2}\right]^{2019} ?$$

(a) 1 **(b)** – 1

(c) $2i$ **(d)** $-2i$

18. If α and β are the roots of $x^2 + x + 1 = 0$, then what is $\sum_{j=0}^{3}\left(\alpha^j + \beta^j\right)$ equal to ?

(a) 8 **(b)** 6

(c) 4 **(d)** 2

19. In a school, 50% students play cricket and 40% play football. If 10% of students play both the games, then what percent of students play neither cricket nor football?

(a) 10% **(b)** 15%

(c) 20% **(d)** 25%

20. If A = {x : $0 \le x \le 2$} and B = {y; y is a prime number}, then what is A ∩ B equal to?

(a) π **(b)** {1}

(c) {2} **(d)** {1, 2}

21. If $x = 1 + i$, then what is the value of $x^6 + x^4 + x^2 + 1$?

(a) $6i - 3$ **(b)** $-6i + 3$

(c) $-6i - 3$ **(d)** $6i + 3$

22. What is the value of

$$2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ldots\infty}}}$$

(a) $\sqrt{2} - 1$ **(b)** $\sqrt{2} + 1$

(c) 3 **(d)** 4

23. If P(n, r) = 2520 and C(n, r) = 21, then what is the value of C($n + 1$, $r + 1$)?

(a) 7 **(b)** 14

(c) 28 **(d)** 56

24. How many terms are there in the expansion of $(1 + 2x + x^2)^5 + (1 + 4y + 4y^2)^5$?

(a) 12 **(b)** 20

(c) 21 **(d)** 22

25. If the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{2n}$ is $184756x^{10}$, then what is the value of n?

(a) 10 **(b)** 8
(c) 5 **(d)** 4

26. If $A=\begin{pmatrix}1 & 2\\ 2 & 3\\ 3 & 4\end{pmatrix}$ and $B=\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$, then which one of the following is correct?
(a) Both AB and BA exist
(b) Neither AB nor BA exists
(c) AB exists but BA does not exist
(d) AB does not exist but BA exists

27. If $n!$ has 17 zeroes, then what is the value of n?
(a) 95
(b) 85
(c) 80
(d) No such values of n exists

28. Let $A \cup B = \{x|(x-a)(x-b) > 0$, where $a < b\}$, what are A and B equal to?
(a) $A = \{x|x > a\}$ and $B = \{x|x > b\}$
(b) $A = \{x|x < a\}$ and $B = \{x|x > b\}$
(c) $A = \{x|x < a\}$ and $B = \{x|x < b\}$
(d) $A = \{x|x > a\}$ and $B = \{x|x < b\}$

29. If the constant term in the expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then what can be the values of k?
(a) ± 2 **(b)** ± 3
(c) ± 5 **(d)** ± 9

30. What is C(47, 4) + C(51, 3) + C(50, 3) + C(49, 3) + C(48, 3) + C(47, 3) equal to?
(a) C(47, 4) **(b)** C(52, 5)
(c) C(52, 4) **(d)** C(47, 5)

31. Let a, b, c be in AP and $k \neq 0$ be a real number, Which of the following are correct?
(1) ka, kb, kc are in AP
(2) $k-a, k-b, k-c$ are in AP
(3) $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are in AP
Select the correct answer using the code given below:
(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

32. How many two-digit numbers are divisible by 4?
(a) 21 **(b)** 22
(c) 24 **(d)** 25

33. Let S_n be the sum of the first n terms of an AP. If $S_{2n} = 3n + 14n^2$, then what is the common difference?
(a) 5 **(b)** 6
(c) 7 **(d)** 9

34. If 3rd, 8th and 13th terms of a GP are p, q and r respectively, then which one of the following is correct?
(a) $q^2 = pr$ **(b)** $r^2 = pq$
(c) $pqr = 1$ **(d)** $2q = p + r$

35. What is the solution of $x \leq 4$, $y \geq 0$ and $x \leq -4$, $y \leq 0$?
(a) $x \geq -4, y \leq 0$ **(b)** $x \leq 4, y \geq 0$
(c) $x \leq -4, y = 0$ **(d)** $x \geq -4, y = 0$

36. If $x^{\log_7 x} > 7$ where $x > 0$, then which one of the following is correct?
(a) $x \in (0, \infty)$ **(b)** $x \in \left(\frac{1}{7}, 7\right)$
(c) $x \in \left(0, \frac{1}{7}\right) \cup (7, \infty)$ **(d)** $x \in \left(\frac{1}{7}, \infty\right)$

