NDA/NA

National Defence Academy / Naval Academy

MATHEMATICS I

QUESTION PAPER 2021

Time : 2:30 Hour **Total Marks : 300**

Important Instructions :

1. *This 'test Booklet contains* **120** *items (questions). Each item is printed in* **English**. *Each item comprises four responses (answer's). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ONLY ONE response for each item.*
2. *You have to mark all your responses ONLY on the separate Answer Sheet provided.*
3. **All** *items carry equal marks.*
4. *Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions.*
5. *Penalty for wrong answers:*
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i) *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, one-third of the marks assigned to that question will be deducted as penalty.*
   (ii) *If a candidate gives more than one answer, it will be treated as a wrong answer even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii) *If a question is left blank, i.e., no answer is given by the candidate, there will be no penalty for that question.*

1. The smallest positive integer n for which

$$\left(\frac{1-i}{1+i}\right)^{n^2} = 1$$

where $i = \sqrt{-1}$, is

(a) 2 **(b)** 4
(c) 6 **(d)** 8

2. The value of x, satisfying the equation

$\log_{\cos x} \sin x = 1$, where $0 < x < \frac{\pi}{2}$,

(a) $\frac{\pi}{12}$ **(b)** $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ **(d)** $\frac{\pi}{6}$

3. If Δ is the value of the determinant

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Then what is the value of the following determinant?

$$\begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_3 \end{vmatrix}$$

($p \neq 0$ or 1, $q \neq 0$ or 1)

(a) $p\Delta$ **(b)** $q\Delta$
(c) $(p+q)\Delta$ **(d)** $pq\Delta$

4. If $C_0, C_1, C_2, \ldots C_n$ are the coefficients in the expansion of $(1 + x)^n$, then what is the value of $C_1 + C_2 + C_3 + \ldots + C_n$?

(a) 2^n **(b)** $2^n - 1$
(c) 2^{n-1} **(d)** $2^n - 2$

5. If $a + b + c = 4$ and $ab + bc + ca = 0$, then what is the value of the following determinant?

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

(a) 32 **(b)** – 64
(c) – 128 **(d)** 64

6. The number of integer value of k, for which the equation $2 \sin x = 2k + 1$ has a solution, is

(a) zero **(b)** one
(c) two **(d)** four

7. If $a_1, a_2, a_3, \ldots a_9$ are in GP, then what is the value of the following determinant?

$$\begin{vmatrix} \ln a_1 & \ln a_2 & \ln a_3 \\ \ln a_4 & \ln a_5 & \ln a_6 \\ \ln a_7 & \ln a_8 & \ln a_9 \end{vmatrix}$$

(a) 0 (b) 1
(c) 2 (d) 4

8. If the roots of the quadratic equation $x^2 + 2x + k = 0$ are real, then
(a) $k < 0$ (b) $k \leq 0$
(c) $k < 1$ (d) $k \leq 1$

9. If n = 100!, then what is the value of the following?

$$\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\frac{1}{\log_4 n}+...+\frac{1}{\log_{100} n}$$

(a) 0 (b) 1
(c) 2 (d) 3

10. If Z = 1 + i, where $i=\sqrt{-1}$, then what is the modulus of $Z+\frac{2}{Z}$?
(a) 1 (b) 2
(c) 3 (d) 4

11. If A and B are two matrices such that AB is of order $n \times n$, then which one of the following is correct?
(a) A and B should be square matrices of same order.
(b) Either A or B should be a square matrix.
(c) Both A and B should be of same order.
(d) Orders of A and B need not be the same.

12. How many matrices of different orders are possible with elements comprising all prime numbers less than 30?
(a) 2 (b) 3
(c) 4 (d) 6

13. Let $A=\begin{vmatrix} p & q \\ r & s \end{vmatrix}$

where p, q, r and s are any four different prime numbers less than 20. What is the maximum value of the determinant?
(a) 215 (b) 311
(c) 317 (d) 323

14. If A and B are square matrices of order 2 such that det (AB) = det (BA), then which one of the following is correct?
(a) A must be a unit matrix.
(b) B must be a unit matrix.
(c) Both A and B must be unit matrices.
(d) A and B need not be unit matrices.

15. What is $\cot 2x \cot 4x - \cot 4x \cot 6x - \cot 6x \cot 2x$ equal to?
(a) – 1 (b) 0
(c) 1 (d) 2

16. If $\tan x = -\frac{3}{4}$ and x is in the second quadrant, then what is the value of $\sin x \cos x$?
(a) $\frac{6}{25}$ (b) $\frac{12}{25}$
(c) $-\frac{6}{25}$ (d) $-\frac{12}{25}$

17. What is the value of the following?

$$\text{cosec}\left(\frac{7\pi}{6}\right)\sec\left(\frac{5\pi}{3}\right)$$

(a) $\frac{4}{3}$ (b) 4
(c) – 4 (d) $-\frac{4}{\sqrt{3}}$

18. If the determinant

$$\begin{vmatrix} x & 1 & 3 \\ 0 & 0 & 1 \\ 1 & x & 4 \end{vmatrix}=0$$

then what is x equal to?
(a) – 2 or 2 (b) – 3 or 3
(c) – 1 or 1 (d) 3 or 4

19. What is the value of the following
tan 31° tan 33° tan 35°.... tan 57° tan 59°
(a) – 1 (b) 0
(c) 1 (d) 2

20. If

$$f(x)=\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & 2(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$$

then what is $f(-1) + f(0) + f(1)$ equal to?
(a) 0 (b) 1
(c) 100 (d) – 100

21. The equation $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$ has
(a) no solution
(b) unique solution
(c) two solutions
(d) infinite number of solutions

22. What is the value of the following
(sin 24° + cos 66°)(sin 24° – cos 66°)
(a) – 1 **(b)** 0
(c) 1 **(d)** 2

23. A chord subtends an angle 120° at the centre of a unit circle. What is the length of the chord ?
(a) $\sqrt{2}-1$ units **(b)** $\sqrt{3}-1$ units
(c) $\sqrt{2}$ units **(d)** $\sqrt{3}$ units

24. What is (1 + cot θ – cosec θ) (1 + tan θ + sec θ) equal to ?
(a) 1 **(b)** 2
(c) 3 **(d)** 4

25. What is $\frac{1+\tan^2\theta}{1+\cot^2\theta}-\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2$ equal to ?
(a) 0 **(b)** 1
(c) 2 tan θ **(d)** 2 cot θ

26. What is the interior angle of a regular octagon of side length 2 cm ?
(a) $\frac{\pi}{2}$ **(b)** $\frac{3\pi}{4}$
(c) $\frac{3\pi}{5}$ **(d)** $\frac{3\pi}{8}$

27. If 7 sin θ + 24 cos θ = 25, then what is the value of (sin θ + cos θ) ?
(a) 1 **(b)** $\frac{26}{25}$
(c) $\frac{6}{5}$ **(d)** $\frac{31}{25}$

28. A ladder 6 m long reaches a point 6 m below the top of a vertical flagstaff. From the foot of the ladder, the elevation of the top of the flagstaff is 75°. What is the height of the flagstaff ?
(a) 12 m **(b)** 9 m
(c) $(6+\sqrt{3})$m **(d)** $(6+3\sqrt{3})$m

29. The shadow of a tower is found to be x metre longer, when the angle of elevation of the sun changes from 60° to 45°. If the height of the tower is $5(3+\sqrt{3})$m , then what is x equal to ?
(a) 8 m **(b)** 10 m
(c) 12 m **(d)** 15 m

30. If 3 cos θ = 4 sin θ, then what is the value of tan (45° + θ) ?
(a) 10 **(b)** 7
(c) $\frac{7}{2}$ **(d)** $\frac{7}{4}$

31. $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$ holds, when
(a) $x \in$ R **(b)** $x \in$ R – (– 1, 1) only
(c) $x \in$ R – {0} only **(d)** $x \in$ R – [– 1, 1] only

32. If tan A = $\frac{1}{7}$, then what is cos 2A equal to ?
(a) $\frac{24}{25}$ **(b)** $\frac{18}{25}$
(c) $\frac{12}{25}$ **(d)** $\frac{6}{25}$

33. The sides of a triangle are m, n and $\sqrt{m^2+n^2+mn}$. What is the sum of the acute angles of the triangle ?
(a) 45° **(b)** 60°
(c) 75° **(d)** 90°

34. What is the area of the triangle ABC with sides a = 10 cm, c = 4 cm and angle B = 30° ?
(a) 16 cm^2 **(b)** 12 cm^2
(c) 10 cm^2 **(d)** 8 cm^2

35. Consider the following statements :
(1) A = {1, 3, 5} and B = {2, 4, 7} are equivalent sets.
(2) A = {1, 5, 9} and B = {1, 5, 5, 9, 9} are equal sets.

Which of the above statements is/are correct ?
(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

36. Consider the following statements :
(1) The null set is subset of every set.
(2) Every set is a subset of itself.
(3) If a set has 10 elements, then its power set will have 1024 elements.

Which of the above statements are correct ?
(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

37. Let R be relation defined as xRy if and only if $2x+3y=20$, where $x, y \in$ N. How many elements of the form (x, y) are there in R ?
(a) 2 **(b)** 3
(c) 4 **(d)** 6

38. Consider the following statements :
(1) A function $f : \mathbb{Z} \to \mathbb{Z}$, defined by $f(x) = x + 1$, is one-one as well as onto.
(2) A function $f : \mathbb{N} \to \mathbb{N}$, defined by $f(x) = x + 1$, is one-one but not onto.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

39. Consider the following in respect of a complex number Z :

(1) $\overline{(Z^{-1})} = (\overline{Z})^{-1}$
(2) $ZZ^{-1} = |Z|^2$

Which of the above is/are correct?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

40. Consider the following statements in respect of an arbitrary complex number Z:

(1) The difference of Z and its conjugate is an imaginary number.
(2) The sum of Z and its conjugate is a real number.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

41. What is the modulus of the complex number $i^{2n+1}(-i)^{2n-1}$, where $n \in N$ and $i = \sqrt{-1}$?

(a) -1 **(b)** 1
(c) $\sqrt{2}$ **(d)** 2

42. If α and β are the roots of the equation $4x^2 + 2x - 1 = 0$, then which one of the following is correct?

(a) $\beta = -2\alpha^2 - 2\alpha$ **(b)** $\beta = 4\alpha^2 - 3\alpha$
(c) $\beta = \alpha^2 - 3\alpha$ **(d)** $\beta = -2\alpha^2 + 2\alpha$

43. If one root of $5x^2 + 26x + k = 0$ is reciprocal of the other, then what is the value of k?

(a) 2 **(b)** 3
(c) 5 **(d)** 8

44. In how many ways can a team of 5 players be selected from 8 players so as not to include a particular player?

(a) 42 **(b)** 35
(c) 21 **(d)** 20

45. What is the coefficient of the middle term in the expansion of $(1 + 4x + 4x^2)^5$?

(a) 8064 **(b)** 4032
(c) 2016 **(d)** 1008

46. What is $C(n, 1) + C(n, 2) + ... + C(n, n)$ equal to?

(a) $2 + 2^2 + 2^3 + ... + 2^n$
(b) $1 + 2 + 2^2 + 2^3 + ... +2^n$
(c) $1 + 2 + 2^2 + 2^3 + ... 2^{n-1}$
(d) $2 + 2^2 + 2^3 + ... + 2^{n-1}$

47. What is the sum of the coefficients of first and last terms in the expansion of $(1 + x)^{2n}$, where n is a natural number?

(a) 1 **(b)** 2
(c) n **(d)** $2n$

48. If the first term of an AP is 2 and the sum of the first five terms is equal to one-fourth of the sum of the next five terms then what is the sum of the first ten terms?

(a) – 500 **(b)** – 250
(c) 500 **(d)** 250

49. Consider the following statements:

(1) If each term of a GP is multiplied by same non-zero number, then the resulting sequence is also a GP.
(2) If each term of a GP is divided by same non-zero number, then the resulting sequence is also a GP.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

50. How many 5-digit prime numbers can be formed using the digits 1, 2, 3, 4, 5 if the repetition of digits is not allowed?

(a) 5 **(b)** 4
(c) 3 **(d)** 0

51. If $f(x + 1) = x^2 - 3x + 2$, then what is $f(x)$ equal to?

(a) $x^2 - 5x + 4$ **(b)** $x^2 - 5x + 6$
(c) $x^2 + 3x + 3$ **(d)** $x^2 - 3x + 1$

52. If x^2, x, -8 are in AP, then which one of the following is correct?

(a) $x \in \{-2\}$ **(b)** $x \in \{4\}$
(c) $x \in \{-2, 4\}$ **(d)** $x \in \{-4, 2\}$

53. The third term of a GP is 3. What is the product of its first five terms?

(a) 81
(b) 243
(c) 729
(d) Cannot be determined due to insufficient data

54. The element in the i^{th} row and j^{th} column of a determinant of third order is equal to $2(i + j)$. What is the value of the determinant?

(a) 0 **(b)** 2
(c) 4 **(d)** 6

55. With the numbers 2, 4, 6, 8 all the possible determinants with these four different elements are constructed. What is the sum of the values of all such determinants ?

(a) 128 **(b)** 64
(c) 32 **(d)** 0

56. What is the radius of the circle $4x^2 + 4y^2 - 20x + 12y - 15 = 0$?

(a) 14 units **(b)** 10.5 units
(c) 7 units **(d)** 3.5 units

57. A parallelogram has three consecutive vertices (– 3, 4), (0, – 4) and (5, 2). The fourth vertex is

(a) (2, 10) **(b)** (2, 9)
(c) (3, 9) **(d)** (4, 10)

58. If the lines $y + px = 1$ and $y - qx = 2$ are perpendicular, then which one of the following is correct ?

(a) $pq + 1 = 0$ **(b)** $p + q + 1 = 0$
(c) $pq - 1 = 0$ **(d)** $p - q + 1 = 0$

59. If A, B and C are in A·P, then the straight line $Ax + 2By + C = 0$ will always pass through a fixed point. The fixed point is

(a) (0, 0) **(b)** (– 1, 1)
(c) (1, – 2) **(d)** (1, – 1)

60. If the image of the point (– 4, 2) by a line mirror is (4, – 2), then what is the equation of the line mirror ?

(a) $y = x$ **(b)** $y = 2x$
(c) $4y = x$ **(d)** $y = 4x$

61. Consider the following statements in respect of the points $(p, p - 3)$, $(q + 3, q)$ and (6, 3) :

(1) The points lie on a straight line.
(2) The points always lie in the first quadrant only for any value of p and q.

Which of the above statements is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

62. What is the acute angle between the lines $x - 2 = 0$ and $\sqrt{3}x - y - 2 = 0$?

(a) 0° **(b)** 30°
(c) 45° **(d)** 60°

63. The point of intersection of diagonals of a square ABCD is at the origin and one of its vertices is at A(4, 2). What is the equation of the diagonal BD ?

(a) $2x + y = 0$ **(b)** $2x - y = 0$
(c) $x + 2y = 0$ **(d)** $x - 2y = 0$

64. If any point on a hyperbola is (3 tan θ, 2 sec θ), then what is the eccentricity of the hyperbola ?

(a) $\frac{3}{2}$ **(b)** $\frac{5}{2}$
(c) $\frac{\sqrt{11}}{2}$ **(d)** $\frac{\sqrt{13}}{2}$

65. Consider the following with regard to eccentricity (e) of a conic section :

(1) $e = 0$ for circle
(2) $e = 1$ for parabola
(3) $e < 1$ for ellipse

Which of the above are correct ?

(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

66. What is the angle between the two lines having direction ratios $\langle 6,3,6 \rangle$ and $\langle 3,3,0 \rangle$?

(a) $\frac{\pi}{6}$ **(b)** $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ **(d)** $\frac{\pi}{2}$

67. if l, m, n are the direction cosines of the line $x - 1 = 2(y + 3) = 1 - z$, then what is $l^4 + m^4 + n^4$ equal to ?

(a) 1 **(b)** $\frac{11}{27}$
(c) $\frac{13}{27}$ **(d)** 4

68. What is the projection of the line segment joining (1, 7, –5) and B (–3, 4, –2) on y-axis ?

(a) 5 **(b)** 4
(c) 3 **(d)** 2

69. What is the number of possible values of k for which the line joining the points (k, 1, 3) and (1, – 2, k + 1) also passes through the point (15, 2, – 4) ?

(a) Zero **(b)** One
(c) Two **(d)** Infinite

70. The foot of the perpendicular drawn from the origin to the plane $x + y + z = 3$ is

(a) (0, 1, 2) **(b)** (0, 0, 3)
(c) (1, 1, 1) **(d)** (– 1, 1, 3)

71. A vector $\vec{r} = a\hat{i} + b\hat{j}$ is equally inclined to both x and y axes. If the magnitude of the vector is 2 units, then what are the values of a and b respectively ?

(a) $\frac{1}{2}, \frac{1}{2}$ **(b)** $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

(c) $\sqrt{2}, \sqrt{2}$ **(d)** 2, 2

72. Consider the following statements in respect of a vector $\vec{c} = \vec{a} + \vec{b}$, where $\vec{a} = |\vec{b}| \neq 0$:

(1) $\vec{c}$ is perpendicular to $(\vec{a} - \vec{b})$.

(2) $\vec{c}$ is perpendicular to $(\vec{a} \times \vec{b})$.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

73. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}| = 4$, then which one of the following is correct?

(a) $\vec{a}$ and $\vec{b}$ must be unit vectors.

(b) $\vec{a}$ must be parallel to $\vec{b}$.

(c) $\vec{a}$ must be perpendicular to $\vec{b}$.

(d) $\vec{a}$ must be equal to $\vec{b}$.

74. If $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar, then what is $(2\vec{a} \times 3\vec{b}).4\vec{c} + (5\vec{b} \times 3\vec{c}).6\vec{a}$ equal to?

(a) 114 **(b)** 66

(c) 0 **(d)** –66

75. Consider the following statements :

(1) The cross product of two unit vectors is always a unit vector.

(2) The dot product of two unit vectors is always unity.

(3) The magnitude of sum of two unit vectors is always greater than the magnitude of their difference.

Which of the above statements are **not** correct?

(a) 1 and 2 only **(b)** 2 and 3 only

(c) 1 and 3 only **(d)** 1, 2 and 3

76. If $\lim_{x \to a} \frac{a^x - x^a}{x^a - a^a} = -1$ then what is the value of a?

(a) – 1 **(b)** 0

(c) 1 **(d)** 2

77. A particle starts from origin with a velocity (in m/s) given by the equation $\frac{dx}{dt} = x + 1$. The time (in second) taken by the particle to traverse a distance of 24 m is

(a) ln 24 **(b)** ln 5

(c) 2ln 5 **(d)** 2ln 4

78. What is $\int_0^a \frac{f(a-x)}{f(x) + f(a-x)} dx$ equal to?

(a) a **(b)** $2a$

(c) 0 **(d)** $\frac{a}{2}$

79. What is $\lim_{x \to -1} \frac{x^3 + x^2}{x^2 + 3x + 2}$ equal to?

(a) 0 **(b)** 1

(c) 2 **(d)** 3

80. If $\int_0^a [f(x) + f(-x)] dx = \int_{-a}^a g(x) dx$ then what is $g(x)$ equal to?

(a) $f(x)$ **(b)** $f(-x) + f(x)$

(c) $-f(x)$ **(d)** None of the above

81. What is the area bounded by $y = \sqrt{16 - x^2}$, $y \geq 0$ and the x-axis?

(a) 16π square units **(b)** 8π square units

(c) 4π square units **(d)** 2π square units

82. The curve $y = -x^3 + 3x^2 + 2x - 27$ has the maximum slope at

(a) $x = -1$ **(b)** $x = 0$

(c) $x = 1$ **(d)** $x = 2$

83. A 24 cm long wire is bent to form a triangle with one of the angles as 60°. What is the altitude of the triangle having the greatest possible area?

