NDA/NA
National Defence Academy / Naval Academy

MATHEMATICS

II

QUESTION PAPER 2021

Time : 2:30 Hour **Total Marks : 300**

Important Instructions :

1. *This 'test Booklet contains* **120** *items (questions). Each item is printed in* **English**. *Each item comprises four responses (answer,s). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ONLY ONE response for each item.*
2. *You have to mark all your responses ONLY on the separate Answer Sheet provided.*
3. **All** *items carry equal marks.*
4. *Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions.*
5. *Penalty for wrong answers:*
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i) *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, one·third of the marks assigned to that question will be deducted as penalty.*
   (ii) *If a candidate gives more than one answer, it will be treated as a wrong answer even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii) *If a question is left blank, i.e., no answer is given by the candidate, there will be no penalty for that question.*

1. If $x^2 + x + 1 = 0$, then what is the value of $x^{199} + x^{200} + x^{201}$?
(a) -2 **(b)** 0
(c) 1 **(d)** 3

2. If x, y, z are in GP, then which of the following is/are correct?
(1) $\ln(3x), \ln(3y), \ln(3z)$ are in AP
(2) $xyz + \ln(x), xyz + \ln(y), xyz + \ln(z)$ are in HP
Select the correct answer using the code given below:
(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

3. If $\log_{10} 2, \log_{10} (2^x - 1), \log_{10} (2^x + 3)$ are in AP, then what is x equal to?
(a) 0 **(b)** 1
(c) $\log_2 5$ **(d)** $\log_5 2$

4. Let S = {2, 3, 4, 5, 6, 7, 9}. How many different 3-digit numbers (with all digits different) from S can be made which are less than 500?
(a) 30 **(b)** 49
(c) 90 **(d)** 147

5. If p = (1111 …up to n digits), then what is the value of $9p^2 + p$?
(a) $10^n p$ **(b)** $2p \cdot 10^n$
(c) $10^n p - 1$ **(d)** $10^n p + 1$

6. The quadratic equation
$3x^2 - (k^2 + 5k)x + 3k^2 - 5k = 0$
has real roots of equal magnitude and opposite sign. Which one of the following is correct?
(a) $0 < k < \frac{5}{3}$
(b) $0 < k < \frac{3}{5}$ only
(c) $\frac{3}{5} < k < \frac{5}{3}$
(d) No such value of k exists

7. If $a_n = n(n!)$, then what is $a_1 + a_2 + a_3 + \ldots + a_{10}$ equal to?
(a) $10! - 1$ **(b)** $11! + 1$
(c) $10! + 1$ **(d)** $11! - 1$

8. If p and q are the non-zero roots of the equation $x^2 + px + q = 0$, then how many possible values can q have?

(a) Nil **(b)** One
(c) Two **(d)** Three

9. If $\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ then what is

$\begin{vmatrix} 3d+5g & 4a+7g & 6g \\ 3e+5h & 4b+7h & 6h \\ 3f+5i & 4c+7i & 6i \end{vmatrix}$ equal to?

(a) Δ **(b)** 7Δ
(c) 72Δ **(d)** -72Δ

10. If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in HP, then which of the following is/are correct?

(1) a, b, c are in AP
(2) $(b+c)^2, (c+a)^2, (a+b)^2$ are in GP

Select the correct answer using the code given below:

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

11. If $A = \begin{vmatrix} 1 & a \\ 0 & 1 \end{vmatrix}$ where a $\in \mathbb{N}$, then what is

$A^{100} - A^{50} - 2A^{25}$ equal to?

(a) $-2I$ **(b)** $-I$
(c) $2I$ **(d)** I

where I is the identity matrix.

12. If $\begin{vmatrix} a & -b & a-b-c \\ -a & b & -a+b-c \\ -a & -b & -a-b+c \end{vmatrix} -- kabc = 0$ $(a \neq 0, b \neq 0, c \neq 0)$

then what is the value of k?

(a) -4 **(b)** -2
(c) 2 **(d)** 4

13. What is $\sum_{n=1}^{8n+7} i^n$ equal to, where $i = \sqrt{-1}$?

(a) -1 **(b)** 1
(c) i **(d)** $-i$

14. If $z = x + iy$, where $i = \sqrt{-1}$, then what does the equation $z\bar{z} + +|z|^2 + 4(z + \bar{z}) - 48 = 0$ represent?

(a) Straight line **(b)** Parabola
(c) Circle **(d)** Pair of straight lines

15. Which one of the following is a square root of $2a + 2\sqrt{a^2 + b^2}$, where $a, b \in R$?

(a) $\sqrt{a+ib} + \sqrt{a-ib}$ **(b)** $\sqrt{a+ib} - \sqrt{a-ib}$
(c) $2a + ib$ **(d)** $2a - ib$

where $i = \sqrt{-1}$.

16. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 + bx + c = 0$, then which one of the following is correct?

(a) $a^2 + b^2 - 2ac = 0$
(b) $-a^2 + b^2 + 2ac = 0$
(c) $a^2 - b^2 + 2ac = 0$
(d) $a^2 + b^2 + 2ac = 0$

17. If $C(n, 4)$, $C(n, 5)$ and $C(n, 6)$ are in AP, then what is the value of n?

(a) 7 **(b)** 8
(c) 9 **(d)** 10

18. How many 4-letter words (with or without meaning) containing two vowels can be constructed using only the letters (without repetition) of the word "LUCKNOW"?

(a) 240 **(b)** 200
(c) 150 **(d)** 120

19. Suppose 20 distinct points are placed randomly on a circle. Which of the following statements is/are correct?

1. The number of straight lines that can be drawn by joining any two of these points is 380.

2. The number of triangles that can be drawn by joining any three of these points is 1140.

Select the correct answer using the code given below.

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

20. How many terms are there in the expansion of

$$\left(\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2\right)^{21}$$

where $a \neq 0, b \neq 0$?

(a) 21 **(b)** 22
(c) 42 **(d)** 43

21. For what values of k is the system of equations $2k^2x + 3y - 1 = 0$, $7x - 2y + 3 = 0$, $6kx + y + 1 = 0$ consistent?

(a) $\frac{3 \pm \sqrt{11}}{10}$ **(b)** $\frac{21 \pm \sqrt{161}}{10}$

(c) $\frac{3 \pm \sqrt{7}}{10}$ **(d)** $\frac{4 \pm \sqrt{11}}{10}$

22. The inverse of a matrix A is given by

$$\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

What is A^{-1} equal to?

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ **(d)** $\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$

23. What is the period of the function $f(x) = \ln(2 + \sin^2 x)$?

(a) $\frac{\pi}{2}$ **(b)** π

(c) 2π **(d)** 3π

24. If $\sin(A + B) = 1$ and $2 \sin(A - B) = 1$, where $0 < A, B < \frac{\pi}{2}$, then what is tan A : tan B equal to?

(a) 1 : 2 **(b)** 2 : 1

(c) 1 : 3 **(d)** 3 : 1

25. Consider a regular polygon with 10 sides. What is the number of triangles that can be formed by joining the vertices which have no common side with any of the sides of the polygon?

(a) 25 **(b)** 50

(c) 75 **(d)** 100

26. Consider all the real roots of the equation $x^4 - 10x^2 + 9 = 0$. What is the sum of the absolute values of the roots?

(a) 4 **(b)** 6

(c) 8 **(d)** 10

27. Consider the expansion of $(1 + x)^n$. Let p, q, r and s be the coefficients of first, second, n^{th} and $(n + 1)^{th}$ terms respectively. What is $(ps + qr)$ equal to?

(a) $1 + 2n$ **(b)** $1 + 2n^2$

(c) $1 + n^2$ **(d)** $1 + 4n$

28. Let $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$ for $0 \le x, y, z \le 1$. What is the value of $x^{1000} + y^{1001} + z^{1002}$?

(a) 0 **(b)** 1

(c) 3 **(d)** 6

29. Let $\sin x + \sin y = \cos x + \cos y$ for all $x, y \in R$.

What is $\tan\left(\frac{x}{2} + \frac{y}{2}\right)$ equal to?

(a) 1 **(b)** 2

(c) $\sqrt{2}$ **(d)** $2\sqrt{2}$

30. Let $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ and $(mI + nA)^2 = A$ where m, n are positive real numbers and I is the identity matrix. What is $(m + n)$ equal to?

(a) 0 **(b)** $\frac{1}{2}$

(c) 1 **(d)** $\frac{3}{2}$

31. What is the value of the following?

$$\cot\left[\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right]$$

(a) $\frac{6}{17}$ **(b)** $\frac{7}{16}$

(c) $\frac{16}{7}$ **(d)** $\frac{17}{6}$

32. Let $4\sin^2 x = 3$, where $0 \le x \le \pi$. What is $\tan 3x$ equal to?

(a) -2 **(b)** -1

(c) 0 **(d)** 1

33. Let p, q and 3 be respectively the first, third and fifth terms of an AP. Let d be the common difference. If the product (pq) is minimum, then what is the value of d?

(a) 1 **(b)** $\frac{3}{8}$

(c) $\frac{9}{8}$ **(d)** $\frac{9}{4}$

34. Consider the following statements in respect of the roots of the equation $x^3 - 8 = 0$:

1. The roots are non-collinear.

2. The roots lie on a circle of unit radius.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

35. Let the equation sec x × cosec $x = p$ have a solution, where p is a positive real number. What should be the smallest value of p?

(a) $\frac{1}{2}$

(b) 1

(c) 2

(d) Minimum does not exist

36. For what value of θ, where $0 < \theta < \frac{\pi}{2}$, does $\sin\theta + \sin\theta\cos\theta$ attain maximum value?

(a) $\frac{\pi}{2}$ **(b)** $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ **(d)** $\frac{\pi}{6}$

37. Consider the following statements in respect of sets :

1. The union over intersection of sets is distributive.
2. The complement of union of two sets is equal to intersection of their complements.
3. If the difference of two sets is equal to empty set, then the two sets must be equal.

Which of the above statements are correct?

(a) 1 and 2 only **(b)** 2 and 3 only

(c) 1 and 3 only **(d)** 1, 2 and 3

38. Consider three sets X, Y and Z having 6, 5 and 4 elements respectively. All these 15 elements are distinct. Let S = (X – Y) ∪ Z. How many proper subsets does S have?

(a) 255 **(b)** 256

(c) 1023 **(d)** 1024

39. Consider the following statements in respect of relations and functions :

1. All relations are functions but all functions are not relations.
2. A relation from A to B is a subset of Cartesian product A × B.
3. A relation in A is a subset of Cartesian product A × A.

Which of the above statements are correct?

(a) 1 and 2 only **(b)** 2 and 3 only

(c) 1 and 3 only **(d)** 1, 2 and 3

40. If $\log_{10} 2 \log_2 10 + \log_{10} (10^x) = 2$, then what is the value of x?

(a) 0 **(b)** 1

(c) $\log_2 10$ **(d)** $\log_5 2$

41. Let ABC be a triangle.

If cos2A + cos2B + cos2C = –1 then which one of the following is correct?

(a) sin A sin B sin C = 0

(b) sin A sin B cos C = 0

(c) cos A sin B sin C= 0

(d) cos A cos B cos C = 0

42. What is the value of the following determinant?

$$\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$$

(a) –1 **(b)** 0

(c) 2 tan A sin B sin C **(d)** 2 tan A sin B sin C

43. Suppose set A consists of first 250 natural numbers that are multiples of 3 and set B consists of first 200 even natural numbers. How many elements does A ∪ B have?

(a) 324 **(b)** 364

(c) 384 **(d)** 400

44. Let S_k denote the sum of first k terms of an AP.

What is $\frac{S_{30}}{S_{20} - S_{10}}$ equal to?

(a) 1 **(b)** 2

(c) 3 **(d)** 4

45. If the roots of the equation $4x^2 - (5k+1)x + 5k = 0$ differ by unity, then which one of the following is a possible value of k?

(a) –3 **(b)** –1

(c) $-\frac{1}{5}$ **(d)** $-\frac{3}{5}$

46. Consider the digits 3, 5, 7, 9. What is the number of 5-digit numbers formed by these digits in which each of these four digits appears?

(a) 240 **(b)** 180

(c) 120 **(d)** 60

47. How many distinct matrices exist with all four entries taken from {1, 2}?

(a) 16 **(b)** 24

(c) 32 **(d)** 48

48. If $i = \sqrt{-1}$, then how many values does i^{-2n} have for different $n \in Z$?

(a) One **(b)** Two

(c) Four **(d)** Infinite

49. If $x = \frac{a}{b-c}, y = \frac{b}{c-a}, z = \frac{c}{a-b}$ then what is the value of the following?

$$\begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix}$$

(a) 0 **(b)** 1

(c) abc **(d)** $ab + bc + ca$

50. Consider the following in respect of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

1. Inverse of A does not exist

2. $A^3 = A$

3. $3A = A^2$

Which of the above are correct?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

Direction : Consider the following for the next two (02) items that follow.