37. How many real roots does the equation $x^2 + 3|x| + 2 = 0$ have?
(a) Zero **(b)** One
(c) Two **(d)** Four

38. Consider the following statements in respect of the quadratic equation $4(x-p)(x-q) - r^2 = 0$, where p, q and r are real numbers:
(1) The roots are real
(2) The roots are equal if $p = q$ and $r = 0$
Which of the above statements is/are correct?
(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

39. Let $S = \{2, 4, 6, 8, ...20\}$, What is the maximum number of subsets does S have?
(a) 10 **(b)** 20
(c) 512 **(d)** 1024

40. A binary number is represented by $(cdccddcccddd)_2$, where $c > d$, what is its decimal equivalent?
(a) 1848 **(b)** 2048
(c) 2842 **(d)** 2872

41. If $\text{cosec}\,\theta = \frac{29}{21}$ where $0 < \theta < 90°$, then what is the value of $4\sec\theta + 4\tan\theta$?
(a) 5 **(b)** 10
(c) 15 **(d)** 20

42. Consider the following statements:

(1) $\cos\theta + \sec\theta$ can never be equal to 1.5.

(2) $\tan\theta + \cot\theta$ can never be less than 2.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

43. A ladder 9 m long reaches a point 9 m below the top of a vertical flagstaff. From the foot of the ladder, the elevation of the flagstaff is 60°. What is the height of the flagstaff?

(a) 9 m **(b)** 10.5 m

(c) 13.5 m **(d)** 15 m

44. What is the length of the chord of a unit circle which subtends an angle θ at the centre?

(a) $\sin\left(\frac{\theta}{2}\right)$ **(b)** $\cos\left(\frac{\theta}{2}\right)$

(c) $2\sin\left(\frac{\theta}{2}\right)$ **(d)** $2\cos\left(\frac{\theta}{2}\right)$

45. What is $\tan\left\{2\tan^{-1}\left(\frac{1}{3}\right)\right\}$ equal to?

(a) $\frac{2}{3}$ **(b)** $\frac{3}{4}$

(c) $\frac{3}{8}$ **(d)** $\frac{1}{9}$

46. What is the scalar projection of $\vec{a} = \vec{i} - 2\vec{j} + \vec{k}$ on $\vec{b} = 4\vec{i} - 4\vec{j} + 7\vec{k}$

(a) $\frac{\sqrt{6}}{9}$ **(b)** $\frac{19}{9}$

(c) $\frac{9}{19}$ **(d)** $\frac{\sqrt{6}}{19}$

47. If the magnitude of the sum of two non-zero vectors is equal to the magnitude of their difference, then which one of the following is correct?

(a) The vectors are parallel

(b) The vectors are perpendicular

(c) The vectors are anti-parallel

(d) The vectors must be unit vectors

48. Consider the following equations for two vectors $\vec{a}$ and $\vec{b}$:

(1) $\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right) = \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2$

(2) $\left(\left|\vec{a}+\vec{b}\right|\right)\left(\left|\vec{a}-\vec{b}\right|\right) = \left|\vec{a}\right|^2 - \left|\vec{b}\right|^2$

(3) $\left|\vec{a}.\vec{b}\right|^2 + \left|\vec{a}\times\vec{b}\right|^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2$

Which of the above statements are correct?

(a) 1, 2 and 3 **(b)** 1 and 2 only

(c) 1 and 3 only **(d)** 2 and 3 only

49. Consider the following statements:

(1) The magnitude of $\vec{a}\times\vec{b}$ is same as the area of a triangle with sides $\vec{a}$ and $\vec{b}$

(2) If $\vec{a}\times\vec{b} = \vec{0}$ where $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$, then $\vec{a} = \lambda\vec{b}$

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

50. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then what is $\sin^2\left(\frac{\theta}{2}\right)$ equal to?