(a) $4\sqrt{3}$ cm **(b)** $2\sqrt{3}$ cm

(c) 6 cm **(d)** 3 cm

84. If $f(x) = e^{|x|}$, which one of the following is correct?

(a) $f'(0) = 1$ **(b)** $f'(0) = -1$

(c) $f'(0) = 0$ **(d)** $f'(0)$ does not exist

85. What is $\int \frac{dx}{\sec x + \tan x}$ equal to?

(a) $\ln(\sec x) + \ln|\sec x + \tan x| + c$

(b) $\ln(\sec x) - \ln|\sec x + \tan x| + c$

(c) $\text{sex } x \tan x - \ln|\sec x - \tan x| + c$

(d) $\ln|\sec x + \tan x| - \ln|\sec x| + c$

86. What is $\int \frac{dx}{\sec^2(\tan^{-1} x)}$ equal to?

(a) $\sin^{-1} x + c$ **(b)** $\tan^{-1} x + c$

(c) $\sec^{-1} x + c$ **(d)** $\cos^{-1} x + c$

87. If $x + y = 20$ and $P = xy$, then what is the maximum value of P?

(a) 100 **(b)** 96

(c) 84 **(d)** 50

88. What is the derivative of

$$\sin(\ln x) + \cos(\ln x)$$

with respect to x at $x = e$?

(a) $\frac{\cos 1 - \sin 1}{e}$ **(b)** $\frac{\sin 1 - \cos 1}{e}$

(c) $\frac{\cos 1 + \sin 1}{e}$ **(d)** 0

89. If $x = e^t \cos t$ and $y = e^t$ sint, then what is $\frac{dx}{dy}$ at $t = 0$ equal to?

(a) 0 **(b)** 1

(c) $2e$ **(d)** – 1

90. What is the maximum value of $\sin 2x \cos 2x$?

(a) $\frac{1}{2}$ **(b)** 1

(c) 2 **(d)** 4

91. What is the derivative of e^x with respect to x^e

(a) $\frac{xe^x}{ex^e}$ **(b)** $\frac{e^x}{x^e}$

(c) $\frac{xe^x}{x^e}$ **(d)** $\frac{e^x}{ex^e}$

92. If a differentiable function $f(x)$ satisfies

$$\lim_{x \to -1} \frac{f(x)+1}{x^2 - 1} = -\frac{3}{2}$$

then what is $\lim_{x \to -1} f(x)$ equal to?

(a) $-\frac{3}{2}$ **(b)** – 1

(c) 0 **(d)** 1

93. If the function

$$f(x) = \begin{cases} a + bx, & x < 1 \\ 5, & x = 1 \\ b - ax, & x > 1 \end{cases}$$

is continuous, then what is the value of $(a + b)$?

(a) 5 **(b)** 10

(c) 15 **(d)** 20

94. Consider the following statements in respect of the function $f(x) = \sin x$:

(1) $f(x)$ increases in the interval $(0, \pi)$.

(2) $f(x)$ decreases in the interval $\left(\frac{5\pi}{2}, 3\pi\right)$.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

95. What is the domain of the function $f(x) = 3^x$?

(a) $(-\infty, \infty)$ **(b)** $(0, \infty)$

(c) $[0, \infty)$ **(d)** $(-\infty, \infty) - \{0\}$

96. If the general solution of a differential equation is $y^2 + 2cy - cx + c^2 = 0$, where c is an arbitrary constant, then what is the order of the differential equation?

(a) 1 **(b)** 2

(c) 3 **(d)** 4

97. What is the degree of the following differential equation?

$$x = \sqrt{1 + \frac{d^2y}{dx^2}}$$

(a) 1

(b) 2

(c) 3

(d) Degree is not defined

98. Which one of the following differential equations has the general solution $y = ae^x + be^{-x}$?

(a) $\frac{d^2y}{dx^2} + y = 0$ **(b)** $\frac{d^2y}{dx^2} - y = 0$

(c) $\frac{d^2y}{dx^2} + y = 1$ **(d)** $\frac{dy}{dx} - y = 0$

99. What is the solution of the following differential equation?

$$\ln\left(\frac{dy}{dx}\right) + y = x$$

(a) $e^x + e^y = c$ **(b)** $e^{x+y} = c$

(c) $e^x - e^y = c$ **(d)** $e^{x-y} = c$

100. What is $\int e^{\left(2 \ln x + \ln x^2\right)} dx$ equal to?

(a) $\frac{x^4}{4} + c$ **(b)** $\frac{x^3}{3} + c$

(c) $\frac{2x^5}{5} + c$ **(d)** $\frac{x^5}{5} + c$

101. Consider the following measures of central tendency for a set of N numbers :

(1) Arithmetic mean

(2) Geometric mean

Which of the above uses/use all the data?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

102. The numbers of Science, Arts and Commerce graduate working in a company are 30, 70 and 50 respectively. If these figures are represented by a pie chart, then what is the angle corresponding to Science graduates?

(a) 36° **(b)** 72°

(c) 120° **(d)** 168°

103. For a histogram based on a frequency distribution with unequal class intervals, the frequency of a class should be proportional to
(a) the height of the rectangle
(b) the area of the rectangle
(c) the width of the rectangle
(d) the perimeter of the rectangle

104. The coefficient of correlation is independent of
(a) change of scale only
(b) change of origin only
(c) both change of scale and change of origin
(d) neither change of scale nor change of origin

105. The following table gives the frequency distribution of number of peas per pea pod of 198 pods :

Number of peass	Frequency
1	4
2	33
3	76
4	50
5	26
6	8
7	1

What is the median of this distribution?
(a) 3 **(b)** 4
(c) 5 **(d)** 6

106. If M is the mean of n observations
$x_1 - k, x_2 - k, x_3 - k, x_n - ... k$, where k is any real number, then what is the mean of
$x_1, x_2, x_3, ..., x_n$?
(a) M **(b)** M + k
(c) M – k **(d)** kM

107. What is the sum of deviations of the variate values 73, 85, 92, 105, 120 from their means?
(a) – 2 **(b)** – 1
(c) 0 **(d)** 5

108. Let x be the HM and y be the GM of two positive numbers m and n, if $5x = 4y$, then which one of the following is correct?
(a) $5m = 4n$ **(b)** $2m = n$
(c) $4m = 5n$ **(d)** $m = 4n$

109. If the mean of a frequency distribution is 100 and the coefficient of variation is 45%, then what is the value of the variance?
(a) 2025 **(b)** 450
(c) 45 **(d)** 4.5

110. Let two events A and B be such that P(A) = L and P(B) = M. Which one of the following is correct?
(a) $P(A|B) < \frac{L+M-1}{M}$
(b) $P(A|B) > \frac{L+M-1}{M}$
(c) $P(A|B) \geq \frac{L+M-1}{M}$
(d) $P(A|B) = \frac{L+M-1}{M}$

111. For which of the following sets of numbers do the mean, median and mode have the same value?
(a) 12, 12, 12, 12, 24 **(b)** 6, 18, 18, 18, 30
(c) 6, 6, 12, 30, 36 **(d)** 6, 6, 6, 12, 30

112. The mean of 12 observation is 75. If two observations are discarded, then the mean of the remaining observations is 65. What is the mean of the discarded observations?
(a) 250
(b) 125
(c) 120
(d) Cannot be determined due to insufficient data

113. If k is one of the roots of the equation
$x(x + 1) + 1 = 0$, then what is its other root?
(a) 1 **(b)** $-k$
(c) k^2 **(d)** $-k^2$

114. The geometric mean of a set of observation is computed as 10. The geometric mean obtained when each observation x_i is replaced by $3x_i^4$ is
(a) 810 **(b)** 900
(c) 30000 **(d)** 81000

115. If $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\overline{A}) = \frac{1}{2}$, then which of the following is/are correct?
(1) A and B are independent events.
(2) A and B are mutually exclusive events.
Select the correct answer using the code given below.
(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

116. The average of a set of 15 observations is recorded, but later it is found that for one observation, the digit in the tens place was wrongly recorded as 8 instead of 3. After correcting the observation, the average is

(a) reduced by $\frac{1}{3}$ (b) increased by $\frac{10}{3}$

(c) reduced by $\frac{10}{3}$ (d) reduced by 50

117. A coin is tossed twice. If E and F denote occurrence of head on first toss and second toss respectively, then what is $P(E \cup F)$ equal to?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) $\frac{1}{3}$

118. In a binomial distribution, the mean is $\frac{2}{3}$ and variance is $\frac{5}{9}$. What is the probability that random variable $X = 2$?

(a) $\frac{5}{36}$ (b) $\frac{25}{36}$

(c) $\frac{25}{54}$ (d) $\frac{25}{216}$

119. If the mode of the scores 10, 12, 13, 15, 15, 13, 12, 10, x is 15, then what is the value of x?

(a) 10 (b) 12

(c) 13 (d) 15

120. If A and B are two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$, then consider the following statements :

(1) The minimum value of $P(A \cup B)$ is $\frac{3}{4}$.

(2) The maximum value of $P(A \cap B)$ is $\frac{5}{8}$.

Which of the above statements is/are correct/

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

Answers

Q. No.	Answer Key	Topic Name	Chapter Name
1	(a)	Algebra of Complex Numbers	Complex Numbers
2	(c)	Properties of Logarithms	Logarithm and its Applications
3	(d)	Properties of Determinants	Matrices and Determinants
4	(b)	Properties of Binomial Coefficients	Binomial Theorem
5	(b)	Properties of Determinants	Matrices and Determinants
6	(c)	Trigonometric Equations	Trigonometric Equations and Inequations
7	(a)	Properties of Determinants	Matrices and Determinants
8	(d)	Nature of Roots	Theory of Equation
9	(b)	Properties of Logarithms	Logarithm and its Applications
10	(b)	Algebra of Complex Numbers	Complex Numbers
11	(d)	Algebra of Matrices	Matrices and Determinants
12	(c)	Basics of Matrices	Matrices and Determinants
13	(c)	Determinant of a Square Matrix	Matrices and Determinants
14	(d)	Properties of Determinants	Matrices and Determinants
15	(c)	Trigonometric Identities	Trigonometric Ratios and Identities
16	(d)	Trigonometric Functions and Properties	Trigonometric Ratios and Identities
17	(c)	Trigonometric Functions and Properties	Trigonometric Ratios and Identities
18	(c)	Determinant of a Square Matrix	Matrices and Determinants
19	(c)	Trigonometric Identities	Trigonometric Ratios and Identities
20	(a)	Properties of Determinants	Matrices and Determinants
21	(b)	Inverse Trigonometric Equations and Inequalities	Inverse Trigonometric Functions
22	(b)	Trigonometric Functions and Properties	Trigonometric Ratios and Identities
23	(d)	Relations between Sides and Angles of a Triangle	Properties of Triangle
24	(b)	Trigonometric Identities	Trigonometric Ratios and Identities
25	(a)	Trigonometric Identities	Trigonometric Ratios and Identities
26	(b)	Measure of an Interior Angle of a Regular Polygon	Polygon
27	(d)	Trigonometric Identities	Trigonometric Ratios and Identities
28	(d)	Heights and Distance	Heights and Distance
29	(b)	Heights and Distance	Heights and Distance
30	(b)	Trigonometric Identities	Trigonometric Ratios and Identities
31	(a)	Identities of Inverse Trigonometric Functions	Inverse Trigonometric Functions

Q. No.	Answer Key	Topic Name	Chapter Name
32	(a)	Trigonometric Identities	Trigonometric Ratios and Identities
33	(b)	Relations between Sides and Angles of a Triangle	Properties of Triangle
34	(c)	Half-Angle Formula and the Area of a Triangle	Properties of Triangle
35	(c)	Types of Sets	Set Theory and Relations
36	(d)	Subsets and Power Set	Set Theory and Relations
37	(b)	Basics of Relations	Set Theory and Relations
38	(c)	Types of Functions	Functions
39	(a)	Properties of Conjugate, Modulus and Argument of Complex Numbers	Complex Numbers
40	(c)	Algebra of Complex Numbers	Complex Numbers
41	(b)	Basics of Complex Numbers	Complex Numbers
42	(a)	Quadratic Equation and Its Solutions	Theory of Equation
43	(c)	Quadratic Equation and Its Solutions	Theory of Equation
44	(c)	Combination	Permutation and Combination
45	(a)	General Term, Middle Term and Greatest Term in Binomial Expansion	Binomial Theorem
46	(c)	Properties of Binomial Coefficients	Binomial Theorem
47	(b)	Binomial Theorem for Positive Integral Index	Binomial Theorem
48	(b)	Arithmetic Progression	Sequence and Series
49	(c)	Geometric Progression	Sequence and Series
50	(d)	Fundamental Principles of Counting	Permutation and Combination
51	(b)	Basics of Functions	Functions
52	(c)	Arithmetic Progression	Sequence and Series
53	(b)	Geometric Progression	Sequence and Series
54	(a)	Properties of Determinants	Matrices and Determinants
55	(d)	Determinant of a Square Matrix	Matrices and Determinants
56	(d)	Basics of Circle	Circle
57	(a)	Basics of 2D Coordinate Geometry	Straight Line
58	(c)	Interaction between Two Straight Lines	Straight Line
59	(d)	Family of Straight Lines	Straight Line
60	(b)	Straight Line and a Point	Straight Line
61	(a)	Basics of 2D Coordinate Geometry	Straight Line
62	(b)	Interaction between Two Straight Lines	Straight Line
63	(a)	Interaction between Two Straight Lines	Straight Line
64	(d)	Basics of Hyperbola	Hyperbola
65	(d)	Basics of Ellipse	Ellipse
66	(b)	Interaction between Two Lines in 3D	Three Dimensional Geometry

Q. No.	Answer Key	Topic Name	Chapter Name
67	(b)	Basics of 3D Geometry	Three Dimensional Geometry
68	(c)	Dot Product of Two Vectors	Vector Algebra
69	(c)	Equation of a Line in 3D	Three Dimensional Geometry
70	(c)	Plane and a Point	Three Dimensional Geometry
71	(c)	Dot Product of Two Vectors	Vector Algebra
72	(c)	Scalar and Vector Triple Products	Vector Algebra
73	(c)	Dot Product of Two Vectors	Vector Algebra
74	(c)	Scalar and Vector Triple Products	Vector Algebra
75	(d)	Cross Product of Two Vectors	Vector Algebra
76	(c)	Methods of Evaluation of Limits	Limits
77	(c)	Variable Separable Form	Differential Equation
78	(d)	Properties of Definite Integral	Definite Integration
79	(b)	Methods of Evaluation of Limits	Limits
80	(a)	Properties of Definite Integral	Definite Integration
81	(b)	Area under the Curve	Area under Curves
82	(c)	Tangent and Normal	Application of Derivatives
83	(a)	Maxima and Minima	Application of Derivatives
84	(d)	Differentiability of a Function	Continuity and Differentiability
85	(d)	Integration using Trigonometric Identities	Indefinite Integration
86	(b)	Integration using Trigonometric Identities	Indefinite Integration
87	(a)	A.M., G.M., H.M. and their Relations	Sequence and Series
88	(a)	Basics of Differentiation	Differential Coefficient
89	(b)	Methods of Differentiation	Differential Coefficient
90	(a)	Trigonometric Identities	Trigonometric Ratios and Identities
91	(a)	Methods of Differentiation	Differential Coefficient
92	(b)	Basics of Limits	Limits
93	(a)	Continuity of a Function	Continuity and Differentiability
94	(b)	Monotonicity	Application of Derivatives
95	(a)	Basics of Functions	Functions
96	(a)	Basics of Differential Equations	Differential Equation
97	(a)	Basics of Differential Equations	Differential Equation
98	(b)	Basics of Differential Equations	Differential Equation
99	(c)	Variable Separable Form	Differential Equation
100	(d)	Methods of Integration	Indefinite Integration
101	(c)	Measures of Central Tendency	Statistics
102	(b)	Basics of Statistics	Statistics
103	(b)	Basics of Statistics	Statistics
104	(c)	Correlation and Covariance	Statistics
105	(a)	Measures of Central Tendency	Statistics
106	(b)	Measures of Central Tendency	Statistics

Q. No.	Answer Key	Topic Name	Chapter Name
107	**(c)**	Measures of Central Tendency	Statistics
108	**(d)**	Measures of Central Tendency	Statistics
109	**(a)**	Measures of Dispersion	Statistics
110	**(c)**	Conditional Probability	Probability
111	**(b)**	Measures of Central Tendency	Statistics
112	**(b)**	Measures of Central Tendency	Statistics
113	**(c)**	Roots of Unity	Complex Numbers
114	**(c)**	Measures of Central Tendency	Statistics
115	**(a)**	Algebra of Probability	Probability
116	**(c)**	Measures of Central Tendency	Statistics
117	**(c)**	Algebra of Probability	Probability
118	**(d)**	Bernoulli Trials and Binomial Distribution	Probability
119	**(d)**	Measures of Central Tendency	Statistics
120	**(c)**	Algebra of Probability	Probability

ANSWERS WITH EXPLANATION

1. **Option (a) is correct.**

Explanation:

Given:

$$\left(\frac{1-i}{1+i}\right)^{n^2} = 1$$

$$\Rightarrow \left(\frac{1-i}{1+i}\times\frac{1-i}{1-i}\right)^{n^2} = 1$$

$$\Rightarrow \left(\frac{1+(i)^2-2i}{1^2-(i)^2}\right)^{n^2} = 1$$

$$\Rightarrow \left(\frac{1-1-2i}{1-(-1)}\right)^{n^2} = 1 \qquad \{\because i^2=-1\}$$

$$\Rightarrow \left(\frac{-2i}{2}\right)^{n^2} = 1$$

$$\Rightarrow (-i)^{n^2} = 1$$

$$\Rightarrow n^2 = 4$$

$$\Rightarrow n = 2$$

Hint:

- Rationalize the $\frac{1-i}{1+i}$ and solve further using $i^2=-1$

2. **Option (c) is correct.**

Explanation:

Given:

$$\log_{\cos x}\sin x = 1;\ 0 < x < \frac{\pi}{2}$$

As we know

$$\log_a a = 1$$

$$\therefore \cos x = \sin x$$

$$\Rightarrow \sin\left(\frac{\pi}{2}-x\right) = \sin x$$

$$\Rightarrow \frac{\pi}{2}-x = x$$

$$\Rightarrow x = \frac{\pi}{4}$$

Shortcut:

Given:

$$\log_{\cos x}\sin x = 1$$

$$\Rightarrow \cos x = \sin x \qquad \{\because \log_a a = 1\}$$

$$\Rightarrow x = \frac{\pi}{4}$$

Hint:

- Use $\log_a a = 1$

3. **Option (d) is correct.**

Explanation:

Given: $$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Now,

$$\begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_3 \end{vmatrix} = p\begin{vmatrix} a_1 & b_1 & qc_1 \\ a_2 & b_2 & qc_2 \\ a_3 & b_3 & qc_3 \end{vmatrix}$$

{$\because$ If the entries of a row or column in a square matrix 'A' are multiplied by a number K $\in$ R, then the determinant of the resultant matrix is K|A|}

$$\Rightarrow \begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_3 \end{vmatrix} = pq\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_3 \end{vmatrix} = pq\Delta$$

Shortcut:

Given: $$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} pa_1 & b_1 & qc_1 \\ pa_2 & b_2 & qc_2 \\ pa_3 & b_3 & qc_3 \end{vmatrix} = pq\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= pq\Delta$$

Hint:

- If A_1 is a matrix obtained by multiplying a row or column of a matrix A by a real number $K \in R$, then $\det A_1 = K \det A$.