A circle is passing through the points (5, –8), (–2, 9) and (2, 1).

51. What are the coordinates of the centre of the circle?

(a) (–2, –50) **(b)** (–50, –20)

(c) (–24, –58) **(d)** (–58, –24)

52. If r is the radius of the circle, then which one of the following is correct?

(a) $r < 10$ **(b)** $10 < r < 30$

(c) $30 < r < 60$ **(d)** $r > 60$

Direction : Consider the following for the next two (02) items that follow.

The two vertices of an equilateral triangle are (0, 0) and (2, 2).

53. Consider the following statements :

1. The third vertex has at least one irrational coordinate.

2. The area is irrational.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

54. The difference of coordinates of the third vertex is

(a) 0 **(b)** $\sqrt{3}$

(c) $2\sqrt{2}$ **(d)** $2\sqrt{3}$

Direction : Consider the following for the next two (02) items that follow.

The coordinates of three consecutive vertices of a parallelogram ABCD are A(1, 3), B(–1, 2) and C(3, 5).

55. What is the equation of the diagonal BD?

(a) $2x - 3y + 2 = 0$

(b) $3x - 2y + 5 = 0$

(c) $2x - 3y + 8 = 0$

(d) $3x - 2y - 5 = 0$

56. What is the area of the parallelogram?

(a) 1 square unit **(b)** $\frac{3}{2}$ square units

(c) 2 square units **(d)** $\frac{5}{2}$ square units

Direction : Consider the following for the next two (02) items that follow.

The equations of the sides AB, BC and CA of a triangle ABC are $x - 2 = 0$, $y + 1 = 0$ and $x + 2y - 4 = 0$ respectively.

57. What is the equation of the altitude through B on AC?

(a) $x - 3y + 1 = 0$ **(b)** $x - 3y + 4 = 0$

(c) $2x - y + 4 = 0$ **(d)** $2x - y - 5 = 0$

58. What are the coordinates of circumcentre of the triangle?

(a) (4, 0) **(b)** (2, 1)

(c) (0, 4) **(d)** (2, –1)

Direction : Consider the following for the next two (02) items that follow.

The two ends of the latus rectum of a parabola are (–2, 4) and (–2,– 4).

59. What is the maximum number of parabolas that can be drawn through these two points as end points of latus rectum?

(a) Only one **(b)** Two

(c) Four **(d)** Infinite

60. Consider the following statements in respect of such parabolas :

1. One of the parabolas passes through the origin (0, 0).

2. The focus of one of the parabolas lies at (–2, 0).

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

61. The locus of a point P(x, y, z) which moves in such a way that $z = 7$ is *a*

(a) line parallel to x-axis

(b) line parallel to y-axis

(c) line parallel to z-axis

(d) plane parallel to xy-plane

62. Consider the following statements :

1. A line in space can have infinitely many direction ratios.

2. It is possible for certain line that the sum of the squares of direction cosines can be equal to sum of its direction cosines.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

63. The xy-plane divides the line segment joining the points (–1, 3, 4) and (2, –5, 6)

(a) internally in the ratio 2 : 3

(b) internally in the ratio 3 : 2

(c) externally in the ratio 2 : 3

(d) externally in the ratio 2 : 1

64. The number of spheres of radius r touching the coordinate axes is

(a) 4 **(b)** 6

(c) 8 **(d)** infinite

65. ABCDEFGH is a cuboid with base ABCD. Let A(0, 0, 0), B(12, 0, 0), C(12, 6, 0) and G(12, 6, 4) be the vertices. If α is the angle between AB and AG; β is the angle between AC and AG, then what is the value of $\cos 2\alpha + \cos 2\beta$?

(a) $\frac{40}{49}$ **(b)** $\frac{64}{49}$

(c) $\frac{120}{49}$ **(d)** $\frac{160}{49}$

66. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a}\times\vec{b}$ is perpendicular to $\vec{c}$. If θ is the angle between $\vec{a}$ and $\vec{b}$, then which of the following is/are correct?

1. $\vec{a}\times\vec{b} = \sin\theta\,\vec{c}$

2. $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right) = \theta$

Select the correct answer using the code given below.

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

67. If $\vec{a} + 3\vec{b} = 3\hat{i} - \hat{j}$ and $2\vec{a} + \vec{b} = \hat{i} - 2\hat{j}$, then what is the angle between $\vec{a}$ and $\vec{b}$?

(a) 0 **(b)** $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ **(d)** $\frac{\pi}{2}$

68. If $\left(\vec{a} + \vec{b}\right)$ is perpendicular to $\vec{a}$ and magnitude of $\vec{b}$ is twice that of $\vec{a}$, then what is the value of $\left(4\vec{a} + \vec{b}\right)\cdot\vec{b}$ equal to?

(a) 0 **(b)** 1

(c) $8|\vec{a}|^2$ **(d)** $8|\vec{b}|^2$

69. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar. Which of the following is / are correct?

1. $\left(\vec{a}\times\vec{b}\right)\times\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$

2. $\left(\vec{a}\times\vec{b}\right)\times\vec{c}$ is perpendicular to $\vec{a}\times\vec{b}$

Select the correct answer using the code given below.

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

70. If the position vectors of A and B are $\left(\sqrt{2}-1\right)\hat{i} - \hat{j}$ and $\hat{i} + \left(\sqrt{2}+1\right)\hat{j}$ respectively, then what is the magnitude of $\overline{\text{AB}}$?

(a) $2\sqrt{2}$ **(b)** $3\sqrt{2}$

(c) $2\sqrt{3}$ **(d)** $3\sqrt{3}$

71. If $y = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$ then what is $\frac{dy}{dx}$ at $x = 0$ equal to?

(a) 0 **(b)** 1
(c) 2 **(d)** 4

72. If $y = \cos x \cos 4x \cos 8x$, then what is $\frac{1}{y}\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ equal to?

(a) –1 **(b)** 0
(c) 1 **(d)** 3

73. Let $f(x)$ be a polynomial function such that $f(x) = x^4$. What is $f'(1)$ equal to?

(a) 0 **(b)** 1
(c) 2 **(d)** 4

74. What is $\lim_{n\to\infty} \frac{a^n + b^n}{a^n - b^n}$ where $a > b > 1$, equal to?

(a) –1 **(b)** 0
(c) 1 **(d)** Limit does not exist

75. Let $f(x) = \begin{cases} 1 + \frac{x}{2k}, & 0 < x < 2 \\ kx, & 2 \le x < 4 \end{cases}$

If $\lim_{x\to 2} f(x)$ exists, then what is the value of k?

(a) –2 **(b)** –1
(c) 0 **(d)** 1

76. Consider the following statements in respect of $f(x) = |x| - 1$:

1. $f(x)$ is continuous at $x = 1$.
2. $f(x)$ is differentiable at $x = 0$.

Which of the above statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

77. If $f(x) = \frac{[x]}{|x|}$, $x \neq 0$, where $[\cdot]$ denotes the greatest integer function, then what is the right-hand limit of $f(x)$ at $x = 1$?

(a) –1
(b) 0
(c) 1
(d) Right-hand limit of $f(x)$ at $x = 1$ does not exist

78. Consider the following statements in respect of the function $f(x) = \sin\left(\frac{1}{x^2}\right)$, $x \neq 0$:

1. It is continuous at $x = 0$, if $f(0) = 0$.
2. It is continuous at $x = \frac{2}{\sqrt{\pi}}$.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

79. What is the range of the function $f(x) = 1 - \sin x$ defined on entire real line?

(a) (2, 0) **(b)** [0, 2]
(c) (1, 1) **(d)** [–1, 1]

80. What is the slope of the tangent of $y = \cos^{-1}(\cos x)$ at $x = -\frac{\pi}{4}$?

(a) –1 **(b)** 0
(c) 1 **(d)** 2

81. What is the integral of $f(x) = 1 + x^2 + x^4$ with respect to x^2?

(a) $x + \frac{x^3}{3} + \frac{x^5}{5} + C$ **(b)** $\frac{x^3}{3} + \frac{x^5}{5} + C$
(c) $x^2 + \frac{x^4}{4} + \frac{x^6}{6} + C$ **(d)** $x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$

82. Consider the following statements in respect of the function $f(x) = x^2 + 1$ in the interval (1, 2) :

1. The maximum value of the function is 5.
2. The minimum value of the function is 2.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

83. If $f(x)$ satisfies $f(1) = f(4)$, then what is $\int_1^4 f'(x)\,dx$ equal to?

(a) –1 **(b)** 0
(c) 1 **(d)** 2

84. What is $\int_0^{\frac{\pi}{2}} e^{\ln(\cos x)}\,dx$ equal to?

(a) –1 **(b)** 0
(c) 1 **(d)** 2

85. If $\int \sqrt{1 - \sin 2x}\,dx = A \sin x + B \cos x + C$, where $0 < x < \frac{\pi}{4}$, then which one of the following is correct?

(a) $A + B = 0$ **(b)** $A + B - \ 2 = 0$
(c) $A + B + 2 = 0$ **(d)** $A + B - 1 = 0$

86. What is the order of the differential equation of all ellipses whose axes are along the coordinate axes?

(a) 1 **(b)** 2
(c) 3 **(d)** 4

87. What is the degree of the differential equation of all circles touching both the coordinate axes in the first quadrant?

(a) 1 **(b)** 2
(c) 3 **(d)** 4

88. What is the differential equation $y = A - \frac{B}{x}$?

(a) $xy_2 + y_1 = 0$ **(b)** $xy_2 + 2y_1 = 0$
(c) $xy_2 - 2y_1 = 0$ **(d)** $2\,xy_2 + y_1 = 0$

89. What is $\int_0^{\pi} \ln\left(\tan\frac{x}{2}\right) dx$ equal to?

(a) 0 **(b)** $\frac{1}{2}$
(c) 1 **(d)** 2

90. Where does the tangent to the curve $y = e^x$ at the point (0, 1) meet x-axis?

(a) (1, 0) **(b)** (–1, 0)
(c) (2, 0) **(d)** $\left(-\frac{1}{2}, 0\right)$

91. Consider the following statements in respect of the function $f(x) = x + \frac{1}{x}$:

1. The local maximum value of $f(x)$ is less than its local minimum value.
2. The local maximum value of $f(x)$ occurs at $x = 1$.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

92. What is the maximum area of a rectangle that can be inscribed in a circle of radius 2 units?

(a) 4 square units **(b)** 6 square units
(c) 8 square units **(d)** 16 square units

93. What is $\int \frac{dx}{x\left(x^2+1\right)}$ equal to?

(a) $\frac{1}{2}\ln\left(\frac{x^2}{x^2+1}\right)+C$ **(b)** $\ln\left(\frac{x^2}{x^2+1}\right)+C$
(c) $\frac{3}{2}\ln\left(\frac{x^2}{x^2+1}\right)+C$ **(d)** $\frac{1}{2}\ln\left(\frac{x^2+1}{x^2}\right)+C$

94. What is the derivative of e^{e^x} with respect to e^x?

(a) e^{e^x} **(b)** e^x
(c) $e^{e^x} e^x$ **(d)** ee^x

95. What is the condition that $f(x) = x^3 + x^2 + kx$ has no local extremum?

(a) $4k < 1$ **(b)** $3k > 1$
(c) $3k < 1$ **(d)** $3k \leq 1$

96. If $f(x) = 2^x$, then what is $\int_2^{10} \frac{f'(x)}{f(x)} dx$ equal to?

(a) 4 In 2 **(b)** In 4
(c) In 5 **(d)** 8 In 2

97. If $\int_{-2}^{0} f(x)\, dx = k$, then $\int_{-2}^{0} |f(x)|\, dx$ is

(a) less than k
(b) greater than k
(c) less than or equal to k
(d) greater than or equal to k

98. If the function $f(x) = x^2 - kx$ is monotonically increasing in the interval $(1, \infty)$, then which one of the following is correct?

(a) $k < 2$ **(b)** $2 < k < 3$
(c) $3 < k < 4$ **(d)** $k > 4$

99. What is the area bounded by $y = [x]$, where [•] is the greatest integer function, the x-axis and the lines $x = -1.5$ and $x = -1.8$?