(a) $\frac{\left|\vec{a}+\vec{b}\right|^2}{4}$ **(b)** $\frac{\left|\vec{a}-\vec{b}\right|^2}{4}$

(c) $\frac{\left|\vec{a}+\vec{b}\right|^2}{2}$ **(d)** $\frac{\left|\vec{a}-\vec{b}\right|^2}{2}$

51. The equation $ax + by + c = 0$ represents a straight line

(a) for all real numbers a, b and c

(b) only when $a \neq 0$

(c) only when $b \neq 0$

(d) only when at least one of a and b is non-zero

52. What is the angle between the lines

$x\cos\alpha + y\sin\alpha = a$ and

$x\sin\beta - y\cos\beta = a$?

(a) $\beta - \alpha$ **(b)** $\pi + \beta - \alpha$

(c) $\frac{(\pi + 2\beta - 2\alpha)}{2}$ **(d)** $\frac{(\pi - 2\beta + 2\alpha)}{2}$

53. What is the distance between the points P($m\cos 2\alpha$, $m\sin 2\alpha$) and Q($m\cos 2\beta$, $m\sin 2\beta$)?

(a) $|2m\sin(\alpha - \beta)|$ **(b)** $|2m\cos(\alpha - \beta)|$

(c) $|m\sin(2\alpha - 2\beta)|$ **(d)** $|m\sin(2\alpha - 2\beta)|$

54. An equilateral triangle has one vertex at $(-1, -1)$ and another vertex at $\left(-\sqrt{3}, \sqrt{3}\right)$. The third vertex may lie on

(a) $\left(-\sqrt{2}, \sqrt{2}\right)$ **(b)** $\left(\sqrt{2}, -\sqrt{2}\right)$

(c) $(1, 1)$ **(d)** $(1, -1)$

55. If the angle between the lines joining the end points of minor axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with one of its foci is $\frac{\pi}{2}$, then what is the eccentricity of the ellipse?

(a) $\frac{1}{2}$ **(b)** $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$ **(d)** $\frac{1}{2\sqrt{2}}$

56. A point on a line has coordinates $(p+1, p-3, \sqrt{2}p)$ where p is any real number. What are the direction cosines of the line?

(a) $\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$

(b) $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}, \frac{1}{2}, -\frac{1}{2}$

(d) Cannot be determined due to insufficient data

57. A point on the line $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z+2}{7}$ has coordinates

(a) (3, 5, 4) **(b)** (2, 5, 5)

(c) (–1, –1, 5) **(d)** (2, –1, 0)

58. If the line $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies on the plane $2x-4y+z=7$, then what is the value of k?

(a) 2 **(b)** 3

(c) 5 **(d)** 7

59. A straight line passes through the point (1, 1, 1) makes an angle 60° with the positive direction of z-axis, and the cosine of the angles made by it with positive directions of the y-axis and the x-axis are in the ratio $\sqrt{3}:1$. What is the acute angle between the two possible positions of the line?

(a) 90° **(b)** 60°

(c) 45° **(d)** 30°

60. If the points $(x, y, -3)$, (2, 0, – 1) and (4, 2, 3) lie on a straight line, then what are the values of x and y respectively?

(a) 1, – 1 **(b)** – 1, 1

(c) 0, 2 **(d)** 3, 4

61. What is the minimum value of?

$\frac{a^2}{\cos^2 x}+\frac{b^2}{\sin^2 x}$ where $a>0$ and $b>0$?