4. **Option (b) is correct.**

Explanation:

$$\because (1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots {}^nC_nx^n$$

Put $x = 1$ in the above equation, we get

$$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots {}^nC_n$$

$$\Rightarrow \quad 2^n = 1 + {}^nC_1 + {}^nC_2 + \ldots {}^nC_n$$

$$\Rightarrow \quad {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n - 1$$

Hint:

- Use $(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots {}^nC_nx^n$, put $x = 1$ and solve further.

5. **Option (b) is correct.**

Explanation:

Given:

$$a + b + c = 4, ab + bc + ca = 0$$

Let $$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \quad \Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$\Rightarrow \quad \Delta = (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$\Rightarrow \quad \Delta = 4[1(bc - a^2) - 1(b^2 - ac) + 1(ab - c^2)]$$

$$\Rightarrow \quad \Delta = 4[ab + bc + ca - (a^2 + b^2 + c^2)]$$

$$\Rightarrow \quad \Delta = 4[0 - \{(a+b+c)^2 - 2(ab+bc+ca)\}]$$

$$\Rightarrow \quad \Delta = 4[-\{16 - 0\}] \quad [\because a + b + c = 4]$$

$$\Rightarrow \quad \Delta = -64$$

Hint:

- Simplify using applying row operation $R_1 \to R_1 + R_2 + R_3$ and solve further using the property of determinant.

6. **Option (c) is correct.**

Explanation:

$$2 \sin x = 2K + 1$$

As we know $-1 \le \sin x \le 1$

$$\therefore \quad -2 \le 2 \sin x \le 2$$

$$\Rightarrow \quad -2 \le 2K + 1 \le 2$$

$$\Rightarrow -2 - 1 \le 2K + 1 - 1 \le 2 - 1$$

$$\Rightarrow \quad -3 \le 2K \le 1$$

$$\Rightarrow \quad -\frac{3}{2} \le K \le \frac{1}{2}$$

So, integer value of $K = -1, 0$

$\therefore$ Number of integer values of K is 2 for which $2 \sin x = 2K + 1$ has a solution.

Hint:

- Use $-1 \le \sin x \le 1$ and solve further.

7. **Option (a) is correct.**

Explanation:

Given: $a_1, a_2, a_3, \ldots, a_9$ are in G.P.

Let the common ratio be 'r'.

$\therefore a_2 = a_1r, a^3 = a_1r^2, a_4 = a_1r^3, a_5 = a_1r^4,$
$a_6 = a_1r^5, a_7 = a_1r^6, a_8 = a_1r^7, a_9 = a_1r^8$

Let $$\Delta = \begin{vmatrix} \ln a_1 & \ln a_2 & \ln a_3 \\ \ln a_4 & \ln a_5 & \ln a_6 \\ \ln a_7 & \ln a_8 & \ln a_9 \end{vmatrix}$$

$$\Rightarrow \quad \Delta = \begin{vmatrix} \ln a_1 & \ln a_1r & \ln a_1r^2 \\ \ln a_1r^3 & \ln a_1r^4 & \ln a_1r^5 \\ \ln a_1r^6 & \ln a_1r^7 & \ln a_1r^8 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} \ln a_1 & \ln a_1 + \ln r & \ln a_1 + 2\ln r \\ \ln a_1 + 3\ln r & \ln a_1 + 4\ln r & \ln a_1 + 5\ln r \\ \ln a_1 + 6\ln r & \ln a_1 + 7\ln r & \ln a_1 + 8\ln r \end{vmatrix}$$

$$\Rightarrow \quad \Delta = \begin{vmatrix} \ln a_1 & \ln r & \ln r \\ \ln a_1 + 3\ln r & \ln r & \ln r \\ \ln a_1 + 6\ln r & \ln r & \ln r \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_2$]

$$\Rightarrow \quad \Delta = 0$$

[Since, two columns are identical]

Hint:

- Let the common ratio of GP be r & then find terms of G.P.
- Simplify the determinant using operations on row/column and use if two rows/columns are identical, then determinant is zero.

8. **Option (d) is correct.**

Explanation:

Given: $x^2 + 2x + k = 0$

As we know $ax^2 + bx + c = 0$ has real roots if $b^2 - 4ac \geq 0$

Comparing given equation with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 2, c = k$$

$\therefore$ For real roots, $(2)^2 - 4(1)(k) \geq 0$

$$\Rightarrow \quad 4 - 4k \geq 0$$

$$4k \leq 4$$

$$\Rightarrow \quad k \leq 1$$

Hint:

- Quadratic equation $ax^2 + bx + c = 0$ has real roots if $b^2 - 4ac \geq 0$.

9. **Option (b) is correct.**

Explanation:

Given: $n = 100!$

Let $$A = \frac{1}{\log_2 n} + \frac{1}{\log_3 n} + ... + \frac{1}{\log_{100} n}$$

$$\Rightarrow \quad A = \log_n 2 + \log_n 3 + ... + \log_n 100$$

$$\left[\because \log_a b = \frac{1}{\log_b a}\right]$$

$$\Rightarrow \quad A = \log_n 2\cdot3\cdot4\cdot5\ldots100$$

$$[\because \log m + \log n = \log mn]$$

$$\Rightarrow \quad A = \log_n 100!$$

$$[\because n! = n\cdot(n-1)\cdot(n-2)\ldots 2\cdot1]$$

$$\Rightarrow \quad A = \log_n n$$

$$\Rightarrow \quad A = 1 \quad [\because \log_a a = 1]$$

Hint:

- Simplify using the properties of logarithm $\log_a b = \dfrac{1}{\log_b a}$ and $\log m + \log n = \log mn$.

10. **Option (b) is correct.**

Explanation:

Given: $Z = 1 + i$

Now, $$Z + \frac{2}{Z} = 1 + i + \frac{2}{1+i}$$

$$\Rightarrow \quad Z + \frac{2}{Z} = 1 + i + \frac{2}{1+i} \times \frac{1-i}{1-i}$$

$$\Rightarrow \quad Z + \frac{2}{Z} = 1 + i + \frac{2(1-i)}{1-i^2}$$

$$\Rightarrow \quad Z + \frac{2}{Z} = 1 + i + \frac{2(1-i)}{1-(-1)} \quad [\because i^2 = -1]$$

$$\Rightarrow \quad Z + \frac{2}{Z} = 1 + i + 1 - i$$

$$\Rightarrow \quad Z + \frac{2}{Z} = 2 + 0i$$

$$\therefore \quad \left|Z + \frac{2}{Z}\right| = \sqrt{2^2 + 0^2} = 2$$

Hint:

- Use if $Z = a + ib$, then $|Z| = \sqrt{a^2 + b^2}$

11. **Option (d) is correct.**

Explanation:

As we know two matrices P and Q are conformable. For the product PQ when the number of columns in P equal to the number of rows in Q.

If $P = [a_{ij}]$ be an $r \times 5$ matrix & $Q = [b_{ij}]$ be an 5×4 matrix, then $(PQ)_{ij} = \sum_{r=1}^{n} a_{ir} b_{rj}$ is a matrix of order $r \times 4$.

Here AB is of order $n \times n$

$\Rightarrow$ A matrix has 'n' rows & B matrix has 'n' columns.

$\therefore$ For product AB, number of column in A = number of rows in B.

Let number of columns in A = number of rows in B = x

Then, matrix A has order $n \times x$ and order of matrix B will be $x \times n$.

Since, x may take any positive integer.

$\therefore$ Orders of A & B need not be the same.

Hint:

- Use if $P = [a_{ij}]$ be an $r \times 5$ matrix & $Q = [b_{ij}]$ be an 5×4 matrix, then $(PQ)_{ij} = \sum_{r=1}^{n} a_{ir} b_{rj}$ is a matrix of order $r \times 4$.

12. Option (c) is correct.

Explanation:

Prime numbers less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

So, there are 10 prime numbers less than 30.

Now, different orders possible including all ten prime numbers are 1×10, 10×1, 2×5, 5×2.

$\therefore$ 4 matrices of different orders are possible with elements comprising all prime numbers less than 30.

Hint:

- There are 10 prime numbers less than 30.
- Number of elements in matrix of order $m \times n$ are mn.

13. Option (c) is correct.

Explanation:

Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.

Now, $A = \begin{vmatrix} p & q \\ r & s \end{vmatrix}$

$\Rightarrow \quad A = ps - qr$

For maximum of A, ps should be maximum and qr should be minimum.

Maximum value of $ps = 19 \times 17 = 323$

& Minimum value of $qr = 2 \times 3 = 6$

$\therefore (A)_{max} = 323 - 6 = 317$

Hint:

- Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19.
- $A = \begin{vmatrix} p & q \\ r & s \end{vmatrix} = ps - qr$

14. Option (d) is correct.

Explanation:

Given: A and B are square matrices of order 2 & det(AB) = det(BA)

As we know from property of matrix multiplication $AB \neq BA$.

But $\quad \det(AB) = \det(A)\cdot\det(B)$

and $\quad \det(BA) = \det(B)\cdot\det(A)$

As we know determinant of matrix is a scalar quantity.

$\therefore \quad \det(A)\cdot\det(B) = \det(B)\cdot\det(A)$

So, for det(AB) = det(BA), A & B need not be unit matrices.

Hint:

- Use $\det(AB) = \det(A)\cdot\det(B)$ and $\det(BA) = \det(B)\cdot\det(A)$
- Determinant of matrix is a scalar quantity.

15. Option (c) is correct.

Explanation:

$$\cot 6x = \cot(4x + 2x)$$

As we know

$$\cot(A + B) = \frac{\cot A \cdot \cot B - 1}{\cot A + \cot B}$$

$$\Rightarrow \quad \cot 6x = \frac{\cot 4x \cdot \cot 2x - 1}{\cot 4x + \cot 2x}$$

$$\Rightarrow \cot 6x \cdot \cot 4x + \cot 6x \cdot \cot 2x = \cot 4x \cdot \cot 2x - 1$$

$$\Rightarrow \cot 2x \cdot \cot 4x - \cot 4x \cdot \cot 6x - \cot 6x \cdot \cot 2x = 1$$

Hint:

- Use $\cot 6x = \cot(4x + 2x)$ and solve further using the formula $\cot(A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$.

16. Option (d) is correct.

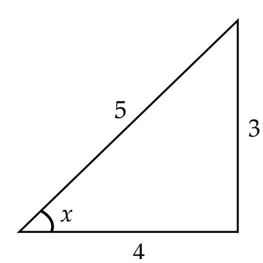
Explanation:

Given:

$$\tan x = -\frac{3}{4} \text{ and}$$

x is in the second quadrant.

$\therefore$ Sign of $\sin x$ will be +ve and sign of $\cos x$ will be –ve.



$$\Rightarrow \quad \sin x = \frac{3}{5} \text{ and } \cos x = -\frac{4}{5}$$

$$\left[\because \sin x = \frac{\text{Perpendicular}}{\text{Hypotenuse}} \text{ and } \cos x = \frac{\text{Base}}{\text{Hypotenuse}}\right]$$

$\therefore \quad \sin x \cos x = \left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = \frac{-12}{25}$

Hint:

- Recall sign of trigonometric functions in second quadrant and find the value of $\sin x$ & $\cos x$ using basic right angle identities.

17. Option (c) is correct.

Explanation:

Let $A = \text{cosec}\left(\frac{7\pi}{6}\right) \cdot \sec\left(\frac{5\pi}{3}\right)$

$\Rightarrow A = \text{cosec}\left(\pi + \frac{\pi}{6}\right) \cdot \sec\left(2\pi - \frac{\pi}{3}\right)$

$\Rightarrow A = -\text{cosec}\frac{\pi}{6} \cdot \sec\frac{\pi}{3}$

$[\because \text{cosec}(\pi + \theta) = -\text{cosec}\,\theta \;\&\; \sec(2\pi - \theta) = \sec\theta]$

$\Rightarrow A = -(2)(2) = -4$

Hint:

- Simplify using $\text{cosec}(\pi + \theta) = -\text{cosec}\,\theta$ and $\sec(2\pi - \theta) = \sec\theta$

18. Option (c) is correct.

Explanation:

Given:

$$\begin{vmatrix} x & 1 & 3 \\ 0 & 0 & 1 \\ 1 & x & 4 \end{vmatrix} = 0$$

Expand the determinant along R_2, we get

$-1(x^2 - 1) = 0$

$\Rightarrow (x-1)(x+1) = 0$

$\Rightarrow x = 1 \text{ or } -1$

Hint:

- Expand the determinant along R_2 and solve further.

19. Option (c) is correct.

Explanation:

Let $A = \tan 31° \tan 33° \ldots \tan 57° \tan 59°$

$\Rightarrow A = (\tan 31° \cdot \tan 59°)(\tan 33° \cdot \tan 57°) \ldots \tan 45°$

As we know $\tan A \cdot \tan B = 1$, if $A + B = 90°$

$\Rightarrow A = (1)(1) \ldots (1) = 1$

Hint:

- Use $\tan A \cdot \tan B = 1$, if $A + B = 90°$

20. Option (a) is correct.

Explanation:

Given:

$$f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & 2(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$$

$$\Rightarrow f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x^2 - x & x^2 + x \\ 3x^2 - 3x & 2x^2 - 6x + 4 & x^3 - x \end{vmatrix}$$

Applying $C_3 \rightarrow C_3 - (C_1 + C_2)$

$$\Rightarrow f(x) = \begin{vmatrix} 1 & x & 0 \\ 2x & x^2 - x & 0 \\ 3x^2 - 3x & 2x^2 - 6x + 4 & x^3 - x - (5x^2 - 9x + 4) \end{vmatrix}$$

$$\Rightarrow f(x) = \begin{vmatrix} 1 & x & 0 \\ 2x & x^2 - x & 0 \\ 3x^2 - 3x & 2x^2 - 6x + 4 & x^3 - 5x^2 + 8x - 4 \end{vmatrix}$$

$\Rightarrow f(x) = (x^3 - 5x^2 + 8x - 4)(x^2 - x - 2x^2)$

$\Rightarrow f(x) = -(x-1)(x-2)^2(x^2 + x)$

$\Rightarrow f(x) = -x(x+1)(x-1)(x-2)^2$

$\therefore f(0) = 0, f(1) = 0 \;\&\; f(-1) = 0$

So, $f(0) + f(-1) + f(1) = 0$

Hint:

- Applying the operation $C_3 \rightarrow C_3 - (C_1 + C_2)$ and solve further.

21. Option (b) is correct.

Explanation:

Given equation $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$

As we know $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}x - \left(\frac{\pi}{2} - \sin^{-1}x\right) = \frac{\pi}{6}$

$\Rightarrow 2\sin^{-1}x = \frac{\pi}{6} + \frac{\pi}{2}$

$\Rightarrow 2\sin^{-1}x = \frac{2\pi}{3}$

$\Rightarrow \sin^{-1}x = \frac{\pi}{3}$

$\Rightarrow x = \sin\frac{\pi}{3}$

$\Rightarrow x = \frac{\sqrt{3}}{2}$

$\therefore$ Equation has unique solution.

Hint:

- Simplify using $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ and solve further.

22. Option (b) is correct.

Explanation:

Let $A = (\sin 24° + \cos 66°)(\sin 24° - \cos 66°)$

$\Rightarrow$ $A = [\sin(90° - 66°) + \cos 66°][\sin(90° - 66°) - \cos 66°]$

$\Rightarrow$ $A = [\cos 66° + \cos 66°][\cos 66° - \cos 66°]$ $[\because \sin(90 - \theta) = \cos\theta]$

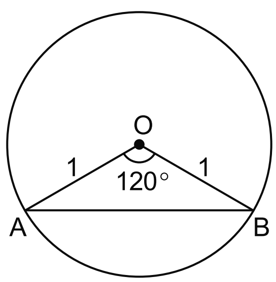
$\Rightarrow$ $A = [2 \cos 66°][0] = 0$

Hint:

- Simplify using $\sin(90° - \theta) = \cos\theta$

23. Option (d) is correct.

Explanation:



Let AB be chord of circle which subtends an angle 120° at the centre.

Applying cosine rule in ΔOAB, we get

$(AB)^2 = (OA)^2 + (OB)^2 - 2\, OA \cdot OB \cos 120°$

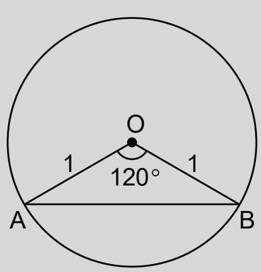
$\Rightarrow$ $(AB)^2 = 1 + 1 - 2(1)(1) \cos 120°$

$\Rightarrow$ $(AB)^2 = 2 - 2\left(-\frac{1}{2}\right)$

$\Rightarrow$ $(AB)^2 = 2 + 1 = 3$

$\Rightarrow$ $AB = \sqrt{3}$ units

Hint:



- Apply cosine rule in ΔOAB & solve further.

24. Option (b) is correct.

Explanation:

Let $A = (1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta)$

$\Rightarrow$ $A = \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$

$\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta},\ \operatorname{cosec}\theta = \frac{1}{\sin\theta},\ \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \sec\theta = \frac{1}{\cos\theta}\right]$

$\Rightarrow$ $A = \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$

$\Rightarrow$ $A = \frac{(\sin\theta + \cos\theta)^2 - 1^2}{\sin\theta \cdot \cos\theta}$ $[\because (a + b)(a - b) = a^2 - b^2]$

$\Rightarrow$ $A = \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta \cdot \cos\theta}$

$\Rightarrow$ $A = \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta \cdot \cos\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow$ $A = 2$

Hint:

- Simplify the given expression using basic trigonometric identities and use $\sin^2\theta + \cos^2\theta = 1$.

25. Option (a) is correct.

Explanation:

Let $A = \frac{1 + \tan^2\theta}{1 + \cot^2\theta} - \left(\frac{1 - \tan\theta}{1 - \cot\theta}\right)^2$

$\Rightarrow$ $A = \frac{\sec^2\theta}{\operatorname{cosec}^2\theta} - \left(\frac{1 - \frac{\sin\theta}{\cos\theta}}{1 - \frac{\cos\theta}{\sin\theta}}\right)^2$

$\left[\because 1 + \tan^2\theta = \sec^2\theta,\ 1 + \cot^2\theta = \operatorname{cosec}^2\theta,\ \frac{\sin\theta}{\cos\theta} = \tan\theta \text{ \& } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

$\Rightarrow$ $A = \frac{\sin^2\theta}{\cos^2\theta} - \left(\frac{\cos\theta - \sin\theta}{\sin\theta - \cos\theta} \cdot \frac{\sin\theta}{\cos\theta}\right)^2$

$\left[\because \cos\theta = \frac{1}{\sec\theta},\ \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$

$\Rightarrow$ $A = \tan^2\theta - \tan^2\theta$

$\Rightarrow$ $A = 0$

Hint:
- Simplify given expression using the trigonometric identities $1 + \tan^2\theta = \sec^2\theta$, $1 + \cot^2\theta = \text{cosec}^2\theta$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$, $\cos\theta \cdot \sec\theta = 1$, $\sin\theta \cdot \text{cosec}\,\theta = 1$

26. Option (b) is correct.

Explanation:

As we know, sum of exterior angle = 360°

∴ Each exterior angle

$$= \frac{360°}{\text{no. of side of regular polygon}}$$

∵ Number of side of regular octagon = 8

∴ Each exterior angle of regular octagon

$$= \frac{360°}{8} = 45°$$

As we know, each exterior angle + each interior angle = 180°

⇒ Interior angle = 180° – exterior angle

⇒ Interior angle = 180° – 45° = 135°

⇒ Interior angle $= \frac{135°}{180°} \times \pi = \frac{3\pi}{4}$

Hint:
- Each exterior angle $= \frac{360°}{\text{No. of side of regular polygon}}$
- Interior angle + exterior angle = 180°

27. Option (d) is correct.

Explanation:

Given:

$$7\sin\theta + 24\cos\theta = 25$$

$$\Rightarrow \frac{7}{25}\sin\theta + \frac{24}{25}\cos\theta = 1$$

Comparing it with $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \sin\theta = \frac{7}{25} \text{ and } \cos\theta = \frac{24}{25}$$

$$\therefore \sin\theta + \cos\theta = \frac{7}{25} + \frac{24}{25}$$

$$\Rightarrow \sin\theta + \cos\theta = \frac{31}{25}$$

Hint:
- Compare the given equation with $\sin^2\theta + \cos^2\theta = 1$.