(a) 0.3 square unit **(b)** 0.4 square unit
(c) 0.6 square unit **(d)** 0.8 square unit

100. The tangent to the curve $x^2 = y$ at (1, 1) makes an angle θ with the positive direction of x-axis. Which one of the following is correct?

(a) $\theta < \frac{\pi}{6}$ **(b)** $\frac{\pi}{6} < \theta < \frac{\pi}{4}$
(c) $\frac{\pi}{4} < \theta < \frac{\pi}{3}$ **(d)** $\frac{\pi}{3} < \theta < \frac{\pi}{2}$

101. Consider the following relations for two events E and F :

1. $P(E \cap F) \geq P(E) + P(F) - 1$
2. $P(E \cup F) = P(E) + P(F) + P(E \cap F)$
3. $P(E \cup F) \leq P(E) + P(F)$

Which of the above relations is/are correct?

(a) 1 only **(b)** 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

102. If $P(A|B) < P(A)$, then which one of the following is correct?

(a) $P(B|A) < P(B)$ **(b)** $P(B|A) > P(B)$

(c) $P(B|A) = P(B)$ **(d)** $P(B|A) > P(A)$

103. When the measure of central tendency is available in the form of mean, which one of the following is the most reliable and accurate measure of variability?

(a) Range

(b) Mean deviation

(c) Standard deviation

(d) Quartile deviation

104. A problem is given to three students A, B and C, whose probabilities of solving the problem independently are $\frac{1}{2}, \frac{3}{4}$ and p respectively. If the probability that the problem can be solved is $\frac{29}{32}$, then what is the value of p?

(a) $\frac{2}{5}$ **(b)** $\frac{2}{3}$

(c) $\frac{1}{3}$ **(d)** $\frac{1}{4}$

105. In a cricket match, a batsman hits a six 8 times out of 60 balls he plays. What is the probability that on a ball played he does not hit a six?

(a) $\frac{2}{3}$ **(b)** $\frac{1}{15}$

(c) $\frac{2}{15}$ **(d)** $\frac{13}{15}$

Direction : Consider the following for the next two (02) items that follow.

Two regression lines are given as $3x - 4y + 8 = 0$ and $4x - 3y - 1 = 0$.

106. Consider the following statements :

1. The regression line of y on x is $y = \frac{3}{4}x + 2$.

2. The regression line of x on y is $x = \frac{3}{4}y + \frac{1}{4}$.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

107. Consider the following statements :

1. The coefficient of correlations r is $\frac{3}{4}$.

2. The means of x and y are 3 and 4 respectively.

Which of the above statements is/are correct?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

Direction : Consider the following for the next two (02) items that follow.

The marks obtained by 60 students in a certain subject out of 75 are given below :

Marks	Number of students
15-20	4
20-25	5
25-30	11
30-35	6
35-40	5
40-45	8
45-50	9
50-55	6
55-60	4
60-65	2

108. What is the median?

(a) 35 **(b)** 38

(c) 39 **(d)** 40

109. What is the mode?

(a) 27.27 **(b)** 27.73

(c) 27.93 **(d)** 28.27

110. What is the mean of natural numbers contained in the interval [15, 64]?

(a) 36.8 **(b)** 38.3

(c) 39.5 **(d)** 40.3

111. For the set of numbers

$$x, x, x + 2, x + 3, x + 10$$

where x is a natural number, which of the following is/are correct?

1. Mean > Mode

2. Median > Mean

Select the correct answer using the code given below.

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

112. The mean of 10 observations is 5.5. If each observation is multiplied by 4 and subtracted from 44, then what is the new mean?

(a) 20 **(b)** 22
(c) 34 **(d)** 44

113. If g is the geometric mean of 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, then which one of the following is correct.

(a) $8 < g < 16$ **(b)** $16 < g < 32$
(c) $32 < g < 64$ **(d)** $g > 64$

114. If the harmonic mean of 60 and x is 48, then what is the value of x?

(a) 32 **(b)** 36
(c) 40 **(d)** 44

115. What is the mean deviation of first 10 even natural numbers?

(a) 5 **(b)** 5.5
(c) 10 **(d)** 10.5

116. If $\sum_{i=1}^{10} x_i = 110$ and $\sum_{i=1}^{10} x_i^2 = 1540$ then what is the variance?

(a) 22 **(b)** 33
(c) 44 **(d)** 55

117. 3-digit numbers are formed using the digits 1, 3, 7 without repetition of digits. A number is randomly selected. What is the probability that the number is divisible by 3?

(a) 0 **(b)** $\frac{1}{3}$
(c) $\frac{1}{4}$ **(d)** $\frac{1}{8}$

118. What is the probability that the roots of the equation $x^2 + x + n = 0$ are real, where $n \in$ N and $n < 4$?

(a) 0 **(b)** $\frac{1}{4}$
(c) $\frac{1}{3}$ **(d)** $\frac{1}{2}$

119. If A and B are two events such that P (not A) = $\frac{7}{10}$, P(not B) = $\frac{3}{10}$ and P(A|B) = $\frac{3}{14}$, then what is P(B|A) equal to?

(a) $\frac{11}{14}$ **(b)** $\frac{9}{14}$
(c) $\frac{1}{4}$ **(d)** $\frac{1}{2}$

120. Seven white balls and three black balls are randomly placed in a row. What is the probability that no two black balls are placed adjacently?

(a) $\frac{7}{15}$ **(b)** $\frac{8}{15}$
(c) $\frac{11}{15}$ **(d)** $\frac{13}{15}$

Answers

Mathematics

Q. No.	Answer Key	Topic Name	Chapter Name
1	**(b)**	Roots of Equation	Theory of Equation
2	**(a)**	Application of Logarithim in AP, GP	Sequence and Series
3	**(c)**	Application of Logarithim in AP, GP	Sequence and Series
4	**(c)**	Fundamental Principle of Counting	Permutation and Combination
5	**(a)**	Value of n Digits	Binomial Theorem
6	**(d)**	Nature of Roots	Quadratic Equation
7	**(d)**	Factorial Notation	Binomial Theorem
8	**(b)**	Relation between Roots and Coefficient	Theory of Equation
9	**(d)**	Properties of Determinants	Matrices and Determinant
10	**(a)**	Harmonic Mean	Sequence and Series
11	**(a)**	Matrix Multiplication	Matrices and Determinant
12	**(a)**	Properties of Determinants	Matrices and Determinant
13	**(a)**	Sum of Complex Number	Complex Number
14	**(c)**	Modulus and Conjugate of Complex	Complex Number
15	**(a)**	Square Roots of Equation	Complex Number
16	**(c)**	Roots of Equation	Theory of Equation
17	**(a)**	Combination	Permutation and Combination
18	**(a)**	Repetition and Non-Repetition Cases	Permutation and Combination
19	**(b)**	Fundamental Principle of Counting	Permutation and Combination
20	**(d)**	Number of Terms	Binomial Theorem
21	**(b)**	System of Equations	Matrices and Determinant
22	**(a)**	Inverse of Matrix	Matrices and Determinant
23	**(b)**	Period of Function	Relation and Function
24	**(d)**	Trigonometric Identities and Values	Trigonometric Ratios and Identities
25	**(b)**	Combination of Sides of Triangle	Permutation and Combination
26	**(c)**	Roots of Quadratic Equation	Theory of Equation
27	**(c)**	Expansion of Terms	Binomial Theorem
28	**(c)**	Inverse Trigonometric Identities	Trigonometric Ratios and Identities
29	**(a)**	Trigonometric Identities and Values	Trigonometric Ratios and Identities
30	**(d)**	Addition and Multiplication of Matrix	Matrices and Determinant
31	**(a)**	Inverse Trigonometric Identities	Inverse Trigonometric Function
32	**(c)**	Trigonometric Identities and Values	Trigonometric Ratios and Identities
33	**(c)**	Maxima and Minima	Application of Derivatives
34	**(a)**	Roots of Equation	Complex Number
35	**(c)**	Trigonometric Identities and Values	Trigonometric Ratios and Identities
36	**(b)**	Maxima and Minima	Application of Derivatives

Q. No.	Answer Key	Topic Name	Chapter Name
37	**(a)**	Union, Intersection, Empty Sets	Set Theory
38	**(c)**	Subsets of Sets	Set Theory
39	**(b)**	Cartesian Product of Relation	Relation and Function
40	**(b)**	Logarithmic Identities	Logarithm and Its Application
41	**(d)**	Trigonometric Equation	Trigonometric Ratios and Identities
42	**(b)**	Properties of Determinant	Matrices and Determinants
43	**(c)**	Union of Sets	Set Theory
44	**(c)**	Sum of n Terms	Sequence and Series
45	**(c)**	Sum and Product of Roots	Theory of Equation
46	**(a)**	Fundamental Principle of Counting	Permutation and Combination
47	**(a)**	Entries of Matrix	Matrices and Determinants
48	**(b)**	Value of Exponent	Complex Number
49	**(a)**	Properties of Determinant	Matrices and Determinants
50	**(c)**	Inverse and Multiplication of Matrix of Matrix	Matrices and Determinants
51	**(d)**	Centre of Circle	Circle
52	**(d)**	Radius of Circle	Circle
53	**(c)**	Equilateral Triangle	Two Dimensional Geometry
54	**(d)**	Difference of Coordinates	Two Dimensional Geometry
55	**(c)**	Scope of Equation of straightion	Straight Lines
56	**(c)**	Area of Triangle	Two Dimensional Geometry
57	**(d)**	Altitude of Triangle	Straight Lines
58	**(a)**	Circumcentre of Triangle	Straight Lines
59	**(b)**	Coordinates of Parabola	Parabola
60	**(c)**	Focus and End Point of Parabola	Parabola
61	**(d)**	Locus of Point	Three Dimensional Geometry
62	**(c)**	Equation of Line in Space	Three Dimensional Geometry
63	**(c)**	Division by Plane	Three Dimensional Geometry
64	**(c)**	3D Figures	Three Dimensional Geometry
65	**(b)**	Angle between Two Lines	Three Dimensional Geometry
66	**(c)**	Dot and Cross Multiply of Vector	Vector Algebra
67	**(d)**	Angle between Two Vectors	Vector Algebra
68	**(a)**	Dot Product of Vectors	Vector Algebra
69	**(c)**	Triple Product of Vectors	Vector Algebra
70	**(c)**	Position of Vector	Vector Algebra
71	**(b)**	Product Rule of Differentiation	Differentiation
72	**(a)**	Product Rule of Differentiation	Differentiation
73	**(c)**	Composite Function Derivative	Differentiation
74	**(c)**	Evaluation of Limits	Limit
75	**(d)**	Condition of Limit	Limit

Q. No.	Answer Key	Topic Name	Chapter Name
76	(a)	Basics of Continuity and Differentiability	Continuity and Differentiability
77	(c)	Limit of Function	Continuity and Differentiability
78	(b)	Continuity of Function	Continuity and Differentiability
79	(b)	Range of Function	Function
80	(a)	Slope of Tangent	Application of Derivative
81	(d)	Integration by Substitution	Integration
82	(b)	Maxima and Minima	Application of Derivative
83	(b)	Basics of Integration	Definite Integration
84	(c)	Integration using Logarithm Property	Definite Integration
85	(b)	Integration using Special Formulae	Integration
86	(b)	Order of Differential Equation	Differential Equation
87	(b)	Degree of Differential Equation	Differential Equation
88	(b)	Formulation of Differential Equation	Differential Equation
89	(a)	Integration using Logarithm Property	Definite Integration
90	(b)	Tangent and Normal	Application of Derivative
91	(a)	Maxima and Minima	Application of Derivative
92	(c)	Maxima and Minima	Application of Derivative
93	(a)	Integration using Special Formulae	Integration
94	(a)	Logarithmic Differentiation	Differentiation
95	(b)	Maxima and Minima	Application of Derivative
96	(d)	Basics of Integration	Definite Integration
97	(d)	Modulus Integration	Definite Integration
98	(a)	Increasing and Decreasing	Application of Derivative
99	(c)	Area Made by Lines	Function
100	(d)	Tangent and Normal	Application of Derivative
101	(c)	Addition Theorem of Probability	Probability
102	(a)	Conditional Probabilities	Probability
103	(c)	Central Tendency	Statistics
104	(d)	Basics of Probability	Probability
105	(d)	Basics of Probability	Probability
106	(c)	Regression of Lines	Statistics
107	(a)	Regression of Lines	Statistics
108	(c)	Median	Statistics
109	(b)	Mode	Statistics
110	(c)	Mean	Statistics
111	(a)	Mean, Median, Mode	Statistics
112	(b)	Mean	Statistics
113	(c)	Geometric Mean	Sequence and Series
114	(c)	Harmonic Mean	Sequence and Series

Q. No.	Answer Key	Topic Name	Chapter Name
115	**(a)**	Mean Deviation	Statistics
116	**(b)**	Variance	Statistics
117	**(a)**	Basic Probability	Probability
118	**(a)**	Classical Definition of Probability	Probability
119	**(d)**	Conditional Probability Theorem	Probability
120	**(a)**	Basic Probability	Probability

ANSWERS WITH EXPLANATION

1. **Option (b) is correct.**

Explanation:

Given: $x^2 + x + 1 = 0$ ...(i)

To find: $x^{199} + x^{200} + x^{201}$

Taking common, we get

$\Rightarrow \quad x^{199}(1 + x^1 + x^2)$

$\Rightarrow \quad x^{199} \times 0$ [using value from (i)]

$\Rightarrow \quad 0$

Hints:

(1) Take common x^{199}.