(a) $(a+b)^2$ **(b)** $(a-b)^2$

(c) a^2+b^2 **(d)** $|a^2+b^2|$

62. If the angles of a triangle ABC are in AP and $b : c = \sqrt{3}:\sqrt{2}$, then what is the measure of angle A?

(a) 30° **(b)** 45°

(c) 60° **(d)** 75°

63. If tan A – tan B = x and cot B – cot A = y, then what is the value of cot (A – B)?

(a) $\frac{1}{x}+\frac{1}{y}$ **(b)** $\frac{1}{y}-\frac{1}{x}$

(c) $\frac{xy}{x+y}$ **(d)** $1+\frac{1}{xy}$

64. What is sin (α + β) – 2sin α cos β + sin (α – β) equal to?

(a) 0 **(b)** 2sin α

(c) 2sin β **(d)** sin α + sin β

65. If 2tan A = 3tan B = 1, then what is tan (A – B) equal to?

(a) $\frac{1}{5}$ **(b)** $\frac{1}{6}$

(c) $\frac{1}{7}$ **(d)** $\frac{1}{9}$

66. What is cos 80° + cos 40° – cos 20° equal to?

(a) 2 **(b)** 1

(c) 0 **(d)** – 19

67. If angle C of a triangle ABC is a right angle, then what is tan A + tan B equal to?

(a) $\frac{a^2-b^2}{ab}$ **(b)** $\frac{a^2}{bc}$

(c) $\frac{b^2}{ca}$ **(d)** $\frac{c^2}{ab}$

68. What is $\cot\left(\frac{A}{2}\right)-\tan\left(\frac{A}{2}\right)$ equal to ?

(a) tan A **(b)** cot A

(c) 2tan A **(d)** 2cot A

69. What is cot A + cosec A equal to?

(a) $\tan\left(\frac{A}{2}\right)$ **(b)** $\cot\left(\frac{A}{2}\right)$

(c) $2\tan\left(\frac{A}{2}\right)$ **(d)** $2\cot\left(\frac{A}{2}\right)$

70. What is

tan 25° tan 15° + tan 15° tan 50° + tan 25° tan 50° equal to?

(a) 0 **(b)** 1

(c) 2 **(d)** 4

71. What is the area of the region bounded by $|x| < 5$, $y = 0$ and $y = 8$?

(a) 40 square units **(b)** 80 square units

(c) 120 square units **(d)** 160 square units

72. Consider the following statements in respect of the function $f(x) = \sin\left(\frac{1}{x}\right)$ for $x \neq 0$ and $f(0) = 0$:

(1) $\lim_{x\to 0} f(x)$ exists

(2) $f(x)$ is continuous at $x = 0$

Which of the above statement is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

73. What is the value of $\lim_{x\to 0} \frac{\sin x^\circ}{\tan 3x^\circ}$?

(a) $\frac{1}{4}$ **(b)** $\frac{1}{3}$

(c) $\frac{1}{2}$ **(d)** 1

74. What is the degree of the differential equation $\frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^2 - x^2\left(\frac{d^4y}{dx^4}\right) = 0$?

(a) 1 **(b)** 2

(c) 3 **(d)** 4

75. Which one of the following is the second degree polynomial function $f(x)$

where $f(0) = 5$, $f(-1) = 10$ and $f(1) = 6$?

(a) $5x^2 - 2x + 5$ **(b)** $3x^2 - 2x - 5$

(c) $3x^2 - 2x + 5$ **(d)** $3x^2 - 10x + 5$

Directions for the following three (03) items:

Read the following information and answer the three items that follow:

A curve $y = me^{mx}$ where $m > 0$ intersects y-axis at a point P.

76. What is the slope of the curve at the point of intersection P?

(a) m **(b)** m^2

(c) $2m$ **(d)** $2m^2$

77. How much angle does the tangent at P make with y-axis?

(a) $\tan^{-1} m^2$ **(b)** $\cot^{-1}(1 + m^2)$

(c) $\sin^{-1}\left(\frac{1}{\sqrt{1+m^4}}\right)$ **(d)** $\sec^{-1}\sqrt{1+m^4}$

78. What is the equation of tangent to the curve at P?

(a) $y = mx + m$ **(b)** $y = -mx + 2m$

(c) $y = m^2x + 2m$ **(d)** $y = m^2x + m$

Directions for the following two (02) items:

Read the following information and answer the two items that follow:

Let $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \text{In } x$.

79. For $x = \frac{\sqrt{\pi}}{2}$, what is the value of $[ho(gof)]\,(x)$?

(a) 0 **(b)** 1

(c) $\frac{\pi}{4}$ **(d)** $\frac{\pi}{2}$

80. What is $[fo(fof)]\,(2)$ equal to?

(a) 2 **(b)** 8

(c) 16 **(d)** 256

81. What is $\int \frac{dx}{2x^2 - 2x + 1}$ equal to?

(a) $\frac{\tan^{-1}(2x-1)}{2} + c$

(b) $2\tan^{-1}(2x - 1) + c$

(c) $\frac{\tan^{-1}(2x+1)}{2} + c$

(d) $\tan^{-1}(2x - 1) + c$

82. What is $\int \frac{dx}{x(1+\text{In } x)^n}$ equal to $(n \neq 1)$?

(a) $\frac{1}{(n-1)(1+\ln x)^{n-1}} + c$

(b) $\frac{1-n}{(1+\ln x)^{1-n}} + c$

(c) $\frac{n+1}{(1+\ln x)^{n+1}} + c$

(d) $-\frac{1}{(n-1)(1+\ln x)^{n-1}} + c$

83. Which one of the following is the differential equation that represents the family of curves $y = \frac{1}{2x^2 - c}$ where c is an arbitrary constant?