28. Option (d) is correct.

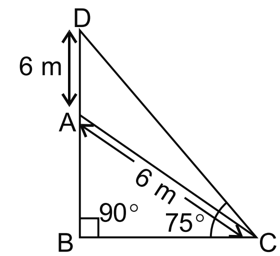
Explanation:

Given: A ladder 6 m long.

Let ladder be AC = 6 m

AD = 6 m.

∠DCB = 75°



In ΔBDC,

$$\angle BDC = 180° - (90° + 75°)$$

$$\Rightarrow \angle BDC = 15°$$

Now, in ΔADC,

$$AD = AC$$

$$\Rightarrow \angle ADC = \angle ACD = 15°$$

$$\Rightarrow \angle ACB = \angle BCD - \angle ACD = 75° - 15° = 60°$$

∴ In ΔABC,

$$\sin 60° = \frac{AB}{AC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AB}{6}$$

$$\Rightarrow AB = 3\sqrt{3} \text{ m}$$

∴ Height of flag staff = BA + AD = $(3\sqrt{3} + 6)$ m

$$\Rightarrow BD = (6 + 3\sqrt{3}) \text{ m}$$

29. Option (b) is correct.

Explanation:

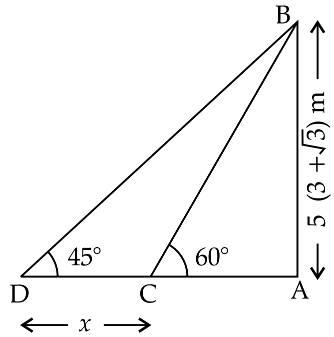
Let AB be the tower. AC be the shadow at 60° angle of elevation and AD at 45°.

⇒ CD = x m.

and ∠ACB = 60°, ∠ADB = 45°

Also,

$$AB = 5(3 + \sqrt{3}) \text{ m}$$



Let $\quad AC = y$ m

In ΔBAC,

$$\tan 60° = \frac{AB}{AC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{5(3+\sqrt{3})}{AC}$$

$$\Rightarrow \quad AC = \frac{5\sqrt{3}(\sqrt{3}+1)}{\sqrt{3}}$$

$$\Rightarrow \quad AC = 5(\sqrt{3}+1) \text{ m}$$

In ΔBAD,

$$\tan 45° = \frac{AB}{AD}$$

$$\Rightarrow \quad 1 = \frac{AB}{AD}$$

$$\Rightarrow \quad AB = AD$$

$$\Rightarrow \quad 5(3+\sqrt{3}) = AC + CD$$

$$\Rightarrow \quad 5(3+\sqrt{3}) = 5(\sqrt{3}+1) + CD$$

$$\Rightarrow \quad CD = 15 + 5\sqrt{3} - 5\sqrt{3} - 5$$

$$\Rightarrow \quad CD = 10 \text{ m}$$

Hint:

- Use $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$ to find the length of shadow at 60° angle of elevation of sun and at 45°.

30. Option (b) is correct.

Explanation:

Given: $\quad 3\cos\theta = 4\sin\theta$

$$\Rightarrow \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3}{4}$$

$$\because \quad \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \quad \tan(45° + \theta) = \frac{\tan 45° + \tan\theta}{1 - \tan 45° \tan\theta}$$

$$\Rightarrow \quad \tan(45° + \theta) = \frac{1+\tan\theta}{1-\tan\theta}$$

$$= \frac{1+\frac{3}{4}}{1-\frac{3}{4}} = \frac{\frac{7}{4}}{\frac{1}{4}}$$

$$\Rightarrow \quad \tan(45° + \theta) = 7$$

Hint:

- Use $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

31. Option (a) is correct.

Explanation:

As we know by property of inverse trigonometric function,

$$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}, x \in R.$$

$$\therefore \quad \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \text{ holds when } x \in R.$$

Hint:

- Recall the property of inverse trigonometric function based on addition of inverse trigonometric functions.

32. Option (a) is correct.

Explanation:

Given: $\quad \tan A = \frac{1}{7}$

As we know,

$$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$\Rightarrow \quad \cos 2A = \frac{1-\left(\frac{1}{7}\right)^2}{1+\left(\frac{1}{7}\right)^2}$$

$$\Rightarrow \quad \cos 2A = \frac{48/49}{50/49}$$

$$\Rightarrow \quad \cos 2A = \frac{48}{50} = \frac{24}{25}$$

Hint:

- Use $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$

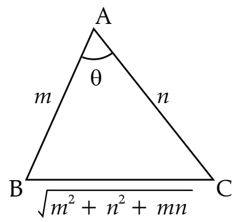
33. Option (b) is correct.

Explanation:

Let In ΔABC,

$$AB = m,\ AC = n \ \&$$

$$BC = \sqrt{m^2 + n^2 + mn}$$



By cosine rule,

$$\cos\theta = \frac{(AB)^2 + (AC)^2 - (BC)^2}{2(AB)(AC)}$$

$$\Rightarrow \quad \cos\theta = \frac{m^2 + n^2 - (m^2 + n^2 + mn)}{2mn}$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \quad \theta = 120°$$

As we know, sum of angles of triangle is equal to 180°.

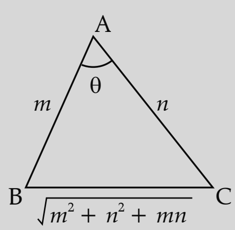
$$\therefore \quad \angle A + \angle B + \angle C = 180°$$

$$\Rightarrow \quad \angle B + \angle C = 180° - \angle A$$

$$= 180° - 120° = 60°$$

$\therefore$ Sum of the acute angles of the triangle is 60°.

Hint:



- Find θ using cosine rule and solve further.

34. Option (c) is correct.

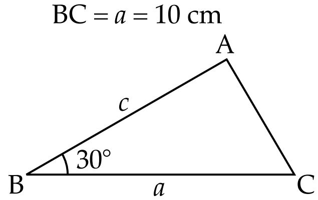
Explanation:

Given:

$$\angle ABC = 30°$$

$$AB = c = 4 \text{ cm}$$

$$BC = a = 10 \text{ cm}$$



$$\text{Area of } \Delta ABC = \frac{1}{2} ac \sin\angle ABC$$

$$= \frac{1}{2} \times 4 \times 10 \times \sin 30°$$

$$= 10 \text{ cm}^2$$

Hint:

- Use area of triangle ABC $= \frac{1}{2} ac \sin B = \frac{1}{2} bc \sin A = \frac{1}{2} ab \sin C$

35. Option (c) is correct.

Explanation:

Statement-I:

Given sets A = {1, 3, 5} & B = {2, 4, 7}

As we know equivalent sets are the sets with equal number of elements in them.

$\because$ Number of element in A = number of element in B = 3

$\therefore$ A & B are equivalent sets.

Statement-II:

Given sets A = {1, 5, 9} and B = {1, 5, 5, 9, 9}

As we know two sets P and Q can be equal only if each element of set P is also the element of the set Q. Also if two sets are the subsets of each other, they are said to be equal.

Here, each element of set A is also the element of the set B and $A \subseteq B$ and $B \subseteq A$.

$\therefore \quad$ A = B (Equal sets)

So, both statements are true.

Hint:

- Two sets A and B are said to be equivalent sets if they have same number of elements.
- Two sets P & Q can be equal only if each element of the set P is also the element of the set Q. Also if two sets are the subset of each other, they are said to be equal.

36. Option (d) is correct.

Explanation:

A set with no elements in it is called a null set. It is denoted by ϕ or { }.

$\therefore$ Null set is a subset of every set.

So, statement 1 is true.

A set A is a subset of another set B if all elements of the set A are elements of the set B.

$\therefore$ Every set is a subset of itself.

So, statement 2 is true.

As we know if a set contains 'n' elements, then number of elements in power set will be 2^n.

Here $n = 10$.

So, number of elements in power set $= 2^{10} = 1024$

So, statement 3 is true.

Hint:

- Recall the property of null set and subset of set.
- If a set contains 'n' elements, then number of elements in power set will be 2^n.

37. Option (b) is correct.

Explanation:

$$R = \{(x, y): 2x + 3y = 20; x, y \in N\}$$

$\because$ $2x + 3y = 20$

$\Rightarrow$ $2x = 20 - 3y$

$\Rightarrow$ $x = \frac{20-3y}{2}$

$\because$ $x, y \in N$

So, possible pairs of (x, y) are (7, 2), (4, 4), (1, 6).

$\therefore$ Number of elements in R is 3.

Hint:

- Use $x = \frac{20-3y}{2}$.
- Find the values of x by putting value of y considering $x, y \in$ N.

38. Option (c) is correct.

Explanation:

Statement-1:

Given:

$$f : Z \to Z, f(x) = x + 1$$

$\Rightarrow$Domain of $f(x) = Z$ and Co-domain of $f(x) = Z$ (Integer)

For one-one:

Let $x_1, x_2 \in Z$

Now, $f(x_1) = x_1 + 1$

and $f(x_2) = x_2 + 1$

Now, $f(x_1) = f(x_2)$

$\Rightarrow$ $x_1 + 1 = x_2 + 1$

$\Rightarrow$ $x_1 = x_2$

$\therefore$ $f(x)$ is one-one function.

For onto:

As we know function is said to be onto function if range of function is equal to the Co-domain of function.

$\because$ $f(x) = x + 1$

For $x \in Z$, range of $f(x)$ will be Z.

So, $f(x)$ is onto function.

So, statement-1 is true.

Statement-2

Given:

$$f : N \to N, f(x) = x + 1$$

$\Rightarrow$ Domain of $f(x) =$ N (natural numbers)

Co-domain of $f(x) =$ N

For one-one:

Let $x_1, x_2 \in N$

$f(x_1) = x_1 + 1$

And $f(x_2) = x_2 + 1$

Now, $f(x_1) = f(x_2)$

$\Rightarrow$ $x_1 + 1 = x_2 + 1$

$\Rightarrow$ $x_1 = x_2$

$\therefore$ $f(x)$ is one-one function.

For onto:

$\because$ $f(x) = x + 1 \geq 2$ for all $x \in N$

So, $f(x)$ does not assume values 1.

So, range of $f(x) \neq$ Co-domain of $f(x)$

$\Rightarrow$ $f(x)$ is not an onto function.

So, statement 2 is also true.

Hint:

- A function $f : x \to y$ is said to be one-one if the image of distinct elements of x under f are distinct i.e. for every $x_1, x_2 \in x, f(x_1) = f(x_2)$ implies $x_1 = x_2$.
- A function is said to be onto if the range of function is equal to the Co-domain of function.

39. Option (a) is correct.

Explanation:

Given: Z is a complex number.

Statement-1:

$$\overline{(Z^{-1})} = (\bar{Z})^{-1}$$

Let $Z = a + ib$

$\bar{Z} = a - ib$

Also we know,

$$Z^{-1} = \frac{\overline{Z}}{|Z|^2}$$

$$\therefore \quad \overline{(Z^{-1})} = \overline{\left(\frac{\overline{Z}}{|Z|^2}\right)} = \overline{\left(\frac{a-ib}{\left(\sqrt{a^2+b^2}\right)^2}\right)}$$

$$= \overline{\left(\frac{a-ib}{a^2+b^2}\right)}$$

$$\Rightarrow \quad \overline{(Z^{-1})} = \frac{a+ib}{a^2+b^2} \quad \ldots(i)$$

Also, $(\overline{Z})^{-1} = (a - ib)^{-1}$

$$= \frac{\overline{a-ib}}{(\sqrt{a^2+b^2})^2}$$

$$\Rightarrow \quad (\overline{Z})^{-1} = \frac{a+ib}{a^2+b^2} \quad \ldots(ii)$$

By (i) and (ii), $\overline{(Z^{-1})} = (\overline{Z})^{-1}$

So, statement (1) is correct.

Statement-2:

$$ZZ^{-1} = |z|^2$$

Let $Z = a + ib$

$$\Rightarrow \quad Z^{-1} = \frac{\overline{Z}}{|Z|^2} = \frac{a-ib}{a^2+b^2}$$

$$\therefore \quad ZZ^{-1} = (a+ib)\frac{(a-ib)}{a^2+b^2} = \frac{a^2+b^2}{a^2+b^2}$$

$$\Rightarrow \quad ZZ^{-1} = 1$$

and $|Z|^2 = (a^2 + b^2)$

$$\therefore \quad ZZ^{-1} \neq |Z|^2$$

So, statement (2) is incorrect.

Hint:

- Use $Z^{-1} = \frac{\overline{Z}}{|Z|^2}$ to simplify both the expressions.
- Assume $Z = a + ib$ and then solve.

40. Option (c) is correct.

Explanation:

Let us assume $Z = x + iy$

Statement-1:

$$Z = x + iy$$

$$\Rightarrow \quad \overline{Z} = x - iy$$

Now, $Z - \overline{Z} = (x + iy) - (x - iy)$

$$= x + iy - x + iy$$

$$= 2iy$$

$\Rightarrow Z - \overline{Z}$ is an imaginary number.

So, statement 1 is true.

Statement-2:

$$Z = x + iy$$

$$\Rightarrow \quad \overline{Z} = x - iy$$

Now, $Z + \overline{Z} = (x + iy) + (x - iy)$

$$= 2x$$

$\Rightarrow Z + \overline{Z}$ is a real number.

So, statement 2 is also true.

Hint:

- Assume $Z = x + iy$ and find its conjugate.

41. Option (b) is correct.

Explanation:

Given: A complex number $i^{2n+1}(-i)^{2n-1}$

Let $Z = i^{2n+1}(-i)^{2n-1}$

As we know, $i^2 = -1$, $i^3 = -i$

$$\Rightarrow \quad Z = (i)^{2n+1}(i^3)^{2n-1}$$

$$\Rightarrow \quad Z = (i)^{2n+1}(i)^{6n-3}$$

$$\Rightarrow \quad Z = i^{8n-2}$$

$$\Rightarrow \quad Z = (i^2)^{4n-1}$$

$$\Rightarrow \quad Z = (-1)^{4n-1}$$

$\because n \in N$, $4n - 1$ is an odd number.

$$\Rightarrow \quad Z = -1$$

So, modulus of given complex number = 1

Hint:

- Use $i^2 = -1$ and $i^3 = -i$ and simplify the given number.

42. Option (a) is correct.

Explanation:

Given: α & β are the roots of equation $4x^2 + 2x - 1 = 0$

As we know, roots of the quadratic equation $ax^2 + bx + c = 0$ is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 4$, $b = 2$, $c = -1$

$$\therefore \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4(4)(-1)}}{2(4)}$$

$$\Rightarrow \quad x = \frac{-2 \pm \sqrt{4+16}}{8}$$

$\Rightarrow \quad x = \frac{-2 \pm 2\sqrt{5}}{8}$

$\Rightarrow \quad x = \frac{-1 \pm \sqrt{5}}{4}$

Let $\alpha = \frac{-1+\sqrt{5}}{4}$ and $\beta = \frac{-(1+\sqrt{5})}{4}$

Now, $-2\alpha^2 - 2\alpha = -2\alpha(\alpha + 1)$

$$= -2\left(\frac{-1+\sqrt{5}}{4}\right)\left(\frac{-1+\sqrt{5}}{4}+1\right)$$

$$= -2\left(\frac{-1+\sqrt{5}}{4}\right)\left(\frac{3+\sqrt{5}}{4}\right)$$

$$= -\left(\frac{-3+3\sqrt{5}-\sqrt{5}+5}{8}\right)$$

$$= -\left(\frac{2+2\sqrt{5}}{8}\right)$$

$$= -\frac{(1+\sqrt{5})}{4}$$

$$= \beta$$

Hint:

- Roots of the quadratic equation

  $ax^2 + bx + c = 0$ is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

43. Option (c) is correct.

Explanation:

Given:

$5x^2 + 26x + k = 0$

Let one root be α then other root will be $\frac{1}{\alpha}$.

As we know, if α and β are the roots of $ax^2 + bx + c = 0$ then, $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$.

$\Rightarrow \alpha + \frac{1}{\alpha} = \frac{-26}{5}$ and $\alpha\frac{1}{\alpha} = \frac{k}{5}$

$\Rightarrow \quad 1 = \frac{k}{5}$

$\Rightarrow \quad k = 5$

Hint:

- Use the relationship with roots and coefficients of a quadratic equation.

44. Option (c) is correct.

Explanation:

Given: Total number of players = 8

Players to be selected = 5

$\because$ One particular player is not included in the team so we have to select 5 players out of 7 players.

$\therefore$ Required ways = ${}^7C_5$

$$= \frac{7 \times 6}{2} = 21 \text{ ways}$$

Hint:

- Use selection of r objects out of n objects is given by ${}^nC_r$.

45. Option (a) is correct.

Explanation:

Given: An expansion $(1 + 4x + 4x^2)^5$

$\because \quad 1 + 4x + 4x^2 = (1 + 2x)^2$

$\Rightarrow \quad (1 + 4x + 4x^2)^5 = (1 + 2x)^{10}$

As we know, number of terms in $(x + y)^n$ are $n + 1$

$\Rightarrow$ No. of terms in $(1 + 2x)^{10}$ are 11.

$\Rightarrow$ The middle term is 6^{th}.

Now, for $(x + y)^n$,

$$T_{r+1} = {}^nC_r x^r y^{n-r}$$

For 6^{th} term,

$$r + 1 = 6 \Rightarrow r = 5$$

$\therefore \quad T_6 = {}^{10}C_5(1)^5(2x)^5$

$$= {}^{10}C_5(2)^5x^5$$

Coefficient of $T_6 = {}^{10}C_5(2)^5$

$$= \frac{10!}{5!5!}(2)^5$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2} \times 2^5$$

$$= 8064$$

Hint:

- Middle term in the binomial expansion of

  $(x+y)^n$ is $\begin{cases} \left(\frac{n}{2}+1\right) \text{term}; & n \text{ even} \\ \left(\left(\frac{n+1}{2}\right)\right)^{th} \text{term}; & n \text{ odd} \end{cases}$

- General term at binomial expansion of $(x + y)^n$ is $T_{r+1} = {}^nC_r x^r y^{n-r}$

- Use ${}^nC_r = \frac{n!}{r!(n-r)!}$

46. Option (c) is correct.

Explanation:

$\because \quad (1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_nx^n$

Put $x = 1$ in the above equation, we get

$$(1+1)^n = 1 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$$

$\Rightarrow$ $2^n - 1 = {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$

$\Rightarrow {}^nC_1 + {}^nC_2 + \ldots {}^nC_n = 1 \cdot \frac{(2^n - 1)}{2-1}$

$\because$ RHS term represents sum of a G.P. of 'n' terms whose first term is 1 and common ratio is 2.