(2) Use value from given equation.

2. **Option (a) is correct.**

Explanation:

Given: x, y, z are in G.P.

Since, in G.P.

$\Rightarrow \quad \sqrt{xz} = y$

$\Rightarrow \quad xz = y^2$

$\Rightarrow \quad 9xz = 9y^2$

[Multiply 9 both sides]

$\Rightarrow \quad 3x \times 3z = (3y)^2$

Taking log both sides

$\Rightarrow \quad \log(3x \times 3z) = \log(3y)^2$

$\Rightarrow \quad \log 3x + \log 3z = 2\log(3y)$

This satisfies condition of A.P.

Hence, $\log 3x$, $\log 3y$, $\log 3z$ are in A.P.

Statement 1 is correct.

Now, for H.P., conditions, if "a, b, c are in A.P., then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in H.P."

Since, $(xyz + \ln x)$, $(xyz + \ln y)$, $(xyz + \ln z)$ are in A.P.

Because $\ln x$, $\ln y$, $\ln z$ are in A.P.

So, they are not in H.P.

Statement 2 is false.

Hint:

(1) Use condition for A.P., G.P., H.P.

3. **Option (c) is correct.**

Explanation:

Given: Three number are in A.P.

Since, they are in A.P.

$\Rightarrow \quad \log_{10} 2 + \log_{10}(2^x + 3) = 2\log_{10}(2^x - 1)$

[put $2^x = y$]

$\Rightarrow \quad \log_{10} 2 + \log_{10}(y + 3) = 2\log_{10}(y - 1)$

[$\because \log(ab) = \log a + \log b$, $\log a^b = b \log a$]

$\Rightarrow \quad \log_{10}(2 \times (y + 3)) = \log_{10}(y - 1)^2$

[on comparing]

$\Rightarrow \quad 2(y + 3) = (y - 1)^2$

$\Rightarrow \quad 2y + 6 = y^2 + 1 - 2y$

$\Rightarrow \quad y^2 - 4y - 5 = 0$

$\Rightarrow \quad y^2 - 5y + y - 5 = 0$

$\Rightarrow \quad y(y - 5) + 1(y - 5) = 0$

$\Rightarrow \quad (y - 5)(y + 1) = 0$

$\Rightarrow \quad y = 5, -1$

Since, $y = 2^x$

$\Rightarrow \quad 2^x = 5, 2^x = -1$

(Negative not possible)

$\Rightarrow \quad 2^x = 5$

$\Rightarrow \quad \log(2)^x = \log 5$

$\Rightarrow \quad x \log 2 = \log 5$

$\Rightarrow \quad x = \log_2 5 \quad \left[\log_a b = \frac{\log b}{\log a}\right]$

Hints:

(1) Use properties of logarithmic

(2) put $2^x = y$

4. Option (c) is correct.

Explanation:

Given: S = {2, 3, 4, 5, 6, 7, 9}

Three digit number need to make smaller than 500, hundredth term will be fill by only 2, 3, 4

2/3/4	Any number	Any number
Hundredth	Tens	Ones

Now, at tens place 6 digits can be fill and then at ones place 5 digits can be fill.

Hence, number possible $= 3 \times 6 \times 5$

$= 90$

> **Hints:**
> **(1)** less than 500.
> **(2)** Digits at hundredth's place can be 2,3 and 4 only.

5. Option (a) is correct.

Explanation:

Consider, $p = (11111 \ldots\ldots\ldots$ up to n digits)

Now,

$p^2 = (11111 \ldots\ldots\ldots \text{ up to } n \text{ digits})^2$

$9p^2 = 9(111 \ldots\ldots\ldots \text{ up to } n \text{ digits})^2$

Add p both sides

$9p^2 + p = 9(111 \ldots\ldots\ldots \text{ up to } n \text{ digits})^2 + p$

$= p[9(111 \ldots\ldots\ldots \text{ up to } n \text{ digits}) + 1]$ …(i)

$= p[9999 \ldots\ldots \text{ up to } n \text{ digits} + 1]$

$= p[1000 \ldots\ldots\ldots \text{ up to } n + 1 \text{ or upto } n \text{ zeroes}]$

$= p \times 10^n$

Hence, $9p^2 + p = p10^n$

> **Hints:**
> **(1)** Use p^2 and multiply 9.
> **(2)** $9p^2 + p = p10^n$.

6. Option (d) is correct.

Explanation:

Given: Magnitude of roots equal but of opposite sign means one positive and second is negative. So, product of roots will be negative and product of roots given by $\frac{c}{a}$ in quadratic equation.

So, for quadratic equation

$3x^2 - (k^2 + 5k)\,x + 3k^2 - 5k = 0$

$\Rightarrow \quad \frac{3k^2 - 5k}{3} < 0$

$\Rightarrow \quad 3k^2 - 5k < 0$

$\Rightarrow \quad k(3k - 5) < 0$

Two conditions will become:

(1) $k < 0$ & $(3k - 5) > 0$

$k < 0$ & $k > \frac{5}{3}$

This not possible

(2) $k > 0$ and $(3k - 5) < 0$

$k > 0$ and $k < \frac{5}{3}$

This satisfies

Hence, $0 < k < \frac{5}{3}$

Roots are equal & opposite in sign

Hence, sum of roots = 0

$\frac{k^2 + 5k}{3} = 0$

$\Rightarrow \quad k(k + 5) = 0$

$\Rightarrow \quad k = 0, k = -5$

Now, we have 3 answers for k

(i) $0 < k < \frac{5}{3}$

(ii) $k = 0, -5$

From (i) & (ii) we don't have any common solution for k.

Hence, no such value of k exists.

> **Shortcut:**
> $\alpha - \alpha = \frac{k^2 + 5k}{3} = 0$
> $\Rightarrow \quad k = 0, k = -5$ …(i)
> Product roots < 0
> $\frac{3k \leq 5k}{3} < 0 \Rightarrow k(3K - 5) < 0$
> + − +
> $-\infty \quad 0 \quad 5/3$
> $k \leftarrow \left(0, \frac{5}{3}\right)$ from(i) and (ii) no any value of k.

7. Option (d) is correct.

Explanation:

Given: $a_n = n(n!)$

$a_1 = 1(1!) = 1$

$a_2 = 2(2!) = 4$

$a_3 = 3(3!) = 18$

$a_4 = 4(4!) = 96$

$a_5 = 5(5!) = 600$

$$a_6 = 6(6!) = 4320$$
$$a_7 = 7(7!) = 35280$$
$$a_8 = 8(8!) = 322560$$
$$a_9 = 9(9!) = 3265920$$
$$a_{10} = 10(10!) = 36288000$$

Now, to find let

$I = a_1 + a_2 + a_3 + \ldots\ldots\ldots a_{10}$

$= 1(1!) + 2(2!) + 3(3!) + \ldots\ldots\ldots\ldots + 10(10!)$ [put all values]

$I = 1 + 4 + 18 + \ldots\ldots\ldots\ldots\ldots + 36288000$

$= 39916799$

$= 39916800 - 1$

$I = 11! - 1$ [value of 11! = 39916800]

Hence,

$a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots a_{10} = 11! - 1$

Hints:

(1) Find value of a_n and add.

(2) Use value of 11!.

Shortcut:

$(1! + 1.1!) + 2.2! + 3.3! + \ldots\ldots + 10.10!) - 1! = (2! + 2.2!) + 3.3! + \ldots\ldots + 10.10! - 1$

$[\because n! + nn! = (n+1)!]$

8. Option (b) is correct.

Explanation:

Quadratic equation is $x^2 + px + q = 0$ ...(i)

Roots given are p, q ...(ii)

We know,

$$\text{Sum of roots} = -\frac{b}{a}$$

$\Rightarrow \quad p + q = -\frac{p}{1}$ [from (i), (ii)]

$\Rightarrow \quad p + q = -p$ ...(iii)

Also, product of roots $= \frac{c}{a}$

$\Rightarrow \quad pq = \frac{q}{1}$ [from (i), (ii)]

$\Rightarrow \quad pq = q$

$\Rightarrow \quad p = 1$

Put in (iii),

$\Rightarrow \quad 1 + q = -1$

$\Rightarrow \quad q = -1 - 1$

$\Rightarrow \quad q = -2$

Only one value of q exist

Hints:

(1) Use product and sum formula.

(2) Solve equations.

9. Option (d) is correct.

Explanation:

Consider,

$$A = \begin{vmatrix} 3d+5g & 4a+7g & 6g \\ 3e+5h & 4b+7h & 6h \\ 3f+5i & 4c+7i & 6i \end{vmatrix}$$

Taking, 6 from C_3

$$A = 6\begin{vmatrix} 3d+5g & 4a+7g & g \\ 3e+5h & 4b+7h & h \\ 3f+5i & 4c+7i & i \end{vmatrix}$$

$C_1 \to C_1 - 5C_3,\ C_2 \to C_2 - 7C_3$

$$A = 6\begin{vmatrix} 3d & 4a & g \\ 3e & 4b & h \\ 3f & 4c & i \end{vmatrix}$$

Taking 3 from C_1, 4 from C_2

$$A = 6 \times 3 \times 4\begin{vmatrix} d & a & g \\ e & b & h \\ f & c & i \end{vmatrix}$$

$$A = 72\begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix}$$

[$\because$ transpose of matrix]

$R_1 \leftrightarrow R_2$

$$A = 72 \times -1\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

[Interchange of Rows]

$A = -72\,\Delta$

[given $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \Delta$

Hints:

(1) Add, subtract of row and column.

(2) Interchange of column.

10. Option (a) is correct.

Explanation:

We know, if p, q, r are in A.P., then $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ are in H.P.

Given, $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ in H.P.

So, $(b + c), (c + a), (a + b)$ are in A.P.

$\Rightarrow \quad (b + c) + (a + b) = 2(c + a)$

$\Rightarrow \quad 2b + a + c = 2c + 2a$

$\Rightarrow \quad 2b = a + c$

Hence, a, b, c are in A.P.

Statement 1 is correct.

Now, $(b + c)^2, (c + a)^2, (a + b)^2$ given to check for G.P. condition.

$\Rightarrow \quad (b + c)^2 \times (a + b)^2 = [(c + a)^2]^2$

$\Rightarrow \quad (b + c)^2 \times (a + b)^2 = [c + a]^4$

Which is not possible since,

$2b = c + a$ from A.P.

Hence, statement 2 is incorrect.

Hint:

(1) Use A.P., H.P. G.P. Properties.

11. Option (a) is correct.

Explanation:

Given: $\quad A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

We know that,

$$A^n = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}^n$$

$$\Rightarrow \quad A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^{100} = \begin{bmatrix} 1 & 100a \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^{50} = \begin{bmatrix} 1 & 50a \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^{25} = \begin{bmatrix} 1 & 25a \\ 0 & 1 \end{bmatrix}$$

Using all values in given equation.

Directly apply

$$\Rightarrow \begin{bmatrix} 1 & 100a \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 50a \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 25a \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 50a \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 50a \\ 0 & 2 \end{bmatrix}$$

$\Rightarrow -2I$

Hint:

(1) Use $A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$

12. Option (a) is correct.

Explanation:

Consider,

$$A = \begin{vmatrix} a & -b & a-b-c \\ -a & b & -a+b-c \\ -a & -b & -a-b+c \end{vmatrix}$$

$R_1 \rightarrow R_1 + R_2$

$$A = \begin{vmatrix} 0 & 0 & -2c \\ -a & b & -a+b-c \\ -a & -b & -a-b+c \end{vmatrix}$$

Expand along R_1 for determinant

$$A = 0 + 0 - 2c\begin{vmatrix} -a & b \\ -a & -b \end{vmatrix}$$

$$A = -2c[ab + ab]$$

Apply in given equation,

$\Rightarrow \quad -4abc - kabc = 0$

$\Rightarrow \quad -4abc = kabc$

$\Rightarrow \quad k = -4$

Hints:

(1) Apply $R_1 \rightarrow R_1 + R_2$

(2) Expand along R_1.