(a) $\frac{dy}{dx} = 4xy^2$ **(b)** $\frac{dy}{dx} = \frac{1}{y}$

(c) $\frac{dy}{dx} = x^2y$ **(d)** $\frac{dy}{dx} = -4xy^2$

Directions for the following two (02) items:

Read the following information and answer the two items that follow:

Consider the equation $x^y = e^{x-y}$

84. What is $\frac{dy}{dx}$ at $x = 1$ equal to?

(a) 0 **(b)** 1

(c) 2 **(d)** 4

85. What is $\frac{d^2y}{dx^2}$ at $x = 1$ equal to?

(a) 0 **(b)** 1

(c) 2 **(d)** 4

Directions for the following three (03) items :

Read the following information and answer the three items that follow:

Consider the function $f(x) = g(x) + h(x)$

where $g(x) = \sin\left(\frac{x}{4}\right)$ and $h(x) = \cos\left(\frac{4x}{5}\right)$

86. What is the period of the function $g(x)$?

(a) π **(b)** 2π

(c) 4π **(d)** 8π

87. What is the period of the function $h(x)$?

(a) π **(b)** $\frac{5\pi}{2}$

(c) $\frac{5\pi}{2}$ **(d)** $\frac{3\pi}{2}$

88. What is the period of the function $f(x)$?

(a) 10π **(b)** 20π

(c) 40π **(d)** 80π

Directions for the following two (02) items:

Read the following information and answer the two items that follow:

Consider the function

$f(x) = 3x^4 - 20x^3 - 12x^2 + 288x + 1$

89. In which one of the following intervals is the function increasing?

(a) $(-2, 3)$ **(b)** $(3, 4)$

(c) $(-3, -2)$ **(d)** $(-4, -3)$

90. In which one of the following intervals is the function decreasing?

(a) $(-2, 3)$ **(b)** $(3, 4)$

(c) $(4, 6)$ **(d)** $(6, 9)$

Directions for the following three (03) items:

Read the following information and answer the three items that follow:

Let $f(x) = x^2 + 2x - 5$ and $g(x) = 5x + 30$

91. What are the roots of the equation $g[f(x)] = 0$?

(a) $1, -1$ **(b)** $-1, -1$

(c) $1, 1$ **(d)** $0, 1$

92. Consider the following statements:

(1) $f[g(x)]$ is a polynomial of degree 3.

(2) $g[g(x)]$ is a polynomial of degree 2.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

93. If $h(x) = 5f(x) - xg(x)$, then what is the derivative of $h(x)$?

(a) -40 **(b)** -20

(c) -10 **(d)** 0

Directions for the following two (02) items :

Read the following information and answer the two items that follow:

Consider the integrals

$$I_1 = \int_0^{\pi} \frac{xdx}{1+\sin x} \text{ and } I_2 = \int_0^{\pi} \frac{(\pi - x)dx}{1-\sin(\pi + x)}$$

94. What is the value of I_1?

(a) 0 **(b)** $\frac{\pi}{2}$

(c) π **(d)** 2π

95. What is the value of $I_1 + I_2$?

(a) 2π **(b)** π

(c) $\frac{\pi}{2}$ **(d)** 0

96. The differential equation which represents the family of curves given by $\tan y = c(1 - e^x)$ is

(a) $e^x \tan y dx + (1 - e^x)dy = 0$

(b) $e^x \tan y dx + (1 - e^x)\sec^2 y dy = 0$

(c) $e^x(1 - e^x)dx + \tan y dy = 0$

(d) $e^x \tan y dy + (1 - e^x)dx = 0$

97. What is the derivative of $2^{(\sin x)^2}$ with respect to $\sin x$?

(a) $\sin x\, 2^{(\sin x)^2} \ln 4$ **(b)** $2\sin x\, 2^{(\sin x)^2} \ln 4$

(c) $\ln(\sin x)\, 2^{(\sin x)^2}$ **(d)** $2\sin x \cos x\, 2^{(\sin x)^2}$

98. For what value of k is the function

$$f(x) = \begin{cases} 2x + \frac{1}{4}, & x < 0 \\ k, & x = 0 \\ \left(x + \frac{1}{2}\right)^2, & x > 0 \end{cases}$$ continuous ?