$\Rightarrow {}^nC_1 + {}^nC_2 + \ldots {}^nC_n = 1 + 2 + 2^2 + \ldots + 2^{n-1}$

Hint:

- Use $(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_nx^n$. Put $x = 1$ & solve further.
- Sum of a G.P of 'n' terms whose first term is 'a' common ratio 'r' is given by $\frac{a(r^n-1)}{r-1}$; $r > 1$

47. Option (b) is correct.

Explanation:

Given: Expansion of $(1 + x)^{2n}$

Now, as we know the coefficient of 1st term $= {}^{2n}C_0$

$$= \frac{2n!}{0!(2n-0)!}$$

$$= \frac{2n!}{2n!} = 1$$

Similarly, last term coefficient $= {}^{2n}C_{2n}$

$$= \frac{2n!}{2n!(2n-2n)!}$$

$$= \frac{2n!}{2n!} = 1$$

$\therefore$ Sum of coefficient of first and last term

$$= 1 + 1 = 2$$

Hint:

- Use coefficient of $(x+1)^{\text{th}}$ term in $(1+a)^n$ is given by ${}^nC_x$.

48. Option (b) is correct.

Explanation:

Let, first term of an A.P. $= a = 2$

Sum of first five terms $= S_5$

Sum of next five terms $= S_{10} - S_5$

Now, $S_5 = \frac{1}{4}(S_{10} - S_5)$

$\Rightarrow$ $4S_5 = S_{10} - S_5$

$\Rightarrow$ $S_{10} = 5S_5$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$ where d = common difference.

$\therefore$ $S_{10} = 5S_5$

$\Rightarrow$ $\frac{10}{2}[2(2)+(10-1)d] = 5 \times \frac{5}{2}[2(2)+(5-1)d]$

$\Rightarrow$ $5[4 + 9d] = \frac{25}{2}[4 + 4d]$

$\Rightarrow$ $2(4 + 9d) = 5(4 + 4d)$

$\Rightarrow$ $8 + 18d = 20 + 20d$

$\Rightarrow$ $2d = -12$

$\Rightarrow$ $d = -6$

$\therefore$ $S_{10} = \frac{10}{2}[2(2) + (10-1)(-6)]$

$= 5[4 - 54]$

$= -250$

Hint:

- Use $S_n = \frac{n}{2}[2a + (n-1)d]$ where n = number of terms, a = first term and d = common difference.
- Solve $S_5 = \frac{1}{4}(S_{10} - S_5)$

49. Option (c) is correct.

Explanation:

Let us assume a G.P. a, b, c, d, e with common ratio be r.

Statement 1: Let the non zero multiplier be s.

New sequence will be as, bs, cs, ds, es.

$\because$ $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = \frac{e}{d} = r$

$\because$ $\frac{bs}{as} = \frac{cs}{bs} = \frac{ds}{cs} = \frac{es}{ds} = r$

$\therefore$ as, bs, cs, ds, es is also a G.P.

Statement 2: Let each of the G.P. is divided by t.

$\Rightarrow$ New sequence will be $\frac{a}{t}, \frac{b}{t}, \frac{c}{t}, \frac{d}{t}, \frac{e}{t}$.

$\because$ $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = \frac{e}{d} = r$

$\Rightarrow$ $\frac{b/t}{a/t} = \frac{c/t}{b/t} = \frac{d/t}{c/t} = \frac{e/t}{d/t} = r$

$\therefore$ $\frac{a}{t}, \frac{b}{t}, \frac{c}{t}, \frac{d}{t}, \frac{e}{t}$ is also a G.P.

So, both the statements are true.

Hint:

- Assume a, b, c, d, e be a G.P. and use the concept that if common ratio of each consecutive terms are equal then the sequence is G.P.

50. Option (d) is correct.

Explanation:

Given: 1, 2, 3, 4, 5 are given digits.

5-digit numbers are formed without repetition of given digits.

$\Rightarrow$ All digits 1, 2, 3, 4, 5 will be present in the number.

Now, prime numbers is a number that has only two factors one and itself.

Also, divisibility rule of 3 says that a number is divisible by 3 when sum of the digits of the number is divisible by 3.

Now, the sum of the digits of 5-digit number made by 1, 2, 3, 4, 5 will be

$1 + 2 + 3 + 4 + 5 = 15$

$\because$ 15 is divisible by 3, so every 5-digit number formed will also be divisible by 3.

$\Rightarrow$ No prime numbers can be formed.

Hint:

- Use the divisibility rule of 3.

51. Option (b) is correct.

Explanation:

Given: $f(x+1) = x^2 - 3x + 2$

Let $x + 1 = t \Rightarrow x = t - 1$

$\Rightarrow \quad f(t) = (t-1)^2 - 3(t-1) + 2$

$\Rightarrow \quad f(t) = t^2 + 1 - 2t - 3t + 3 + 2$

$\Rightarrow \quad f(t) = t^2 - 5t + 6$

or $\quad f(x) = x^2 - 5x + 6$

Shortcut:

$f(x+1) = x^2 - 3x + 2$

$\Rightarrow \quad f(x) = (x-1)^2 - 3(x-1) + 2$

$\Rightarrow \quad f(x) = x^2 - 5x + 6$

Hint:

- Assume $x + 1 = t$ and replace x by $t - 1$ in the given functional equation.

52. Option (c) is correct.

Explanation:

Given: $x^2, x, -8$ are in A.P.

As we common difference of an A.P. is equal.

$\Rightarrow \quad x^2 - x = x - (-8)$

$\Rightarrow \quad x^2 - x = x + 8$

$\Rightarrow \quad x^2 - 2x - 8 = 0$

$\Rightarrow \quad (x-4)(x+2) = 0$

$\Rightarrow \quad x = 4, -2$

$\therefore \quad x \in \{-2, 4\}$

Hint:

- Recall the definition of an A.P.

53. Option (b) is correct.

Explanation:

Given: Third term of a G.P. = 3

Let a be the first term and r be the common ratio.

$\Rightarrow \quad T_3 = ar^{3-1}$

$\Rightarrow \quad T_3 = ar^2 = 3 \quad \text{...[i]}$

So, the first five terms of G.P. = $a, ar, 3, ar^3, ar^4$

$$\begin{aligned}\text{Product} &= (a)(ar)(3)(ar^3)(ar^4)\\ &= 3a^4r^8\\ &= 3(ar^2)^4\\ &= 3(3)^4 = 243 \quad \text{[from (i)]}\end{aligned}$$

Hint:

- Use the n^{th} term of a G.P. with a as first term and r as the common ratio is ar^{n-1}.

54. Option (a) is correct.

Explanation:

Given: $a_{ij} = 2(i + j)$

After put the values of i and j from 1 to 3, matrix A will be.

$$A = \begin{bmatrix} 4 & 6 & 8 \\ 6 & 8 & 10 \\ 8 & 10 & 12 \end{bmatrix}$$

$$\Rightarrow \quad |A| = \begin{vmatrix} 4 & 6 & 8 \\ 6 & 8 & 10 \\ 8 & 10 & 12 \end{vmatrix}$$

$$\Rightarrow \quad |A| = 2^3 \begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_2$, we get

$$|A| = 2^3 \begin{vmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ 4 & 1 & 1 \end{vmatrix}$$

$\Rightarrow \quad |A| = 0 \quad [\because \text{Two columns are identical}]$

Hint:
- Find matrix using $a_{ij} = 2(i + j)$.
- If two columns/rows are identical, then the value of determinant is zero.

55. Option (d) is correct.

Explanation:

Given: 4 elements 2, 4, 6, 8.

Now, total number of determinants possible = 4! = 24 ways.

Let one way be $\begin{vmatrix} 2 & 8 \\ 6 & 4 \end{vmatrix} = (2 \times 4) - (6 \times 8) = -40$

Now, we will get – 40 in three other ways also.

$\begin{vmatrix} 4 & 8 \\ 6 & 2 \end{vmatrix}, \begin{vmatrix} 2 & 6 \\ 8 & 4 \end{vmatrix}, \begin{vmatrix} 4 & 6 \\ 8 & 2 \end{vmatrix}$

So in 4 ways we will get determinant = – 40 and similarly if we interchange any row or column then determinant gets negative.

$\Rightarrow$ As $\begin{vmatrix} 2 & 8 \\ 6 & 4 \end{vmatrix} = -40$

$\Rightarrow \begin{vmatrix} 8 & 2 \\ 4 & 6 \end{vmatrix} = (8) \times (6) - (2)(4) = +40$

So, 4 determinants will have +40 as its value.

Similarly, for every determinant there exists another determinant with negative sign.

$\Rightarrow$ Sum of determinants = 0

Hint:
- Use fundamental principle of counting to find total number of ways.
- Recall the properties of determinants.

56. Option (d) is correct.

Explanation:

Given:

$$4x^2 + 4y^2 - 20x + 12y - 15 = 0$$

$$\Rightarrow \quad x^2 + y^2 - 5x + 3y - \frac{15}{4} = 0$$

As we know radius of circle $x^2 + y^2 + 2gx + 2f + y + c = 0$ is $r = \sqrt{g^2 + f^2 - c}$

Here $g = -\frac{5}{2}, f = \frac{3}{2}, c = -\frac{15}{4}$

$$\therefore \quad \text{radius } r = \sqrt{\frac{25}{4} + \frac{9}{4} + \frac{15}{4}}$$

$$\Rightarrow \quad r = \sqrt{\frac{49}{4}} = \frac{7}{2}$$

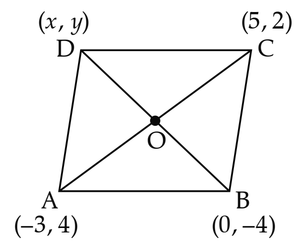
$$\Rightarrow \quad r = 3.5 \text{ units}$$

Hint:
- Radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $\sqrt{g^2 + f^2 - c}$.

57. Option (a) is correct.

Explanation:

Given: Parallelogram has three consecutive vertices (–3, 4), (0, –4) & (5, 2).



Let fourth vertex be (x, y)

As we know diagonals of parallelogram bisect each other.

$\therefore$ Mid point of AC = mid point of BD

$$\Rightarrow \quad \left(\frac{5-3}{2}, \frac{4+2}{2}\right) = \left(\frac{x+0}{2}, \frac{y-4}{2}\right)$$

$$\Rightarrow \quad (1, 3) = \left(\frac{x}{2}, \frac{y-4}{2}\right)$$

$$\Rightarrow \quad \frac{x}{2} = 1 \text{ and } \frac{y-4}{2} = 3$$

$$\Rightarrow \quad x = 2 \text{ and } y - 4 = 6$$

$$\Rightarrow \quad x = 2 \text{ and } y = 10$$

$\therefore$ Coordinates of fourth vertex is (2, 10).

Hint:
- Diagonals of parallelogram bisect each other.

58. Option (c) is correct.

Explanation:

Given lines:

$$y + px = 1 \text{ and } y - qx = 2$$

$$\Rightarrow \quad y = -px + 1 \quad \text{...(i)}$$

$$\text{and} \quad y = qx + 2 \quad \text{...(ii)}$$

Compare line (i) with $y = mx + c$, we get

Slope of line (i) is $m_1 = -p$

Compare line (ii) with $y = mx + c$, we get

Slope of the line (ii) is $m_2 = q$

As we know product of slopes of two perpendicular lines is –1.

$\Rightarrow \quad -pq = -1$

$\Rightarrow \quad pq - 1 = 0$

Hint:

- Use two lines are perpendicular if the product of their slopes is –1.

59. Option (d) is correct.

Explanation:

Given line

$$Ax + 2By + C = 0 \quad \ldots(i)$$

Since, A, B & C are in A.P.

$\Rightarrow \quad 2B = A + C$

Put 2B = A + C in the equation (i), we get

$$Ax + (A + C)y + C = 0$$

$\Rightarrow \quad A(x + y) + C(y + 1) = 0$

$\Rightarrow \quad x + y + \frac{C}{A}(y + 1) = 0 \quad \ldots(ii)$

As we know $L_1 + \lambda L_2 = 0$ represents family of lines passing through the intersection point of $L_1 = 0$ & $L_2 = 0$.

$\therefore$ Equation (ii) passes through intersection point of

$$x + y = 0 \quad \ldots(iii)$$

and $\quad y + 1 = 0 \quad \ldots(iv)$

On solving equation (iii) & (iv), we get

$$x = 1 \text{ and } y = -1$$

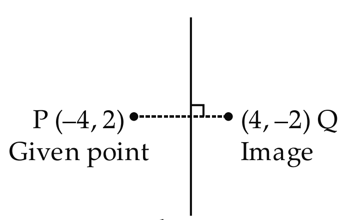
$\therefore$ $Ax + 2By + C = 0$ always passes through the point (1, –1).

Hint:

- If l, m, n are in A.P. then $2m = l + n$.
- $L_1 + \lambda L_2 = 0$ represents family of lines passing through the intersection point of $L_1 = 0$ and $L_2 = 0$.

60. Option (b) is correct.

Explanation:



The line mirror is perpendicular to the line joining two points P (–4, 2) and Q (4, –2) and it passes through the mid point of PQ.

As we know slope of a line joining two points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$

So, slope of a line PQ, $m_1 = \frac{-2-2}{4-(-4)} = \frac{-4}{8} = -\frac{1}{2}$

As we know product of the slopes of perpendicular lines is –1.

$\therefore$ Slope of line mirror = 2

Now, midpoint of PQ = $\left(\frac{4-4}{2}, \frac{2-2}{2}\right) = (0, 0)$

As we know equation of line passing through the point (x_1, y_1) having a slope 'm' is given by $y - y_1 = m(x - x_1)$

$\therefore$ Equation of mirror line is $y - 0 = 2(x - 0)$

$\Rightarrow \quad y = 2x$

Hint:

- Line mirror is perpendicular to the line joining two points P(–4, 2) and Q(4, –2) and it passes through the mid point of PQ.
- If two lines are perpendicular, then product of their slopes is –1.
- Equation of line passing through the point (x_1, y_1) and having a slope 'm' is given by $y - y_1 = m(x - x_1)$

61. Option (a) is correct.

Explanation:

Statement-1:

As we know if three points P (x_1, y_1), Q (x_2, y_2) & R (x_3, y_3) are colinear, then

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Let P $(p, p - 3)$, Q = $(q + 3, q)$ & R = (6, 3)

Now, $\begin{vmatrix} p & p-3 & 1 \\ q+3 & q & 1 \\ 6 & 3 & 1 \end{vmatrix} = p(q - 3) - (p - 3)(q + 3 - 6) + 1(3q + 9 - 6q)$

$= pq - 3p - (pq - 3p - 3q + 9) - 3q + 9$

$= 0$

$\therefore$ Given points lies on a straight line.

So, statement 1 is true.

Statement-2:

Given points $(p, p - 3)$, $(q + 3, q)$ and (6, 3).

As we can see that, for any value of p and q it is not necessary that the points lies in the first quadrant only.

So, statement 2 is false.

Hint:

- Three points P (x_1, y_1), Q (x_2, y_2) & R (x_3, y_3) are colinear if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

62. Option (b) is correct.

Explanation:

Given lines

$$x - 2 = 0 \quad \ldots\text{(i)}$$

$$\sqrt{\ }\, x - y - 2 = 0 \quad \ldots\text{(ii)}$$

Since, line (i) is parallel to y-axis.

$\therefore$ Slope of line (i) is $m_1 \to \infty$

Compare line (ii) with $y = mx + c$, we get

$$m = \sqrt{3}$$

$\therefore$ Slope of line (ii) is $m_2 = \sqrt{3}$

As we know angle between two lines having the slopes m_1 and m_2 is given by

$$\theta = \tan^{-1}\left|\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)\right|$$

$\therefore$ Angle between line 1 and line 2 is

$$\theta = \tan^{-1}\left|\frac{1 - \frac{m_2}{m_1}}{\frac{1}{m_1} + m_2}\right|$$

$$\Rightarrow \quad \theta = \tan^{-1}\left|\frac{1-0}{0+\sqrt{3}}\right| \quad \{\because m_1 \to \infty\}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left|\frac{1}{\sqrt{3}}\right|$$

$$\Rightarrow \quad \theta = 30^\circ \text{ or } 150^\circ$$

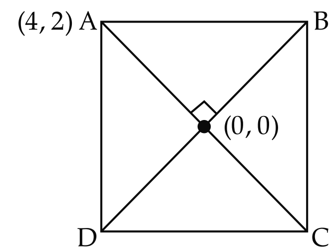
$\therefore$ Acute angle is 30°.

Hint:

- Angle between two lines $y = m_1x + c_1$ and $y = m_2x + c_2$ is given by $\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.
- Parallel lines have equal slope.

63. Option (a) is correct.

Explanation:



As we know diagonals of square bisect each other at right angle.

$\therefore$ AC $\perp$ BD

As we know product of slopes of perpendicular lines is –1.

$$\therefore \quad \text{Slope of BD} = -\frac{1}{\text{Slope of AC}}$$

$$\text{Slope of AC} = \frac{2-0}{4-0} = \frac{1}{2}$$

$$\Rightarrow \quad \text{Slope of BD} = -2$$

As we know equation of line passing through the point (x_1, y_1) and having a slope 'm' is given by $y - y_1 = m(x - x_1)$.

$\therefore$ Equation of line BD is $y - 0 = -2(x - 0)$

$$\Rightarrow \quad y + 2x = 0$$

Hint:

- Diagonals of square bisect each other at right angle.
- If two lines are perpendicular, then product of their slopes is –1.
- Equation of line passing through the point (x_1, y_1) having a slope 'm' is given by $y - y_1 = m(x - x_1)$.

64. Option (d) is correct.

Explanation:

As we know parametric coordinates of any point on the hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ is $(b\tan\theta, a\sec\theta)$.

Here, given point $(3\tan\theta, 2\sec\theta)$.

$$\therefore \quad a = 2, b = 3$$

As we know ecentricity of hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ is given by

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$\therefore \quad e = \sqrt{1+\frac{9}{4}}$

$\Rightarrow \quad e = \sqrt{\frac{13}{4}}$

$\Rightarrow \quad e = \frac{\sqrt{13}}{2}$

Hint:

- Parametric coordinates of any point on the hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ is $(b\tan\theta,\ a\sec\theta)$.
- Eccentricity of hyperbola is $e = \sqrt{1+\frac{b^2}{a^2}}$.

65. Option (d) is correct.

Explanation:

As we know the eccentricity of a circle is 0 *i.e.* $e = 0$ for a circle.

So, statement 1 is true.

We also know that eccentricity of a parabola is 1 *i.e.* $e = 1$ for a parabola.

So, statement 2 is true.

We also know that eccentricity of an ellipse is always less than 1 *i.e.* $e < 1$ for an ellipse.

So, statement 3 is also true.

Hint:

- The eccentricity 'e' of a conic section is defined to be the distance from any point on the conic section to its focus, divided by the perpendicular distance from that point to the nearest direction.

  If $e = 0$, the conic is circle

  If $e = 1$, the conic is parabola

  If $e < 1$, the conic is ellipse

  If $e > 1$, the conic is hyperbola,

66. Option (b) is correct.

Explanation:

Given: Direction ratios of two line {6, 3, 6} and {3, 3, 0}.

As we know if two lines have direction ratios $\{a_1, b_1, c_1\}$ and $\{a_2, b_2, c_2\}$, then angle between two lines is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}$$

Here, $a_1 = 6, b_1 = 3, c_1 = 6, a_2 = 3, b_2 = 3, c_2 = 0$.