13. Option (a) is correct.

Explanation:

Consider, Given $\sum_{n=1}^{8n+7} i^n$

Now, we get by expansion, let

$I = i^1 + i^2 + i^3 + \ldots\ldots\ldots\ldots.. + i^{8n+7}$

We know, if consider

$i^1 + i^2 + i^3 + i^4$, we get:

$\Rightarrow \quad i - 1 - i + 1 = 0$

[value of $i^1 = i, i^2 = -1,$
$i^3 = -i, i^4 = +1$]

Similarly, in expansion consecutive 4 terms get cancel out

So, In $(8n + 7)$ terms we are left with only 3 terms.

$\{(8n + 7)$ divisible by 4 gives remainder 3$\}$

Hence, $I = i^1 + i^2 + i^3$

$= i + (-1) + (-i)$

$= -1$

Hints:
(1) Consecutive 4 terms cancel out.
(2) Remainder 3 terms.

14. Option (c) is correct.

Explanation:

Given: $z = x + iy$ ...(i)

We know, conjugate $\bar{z} = x - iy$ ...(ii)

Now, consider

$$z\bar{z} + |z|^2 + 4(z + \bar{z}) - 48 = 0$$

We know,

$|z|$ = Modulus of complex number

$$|z| = \sqrt{x^2 + y^2} \quad \text{...(iii)}$$

Put (i), (ii), (iii) in equation

$$\Rightarrow (x + iy)(x - iy) + \left(\sqrt{x^2 + y^2}\right)^2 + 4(x + iy + x - iy) - 48 = 0$$

$$\Rightarrow x^2 - (iy)^2 + (x^2 + y^2) + 4(2x) - 48 = 0$$

$\{i^2 = -1\}$

$$\Rightarrow x^2 + y^2 + x^2 + y^2 + 8x - 48 = 0$$

$$\Rightarrow 2x^2 + 2y^2 + 8x - 48 = 0 \quad \text{....(iv)}$$

Compare (iv) with general equation of circle $x^2 + y^2 + 2gx + 2fy + c = 0$

Hence, it is equation of circle.

Hints:
(1) Use conjugate, modulus
(2) Compare with circle

15. Option (a) is correct.

Explanation:

Let $I = 2a + 2\sqrt{a^2 + b^2}$

[Add and Subtract ib, where $i = \sqrt{-1}$]

$$I = (2a + ib - ib) + 2\sqrt{a^2 + b^2}$$

$$I = (a + ib + a - ib) + 2\sqrt{a^2 + b^2}$$

$$I = [(a + ib) + (a - ib)] + 2\sqrt{a^2 + b^2} \quad \text{...(i)}$$

Now, consider $J = (a + ib)(a - ib)$

$J = a(a - ib) + ib(a - ib)$

$J = a^2 - abi + abi - i^2b^2$

$J = a^2 - b^2i^2$

$J = a^2 + b^2$ $\quad [i^2 = -1]$

So,

put $(a + ib)(a - ib) = a^2 + b^2$ in (i), we get

$$I = [(a + ib) + (a - ib)] + 2\sqrt{(a+ib)(a-ib)}$$

$$= \left(\sqrt{a+ib}\right)^2 + \left(\sqrt{a-ib}\right)^2 + 2\sqrt{(a+ib)(a-ib)}$$

$$I = \left[\sqrt{a+ib} + \sqrt{a-ib}\right]^2$$

$\{$using $(a + b)^2 = a^2 + b^2 + 2ab\}$

Hence,

$$2a + 2\sqrt{a^2 + b^2} = \left[\sqrt{a+ib} + \sqrt{a-ib}\right]^2$$

Taking square root both sides, we get

$$\sqrt{2a + 2\sqrt{a^2 + b^2}} = \sqrt{a+ib} + \sqrt{a-ib}$$

Hints:
(1) Add and Subtract *ib.*
(2) $a^2 + b^2 = (a + ib)(a - ib)$.
(3) Use $a^2 + b^2 + 2ab = (a + b)^2$.

16. Option (c) is correct.

Explanation:

Given: Quadratic equation $ax^2 + bx + c = 0$

We know,

$$\text{Product of roots} = \frac{c}{a}$$

$$\text{Sum of roots} = \frac{-b}{a}$$

Since, $\sin\theta$, $\cos\theta$ are roots given, we get

$$\Rightarrow \sin\theta + \cos\theta = \frac{-b}{a} \quad \text{...(i)}$$

$$\Rightarrow \sin\theta\cos\theta = \frac{c}{a} \quad \text{...(ii)}$$

Now, squaring both sides of eq.(i)

$$(\sin\theta + \cos\theta)^2 = \left(\frac{-b}{a}\right)^2$$

$$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{b^2}{a^2}$$

$$\Rightarrow \quad \sin^2\theta + \cos^2\theta + \frac{2c}{a} = \frac{b^2}{a^2}$$

[using (ii)]

$$\Rightarrow \quad 1 + \frac{2c}{a} = \frac{b^2}{a^2}$$

$$\Rightarrow \quad a^2 - b^2 + 2ac = 0$$

Hints:
(1) Use product and Sum of roots.
(2) Squaring of equation.

17. Option (a) is correct.

Explanation:

Given: C(n, 4), C(n, 5), C(n, 6) in A.P.

We know, formula for combination

$$C(n, r) = {}^nC_r = \frac{n!}{r!(n-r)!}\text{, we get}$$

$$\Rightarrow \quad C(n, 4) = \frac{n!}{4!(n-4)!}$$

$$\Rightarrow \quad C(n, 5) = \frac{n!}{5!(n-5)!}$$

$$\Rightarrow \quad C(n, 6) = \frac{n!}{6!(n-6)!}$$

Since, given in A.P.

$$2 \times C(n, 5) = C(n, 4) + C(n, 6)$$

$$2\,{}^nC_5 = {}^nC_4 + {}^nC_6$$

$$\frac{2n!}{(n-5)!5!} = \frac{n!}{(n-4)!4!} + \frac{n!}{(n-6)!6!}$$

$$\Rightarrow \quad \frac{2}{(n-5)!5!} = \frac{1}{(n-4)!4!} + \frac{1}{(n-6)!6!}$$

$$\Rightarrow \quad \frac{2}{(n-5)(n-6)!5!} = \frac{1}{(n-4)(n-5)(n-6)!4!} + \frac{1}{(n-6)!6!}$$

Multiplying $6!(n-6)!$ in all terms

$$\Rightarrow \quad \frac{2\times 6!}{5!\times(n-5)} = \frac{6!}{(n-4)(n-5)4!} + 1$$

$$\Rightarrow \quad \frac{12}{(n-5)} = \frac{30}{(n-4)(n-5)} + 1$$

$$\Rightarrow \frac{12}{(n-5)} - \frac{30}{(n-4)(n-5)} = 1$$

$$\Rightarrow \quad \frac{12n-48-30}{(n-4)(n-5)} = 1$$

$$\Rightarrow \quad 12n - 48 - 30 = n^2 - 9n + 20$$

$$\Rightarrow \quad n^2 - 21n + 98 = 0$$

$$\Rightarrow \quad (n-7)(n-14) = 0$$

$$\Rightarrow \quad n = 7 \text{ or } n = 14$$

Hence, $n = 7$ is correct answer.

$$\Rightarrow \quad \frac{2\times n!}{5!(n-5)!} = \frac{n!}{4!(n-4)!} + \frac{n!}{6!(n-6)!}$$

$$\Rightarrow \quad \frac{2}{5!(n-5)!} = \frac{1}{4!(n-4)!} + \frac{1}{6!(n-6)!}$$

$$\Rightarrow \quad \frac{2}{5!(n-5)!} = \frac{6!(n-6)!+4!(n-4)!}{4!(n-4)!\times 6!(n-6)!}$$

$$\Rightarrow \frac{2\ 4!(n-4)!\times 6!(n-6)!}{5!(n-5)!} = 6!(n-6)! + 4!(n-4)!$$

$$\Rightarrow \frac{2x4!(n-4)!\times 6\times 5!\times(n-6)\times(n-5)!}{5!(n-5)!} = 6!(n-6)! + 4(n-4)!$$

$$\Rightarrow 2[4!(n-4)! \times 6 \times (n-6)\,] = 6!(n-6)! + 4(n-4)!$$

$$\Rightarrow 4!(n-4)! \times 12 \times (n-6) = 6 \times 5 \times 4! \times (n-6) \times (n-5)(n-4)! + 4(n-4)!$$

$$\Rightarrow 4!(n-4)! \times 6 \times (n-6) = 4!(n-4)!\,[6 \times 5(n-6)(n-5) + 1]$$

$$\Rightarrow \quad 12(n-6) = 30(n-6)(n-5) + 1$$

$$\Rightarrow \quad 12n - 72 = 30[n^2 - 5n - 6n + 30] + 1$$

$$\Rightarrow \quad 12n - 72 = 30[n^2 - 11n + 30] + 1$$

$$\Rightarrow \quad 12n - 72 = 30n^2 - 330n + 900 + 1$$

$$\Rightarrow \quad 12n - 72 = 30n^2 - 330n + 901$$

$$\Rightarrow 30n^2 - 342n + 973 = 0\text{, we get}$$

$$\Rightarrow \quad n = 7$$

Hints:
(1) Use combination formula.
(2) Solve equation.

18. Option (a) is correct.

Explanation:

Consider, word LUCKNOW,

It contain 2 vowels (O, U) and 5 constant (L, C, K, N, W)

Now, we have to make 4 letter, it must

Contain 2 vowels and only 2 consonant out of 5, we get

$\Rightarrow$Ways to select consonant = ${}^5C_2$

$\Rightarrow$ Ways to select vowels = ${}^2C_2$

Finally, 4 letter words = ${}^2C_2 \times {}^5C_2 \times 4!$

$= 1 \times 10 \times 24$

$= 240$

Hints:

(1) Select vowel, consonant.

(2) 4 Letter word.

19. Option (b) is correct.

Explanation:

Given: Twenty distinct points on circle.

First, we have join any two points to make line.

Using concept of combination, we get

Number of lines(l) = ${}^{20}C_2$

$$l = \frac{20!}{18! \times 2!}$$

$$\left[{}^nC_r = \frac{n!}{r!(n-r)!}\right]$$

$$l = \frac{20 \times 19}{2}$$

$[n! = n(n-1)(n-2)! \ldots\ldots]$

$l = 190$

Statement 1 is incorrect.

Now, to make triangle, we have to connect 3 points.

Number of triangles (T) = ${}^{20}C_3$

$$T = \frac{20!}{3!(20-3)!}$$

$$T = \frac{20!}{3! \times 17!}$$

$$T = \frac{20 \times 19 \times 18 \times 17!}{3 \times 2 \times 17!}$$

$$T = \frac{20 \times 19 \times 18}{3 \times 2}$$

$T = 1140$

Statement 2 is correct.

Hint:

(1) Use combination concept.

20. Option (d) is correct.

Explanation:

Given: expansion of $\left(\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2\right)^{21} = I$ ....(i)

Now, we know

$$(x + y)^2 = x^2 + y^2 + 2xy$$

Using this, we get

$$\left[\frac{a}{b} + \frac{b}{a}\right]^2 = \frac{a^2}{b^2} + \frac{b^2}{a^2} + 2$$

Putting in (i),

$$\left[\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2\right]^{21} = \left[\left[\frac{a}{b} + \frac{b}{a}\right]^2\right]^{21}, \text{ we get}$$

$$I = \left[\frac{a}{b} + \frac{b}{a}\right]^{42}$$

We know, number of terms in expansion of $(x + y)^n$ is $(n + 1)$

Hence, number of terms = 42 + 1

= 43

Hints:

(1) Use $(a + b)^2$ formula.

(2) Number of terms is $(n + 1)$.