(a) $\frac{1}{4}$ **(b)** $\frac{1}{2}$

(c) 1 **(d)** 2

99. What is the area of the region enclosed between the curve $y^2 = 2x$ and the straight line $y = x$?

(a) $\frac{2}{3}$ square units **(b)** $\frac{4}{3}$ square units

(c) $\frac{1}{3}$ square units **(d)** 1 square unit

100. If $f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 6x + 7$ increases in the interval T and decreases in the interval S, then which one of the following is correct?

(a) T = $(-\infty, 2) \cup (3, \infty)$ and S = (2, 3)

(b) T = ϕ and S = $(-\infty, \infty)$

(c) T = $(-\infty, \infty)$ and S = ϕ

(d) T = (2, 3) and S = $(-\infty, 2) \cup (3, \infty)$

101. A coin is biased so that heads comes up thrice as likely as tails. For three independent tosses of a coin, what is the probability of getting at most two tails?

(a) 0.16 **(b)** 0.48

(c) 0.58 **(d)** 0.98

102. A bag contains 20 books out of which 5 are defective. If 3 of the books are selected at random and removed from the bag in succession without replacement, then what is the probability that all three books are defective?

(a) 0.009 **(b)** 0.016

(c) 0.026 **(d)** 0.047

103. The median of the observations 22, 24, 33, 37, $x+1, x+3$, 46, 47, 57, 58 in ascending order is 42. What are the values of 5^{th} and 6^{th} observations respectively?

(a) 42, 45 **(b)** 41, 43

(c) 43, 46 **(d)** 40, 40

104. Arithmetic mean of 10 observations is 60 and sum of squares of deviations from 50 is 5000. What is the standard deviation of the observations?

(a) 20 **(b)** 21

(c) 22.36 **(d)** 24.70

105. If p and q are the roots of the equation $x^2 - 30x + 221 = 0$,

what is the value of $p^3 + q^3$?

(a) 7010 **(b)** 7110

(c) 7210 **(d)** 7240

106. For the variable x and y, the two regression lines are $6x + y = 30$ and $3x + 2y = 25$. What are the value of $\bar{x}, \bar{y}$ and r respectively?

(a) $\frac{20}{3}, \frac{35}{9}, -0.5$ **(b)** $\frac{20}{3}, \frac{35}{9}, 0.5$

(c) $\frac{35}{9}, \frac{20}{3}, -0.5$ **(d)** $\frac{35}{9}, \frac{20}{3}, 0.5$

107. The class marks in a frequency table are given to be 5, 10, 15, 20, 25, 30, 35, 40, 45, 50. The class limits of the first five classes are

(a) 3–7, 7–13, 13–17, 17–23, 23–27

(b) 2.5–7.5, 7.5–12.5, 12.5–17.5, 17.5–22.5, 22.5–27.5

(c) 1.5–8.5, 8.5–11.5, 11.5–18.5, 18.5–21.5, 21.5–28.5

(d) 2–8, 8–12, 12–18, 18–22, 22–28

108. The mean of 5 observations is 4.4 and variance is 8.24. If three of the five observations are 1, 2 and 6, then what are the other two observations?

(a) 9, 16 **(b)** 9, 4

(c) 81, 16 **(d)** 81, 4

109. If a coin is tossed till the first head appears, then what will be the sample space?

(a) {H}

(b) {TH}

(c) {T, HT, HHT, HHHT, ...}

(d) {H, TH, TTH, TTTH, ...}

110. Consider the following discrete frequency distribution:

x	1	2	3	4	5	6	7	8
f	3	15	45	57	50	36	25	9

What is the value of median of the distribution?

(a) 4 **(b)** 5

(c) 6 **(d)** 7

111. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on them is a prime number?

(a) $\frac{5}{12}$ **(b)** $\frac{1}{2}$

(c) $\frac{7}{12}$ **(d)** $\frac{2}{3}$

112. If 5 of a Company's 10 delivery trucks do not meet emission standards and 3 of them are chosen for inspection, then what is the probability that none of the trucks chosen will meet emission standards?

(a) $\frac{1}{8}$ **(b)** $\frac{3}{8}$

(c) $\frac{1}{12}$ **(d)** $\frac{1}{4}$

113. There are 3 coins in a box. One is a two-headed coin; another is a fair coin; and third is biased coin that comes up heads 75% of time. When one of the three coins is selected at random and flipped, it shows heads. What is the probability that is was the two-headed coin?