Let angle between two lines be θ.

$\therefore \quad \cos\theta = \frac{(6)(3)+(3)(3)+(6)(0)}{\sqrt{36+9+36}\cdot\sqrt{9+9+0}}$

$\Rightarrow \quad \cos\theta = \frac{18+9}{(9)(3\sqrt{2})}$

$\Rightarrow \quad \cos\theta = \frac{27}{9(3\sqrt{2})}$

$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \theta = \frac{\pi}{4}$

Hint:

- If angle between two lines having direction ratios $\{a_1, b_1, c_1\}$ and $\{a_2, b_2, c_2\}$ be θ, the

  $$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}$$

67. Option (b) is correct.

Explanation:

Given line

$$x - 1 = 2(y+3) = 1 - z$$

$\Rightarrow \quad \frac{x-1}{2} = \frac{y+3}{1} = \frac{z-1}{-2}$

$\Rightarrow$ Direction ratios of line = {2, 1, –2}

$\left[\because \text{Direction ratios of line } \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \text{ is } (a, b, c)\right]$

As we know if direction ratios of the line is $\{a, b, c\}$, then direction cosine of the line will be $\{l, m, n\}$

$$= \left\{\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}\right\}$$

$\therefore$ Direction cosine of given line is $\{l, m, n\}$

$$= \left\{\frac{2}{\sqrt{4+1+4}}, \frac{1}{\sqrt{4+1+4}}, \frac{-2}{\sqrt{4+1+4}}\right\}$$

$\Rightarrow \quad \{l, m, n\} = \left\{\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}\right\}$

Now, $l^4 + m^4 + n^4 = \left(\frac{2}{3}\right)^4 + \left(\frac{1}{3}\right)^4 + \left(\frac{-2}{3}\right)^4$

$$= \frac{16}{81} + \frac{1}{81} + \frac{16}{81}$$

$$= \frac{33}{81} = \frac{11}{27}$$

Hint:

- If a, b & c are the direction ratios of the line then direction cosine of the line is $\{l, m, n\}$

$$= \left\{\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}\right\}$$

68. Option (c) is correct.

Explanation:

Given points A (1, 7, –5) and B (–3, 4, –2)

As we know projection of two lines joining the two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) on the line having direction cosines l, m & n is given by

$$P'Q' = |l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)|$$

Now, direction cosine of y-axis is

$$\{l, m, n\} = \{0, 1, 0\}$$

∴ Projection of the line segment joining A & B on y-axis is

$$A'B' = |0(-3 - 1) + 1(4 - 7) + 0(-2 + 5)|$$

$$A'B' = 3$$

Hint:

- Projection of two line joining the two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) on the line having direction cosines l, m & n is given by
  $P'Q' = |l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)|$
- Direction cosine of y-axis is 0, 1, 0.

69. Option (c) is correct.

Explanation:

Given: The line joining the points $(k, 1, 3)$ and $(1, -2, k + 1)$ also passes through the piont (15, 2, –4)

As we know equation of line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

∴ Equation of line passing through the points $(k, 1, 3)$ and $(1, -2, k + 1)$ is

$$\frac{x-k}{1-k} = \frac{y-1}{-2-1} = \frac{z-3}{(k+1)-3}$$

$$\Rightarrow \quad \frac{x-k}{1-k} = \frac{y-1}{-3} = \frac{z-3}{k-2}$$

Since, line passes through the piont (15, 2, –4)

$$\therefore \quad \frac{15-k}{1-k} = \frac{2-1}{-3} = \frac{-4-3}{k-2}$$

$$\Rightarrow \quad \frac{15-k}{1-k} = \frac{-1}{3} = \frac{-7}{k-2}$$

$$\Rightarrow \quad \frac{15-k}{1-k} = \frac{-1}{3} \text{ or } \frac{-7}{k-2} = \frac{-1}{3}$$

$$\Rightarrow \quad 45 - 3k = -1 + k \text{ or } -21 = -k + 2$$

$$\Rightarrow \quad 4k = 46 \text{ or } k = 23$$

$$\Rightarrow \quad k = \frac{23}{2} \text{ or } k = 23$$

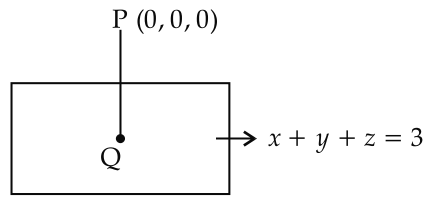
So, there are two possible values of k is possible.

Hint:

- Equation of line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by
  $$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

70. Option (c) is correct.

Explanation:



Let P be the given point and Q be the foot of the perpendicular.

Given: Equation of plane

$$x + y + z = 3$$

∴ Direction ratios of the normal of plane are {1, 1, 1}.

As we know equation of line passing through the point (x_1, y_1, z_1) having direction ratios a, b, c is given by $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

∴ Equation of line PQ is $\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-0}{1}$

$$\Rightarrow \quad x = y = z = \lambda$$

Let the coordinates of point Q be $(\lambda, \lambda, \lambda)$

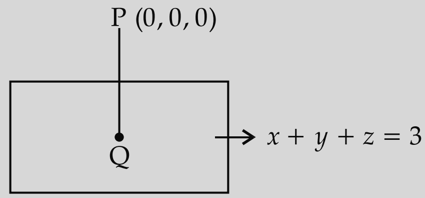
Since, Q lies in the plane $x + y + z = 3$

$$\therefore \quad \lambda + \lambda + \lambda = 3$$

$$\Rightarrow \quad \lambda = 1$$

∴ Coordinate of Q is (1, 1, 1).

Hint:



- Use equation of line PQ is $x = y = z$. Assume the coordinates of point Q on line and find the value of parametric coefficient by satisfying the point Q in the plane.

71. Option (c) is correct.

Explanation:

Given: Vector $\vec{r} = a\hat{i} + b\hat{j}$ is equally inclined to both x and y axes and magnitude of the vector is 2 units.

$$\Rightarrow \quad |\vec{r}| = 2$$

$$\Rightarrow \quad \sqrt{a^2 + b^2} = 2$$

$$\Rightarrow \quad a^2 + b^2 = 4 \quad \ldots(1)$$

$\because$ Vector $\vec{r} = a\hat{i} + b\hat{j}$ is equally inclined to both x and y axes.

Let θ be the angle between the $\dfrac{(a\hat{i} + b\hat{j}).\hat{i}}{\sqrt{a^2 + b^2}.(1)}$ and both x and y axis.

$$\Rightarrow \quad \cos\theta = \frac{(a\hat{i} + b\hat{j}).\hat{i}}{\sqrt{a^2 + b^2}.(1)} = \frac{(a\hat{i} + b\hat{j}).\hat{j}}{\sqrt{a^2 + b^2}.(1)}$$

$$\Rightarrow \quad \cos\theta = \frac{a}{2} = \frac{b}{2}$$

$$\Rightarrow \quad a = b$$

Put $a = b$ in the equation (1), we get

$$a^2 + a^2 = 4$$

$$\Rightarrow \quad 2a^2 = 4$$

$$\Rightarrow \quad a = \pm\sqrt{2}$$

$$\because \quad a = b$$

$$\therefore \quad b = \pm\sqrt{2}$$

So, value of a & b are $\pm\sqrt{2}$ & $\pm\sqrt{2}$ respectively.

Hint:

- Magnitude of vector $\vec{r} = x\hat{i} + y\hat{j}$ is $|\vec{r}| = \sqrt{x^2 + y^2}$.
- If angle between two vectors $\vec{a}$ and $\vec{b}$ is θ then $\cos\theta = \dfrac{a \cdot b}{|a||b|}$

72. Option (c) is correct.

Explanation:

Given: $\vec{c} = \vec{a} + \vec{b}$, $|\vec{a}| = |\vec{b}| \neq 0$

Statement-1: As we know if $\vec{p}$ and $\vec{q}$ are perpendicular, then $\vec{p} \cdot \vec{q} = 0$.

$$\text{Now,} \quad \vec{c} \cdot (\vec{a} - \vec{b}) = (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$$

$$= \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b}$$

$$= |\vec{a}|^2 - \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} - |\vec{b}|^2$$

$$= |\vec{a}|^2 - |\vec{b}|^2$$

$$= 0 \qquad [\because |\vec{a}| = |\vec{b}|]$$

$\Rightarrow \vec{c}$ is perpendicular to $(\vec{a} - \vec{b})$.

So, statement 1 is true.

Statement-2:

$$\text{Now,} \quad \vec{c} \cdot (\vec{a} \times \vec{b}) = (\vec{a} + \vec{b}) \cdot (\vec{a} \times \vec{b})$$

$$= \vec{a} \cdot (\vec{a} \times \vec{b}) + \vec{b} \cdot (\vec{a} \cdot \vec{b})$$

$$= [\vec{a} \quad \vec{a} \quad \vec{b}] + [\vec{b} \quad \vec{a} \quad \vec{b}]$$

$$= 0 + 0$$

$$= 0$$

$\Rightarrow \vec{c}$ is perpendicular to $(\vec{a} \times \vec{b})$.

So, statement 2 is also true.

Hint:

- If $\vec{p}$ and $\vec{q}$ are perpendicular, then $\vec{p} \cdot \vec{q} = 0$.
- Use $\vec{x} \cdot \vec{x} = |\vec{x}|^2$ and $\vec{x} \cdot \vec{y} = \vec{y} \cdot \vec{x}$.
- Use $\vec{a} \cdot (\vec{b} \times \vec{c}) = [\vec{a} \quad \vec{b} \quad \vec{c}]$

73. Option (c) is correct.

Explanation:

$$\textbf{Given:} \quad |\vec{a} + \vec{b}| = |\vec{a} - \vec{b}| = 4$$

$$\Rightarrow \quad |\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

$$\Rightarrow \quad (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) \quad [\because |\vec{x}|^2 = \vec{x} \cdot \vec{x}]$$

$$\Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} = \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b}$$

$$\Rightarrow |\vec{a}|^2 + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} + |\vec{b}|^2 = |\vec{a}|^2 - \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{b} + |\vec{b}|^2$$

$$\Rightarrow \quad 4\vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \quad \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \quad \vec{a} \perp \vec{b}$$

Hint:

- Squaring both the sides of given equation and simplify using $|\vec{x}|^2 = \vec{x}\cdot\vec{x}$ and $\vec{x}\cdot\vec{y} = \vec{y}\cdot\vec{x}$.
- If two vector $\vec{a}$ and $\vec{b}$ are perpendicular then $\vec{a}\cdot\vec{b} = 0$.

74. Option (c) is correct.

Explanation:

Given: $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar.

$\Rightarrow \left[\vec{a} \quad \vec{b} \quad \vec{c}\right] = 0$

Let $\quad A = (2\vec{a}\times 3\vec{b})\cdot 4\vec{c} + (5\vec{b}\times 3\vec{c})\cdot 6\vec{a}$

$\Rightarrow \quad A = \left[2\vec{a} \quad 3\vec{b} \quad 4\vec{c}\right] + \left[5\vec{b} \quad 3\vec{c} \quad 6\vec{a}\right]$

$\left\{\because (\vec{p}\times\vec{q})\cdot\vec{r} = \left[\vec{p} \quad \vec{q} \quad \vec{r}\right]\right\}$

$\Rightarrow \quad A = (2\times 3\times 4)\left[\vec{a}\ \vec{b}\ \vec{c}\right] + (5\times 3\times 6)\left[\vec{b}\ \vec{c}\ \vec{a}\right]$

$\Rightarrow \quad A = 0 + 0 \qquad \left[\because [\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = 0\right]$

Hint:

- If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar, then $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$.
- Use $(\vec{a}\times\vec{b})\cdot\vec{c} = [\vec{a}\ \vec{b}\ \vec{c}]$.

75. Option (d) is correct.

Explanation:

Statement-1: As we know the cross product of two vectors $\vec{p}$ and $\vec{q}$ is given by $\vec{p}\times\vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$.

Let $\quad |\vec{a}| = |\vec{b}| = 1$

$\Rightarrow \quad |\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

$\Rightarrow \quad |\vec{a}\times\vec{b}| = \sin\theta$

As we know the range of $\sin\theta$ is $[-1, 1]$.

So, it is not necessarily true that the cross product of two unit vectors is always a unit vector.

So, statement 1 is false.

Statement-2:

Let, $\vec{a}$ and $\vec{b}$ are two unit vectors.

$\Rightarrow \quad |\vec{a}| = 1\ \&\ |\vec{b}| = 1$

As we know the dot product of two vector $\vec{p}$ and $\vec{q}$ is given by $\vec{p}\cdot\vec{q} = |\vec{p}||\vec{q}|\cos\theta$.

$\therefore \quad \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow \quad |\vec{a}\cdot\vec{b}| = \cos\theta$

$\because \quad -1 \le \cos\theta \le 1$

So, it is not necessarily true that dot product of two vectors is always a unit vector.

So, statement 2 is false.

Statement-3:

Let $\vec{a}$ and $\vec{b}$ are two unit vectors.

$\Rightarrow \quad |\vec{a}| = |\vec{b}| = 1$

Now, $|\vec{a}+\vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}}$

$\Rightarrow \quad |\vec{a}+\vec{b}| = \sqrt{1+1+2\vec{a}\cdot\vec{b}}$

$\Rightarrow \quad |\vec{a}+\vec{b}| = \sqrt{2}\sqrt{1+\vec{a}\cdot\vec{b}} \qquad \ldots(1)$

Now, $|\vec{a}-\vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}}$

$\Rightarrow \quad |\vec{a}-\vec{b}| = \sqrt{1+1-2\vec{a}\cdot\vec{b}}$

$\Rightarrow \quad |\vec{a}-\vec{b}| = \sqrt{2}\sqrt{1-\vec{a}\cdot\vec{b}} \qquad \ldots(2)$

If angle between vectors $\vec{a}$ and $\vec{b}$ is 90°, then $\vec{a}\cdot\vec{b} = 0$.

$\Rightarrow \quad |\vec{a}+\vec{b}| = |\vec{a}-\vec{b}| = \sqrt{2}$

So, the magnitude of sum of two unit vector is not always greater than the magnitude of their difference.

So, statement 3 is also false.

Hint:

- The cross product of two vectors $\vec{p}$ and $\vec{q}$ is given by $\vec{p}\times\vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$.
- The dot product of two vectors $\vec{p}$ and $\vec{q}$ is given by $\vec{p}\cdot\vec{q} = |\vec{p}||\vec{q}|\cos\theta$.
- $|\vec{p}+\vec{q}| = \sqrt{|\vec{p}|^2 + |\vec{q}|^2 + 2\vec{p}\cdot\vec{q}}$

  $|\vec{p}-\vec{q}| = \sqrt{|\vec{p}|^2 + |\vec{q}|^2 - 2\vec{p}\cdot\vec{q}}$

76. Option (c) is correct.

Explanation:

Given: $\lim\limits_{x\to a}\dfrac{a^x - x^a}{x^a - a^a} = -1$

LHS limit has $\dfrac{0}{0}$ indeterminate form.

As we know if $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}$ has $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$ indeterminate form, then $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)} = \lim\limits_{x\to a}\dfrac{f'(x)}{g'(x)}$

$$\therefore \quad \lim_{x\to a}\frac{\frac{d}{dx}(a^x - x^a)}{\frac{d}{dx}(x^a - a^a)} = -1$$

$$\Rightarrow \quad \lim_{x\to a}\frac{a^x \log_e a - ax^{a-1}}{ax^{a-1} - 0} = -1$$

$$\left[\because \frac{d}{dx}(a^x) = a^x \log_e a,\ \frac{d}{dx}(x^n) = nx^{n-1}\right]$$

$$\Rightarrow \quad \lim_{x\to a}\left[\frac{a^x \log_e a}{ax^{a-1}} - 1\right] = -1$$

$$\Rightarrow \quad \lim_{x\to a}\frac{a^x \log_e a}{ax^{a-1}} = -1 + 1 = 0$$

$$\Rightarrow \quad \frac{a^a \log_e a}{a \cdot a^{a-1}} = 0$$

$$\Rightarrow \quad \log_e a = 0$$

$$\Rightarrow \quad a = 1$$

Hint:

- If $\lim_{x\to a}\frac{f(x)}{g(x)}$ has $\frac{0}{0}$ or $\frac{\infty}{\infty}$ indeterminate form then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$.
- Use $\frac{d}{dx}(x^n) = nx^{n-1}$, $\frac{d}{dx}(a^x) = a^x \log_e a$.

77. Option (c) is correct.

Explanation:

Given: $\frac{dx}{dt} = x + 1$

$$\Rightarrow \quad \frac{dx}{x+1} = dt$$

Integrating both sides of above equation, we get

$$\Rightarrow \quad \int \frac{dx}{x+1} = \int dt$$

$\Rightarrow$ $\ln(x + 1) = t + C$

$\because$ At $t = 0$, $x = 0$

$\therefore$ $\ln(1) = 0 + C$

$\Rightarrow$ $C = 0$

$\therefore$ $\ln(x + 1) = t$

Now, at $x = 24$ m

$t = \ln(24 + 1)$

$\Rightarrow$ $t = \ln 25$

$\Rightarrow$ $t = \ln 5^2$

$\Rightarrow$ $t = 2 \ln 5$

Hint:

- Use $\frac{dx}{x+1} = dt$, integrating both the sides and solve further using the basic formulae of indefinite integration from definion.

78. Option (d) is correct.

Explanation:

Let $$I = \int_0^a \frac{f(a-x)}{f(x)+f(a-x)}dx \quad \ldots(i)$$

As we know $\int_0^b g(x)dx = \int g(b \quad x)dx$

$$\therefore \quad I = \int_0^a \frac{f(a-(a-x))}{f(a-x)+f(a-(a-x))}dx$$

$$\Rightarrow \quad I = \int_0^a \frac{f(x)}{f(a-x)+f(x)}dx \quad \ldots(ii)$$

Adding equation (i) & equation (ii), we get

$$2I = \int \frac{f(x)+f(a-x)}{f(x)+f(a-x)}dx$$

$$\Rightarrow \quad 2I = \int_0^a 1\cdot dx$$

$$\Rightarrow \quad 2I = [x]_0^a$$

$$\Rightarrow \quad 2I = a - 0 = a$$

$$\Rightarrow \quad I = \frac{a}{2}$$

Hint:

- Use $\int_0^b g(x)dx = \int_0^b g(b-x)dx$.
- If $f(x) = F(x)$, then $\int_a^b f(x)dx = F(b) - F(a)$.

79. Option (b) is correct.

Explanation:

Let $$L = \lim_{x\to -1}\frac{x^3 + x^2}{x^2 + 3x + 2}$$

Given limit has $\frac{0}{0}$ indeterminate form.