21. Option (b) is correct.

Explanation:

Consider, system of equations:

$2k^2x + 3y - 1 = 0$ ...(i)

$7x - 2y + 3 = 0$ ...(ii)

$6kx + y + 1 = 0$ ...(iii)

Since, solution given is consistent

$$\begin{vmatrix} 2k^2 & 3 & -1 \\ 7 & -2 & 3 \\ 6k & 1 & 1 \end{vmatrix} = 0$$

Expand along row 1

$$2k^2\begin{vmatrix} -2 & 3 \\ 1 & 1 \end{vmatrix} - 3\begin{vmatrix} 7 & 3 \\ 6k & 1 \end{vmatrix} - 1\begin{vmatrix} 7 & -2 \\ 6k & 1 \end{vmatrix} = 0$$

$2k^2[-2 \ -3] - 3[7 - 18k] - 1[7 + 12k] = 0$

$2k^2(-5) - 21 + 54k - 7 - 12k = 0$

$-10k^2 + 42k - 28 = 0$

$$-2[5k^2 - 21k + 14] = 0$$

$$\Rightarrow \quad 5k^2 - 21k + 14 = 0$$

For quadratic equation, consider

$a = 5, b = -21, c = 14$

By quadratic formula

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ we get}$$

$$k = \frac{21 \pm \sqrt{161}}{10}$$

Hints:

(1) Use consistent solution condition.

(2) Solve equation.

22. Option (a) is correct.

Explanation:

Consider, $$A = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

We know,

$$A^{-1} = \frac{1}{|A|} \quad \text{[Adjoint of A]} \quad \ldots(i)$$

Now, we know

$$\left\{ \begin{matrix} \text{if } B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ then} \\ \text{Adjoint of } B = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \end{matrix} \right\}$$

By this we get:

$$\text{Adjoint of } A = \begin{bmatrix} \frac{-1}{2} & -1 \\ -\frac{3}{2} & -2 \end{bmatrix}$$

Now, Since $$A = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

$$|A| = \left[-2 \times -\frac{1}{2} - \frac{3}{2} \times 1 \right] = -\frac{1}{2}$$

Hence, $$A^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

23. Option (b) is correct.

Explanation:

Period of composite functions such that $f[g(x)]$ is t if $f(x)$ is non period function and $g(x)$ is period of t.

Since, logarithmic and constant are non-period function. We find period on basis of $\sin^2 x$. ...(i)

Now, we know, period of $\sin x$ is 2π.

We find period of $\sin^2 x$, By trigonometric formula:

$$2\sin^2 x = 1 - \cos 2x$$

$$\Rightarrow \quad \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$$

[period of $\cos x$ is 2π and period of $\cos 2x$ is π]

$\Rightarrow$ period of $\sin^2 x$ is π ...(ii)

Hence,

$$\text{Period of } f(x) = \log(2 + \sin^2 x)$$
$$= \pi \quad \text{[By (i), (ii)]}$$

Hints:

(1) Period of composite function.

(2) Period of $\sin^2 x$ is π.

24. Option (d) is correct.

Explanation:

Given: $\sin(A + B) = 1$

$$\sin(A + B) = \sin \frac{\pi}{2}$$

On comparing

$$A + B = \frac{\pi}{2} \quad \ldots(i)$$

Now, $2\sin(A - B) = 1$

$$\sin(A - B) = \frac{1}{2}$$

$$\sin(A - B) = \sin \frac{\pi}{6}$$

$$A - B = \frac{\pi}{6} \quad \ldots(ii)$$

On solving (i) and (ii), we get

$$A = \frac{\pi}{3}, B = \frac{\pi}{6}$$

Now, to find $$I = \frac{\tan A}{\tan B}$$

$$I = \frac{\tan \frac{\pi}{3}}{\tan \frac{\pi}{6}} = \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = \frac{3}{1}$$

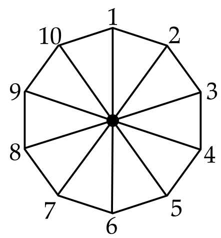
$\Rightarrow$ tan A : tan B = 3 : 1

Hint:

sin 90° = 1.

25. Option (b) is correct.

Explanation:



Number of triangles that can be formet from any 10 vertices $=10C_3 \frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1}=120$

Number of triangles that can be formed with only one side of triangles .....(1, 2, 9), (2, 3, 5) .... (2, 3, 10) .... y $=(10-4)\times 10=60$

Numbe of triangles that can be formed with two consecutive sides of polygon.

e.g., {(1, 2, 3), (2, 3, 4) ...... (10, 1, 2)} =10

$\therefore$Number of triangly that can be formed with no common side with any of the sides of the polygon

$$=120-60-10=50$$

Hints:

(1) Find all triangle,

(2) Find number of triangle with one side common.

(3) Find number o triangle with two side common.

26. Option (c) is correct.

Explanation:

Given: Quadratic equation:

$$x^4-10x^2+9=0$$

Put $x^2 = y$, we get

$\Rightarrow \quad y^2-10y+9=0$

$\Rightarrow \quad y^2-9y-y+9=0$

$\Rightarrow \quad y(y-9)-1(y-9)=0$

$\Rightarrow \quad (y-1)(y-9)=0$

$\Rightarrow \quad y=1, 9$

But $y = x^2$, we get

$\Rightarrow \quad x^2=1, x^2=9$

$\Rightarrow \quad x=+1, -1, +3, -3$

Absolute values of roots = $|x|$, we get

Sum of absolute values:

$$S = |+1| + |-1| + |+3| + |-3|$$

$$S = +1+1+3+3$$

$$S = 8$$

Hints:

(1) Solve quadratic equation

(2) Sum of absolute values

27. Option (c) is correct.

Explanation:

In expansion of $(1+x)^n$:

Coefficient of first term = $^nC_0 = 1$

Coefficient of second term = $^nC_1 = n$

Coefficient of n^{th} term = $^nC_{n-1} = n$

Coefficient of $(n+1)^{th}$ term = $^nC_n = 1$

Given that coefficient's are p, q, r, s:

$\Rightarrow \quad {}^nC_0 = p = 1$

$\Rightarrow \quad {}^nC_1 = q = n$

$\Rightarrow \quad {}^nC_{n-1} = r = n$

$\Rightarrow \quad {}^nC_n = s = 1$

Now, to find let I = $ps + qr$

$$I = 1\times 1 + n\times n$$

$$= 1 + n^2$$

Hints:

(1) Coefficient of first, second, n^{th}, $(n+1)^{th}$ term.

(2) Value of $^nC_r = \frac{n!}{r!(n-r)!}$

28. Option (c) is correct.

Explanation:

Given:

$$\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3}{2}\pi$$

We write

$$\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2}$$

On comparing, we get

$$\sin^{-1}x = \frac{\pi}{2}, \sin^{-1}y = \frac{\pi}{2}, \sin^{-1}z = \frac{\pi}{2}$$

$\Rightarrow x = 1, y = 1, z = 1$ ...(i)

Now, to find I = $x^{1000} + y^{1001} + z^{1002}$

Using values from (i)

$$I = 1 + 1 + 1 = 3$$
$$I = 3$$

Hints:

(1) $\sin 90° = 1$

(2) Split $\frac{3}{2}\pi$ and get value

29. Option (a) is correct.

Explanation:

Given:

$\sin x + \sin y = \cos x + \cos y$

Use formulas, we get

$$\begin{bmatrix} \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\ \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \end{bmatrix}$$

$$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

On concelling out, we get

$$\Rightarrow \sin\left(\frac{x+y}{2}\right) = \cos\left(\frac{x+y}{2}\right)$$

$$\Rightarrow \sin\left[\frac{x}{2}+\frac{y}{2}\right] = \cos\left[\frac{x}{2}+\frac{y}{2}\right]$$

$$\Rightarrow \frac{\sin\left[\frac{x}{2}+\frac{y}{2}\right]}{\cos\left[\frac{x}{2}+\frac{y}{2}\right]} = 1$$

[Divide $\cos\left[\frac{x}{2}+\frac{y}{2}\right]$ both sides]

$$\Rightarrow \tan\left[\frac{x}{2}+\frac{y}{2}\right] = 1$$

$$\left[\because \frac{\sin\theta}{\cos\theta} = \tan\theta\right]$$

Hints:

(1) Use formulae of sin (A + B), cos (A + B)

(2) Use formulae $\frac{\sin\theta}{\cos\theta} = \tan\theta$

30. Option (d) is correct.

Explanation:

Consider, $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

Now, we know

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow mI = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \quad \ldots(i)$$

$$\Rightarrow nA = \begin{bmatrix} 0 & 2n \\ -2n & 0 \end{bmatrix} \quad \ldots(ii)$$

Consider, given equation $(mI + nA)^2 = A$

{By (i), (ii)}

$$\Rightarrow \left(\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} + \begin{bmatrix} 0 & 2n \\ -2n & 0 \end{bmatrix}\right)^2 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} m & 2n \\ -2n & m \end{bmatrix}^2 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} m & 2n \\ -2n & m \end{bmatrix}\begin{bmatrix} m & 2n \\ -2n & m \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} m^2 - 4n^2 & 2mn + 2mn \\ -2mn - 2mn & -4n^2 + m^2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

On comparing, we get

$$m^2 - 4n^2 = 0 \quad \ldots(iii)$$
$$2mn + 2mn = 2 \quad \ldots(iv)$$

Now, from (iv),

$$\Rightarrow 4mn = 2$$

$$\Rightarrow mn = \frac{1}{2}$$

$$\Rightarrow m = \frac{1}{2n} \quad \ldots(v)$$

Put in (iii),

$$\Rightarrow \left(\frac{1}{2n}\right)^2 - 4n^2 = 0$$

$$\Rightarrow \frac{1}{4n^2} - 4n^2 = 0$$

$$\Rightarrow \frac{1-16n^4}{4n^2} = 0$$

$\Rightarrow \quad 1 - 16n^4 = 0$

$\Rightarrow \quad 1 = 16n^4$

$\Rightarrow \quad \frac{1}{16} = n^4$

$\Rightarrow \quad \left(\frac{1}{2}\right)^4 = n^4$

$\Rightarrow \quad n = \frac{1}{2}$

Put in (ii), we get

$m = 1$

Hence, $m + n = 1 + \frac{1}{2} = \frac{3}{2}$

Hints:
(1) Make matrix and Satisfy equations.
(2) Solve by comparing.

31. Option (a) is correct.

Explanation:

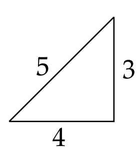
Given: $\cot\left[\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right]$

Let

$I = \cot\left[\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right]$

Now, consider $\sin^{-1}\frac{3}{5} = x$

$\Rightarrow \quad \frac{3}{5} = \sin x$



By pythogoras theorem, we get:

$P^2 + B^2 = H^2$

So, $\quad \tan x = \frac{3}{4}$

$x = \tan^{-1}\frac{3}{4}$

$\text{Cot}^{-1}\frac{3}{2} = \tan^{-1}\frac{2}{3}$

$I = \cot\left[\tan^{-1}\left[\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right]\right]$

$= \cot\left[\tan^{-1}\left[\frac{\frac{9+8}{12}}{1-\frac{1}{2}}\right]\right]$

$= \cot\left[\tan^{-1}\left[\frac{\frac{17}{12}}{\frac{1}{2}}\right]\right]$

$= \cot\left[\tan^{-1}\left[\frac{17}{6}\right]\right]$

$= \cot\left[\cot^{-1}\left(\frac{6}{17}\right)\right]$

$\left[\because \tan^{-1}x = \cot^{-1}\frac{1}{x}\right]$

$= \frac{6}{17}$

Hints:
(1) Comment function in $\tan\theta$.
(2) Use formula $\tan(A + B)$.

32. Option (c) is correct.

Explanation:

Given: $\quad 4\sin^2 x = 3$

$\Rightarrow \quad \sin^2 x = \frac{3}{4}$

$\Rightarrow \quad \sin x = \frac{\sqrt{3}}{2}$

[Taking square root]

$\Rightarrow \quad \sin x = \sin 60°$

$\Rightarrow \quad x = 60°, 120°$

Now, to find $I = \tan 3x$

$I = \tan 3 \times 60°$

$I = \tan 180°$

We can write

$I = \tan(180° + 0°)$

$[\tan(180° + \theta) = \tan\theta]$

$I = \tan 0° = 0$

$I = \tan 3 \times 120°$

$= \tan 360°$

Hints:

(1) Get x by solving.

(2) Put in given equation.