(a) $\frac{2}{9}$ **(b)** $\frac{1}{3}$

(c) $\frac{4}{9}$ **(d)** $\frac{5}{9}$

114. Consider the following statements:

(1) If A and B are mutually exclusive events, then it is possible that P(A) = P(B) = 0.6.

(2) If A and B are any two events such that $P(A|B) = 1$, then $P\left(\overline{B}|\overline{A}\right) = 1$.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

115. If a fair die is rolled 4 times, then what is the probability that there are exactly 2 sixes?

(a) $\frac{5}{216}$ **(b)** $\frac{25}{216}$

(c) $\frac{125}{216}$ **(d)** $\frac{175}{216}$

116. Mean of 100 observations is 50 and standard deviation is 10. If 5 is added to each observation, then what will be the new mean and new standard deviation respectively?

(a) 50, 10 **(b)** 50, 15

(c) 55, 10 **(d)** 55, 15

117. If the range of a set of observations on a variable X is known to be 25 and if Y = 40 + 3X, then what is the range of the set of corresponding observations on Y?

(a) 25 **(b)** 40

(c) 75 **(d)** 115

118. If V is the variance and M is the mean of first 15 natural numbers, then what is $V + M^2$ equal to?

(a) $\frac{124}{3}$ **(b)** $\frac{148}{3}$

(c) $\frac{248}{3}$ **(d)** $\frac{124}{9}$

119. A car travels first 60 km at a speed of $3v$ km/hr and travels next 60 km at $2v$ km/hr. What is the average speed of the car?

(a) $2.5v$ km/hr **(b)** $2.4v$ km/hr

(c) $2.2v$ km/hr **(d)** $2.1v$ km/hr

120. The mean weight of 150 students in a certain class is 60 kg. The mean weight of boys is 70 kg and that of girls is 55 kg. What are the number of boys and girls respectively in the class?

(a) 75 and 75 **(b)** 50 and 100

(c) 70 and 80 **(d)** 100 and 50

Answers

Q. No.	Answer Key	Topic Name	Chapter Name
1	**(c)**	Quadratic Equation	Algebra
2	**(a)**	Sequence and Progression	Algebra
3	**(a)**	Set and Relation	Algebra
4	**(a)**	Sequence and Progression	Algebra
5	**(c)**	Quadratic Equation	Algebra
6	**(c)**	Sequence and Progression	Algebra
7	**(c)**	Function	Algebra
8	**(b)**	Sequence and Progression	Algebra
9	**(b)**	Set and Relation	Calculus
10	**(d)**	Coordinate Geometry	Geometry
11	**(c)**	Determinants	Algebra
12	**(d)**	Determinants	Algebra
13	**(a)**	Determinants	Algebra
14	**(a)**	Matrices	Algebra
15	**(c)**	Sequence and Progression	Algebra
16	**(c)**	Quadratic Equation	Algebra
17	**(c)**	Complex Number	Algebra
18	**(d)**	Complex Number	Algebra
19	**(c)**	Probability	Algebra
20	**(c)**	Set and Relation	Algebra
21	**(c)**	Complex Number	Algebra
22	**(b)**	Quadratic Equation	Algebra
23	**(c)**	Permutation and Combination	Algebra
24	**(c)**	Binomial Expansion	Algebra
25	**(a)**	Binomial Expansion	Algebra
26	**(c)**	Matrices	Algebra
27	**(d)**	Permutation and Combination	Algebra
28	**(b)**	Sets and Relation	Algebra
29	**(b)**	Binomial Expansion	Algebra
30	**(c)**	Permutation and Combination	Algebra
31	**(d)**	Sequence and Progression	Algebra
32	**(b)**	Sequence and Progression	Algebra
33	**(c)**	Sequence and Progression	Algebra
34	**(a)**	Sequence and Progression	Algebra
35	**(c)**	Function	Calculus
36	**(c)**	Function	Calculus
37	**(a)**	Function	Calculus
38	**(c)**	Quadratic Equation	Algebra
39	**(d)**	Set and Relation	Calculus