As we know if $\lim_{x\to a}\frac{f(x)}{g(x)}$ has $\frac{0}{0}$ or $\frac{\infty}{\infty}$ indeterminate form, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

$$\therefore \quad L = \lim_{x\to -1}\frac{\frac{d}{dx}(x^3 + x^2)}{\frac{d}{dx}(x^2 + 3x + 2)}$$

$\Rightarrow \quad L = \lim_{x\to -1}\frac{3x^2+2x}{2x+3} \quad \left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$

$\Rightarrow \quad L = \frac{3(-1)^2+2(-1)}{2(-1)+3}$

$\Rightarrow \quad L = \frac{3-2}{-2+3} = 1$

Shortcut:

Let $L = \lim_{x\to -1}\frac{x^3+x^2}{x^2+3x+2} \quad \left[\frac{0}{0} \text{ form}\right]$

$\Rightarrow \quad L = \lim_{x\to -1}\frac{3x^2+2x}{2x+3}$

[Applying L'hospital rule]

$\Rightarrow \quad L = 1$

Hint:

- If $\lim_{x\to a}\frac{f(x)}{g(x)}$ has $\frac{0}{0}$ or $\frac{\infty}{\infty}$ indeterminate form then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$.
- Use $\frac{d}{dx}(x^n) = nx^{n-1}$

80. Option (a) is correct.

Explanation:

Given: $\int_0^a [f(x)+f(-x)]dx = \int_{-a}^a g(x)dx$

Let $\quad I = \int_{-a}^a g(x)dx$

$\Rightarrow \quad I = \int_{-a}^0 g(x)dx + \int_0^a g(x)dx$

$\left[\because \int_a^b h(x)dx = \int_a^c h(x)dx + \int_c^b h(x)dx;\ a<c<b\right]$

Put $x = -t$ in first integral, we get

$I = \int_a^0 -g(-t)dt + \int_0^a g(x)dx$

$\Rightarrow \quad I = \int_0^a g(-t)dt + \int_0^a g(x)dx$

$\left[\because \int_a^b h(x)dx = -\int_b^a h(x)dx\right]$

$\Rightarrow \quad I = \int_0^a g(-x)dx + \int_0^a g(x)dx$

$\left[\because \int_a^b h(x)dx = \int_a^b h(t)dt\right]$

$\Rightarrow \quad I = \int_0^a [g(x)+g(-x)]dx$

$\therefore \int_0^a [f(x)+f(-x)]dx = \int_0^a [g(x)+g(-x)]dx$

$\Rightarrow \quad g(x) = f(x)$

Hint:

- Use properties of definite integration
  1. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx;\ a<c<b$
  2. $\int_a^b f(x)dx = -\int_b^a f(x)dx$
  3. $\int_a^b f(x)dx = \int_a^b f(t)dt$

81. Option (b) is correct.

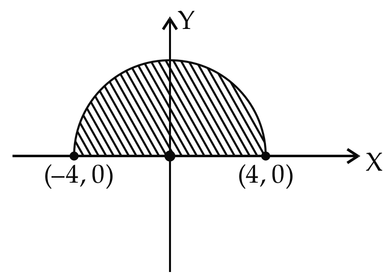
Explanation:

Given: $y = \sqrt{16-x^2}$

$\Rightarrow \quad y^2 = 16 - x^2$

$\Rightarrow \quad x^2 + y^2 = 16$

$\Rightarrow$ It represents circle of radius 4 units with centre at (0, 0).



So, Area bounded by $y = \sqrt{16-x^2}$, $y \geq 0$ and the x-axis is $A = \frac{1}{2}[\pi(4)^2]$.

[$\because$ Area of circle of radius 'r' is πr^2]

$\Rightarrow \quad A = \frac{1}{2}\pi(16) = 8\pi$ sq. units

Hint:

- Draw the graph of curve & identify the required region and use area of circle is πr^2, where 'r' is the radius of circle.

82. Option (c) is correct.

Explanation:

Given: $y = -x^3 + 3x^2 + 2x - 27$

As we know slope of the curve $y = f(x)$ is $\frac{dy}{dx}$.

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}(-x^3 + 3x^2 + 2x - 27)$

$\Rightarrow \quad \frac{dy}{dx} = -3x^2 + 6x + 2$

$$\left[\because \frac{d}{dx}(x^n) = nx^{n-1}\right]$$

As we know quadratic expression $ax^2 + bx + c$;

$a < 0$ has maximum value at $x = -\frac{b}{2a}$.

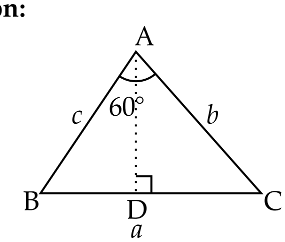
$\therefore \frac{dy}{dx}$ has maximum value of $x = \frac{-6}{2(-3)} = 1$

Hint:

- Slope of the curve $y = f(x)$ is $\frac{dy}{dx}$.
- Quadratic expression $f(x) = ax^2 + bx + c$; $a < 0$ has maximum value at $x = -\frac{b}{2a}$.

83. Option (a) is correct.

Explanation:



Perimeter of $\Delta ABC = 24$

$\Rightarrow a + b + c = 24$

By Heron's formula,

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{a+b+c}{2}$

For maximum area, $s(s - a)(s - b)(s - c)$ should be maximum. Since, s is a constant (because the perimeter is fixed). So, for maximum area, $(s - a)(s - b)(s - c)$ should be maximum. Now, $(s - a) + (s - b) + (s - c) = 3s - (a + b + c) = 36 - 24 = 12$ = constant. As we know by AM-GM inequality, a product xyz is maximum, if sum $x + y + z$ is a constant, when $x = y = z$.

$\therefore$ For maximum area,

$$s - a = s - b = s - c$$

$\Rightarrow a = b = c$

$\Rightarrow a = b = c = 8 \quad [\because a + b + c = 24]$

$\Rightarrow \Delta ABC$ is equilateral triangle.

$\Rightarrow \angle A = \angle B = \angle C = 60°$

Now, length of altitude $= c \sin 60°$

$\Rightarrow$ Length of altitude $= 8 \cdot \frac{\sqrt{3}}{2} = 4\sqrt{3}$ cm

Hint:

- Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$; where $s = \frac{a+b+c}{2}$, & sides of Δ are a, b, c.
- Use if $x + y + z$ = constant, then xyz is maximum when $x = y = z$.

84. Option (d) is correct.

Explanation:

Given: $f(x) = e^{|x|}$

Let us generalize the $f(x)$ using the definition of modulus function.

So, $$f(x) = \begin{cases} e^x; & x \geq 0 \\ e^{-x}; & x < 0 \end{cases}$$

Differentiate the $f(x)$ w.r.t. x, we get

$$f'(x) = \begin{cases} e^x; & x \geq 0 \\ e^{-x}\frac{d}{dx}(-x); & x < 0 \end{cases}$$

$$\left[\because \frac{d}{dx}[f(g(x)) = f'(g(x)) \cdot g'(x)\right]$$

$$\Rightarrow f'(x) = \begin{cases} e^x; & x \geq 0 \\ -e^{-x}; & x < 0 \end{cases}$$

Let us check differentiability of $f(x)$ at $x = 0$.

Now, L.H.D. $= f'(0^-) = -e^{-0} = -1$

R.H.D. $= f'(0^+) = e^0 = 1$

$\because$ L.H.D. $\neq$ R.H.D.

$\therefore f'(0)$ does not exist.

Hint:

- Use $|x| = \begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$
- Function $f(x)$ is differentiable at $x = a$ if L.H.D. = R.H.D. = Finite at $x = a$.
- $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

85. Option (d) is correct.

Explanation:

Let $$I = \int \frac{dx}{\sec x + \tan x}$$

$$\Rightarrow I = \int \frac{dx}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}$$

$$\Rightarrow I = \int \frac{\cos x}{1 + \sin x} dx$$

Let $1+\sin x = t \Rightarrow \cos x dx = dt$

$\therefore \quad I = \int \frac{1}{t} dt$

$\Rightarrow \quad I = \ln|t| + C$

$\Rightarrow \quad I = \ln|1+\sin x| + C$

$\Rightarrow \quad I = \ln\left|\frac{1+\sin x}{\cos x}\cdot \cos x\right| + C$

$\Rightarrow \quad I = \ln\left|(\sec x + \tan x)\cdot \cos x\right| + C$

$\Rightarrow \quad I = \ln\left|(\sec x + \tan x)\right| + \ln|\cos x| + C$

$\Rightarrow \quad I = \ln|\sec x + \tan x| + \ln\left|\frac{1}{\sec x}\right| + C$

$\Rightarrow \quad I = \ln|\sec x + \tan x| - \ln|\sec x| + C$

Hint:
- Simplify the given integral using trigonometric identities $\sec\theta = \frac{1}{\cos\theta}$ & $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and then solve further using substitution method.
- Use $\ln(ab) = \ln a + \ln b$ and $\ln a^b = b\ln a$.

86. Option (b) is correct.

Explanation:

Let $\quad I = \int \frac{dx}{\sec^2(\tan^{-1} x)}$

As we know $1 + \tan^2\theta = \sec^2\theta$

$\therefore \quad I = \int \frac{dx}{1+\tan^2(\tan^{-1} x)}$

$\Rightarrow \quad I = \int \frac{dx}{1+[\tan(\tan^{-1} x)]^2}$

$\Rightarrow \quad I = \int \frac{dx}{1+x^2} \quad [\because \tan(\tan^{-1}x) = x]$

$\Rightarrow \quad I = \tan^{-1}x + C$

Hint:
- Simplify using $1 + \tan^2\theta = \sec^2\theta$.
- Use $\tan(\tan^{-1}x) = x$.
- Recall basic formulae of indefinite integration from definition.

87. Option (a) is correct.

Explanation:

Given: $\quad x + y = 20$

$P = xy$

As we know, A.M. ≥ G.M.

$\Rightarrow \quad \frac{x+y}{2} \geq \sqrt{xy}$

$\Rightarrow \quad \frac{20}{2} \geq (P)^{1/2}$

$\Rightarrow \quad 10 \geq (P)^{1/2}$

$\Rightarrow \quad P \leq 100$

$\therefore$ Maximum value of P is 100.

Hint:
- Use A.M. ≥ G.M.

88. Option (a) is correct.

Explanation:

Let $\quad f(x) = \sin(\ln x) + \cos(\ln x)$

$\Rightarrow \quad f'(x) = \frac{d}{dx}[\sin(\ln x) + \cos(\ln x)]$

$\Rightarrow \quad f'(x) = \frac{d}{dx}\{\sin(\ln x)\} + \frac{d}{dx}\{\cos(\ln x)\}$

$\Rightarrow \quad f'(x) = \cos(\ln x)\frac{d}{dx}(\ln x) - \sin(\ln x)\frac{d}{dx}(\ln x)$

$\left\{\because \frac{d}{dx}[f(g(x))] = f'(g(x))\cdot g'(x)\right\}$

$\Rightarrow \quad f'(x) = \cos(\ln x)\left(\frac{1}{x}\right) - \sin(\ln x)\cdot\left(\frac{1}{x}\right)$

$\Rightarrow \quad f'(x) = \frac{\cos(\ln x) - \sin(\ln x)}{x}$

Put $x = e$ in the above equation, we get

$f'(e) = \frac{\cos(\ln e) - \sin(\ln e)}{e}$

$\Rightarrow \quad f'(e) = \frac{\cos 1 - \sin 1}{e} \quad [\because \ell ne = 1]$

Hint:
- Use $\frac{d}{dx}[f(g(x))] = f'(g(x))\cdot g'(x)$
- Use $\frac{d}{dx}(\sin x) = \cos x$, $\frac{d}{dx}(\cos x) = -\sin x$ & $\frac{d}{dx}(\ln x) = \frac{1}{x}$

89. Option (b) is correct.

Explanation:

Given: $\quad x = e^t \cos t$ & $y = e^t \sin t$

Now, $\quad \frac{dx}{dy} = \frac{dx/dt}{dy/dt}$

Now, $\frac{dx}{dt} = \frac{d}{dt}[e^t \cos t]$

$$\Rightarrow \frac{dx}{dt} = e^t \frac{d}{dt}(\cos t) + \cos t \frac{d}{dt} e^t$$

$$[\because (uv)' = u'v + v'u]$$

$$\Rightarrow \frac{dx}{dt} = -e^t \sin t + \cos t \cdot e^t$$

$$\Rightarrow \frac{dx}{dt} = e^t(\cos t - \sin t)$$

Now, $\frac{dy}{dt} = \frac{d}{dt}[e^t \sin t]$

$$\Rightarrow \frac{dy}{dt} = e^t \frac{d}{dt}(\sin t) + \sin t \frac{d}{dt}(e^t)$$

$$\Rightarrow \frac{dy}{dt} = e^t \cos t + \sin t \cdot e^t$$

$$\Rightarrow \frac{dy}{dt} = e^t(\sin t + \cos t)$$

$$\therefore \frac{dx}{dy} = \frac{e^t(\cos t - \sin t)}{e^t(\sin t + \cos t)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{\cos t - \sin t}{\sin t + \cos t}$$

$$\Rightarrow \left(\frac{dx}{dy}\right)_{t=0} = \frac{\cos 0 - \sin 0}{\sin 0 + \cos 0} = 1$$

Hint:
- Use $\frac{dx}{dy} = \frac{dx/dt}{dy/dt}$
- Use product rule of differentiation $(uv)' = uv' + vu'$.

90. Option (a) is correct.

Explanation:

Let $f(x) = \sin 2x \cdot \cos 2x$

$$\Rightarrow f(x) = \frac{1}{2} \times (2 \sin 2x \cdot \cos 2x)$$

$$\Rightarrow f(x) = \frac{1}{2} \sin 4x [\because \sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha]$$

As we know the range of $\sin x$ is [–1, 1]

$$\therefore [f(x)]_{\max} = \frac{1}{2}$$

Hint:
- Simplify the given expression using trigonometric formula $\sin 2\alpha = 2 \sin\alpha \cdot \cos\alpha$.
- Range of $\sin x$ is [–1, 1].

91. Option (a) is correct.

Explanation:

$$\frac{d(e^x)}{d(x^e)} = \frac{\frac{d(e^x)}{dx}}{\frac{d}{dx}(x^e)}$$

$$\Rightarrow \frac{d(e^x)}{d(x^e)} = \frac{e^x}{ex^{e-1}}$$

$$\left[\because \frac{d}{dx}(e^x) = e^x \text{ and } \frac{d}{dx}(x^n) = nx^{n-1}\right]$$

$$\Rightarrow \frac{d(e^x)}{d(x^e)} = \frac{e^x}{ex^e \cdot x^{-1}}$$

$$\Rightarrow \frac{d(e^x)}{d(x^e)} = \frac{xe^x}{ex^e}$$

Hint:
- Use $\frac{d(e^x)}{d(x^e)} = \frac{\frac{d(e^x)}{dx}}{\frac{d(x^e)}{dx}}$
- Use $\frac{d}{dx}(e^x) = e^x$ & $\frac{d}{dx}(x^n) = nx^{n-1}$

92. Option (b) is correct.

Explanation:

Given: $\lim_{x\to -1} \frac{f(x)+1}{x^2-1} = -\frac{3}{2}$

$$\Rightarrow \frac{\lim_{x\to -1}\{f(x)+1\}}{\lim_{x\to -1}\{x^2-1\}} = -\frac{3}{2}$$

$$\Rightarrow \lim_{x\to -1}\{f(x)+1\} = -\frac{3}{2}\left\{\lim_{x\to -1}(x^2-1)\right\}$$

$$\Rightarrow \lim_{x\to -1} f(x)+1 = -\frac{3}{2}\{(-1)^2-1\}$$

$$\Rightarrow \lim_{x\to -1} f(x)+1 = 0$$

$$\Rightarrow \lim_{x\to -1} f(x) = -1$$

Hint:
- Simplify the given limit using algebra of limits and solve further.

93. Option (a) is correct.

Explanation:

Given: $f(x) = \begin{cases} a+bx, & x<1 \\ 5, & x=1 \\ b-ax, & x>1 \end{cases}$

As we know if function is continuous at a point then limiting value of a function is equal to the functional value at that point.

So, for $f(x)$ to be continuous at $x = 1$

$$\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = f(1)$$

$$\Rightarrow \lim_{x\to1^-}(a+bx) = \lim_{x\to1^+}(b-ax) = 5$$

$$\Rightarrow a+b = b-a = 5$$

$$\therefore a+b = 5$$

Hint:
- If $g(x)$ is a continuous at a point then limiting value of $g(x)$ is equal to the functional value of $g(x)$ at that point.

94. Option (b) is correct.

Explanation:

Given: $f(x) = \sin x$

$\Rightarrow \quad f'(x) = \cos x$

$\because f'(x) > 0, \forall x \in \left(0, \frac{\pi}{2}\right)$ and $f'(x) < 0, \forall x \in \left(\frac{\pi}{2}, \pi\right)$

$\therefore$ $f(x)$ is not strictly increasing function in the interval $(0, \pi)$.

So, statement-1 is false.

$\because f'(x) < 0 \ \forall x \in \left(\frac{5\pi}{2}, 3\pi\right)$

$\therefore$ $f(x)$ decreases in the interval $x \in \left(\frac{5\pi}{2}, 3\pi\right)$

So, statement 2 is true.

Hint:
- If function is increasing, then first derivative of function is positive.
- If function is decreasing, then first derivative of function is negative.

95. Option (a) is correct.

Explanation:

Given function $\quad f(x) = 3^x$

Let $\quad y = 3^x$

Take logarithm on both sides

$$\log y = \log 3^x$$

$$\Rightarrow \log y = x\log 3$$

$$\Rightarrow x = \frac{\log y}{\log 3}$$

$$\Rightarrow x = \log_3 y$$

Because $\frac{\log a}{\log b} = \log_{(b)}(a)$

From here we can clearly say that domain of $f(x) = 3^x$ is R i.e. $(-\infty, \infty)$

Hint:
- Domain for exponential function, $y = a^x$ is R where $a \in R^+$ & $a \neq 1$.
- Use properties of logarithm:

  1. $\frac{\log a}{\log b} = \log_b a$

  2. $\log a^m = m \log a$

96. Option (a) is correct.

Explanation:

Order of differential equation is equal to the number of arbitrary (independent) constants in it.

Given, solutions of differential equation is $y^2 + 2cy - cx + c^2 = 0$

Here number of arbitrary constant is 1 which is c.

$\therefore$ Order of differential equation is 1.

Hint:
- Order of differential equation is equal to the number of arbitrary constant in it.

97. Option (a) is correct.

Explanation:
- Degree of differential equation is defined when differential equation can be expressed in the form of a polynomial.
- When differential equation is expressed in form of polynomial, then degree is the power of highest order derivative in the differential equation.

Given differential equation,

$$x = \sqrt{1 + \frac{d^2y}{dx^2}}$$

Squaring both the sides, we get

$$x^2 = 1 + \frac{d^2y}{dx^2}$$

$\because$ Power of highest order derivative is 1.

$\therefore$ Degree of differential equation is 1.

Hint:

Degree of differential equation is the power of highest order derivative when differential equation express in form of polynomial.

98. Option (b) is correct.

Explanation:

Given: $y = ae^x + be^{-x}$

Differentiating both the sides of the above equation w.r.t. x,

$$\frac{dy}{dx} = \frac{d}{dx}(ae^x) + \frac{d}{dx}(be^{-x})$$

$$\Rightarrow \quad \frac{dy}{dx} = ae^x - be^{-x}$$

Again differentiating above equation w.r.t. x,

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(ae^x) - \frac{d}{dx}(be^{-x})$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = ae^x + be^{-x}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - y = 0$$

Hint:

- Use the method of eliminating the arbitrary constants by differentiation it two times (as two constants are present).

99. Option (c) is correct.

Explanation:

Given: Differential equation $\ln\left(\frac{dy}{dx}\right) + y = x$.

$$\Rightarrow \quad \ln\left(\frac{dy}{dx}\right) = x - y$$

$$\Rightarrow \quad \frac{dy}{dx} = e^{x-y}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^x}{e^y}$$

$$\Rightarrow \quad e^y dy = e^x dx$$

Integrating both the sides, we get

$$\int e^y dy = \int e^x dx$$

$$\Rightarrow \quad e^y + C_1 = e^x + C_2$$

$$\Rightarrow \quad e^x - e^y = C \qquad [\text{where } C = C_1 - C_2]$$

This is the required solution.

Hint:

- Use variable separable method to find the required solution.