33. Option (c) is correct.

Explanation:

Given: for AP, first term = p

third term = q

fifth term = 3

Let for A.P. first term = a, difference = d

Now, for n^{th} term

$$a_n = a + (n-1)d$$

$\Rightarrow$ $a_1 = a = p$

Third term, we get:

$\Rightarrow$ $a_3 = a + (3-1)d = q$

$a_3 = a + 2d = q$ …(i)

Fifth term, we get

$\Rightarrow$ $a_5 = a + (5-1)d = 3$

$\Rightarrow$ $a_5 = p + 4d = 3$ …(ii)

Now, to find Let I = pq

I = pq

I = $(3-4d)(a+2d)$ [from (i) & (ii)]

$= (3-4d)(3-4d+2d)$

$= (3-4d)(3-2d)$

$= 3(3-2d) - 4d(3-2d)$

$= 9 - 6d - 12d + 8d^2$

$= 9 - 18d + 8d^2$

Now, we have to find I minimum

$$I = 9 - 18d + 8d^2$$

Differentiate with respect to d

$$\frac{dI}{dd} = 0 - 18 + 8 \times 2d$$

$$\frac{dI}{dd} = -18 + 16d$$

Put $\frac{dI}{dd} = 0$

$-18 + 16d = 0$

$16d = 18$

$$d = \frac{18}{16} = \frac{9}{8}$$

At $d = \frac{9}{8}$, critical point minimum, value of I

$$I = 9 - 18\left(+\frac{9}{8}\right) + 8\left(+\frac{9}{8}\right)^2$$

$I = -\frac{9}{8}$, to check

$\frac{d^2I}{d^2d} = +16$, which is Positive

Hints:

(1) Use a_n formula for n^{th} term.

(2) Find pq, make in respect of d

(3) Differentiate to find critical point

34. Option (a) is correct.

Explanation:

Given:

Equation is $x^3 - 8 = 0$

Now, $x^3 - 8 = 0$

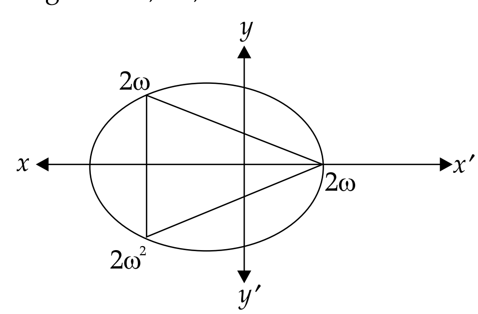
$\Rightarrow$ $x^3 = 8$

$\Rightarrow$ $x^3 = 2^3$

$\Rightarrow$ $x = 2$

By property of complex number, that says for $x^3 - 1 = 0$, roots are 1, ω, ω^2

We get: $x = 2, 2\omega, 2\omega^2$



Statement 1: Roots are non-collinear correct. We can see from figure, roots not on one line.

So, true.

Statement 2: Roots lie on circle inside this triangle but circle is of Radius 2.

As, complex roots is 2. So, false.

Hence, statement 1 correct only.

Hints:

(1) Use $x^3 - 1 = 0$.

(2) Find roots, Make figure.

35. Option (c) is correct.

Explanation:

Given: $\sec x \cdot \operatorname{cosec} x = p$ …(i)

We know,

$$\sec x = \frac{1}{\cos x}$$

$$\text{cosec } x = \frac{1}{\sin x}$$

Put value in (i),

$$\Rightarrow \quad \frac{1}{\cos x} \times \frac{1}{\sin x} = p$$

$$\Rightarrow \quad \frac{1}{\sin x \cos x} = p$$

[Multiple and Divide by 2]

$$\Rightarrow \quad \frac{2}{2\sin x \cos x} = p$$

$$\Rightarrow \quad \frac{2}{\sin 2x} = p$$

[using $\sin 2x = 2\sin x \cos x$]

Now, we know value of sine function maximum is +1 and minimum is –1.

$\Rightarrow$ $\sin 2x$, maximum is +1 and minimum is –1

Now, $\quad p = \dfrac{2}{\sin 2x}$

Hints:

(1) Use trigonometric formulae.

(2) Sin x value maximum and minimum.

36. Option (b) is correct.

Explanation:

$\sin\theta + \sin\theta\cos\theta$

Let $\quad y = \sin\theta + \sin\theta\cos\theta$

We have to find maximum value of y.

Differentiate with respect to θ.

$$\Rightarrow \quad \frac{dy}{d\theta} = \cos\theta + [\sin\theta \times -\sin\theta + \cos\theta \times + \cos\theta]$$

$$\Rightarrow \quad \frac{dy}{d\theta} = \cos\theta + [-\sin^2\theta + \cos^2\theta]$$

$$\Rightarrow \quad \frac{dy}{d\theta} = \cos\theta - \sin^2\theta + \cos^2\theta \quad \text{...(i)}$$

Put $\quad \dfrac{dy}{d\theta} = 0$

$$\Rightarrow \quad \cos\theta - \sin^2\theta + \cos^2\theta = 0$$

$$\Rightarrow \quad \cos\theta - (1 - \cos^2\theta) + \cos^2\theta = 0$$

[Using $\sin^2\theta + \cos^2\theta = 1$]

$$\Rightarrow \quad 2\cos^2\theta + \cos\theta - 1 = 0$$

$$\Rightarrow \quad 2\cos^2\theta + 2\cos\theta - \cos\theta - 1 = 0$$

[Middle term split]

$$\Rightarrow \quad 2\cos\theta(\cos\theta + 1) - 1(\cos\theta + 1) = 0$$

$$\Rightarrow \quad (\cos\theta + 1)(2\cos\theta - 1) = 0$$

$\Rightarrow \cos\theta = -1$ or $\cos\theta = \dfrac{1}{2}$, we get:

$$\theta = \pi \text{ or } \frac{\pi}{3}$$

But, given that $0 < \theta < \dfrac{\pi}{2}$

So, $\quad \theta = \dfrac{\pi}{3}$

Now, to check from (i),

$$\frac{d^2y}{d\theta^2} = -\sin\theta - 2\sin\theta\cos\theta + 2\cos\theta \times -\sin\theta$$

$$= -\sin\theta - \sin 2\theta - \sin 2\theta$$

$$= -\sin\theta - 2\sin 2\theta$$

At $\theta = \dfrac{\pi}{3}$

$$= -\sin\frac{\pi}{3} - 2\sin\frac{2\pi}{3}$$

$$= \frac{-\sqrt{3}}{2} - 2 \times \frac{\sqrt{3}}{2}$$

$$= \frac{-\sqrt{3}}{2} - \sqrt{3}$$

$\dfrac{d^2y}{d\theta^2}$ is negative at $\theta = \dfrac{\pi}{3}$ means maximum.

Hints:

(1) Differentiate and find θ.

(2) Use trigonometric formula.

37. Option (a) is correct.

Explanation:

Statement 1: Union over intersection is distributive correct.

Ex. $A \cup (B \cap C) \Rightarrow A \cap (B \cup C)$

Statement 2: It is property of sets of complement of two sets. So, correct.

Ex. $\quad (A \cup B)' = A' \cap B'$

Statement 3: It is false, prove by example

Let $A = \phi$, $B = [1, 2, 3]$

$\Rightarrow \quad A - B = \phi$

So, A – B is void but A and B is not equal.

Hint:

(1) Use property of sets.

38. Option (c) is correct.

Explanation:

Given: X set = 6 elements

Y set = 5 elements

Z set = 4 elements

It is also given that all 15 elements are distinct.

Now, to find elements in S we get step by step.

Elements in X – Y = X = 6 elements

[∵ No elements common in X, so none get cancel out]

Now,

Element in $S = (X - Y) \cup Z$

$\Rightarrow \quad S = \text{element in } (X - Y) \cup \text{Element in } Z$

$\Rightarrow \quad S = 6 \text{ element} \cup 4 \text{ element}$

$\Rightarrow \quad S = 10 \text{ element}$

[∵ No element common, So, all add]

Now, S have 10 element, we have to find proper subsets.

Formula of proper subsets = $2^n - 1$

$\Rightarrow$ proper subsets $= 2^{10} - 1 = 1024 - 1 = 1023$

Hints:

(1) No element common, so find total element.

(2) Use formula of proper subsets.

39. Option (b) is correct.

Explanation:

Statement 1: All relation are not function because in function need only one image and in relation more than one image possible. So, false.

Statement 2: It is correct. Relation is subset of A × B.

Statement 3: It is correct. When relation A × A is possible. So, elements is subset.

Hints:

(1) All relation not function.

(2) Use definition of relation.

40. Option (b) is correct.

Explanation:

Given:

$\log_{10} 2 \log_2 10 + \log_{10}(10^x) = 2$

We know that,

$$\log_a b = \frac{\log b}{\log a} \quad \ldots(i)$$

and $\log (a)^b = b \log a \quad \ldots(ii)$

Using (i) and (ii) formula in given equation we get:

$$\Rightarrow \frac{\log 2}{\log 10} \times \frac{\log 10}{\log 2} + \frac{x \times \log 10}{\log 10} = 2$$

$$\Rightarrow 1 + \frac{x \times \log 10}{\log 10} = 2$$

$\Rightarrow \quad 1 + x \times 1 = 2$

$\Rightarrow \quad 1 + x = 2$

$\Rightarrow \quad x = 1$

Hints:

(1) Use logarithmic formula.

(2) Solve and cancel out same terms.

41. Option (d) is correct.

Explanation:

Given: ABC is triangle

and cos 2A + cos 2B + cos 2C = –1

Now, we know

Sum of angles of triangle = 180°

$\Rightarrow \quad A + B + C = 180° \quad \ldots(i)$

Now, using cos 2A + cos 2B + cos 2C = –1

$\Rightarrow \quad 1 + \cos 2A + \cos 2B + \cos 2C = 0$

$\Rightarrow \quad 2\cos^2 A + \cos 2B + \cos 2C = 0$

$[\because 1 + \cos 2A = 2\cos^2 A]$

$$\left[\text{using } \cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right) \quad 2\cos\left(\frac{x-y}{2}\right)\right]$$

$$\Rightarrow 2\cos^2 A + \left[2\cos\left(\frac{2B+2C}{2}\right)\cos\left(\frac{2B-2C}{2}\right)\right] = 0$$

$\Rightarrow \quad 2\cos^2 A + [2\cos (B + C) \cos (B - C)] = 0$

[using A + B + C = 180°]

$\Rightarrow \quad 2\cos^2 A + 2\cos(180° - A) \cos(B - C) = 0$

$\Rightarrow \quad 2\cos^2 A + 2 \times -\cos A \cos (B - C) = 0$

$[\because \cos (\pi - \theta) = -\cos \theta]$

$\Rightarrow \quad 2\cos^2 A - 2\cos A \cos (B - C) = 0$

$\Rightarrow \quad 2\cos A [\cos A - \cos (B - C)] = 0$

[using A + B + C = 180°]

$\Rightarrow$ 2cos A[cos(180 – (B + C) – cos(B – C)] = 0

$\Rightarrow$ 2cos A[–cos(B + C) – cos(B – C)] = 0

[$\because$ cos (π – θ) = – cos θ]

$\Rightarrow$ –2cos A[cos(B + C) + cos(B – C)] = 0

[using $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$]

$\Rightarrow$ –2cos A

$$\left[2\cos\left(\frac{B+C+B-C}{2}\right)\cos\left(\frac{B+C-B+C}{2}\right)\right] = 0$$

$\Rightarrow$ –2 cos A × 2cos B cos C = 0

$\Rightarrow$ cos A cos B cos C = 0

Hints:

(1) Use A + B + C = 180° to change function.

(2) Use Trigonometric formulae.

42. Option (b) is correct.

Explanation:

Consider,

$$A = \begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$$

Expand along R_1

$$|A| = \cos C\begin{vmatrix} 0 & -\tan A \\ \sin B & \cos C \end{vmatrix} - \tan A\begin{vmatrix} \sin B & -\tan A \\ 0 & \cos C \end{vmatrix} + 0$$

= cos C[0 + tan A sin B] – tan A[sin B cos C – 0]

= cos C tan A sin B – tan A sin B cos C = 0

$\Rightarrow$ |A| = 0

Hint:

(1) Expand determinant along Row 1.

43. Option (c) is correct.

Explanation:

Given: Set A and Set B

Set A is first 250 Natural Number and Multiple of 3.

$\Rightarrow$ A = {3, 6, 9 …………. 750}

Set B is first 200 Natural number and even.

$\Rightarrow$ B = {2, 4, 8……………396}

Now, common elements in set A and set B are those that Divisible of 6.

$\Rightarrow$ A ∩ B = {6, 12……………….396}

Now, we find elements number in A ∩ B.