Q. No.	Answer Key	Topic Name	Chapter Name
40	**(d)**	Set and Relation	Calculus
41	**(b)**	Trigonometric Ratios and Identities	Trigonometry
42	**(c)**	Trigonometric Ratios and Identities	Trigonometry
43	**(c)**	Height and Distance	Trigonometry
44	**(c)**	Trigonometric Ratios and Identities	Trigonometry
45	**(b)**	Trigonometric Ratios and Identities	Trigonometry
46	**(b)**	Vector Algebra	Algebra
47	**(b)**	Vector Algebra	Algebra
48	**(a)**	Vector Algebra	Algebra
49	**(b)**	Vector Algebra	Algebra
50	**(b)**	Vector Algebra	Algebra
51	**(d)**	Point and Straight line	Coordinate Geometry
52	**(c)**	Trigonometric Ratio & Identities	Trigonometry
53	**(a)**	Point and Straight line	Coordinate Geometry
54	**(c)**	Point and Straight line	Coordinate Geometry
55	**(b)**	Ellipse	Coordinate Geometry
56	**(a)**	Point and Straight line	Coordinate Geometry
57	**(b)**	3D	Coordinate Geometry
58	**(d)**	3D	Coordinate Geometry
59	**(b)**	3D	Coordinate Geometry
60	**(a)**	3D	Coordinate Geometry
61	**(a)**	Trigonometric Ratios and Identities	Trigonometry
62	**(d)**	Trigonometric Ratios and Identities	Trigonometry
63	**(a)**	Trigonometric Ratios and Identities	Trigonometry
64	**(a)**	Trigonometric Ratios and Identities	Trigonometry
65	**(c)**	Trigonometric Ratios and Identities	Trigonometry
66	**(c)**	Trigonometric Ratios and Identities	Trigonometry
67	**(d)**	Trigonometric Ratios and Identities	Trigonometry
68	**(d)**	Trigonometric Ratios and Identities	Trigonometry
69	**(b)**	Trigonometric Ratios and Identities	Trigonometry
70	**(b)**	Trigonometric Ratios and Identities	Trigonometry
71	**(b)**	Application of Integral	Calculus
72	**(d)**	Limit	Calculus
73	**(b)**	Limit	Calculus
74	**(a)**	Differential Equation	Calculus
75	**(c)**	Quadratic Equation	Algebra
76	**(b)**	Application of Derivative	Calculus
77	**(c)**	Application of Derivative	Calculus
78	**(d)**	Application of Derivative	Calculus
79	**(a)**	Function	Calculus
80	**(d)**	Function	Calculus

Q. No.	Answer Key	Topic Name	Chapter Name
81	**(d)**	Integration	Calculus
82	**(d)**	Integration	Calculus
83	**(d)**	Differential Equation	Calculus
84	**(a)**	Continuity and Differentiability	Calculus
85	**(b)**	Continuity and Differentiability	Calculus
86	**(d)**	Function	Calculus
87	**(c)**	Function	Calculus
88	**(d)**	Function	Calculus
89	**(a)**	Application of Derivative	Calculus
90	**(b)**	Application of Derivative	Calculus
91	**(b)**	Function	Calculus
92	**(d)**	Function	Calculus
93	**(b)**	Differentiation	Calculus
94	**(c)**	Integration	Calculus
95	**(a)**	Integration	Calculus
96	**(b)**	Differential Equation	Calculus
97	**(a)**	Continuity and Differentiability	Calculus
98	**(a)**	Continuity and Differentiability	Calculus
99	**(a)**	Application of Derivative	Calculus
100	**(a)**	Application of Derivative	Calculus
101	**(d)**	Probability	Algebra
102	**(a)**	Probability	Algebra
103	**(b)**	Statistics	Algebra
104	**(a)**	Statistics	Algebra
105	**(b)**	Nature Equation	Algebra
106	**(d)**	Statistics	Algebra
107	**(b)**	Statistics	Algebra
108	**(b)**	Statistics	Algebra
109	**(d)**	Probability	Algebra
110	**(a)**	Statistics	Algebra
111	**(a)**	Probability	Algebra
112	**(c)**	Probability	Algebra
113	**(c)**	Probability	Algebra
114	**(b)**	Probability	Algebra
115	**(b)**	Probability	Algebra
116	**(c)**	Statistics	Algebra
117	**(c)**	Statistics	Algebra
118	**(c)**	Statistics	Algebra
119	**(b)**	Applied Mathematics	Algebra
120	**(b)**	Applied Mathematics	Algebra