100. Option (d) is correct.

Explanation:

Let $\quad I = \int e^{(2\ln x + \ln x^2)} dx$

$$\Rightarrow \quad I = \int e^{(\ln x^2 + \ln x^2)} dx \quad [\because \log_a b^m = m\log_a b]$$

$$\Rightarrow \quad I = \int e^{(\ln(x^2 \cdot x^2))} dx$$

$$[\because \log_a b + \log_a c = \log_a bc]$$

$$\Rightarrow \quad I = \int e^{\ln(x^4)} dx$$

$$\Rightarrow \quad I = \int x^4 dx \qquad [\because {}_a\log_a x = x]$$

$$\Rightarrow \quad I = \frac{x^5}{5} + C$$

Hint:

- Use principle properties of logarithm, $\log_a b^m = m\log_a b$, $\log_a b + \log_a c = \log_a bc$ and $a\log a^x = x$.

101. Option (c) is correct.

Explanation:

Arithmetic mean of n numbers is,

$$\text{A.M.} = \frac{\Sigma a_i}{n} = \frac{a_1 + a_2 + \ldots + a_n}{n}$$

Geometric mean of n numbers is,

$$\text{G.M.} = n\sqrt{\prod_{i=1}^{n} a_i} = n\sqrt{a_1 \cdot a_2 \cdot \ldots \cdot a_n}$$

So, we can clearly see that both A.M. and G.M. uses all the data.

Shortcut:

A.M. of n numbers $= \frac{\Sigma a_i}{n}$

G.M. of n numbers $= n\sqrt{\Pi a_i}$

So, both uses all the data.

Hint:
- Recall the formula of A.M. of n numbers and G.M. of n numbers.

102. Option (b) is correct.

Explanation:

Given:

	Science	Arts	Commerce
No. of graduates	30	70	50

Total number of graduates = 30 + 70 + 50
= 150

Now, when these are represents on pie chart, then 150 graduates are represented by 360°

1 graduate is represented by $\frac{360^\circ}{150} = \frac{12^\circ}{5}$

30 graduates are represented by $\frac{12}{5} \times 30^\circ = 72^\circ$

Hint:
- Recall the concept of representation of data on a pie chart.

103. Option (b) is correct.

Explanation:

In a histogram, it is the area of the bar that denotes the value. While constructing a histogram with non-uniform (unequal) class intervals (widths), we must ensure that the area of the rectangles are proportional to the class frequency.

⇒ The frequency of a class should be proportional to the area of the rectangle.

Shortcut:
From the definition of histogram, the frequency of a class is proportional to the area of rectangle and not its height.

Hint:
- Recall the properties of histogram.

104. Option (c) is correct.

Explanation:

In simple regression analysis, the coefficient of correlation, is a statistic which indicates an association between the independent variables and dependent variables.

By the properties of coefficient of correlation.

1. The coefficient of correlation lies between –1 and 1.
2. The coefficient of correlation is independent of change of scale, which means some value is multiplied or divided to observations.
3. The coefficient of correlation is independent of change of origin of the variable X and Y, which means some value has been added or subtracted in observations. So, it is independent of both change of scale and change of origin.

Hint:
- Recall the properties of coefficient of correlation.

105. Option (a) is correct.

Explanation:

Given: $\Sigma f_i = 198$

For odd Σf, median = $\frac{n+1}{2}$

For even Σf, median = $\frac{n}{2}$

$$\text{Median} = \frac{198}{2} = 99$$

99 is closest to frequency 76 which is for 3 number of peas.

∴ Median = 3.

Hint:
- Recall the definition of median of frequency distribution.

106. Option (b) is correct.

Explanation:

Given = M is mean of $x_1, -k, x_2 - k, \ldots, x_n - k$

$$\Rightarrow \quad M = \frac{\sum_{i=1}^{n}(x_i - k)}{n}$$

$$\Rightarrow \quad M = \frac{(x_1 - k) + (x_2 - k) + \ldots + (x_n - k)}{n}$$

$$\Rightarrow \quad M = \frac{x_1 + x_2 + \ldots + x_n}{n} - \frac{nk}{n}$$

$$\Rightarrow \quad M = M' - k$$

Now, required mean = $M' = \frac{x_1 + x_2 + \ldots + x_n}{n}$

$$\Rightarrow \quad M' = M + k$$

Hint:
- Use mean = $\frac{\text{Sum of observations}}{\text{No. of observations}}$

107. Option (c) is correct.

Explanation:

Given: Values are 73, 85, 92, 105, 120

$$\text{Mean} = \bar{x} = \frac{73+85+92+105+120}{5}$$

$$\Rightarrow \quad \bar{x} = \frac{475}{5}$$

$$\Rightarrow \quad \bar{x} = 95$$

$$\text{Required value} = \sum_{i=1}^{5}(x_i - \bar{x})$$

$$= (x_1 - \bar{x}) + (x_2 - \bar{x}) + (x_3 - \bar{x}) + (x_4 - \bar{x}) + (x_5 - \bar{x})$$

$$= (73-95)+(85-95)+(92-95)+(105-95)+(120-95)$$

$$= -22 - 10 - 3 + 10 + 25$$

$$= 0$$

Shortcut:

Sum of deviation from mean is zero.

Hint:

- Use mean $= \bar{x} = \frac{\Sigma x_i}{n}$.

108. Option (d) is correct.

Explanation:

Given: H.M. of m & $n = x$

G.M. of m & $n = y$

$$5x = 4y$$

$$\Rightarrow \quad \frac{x}{y} = \frac{4}{5}$$

$\because$ H.M. of m & $n = \frac{2mn}{m+n} = x$.

and G.M. of m & $n = \sqrt{mn} = y$

$$\frac{x}{y} = \frac{2mn}{(m+n)\sqrt{mn}}$$

$$\Rightarrow \quad \frac{4}{5} = \frac{2mn}{(m+n)\sqrt{mn}}$$

$$\Rightarrow \quad \frac{2}{5} = \frac{\sqrt{mn}}{m+n}$$

$$\Rightarrow \left(\frac{m}{\sqrt{mn}} + \frac{n}{\sqrt{mn}}\right) = \frac{5}{2}$$

$$\Rightarrow \quad \sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \frac{5}{2}$$

Let $\quad \sqrt{\frac{m}{n}} = t$

$$\Rightarrow \quad t + \frac{1}{t} = \frac{5}{2}$$

$$\Rightarrow \quad 2t^2 - 5t + 2 = 0$$

$$\Rightarrow \quad 2t^2 - 4t - t + 2 = 0$$

$$\Rightarrow \quad (t-2)(2t-1) = 0$$

$$\Rightarrow \quad t = 2 \text{ or } \frac{1}{2}$$

$$\Rightarrow \quad \sqrt{\frac{m}{n}} = 2 \text{ or } \frac{1}{2}$$

$$\Rightarrow \quad \frac{m}{n} = 4 \text{ or } \frac{1}{4}$$

$$\Rightarrow \quad m = 4n \text{ or } n = 4m$$

Hint:

- Use harmonic mean of x & $y = \frac{2xy}{x+y}$ and geometric mean of x and $y = \sqrt{xy}$.

109. Option (a) is correct.

Explanation:

Given: Mean = 100

Coefficient of variance = 45%

$$\because \quad CV = \frac{\sigma}{\mu}$$

[σ = standard deviation, μ = mean]

$$\sigma = C.V. \times \mu$$

$$\Rightarrow \quad \sigma = \frac{45}{100} \times 100$$

$$\Rightarrow \quad \sigma = 45$$

$$\text{Variance} = \sigma^2 = (45)^2$$

$$= 2025$$

Hint:

- Use coefficient of variance $= \frac{\text{Standard deviation}}{\text{Mean}}$

110. Option (c) is correct.

Explanation:

Given: $P(A) = L, P(B) = M$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

By addition theorem of probability

$$P\left(\frac{A}{B}\right) = \frac{P(A) + P(B) - P(A \cup B)}{P(B)}$$

By Axiomatic approach of probability,

$$P(A \cup B) \in [0, 1]$$

$$\Rightarrow \quad P(A \cup B) \leq 1$$

$$\Rightarrow \quad P\left(\frac{A}{B}\right) \geq \frac{L+M-1}{M}$$

Hint:

- Use conditional probability, $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$ and addition theorem of probability, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- Use the Axiomatic approach of probability, probability lies between 0 and 1.

111. Option (b) is correct.

Explanation:

As we know, the mean or average of a data set is found by adding all the numbers in the data set and then dividing by number of values in the set.

$$\bar{x} = \frac{\Sigma x_i}{n}$$

Median is the middle value when a data set is in ordered form (least to greatest)

Mode is the number that occurs most often in the data set.

(1) 12, 12, 12, 12, 24

$$\text{Mean} = \frac{12+12+12+12+24}{5} = \frac{72}{5}$$

Median = 12

Mode = 12

(2) 6, 18, 18, 18, 30

$$\text{Mean} = \frac{6+18+18+18+30}{5} = \frac{90}{5} = 18$$

Median = 3^{rd} value = 18

Mode = 18

Mean = Mode = Median

Hint:

- Mean is found by adding all the numbers of data set and then dividing by number of values in the set.
- Median is the middle value when a data set is in ascending order.
- Mode is the number that occurs most often in data set.

112. Option (b) is correct.

Explanation:

Given: Mean of 12 observations = 75

Let two discarded observations be a and b.

$$\bar{x} = \frac{\Sigma x_i}{12}$$

$$\Rightarrow \quad 75 = \frac{\Sigma x_i}{12}$$

$$\Rightarrow \quad \Sigma x_i = 75 \times 12$$

Mean after observation a and b are discarded = 65.

$$\Rightarrow \quad 65 = \frac{\Sigma x_i - (a+b)}{10}$$

$$\Rightarrow \quad \Sigma x_i - (a+b) = 650$$

$$\Rightarrow \quad (a+b) = (75 \times 12) - 650$$

$$\Rightarrow \quad a + b = 250$$

$$\text{Mean of } a \,\&\, b = \frac{a+b}{2} = \frac{250}{2}$$

$$\Rightarrow \quad \text{Mean} = 125$$

Hint:

- Use mean = $\bar{x} = \frac{\Sigma x_i}{n}$ where n is number of observations.

113. Option (c) is correct.

Explanation:

Given: k is one root of $x(x+1)+1=0$

$$\Rightarrow \quad x^2 + x + 1 = 0$$

By Sridharacharya rule, roots of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$\Rightarrow \quad x = \frac{-1 \pm i\sqrt{3}}{2}$$

Let $k = \frac{-1 + i\sqrt{3}}{2}$ and $\beta = \frac{-1 - \sqrt{3}i}{2}$

$$k^2 = \frac{(-1 + i\sqrt{3})^2}{4}$$

$$\Rightarrow \quad k^2 = \frac{1 - 3 - 2\sqrt{3}i}{4}$$

$$\Rightarrow \quad k^2 = \frac{-2 - 2\sqrt{3}i}{4}$$

$$\Rightarrow \quad k^2 = \frac{-1 - \sqrt{3}i}{2}$$

$$\Rightarrow \quad k^2 = \beta$$

Shortcut:

$$x(x+1)+1=0$$

$$\Rightarrow \quad x^2+x+1=0$$

$$\Rightarrow \quad x=\frac{-1\pm\sqrt{1-4}}{2}$$

$$=\frac{-1\pm i\sqrt{3}}{2}$$

$\Rightarrow$ Roots are ω and ω^2

Let $k=\omega^2$ and $\beta=\omega$

$$k^2=(\omega^2)^2=\omega^3\omega=\omega$$

$$k^2=\beta$$

Hint:

- Roots of $ax^2+bx+c=0$ are

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

114. Option (c) is correct.

Explanation:

Let number of observation = n

$$\text{G.M.} = \sqrt[n]{x_1\cdot x_2\ldots x_n}$$

$$\Rightarrow \quad 10=\sqrt[n]{x_1\cdot x_2\cdot\ldots x_n}$$

New observations are $3x_1^4, 3x_2^4, 3x_3^4, \ldots 3x_n^4$

$$\text{New G.M.} = \sqrt[n]{3x_1^4\cdot 3x_2^4\cdot 3x_3^4\cdot\ldots\cdot 3x_n^4}$$

$$=\sqrt[n]{(3)^n\cdot(x_1\cdot x_2\cdot x_3\cdot\ldots\cdot x_n)^4}$$

$$=3\left(\sqrt[n]{x_1\cdot x_2\cdot x_3\cdot\ldots .x_n}\right)^4$$

$$=3(10)^4$$

$$=30000$$

Hint:

- Use $\text{G.M.} = \sqrt[n]{x_1\cdot x_2\cdot\ldots\cdot x_n}$

115. Option (a) is correct.

Explanation:

$$\text{Given: } P(A\cup B)=\frac{5}{6}$$

$$P(A\cap B)=\frac{1}{3}$$

$$P(\ \bar{}\ )=\frac{1}{2}$$

By complement rule of probability,

$$P(A)=1-P(\bar{A})$$

$$\Rightarrow \quad P(A)=1-\frac{1}{2}=\frac{1}{2}$$

By addition theorem of probability,

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$\Rightarrow \quad \frac{5}{6}=\frac{1}{2}+P(B)-\frac{1}{3}$$

$$\Rightarrow \quad P(B)=\frac{5}{6}-\frac{1}{2}+\frac{1}{3}=\frac{5-3+2}{6}=\frac{4}{6}=\frac{2}{3}$$

Statement 1: A and B are independent events.

As we know, if $P(A\cap B)=P(A)\cdot P(B)$ then A and B are independent events.

$$P(A)\cdot P(B)=\frac{1}{2}\times\frac{2}{3}=\frac{1}{3}=P(A\cap B)$$

$\therefore$ A and B are independent events.

So, statement 1 is correct.

Statement-2: A and B are mutually exclusive events.

As we know, if $P(A\cap B)=0$ then A and B are mutually exclusive events.

$$\because \quad P(A\cap B)=\frac{1}{3}\neq 0$$

So, A and B are not mutually exclusive events.

So, statement 2 is incorrect.

Hint:

- Use the definition of independent events and mutually exclusive events.
- Use addition theorem of probability and complement rule of probability to find P(A) and P(B).

116. Option (c) is correct.

Explanation:

Given: Set of 15 observations

$$n=15$$

As we know,

$$\text{Average} = \bar{x}=\frac{\Sigma x_i}{n}$$

$$\Rightarrow \quad \bar{x}=\frac{\Sigma x_i}{15}$$

$$\Rightarrow \quad \Sigma x_i=15\bar{x}$$

Let x_a was the observation recorded wrongly x_a' was corrected observation.

$$\text{Let new average} = \bar{x}'=\frac{\Sigma x_i'}{n}=\frac{\Sigma x_i'}{15}$$

$$=\frac{\Sigma x_i-x_a+x_a'}{15}$$

As x_a' and x_a has difference of only tens digit, it must be 3 instead of 8.

$\Rightarrow \quad x_a - x_a' = 8(10) - 3(10) = 50$

$\therefore \quad \bar{x}' = \frac{\Sigma x_i}{15} - \frac{(x_a - x_a')}{15}$

$\Rightarrow \quad \bar{x}' = \bar{x} - \frac{50}{15}$

$\Rightarrow \quad \bar{x}' = \bar{x} - \frac{10}{3}$

$\therefore$ New average is reduced by $\frac{10}{3}$.

Shortcut:

Let $\bar{x} = \frac{\Sigma x_i}{15}$

New $\bar{x} = \bar{x}' = \frac{\Sigma x_i'}{15}$

Difference between old and new observation $= x_a - x_a'$

$= 8(10) - 3(10)$

$= 50$

$\Rightarrow \quad \bar{x}' = \frac{\Sigma x_i}{15} - \frac{(x_a - x_a')}{15}$

$\Rightarrow \quad \bar{x}' = \bar{x} - \frac{50}{15}$

$\Rightarrow \quad \bar{x}' = \bar{x} - \frac{10}{3}$

So, new average is reduced by $\frac{10}{3}$.

Hint:

- Average $= \bar{x} = \frac{\Sigma x_i}{n}$
- Difference between old and new observation = 8(10) – 3(10) = 50.

117. Option (c) is correct.

Explanation:

Given:

E : Head on first toss

F : Head on second toss

Coin is tossed twice.

Sample space = {(H, H), (T, H), (H, T), (T, T)}

E = {(H, H), (H, T)}

$P(E) = \frac{2}{4} = \frac{1}{2}$

F = {(H, H), (T, H)}

$P(F) = \frac{2}{4} = \frac{1}{2}$

$E \cap F = \{(H, H)\}$

$P(E \cap F) = \frac{1}{4}$

By addition theorem of probability,

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$

Shortcut:

Sample space = {(H, H), (H, T), (T, H), (T, T)}

$E \cup F = \{(H, H), (H, T), (T, H)\}$

$P(E \cup F) = \frac{3}{4}$

Hint:

- Use addition theorem of probability,
  $P(A) + P(B) - P(A \cap B) = P(A \cup B)$

118. Option (d) is correct.

Explanation:

Given: Mean $= \frac{2}{3}$, Variance $= \frac{5}{9}$

Let n = number of events,

p = probability of occurrence of events

q = probability of non-occurrence of event

$\because \quad$ Mean $= np = \frac{2}{3}$

Variance $= npq = \frac{5}{9}$

$\Rightarrow \quad q = \frac{5/9}{2/3} = \frac{5}{6}$

Also, $\quad p = 1 - q = 1 - \frac{5}{6} = \frac{1}{6}$

and $\quad np = \frac{2}{3}$

$\Rightarrow \quad n = \frac{2/3}{1/6} = 4$

$\because \quad P(X = 2) = {}^nC_2(p)^2(q)^{n-2}$

$= {}^4C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$

$= \frac{4 \times 3}{2} \times \frac{1}{6 \times 6} \times \frac{5 \times 5}{6 \times 6}$

$$= \frac{25}{216}$$

Required probability = $\frac{25}{216}$

Hint:

- For binomial distribution, mean = np and variance = npq where n is number of trails, p is probability of success and q is probability of failure.
- $P(X = r) = {}^nC_r(p)^r(q)^{n-r}$

119. Option (d) is correct.

Explanation:

Given: Scores are 10, 12, 13, 15, 15, 13, 12, 10, x.

Mode is 15.

As we know, the mode of n observations is the number that has the highest frequency.

Here, frequency of score '12' = 2

Frequency of score '13' = 2

Frequency of score '15' = 2

So, for mode to be 15, its frequency must be the highest so $x = 15$.

$\Rightarrow$ Frequency of score '15' = 3

$\therefore \quad x = 15$

Shortcut:

For mode to be '15', its frequency must be highest

$\Rightarrow \quad x = 15.$

Hint:

- Recall that mode of n observations is the number that has the highest frequency.

120. Option (c) is correct.

Explanation:

Given:

$$P(A) = \frac{3}{4} \text{ and } P(B) = \frac{5}{8}$$

Statement 1:

As we know

$\min\cdot(P(A \cup B)) = \max\cdot(P(A), P(B))$

$\because \quad P(A) > P(B)$

$\Rightarrow \quad \min\cdot(P(A \cup B)) = \frac{3}{4}$

So, statement 1 is correct.

Statement 2:

As we know, $\max\cdot(P(A \cap B)) = \min\cdot(P(A), P(B))$

$\because \quad P(B) < P(A)$

$\Rightarrow \quad \max.(P(A \cap B)) = \frac{5}{8}$

So, statement 2 is also correct.

Hint:

- Use $\min(P(A \cup B) = \max\cdot(P(A), P(B))$ and $\max\cdot(P(A \cap B)) = \min\cdot(P(A), P(B))$