This is A.P. = 6, 12, 18…………….. 400

$\Rightarrow$ $a = 6, d = 6$

{using formula of n^{th} term}

$$a_n = a + (n-1)d$$

$\Rightarrow$ $396 = 6 + (n-1)6$

$\Rightarrow$ $390 = (n-1)6$

$\Rightarrow$ $n = 66$

So, A ∩ B have 66 terms

Now, finally terms in A ∪ B

$\Rightarrow$ $n(A \cup B) = n|A| + n|B| - n(A \cap B)$

$= 250 + 200 - 66$

$= 384$

Hints:

(1) Make set A and set B.

(2) Find element in A ∩ B.

44. Option (c) is correct.

Explanation:

Given: S_k in sum of k terms.

We know, sum of first k terms of any A.P.

$$S_k = \frac{k}{2}[2a + (k-1)d]$$

Now,

$\Rightarrow$ $S_{30} = \frac{30}{2}[2a + (30-1)d]$

$= 15(2a + 29d)$

$S_{20} = \frac{20}{2}[2a + (20-1)d]$

$= 10(2a + 19d)$

$S_{10} = \frac{10}{2}[2a + (10-1)d$

$= 5(2a + 9d)$

$\therefore$ $\frac{S_{30}}{S_{20} - S_{10}} = \frac{15(2a+29d)}{10(2a+19d) - 5(2a+9d)}$

$= \frac{15(2a+29d)}{5(4a+38d-2a-9d)}$

$= \frac{15}{5} \times \frac{(2a+29d)}{(2a+29d)}$

$= 3$

Hints:

(1) Use formula of n terms.

(2) Find value.

45. Option (c) is correct.

Explanation:

Given: Quadratic equation

$4x^2 - (5k+1)x + 5k = 0$

For any quadratic equation:

Let $a = 4$, $b = -(5k+1)$, $c = 5k$ …(i)

By quadratic formula:

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

So, two roots are:

$$x = \frac{-b+\sqrt{D}}{2a}, \frac{-b-\sqrt{D}}{2a}$$

Given, difference of roots is unity

$$\Rightarrow \left(\frac{-b+\sqrt{D}}{2a}\right) - \left(\frac{-b-\sqrt{D}}{2a}\right) = 1$$

$$\Rightarrow \frac{-b+\sqrt{D}+b+\sqrt{D}}{2a} = 1$$

$$\Rightarrow \frac{\sqrt{D}}{a} = 1$$

$[D = b^2 - 4ac]$

$$\Rightarrow \frac{\sqrt{b^2-4ac}}{a} = 1$$

$$\Rightarrow \sqrt{b^2-4ac} = a$$

Square both sides

$$\Rightarrow b^2 - 4ac = a^2$$

$$\Rightarrow (5k+1)^2 - 4 \times 4 \times 5k = 4^2$$

[from (i)]

$$\Rightarrow 25k^2 + 1 + 10k - 80k = 16$$

$$\Rightarrow 25k^2 - 70k + 1 - 16 = 0$$

$$\Rightarrow 25k^2 - 70k - 15 = 0$$

$$\Rightarrow 5(5k^2 - 14k - 3) = 0$$

$$\Rightarrow 5k^2 - 14k - 3 = 0$$

$$\Rightarrow 5k^2 - 15k + k - 3 = 0$$

(Middle term split)

$$\Rightarrow 5k(k-3) + 1(k-3) = 0$$

$$\Rightarrow (k-3)(5k+1) = 0$$

$$\Rightarrow k = +3, \frac{-1}{5}$$

Only option is (c) $\frac{-1}{5}$.

Hints:

(1) Use quadratic equation.

(2) Use given usity condition.

46. Option (a) is correct.

Explanation:

Given: five digits number need to make from 3, 5, 7, 9.

For 5 digit number from 4 digit, one number will get repeat and it is given all 4 digits must appear.

So, we get:

Number of ways for 5 Digit number where 2 Digit Repeat $= \frac{5!}{2!}$

Now, finally all digits use so multiply by 4.

Hence,

$$\text{Number formed} = \frac{5!}{2!} \times 4 = 240$$

Hints:

(1) 5 Digit number but 2 Repeat.

(2) All four used, so multiply.

47. Option (a) is correct.

Explanation:

Given: Matrix made with four entries.

So, it matrix of 2×2

$\Rightarrow$ Every digit fill by (1, 2). So, entries allowed in each is two

$$\Rightarrow \quad A = \begin{bmatrix} - & - \\ - & - \end{bmatrix}_{2\times 2}$$

Hence, number of matrix $= 2 \times 2 \times 2 \times 2$

$= 2^4$

$= 16$

Hints:

(1) Fill each entry with two number.

(2) Four entries have to fill.

48. Option (b) is correct.

Explanation:

We know, polar form

$$i = r(\cos\theta + i\sin\theta)$$

Consider, $r = 1, \theta = \frac{\pi}{2}$, we get

$$(i)^{-2n} = 1\left[\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right]$$

[By polar form]

$$= \left[\cos(-2n)\frac{\pi}{2} + i\sin(-2n)\frac{\pi}{2}\right]$$

$$= [\cos(-n\pi) + i\sin(-n\pi)]$$

$\{\sin n\pi = 0\}$

$$= \cos(-n\pi)$$

$$= \cos n\pi \quad [\cos(-\theta) = \cos\theta]$$

$$= +1 \text{ or } -1 \quad [n \in Z \text{ given}]$$

Hence, we get two values.

Hints:
(1) Use polar form.
(2) Put trigonometric formula.

Shortcut:

$$I = i - 2n = (-1)^{-n} = \frac{1}{(-1)^n}$$

So, I = 1 (When n is even)

I = 2 (When n is odd)

49. Option (a) is correct.

Explanation:

Given:

$$I = \begin{vmatrix} 1 & -x & x \\ 1 & 1 & -y \\ 1 & z & 1 \end{vmatrix}$$

Expand along R_1

$$I = 1\begin{vmatrix} 1 & -y \\ z & 1 \end{vmatrix} - (-x)\begin{vmatrix} 1 & -y \\ 1 & 1 \end{vmatrix} + x\begin{vmatrix} 1 & 1 \\ 1 & z \end{vmatrix}$$

$$I = 1[1 + yz] + x[1 + y] + x[z - 1]$$

$$I = 1 + yz + x + xy + xz - x$$

$$I = 1 + yz + xy + xz \quad \ldots\text{(i)}$$

Now, given values

$$x = \frac{a}{b-c}, y = \frac{b}{c-a}, z = \frac{c}{a-b}$$

Put values in (i),

$$I = 1 + \frac{b}{c-a}\times\frac{c}{a-b} + \frac{a}{b-c}\times\frac{b}{c-a} + \frac{a}{b-c}\times\frac{c}{a-b}$$

$$I = 1 + \frac{bc}{(c-a)(a-b)} + \frac{ab}{(b-c)(c-a)} + \frac{ac}{(b-c)(a-b)}$$

$$I = \frac{(a-b)(b-c)(c-a) + bc(b-c) + ab(a-b) + ac(c-a)}{(a-b)(b-c)(c-a)}$$

$$I = \frac{(ab - ac - b^2 + bc)(c-a) + b^2c - bc^2 + a^2b - ab^2 + ac^2 - a^2c}{(a-b)(b-c)(c-a)}$$

$$I = \frac{abc - ac^2 - b^2c + bc^2 - a^2b + a^2c + ab^2 - abc + b^2c - bc^2 + a^2b - ab^2 + ac^2 - a^2c}{(a-b)(b-c)(c-a)}$$

$I = 0$ (after solving numerator)

Hints:
(1) Expand along R_1.
(2) Use given values of x, y, z.
(3) Solve numerator.

50. Option (c) is correct.

Explanation:

Consider,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Expand along R_1

$$|A| = 1\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= 1[1 - 1] - 1[1 - 1] + [1 - 1]$$

$$= 0 - 0 + 0$$

$$= 0$$

We know,

$$A^{-1} = \frac{1}{|A|}$$

[Transpose of co-factor matrix]

Since, $|A| = 0$

So, A^{-1} = Not exist

Statement 1 correct.

Now,

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

[Multiply of matrix]

$$\Rightarrow \quad A^3 = A^2 \times A$$

$$= \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix}$$

[Again, multiply]

So, we get

$$A^3 \neq A$$

Statement 2 is incorrect.

But, $$A^2 = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = 3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = 3A$$

Hence,
Statement 3 is correct.

Hint:
(1) Matrix multiplication.

51. Option (d) is correct.

Explanation:

Consider, General equation of circle

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \ldots\text{(i)}$$

Satisfy all three given points in (i), we get:

By (5, –8):

$$10g - 16f + c + 89 = 0 \quad \ldots\text{(ii)}$$

By (–2, 9):

$$-4g + 18f + c + 85 = 0 \quad \ldots\text{(iii)}$$

By (2, 1):

$$4g + 2f + c + 5 = 0 \quad \ldots\text{(iv)}$$

Now, subtract (iii) from (ii), we get

$$14g - 34f + 4 = 0$$

$$\Rightarrow \quad 2(7g - 17f + 2) = 0$$

$$\Rightarrow \quad 7g - 17f + 2 = 0 \quad \ldots\text{(v)}$$

Now, subtract (iii) from (iv), we get

$$\Rightarrow \quad 8g - 16f - 80 = 0$$

$$\Rightarrow \quad 8(g - 2f - 10) = 0$$

$$\Rightarrow \quad g - 2f - 10 = 0 \quad \ldots\text{(vi)}$$

Solving, (v) and (vi),

By substitution method, we get

$$g = 58, f = 24$$

We know, centre of circle is given by

$$(-g, -f) = (-58, -24)$$

Hints:
(1) Satisfy given equation values.
(2) Use formula of centre and radius.

52. Option (d) is correct.

Explanation:

Now, put value of $g = 58, f = 24$ in (iv), we get

$$232 + 48 + c + 5 = 0$$

$$c = -285$$

$$\text{Radius} = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{(-58)^2 + (24)^2 + 285}$$

$$= \sqrt{3364 + 576 + 285}$$

$$= \sqrt{4225}$$

$$= 65$$

Hence, $r > 60$

Hint:
Radius is $\sqrt{g^2 + f^2 - c}$

53. Option (c) is correct.

Explanation:

Given: Two coordinate are A(0, 0), B(2, 2)

Let third coordinate is C(x, y)

Since, equilateral triangle given:

$$AB = BC = AC$$

Now,

Length of AB:

$$AB = \sqrt{(2-0)^2 + (2-0)^2}$$

$$AB = \sqrt{8} \quad \ldots\text{(i)}$$

Length of BC:

$$BC = \sqrt{(x-2)^2 + (y-2)^2} \quad \text{...(ii)}$$

Length of AC:

$$AC = \sqrt{(x-0)^2 + (y-0)^2}$$

$$AC = \sqrt{x^2 + y^2} \quad \text{...(iii)}$$

Now, put AB = AC

$$\sqrt{8} = \sqrt{x^2 + y^2}$$

[By (i), (iii)]

Square both sides

$$\Rightarrow \quad 8 = x^2 + y^2 \quad \text{...(iv)}$$

Now, put BC = AC

$$\sqrt{(x-2)^2 + (y-2)^2} = \sqrt{x^2 + y^2}$$

Square both sides

$$\Rightarrow \quad (x-2)^2 + (y-2)^2 = x^2 + y^2$$

$$\Rightarrow \quad x^2 + 4 - 4x + y^2 + 4 - 4y = x^2 + y^2$$

$$\Rightarrow \quad 8 - 4x - 4y = 0$$

$$\Rightarrow \quad -(4x + 4y) = -8$$

$$\Rightarrow \quad x + y = 2 \quad \text{...(v)}$$

Now, put $y = 2 - x$ in (iv), we get

$$\Rightarrow \quad 8 = x^2 + (2-x)^2$$

$$\Rightarrow \quad 8 = x^2 + 4 + x^2 - 4x$$

$$\Rightarrow \quad 8 = 2x^2 - 4x + 4$$

$$\Rightarrow \quad 2x^2 - 4x - 4 = 0$$

$$\Rightarrow \quad 2(x^2 - 2x - 2) = 0$$

$$\Rightarrow \quad x^2 - 2x - 2 = 0$$

By quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times -2}}{2a}$$

$$x = \frac{2 \pm \sqrt{4+8}}{2 \times 1}$$

$$x = \frac{2 \pm \sqrt{12}}{2}$$

$$x = \frac{2 \pm 2\sqrt{3}}{2}$$

$$x = 1 \pm \sqrt{3}$$

put $x = 1 \pm \sqrt{3}$ in (v), we get

$$y = 1 \mp \sqrt{3}$$

Hence, $(x, y) = (1 \pm \sqrt{3},\ 1 \mp \sqrt{3})$

Statement 1 is correct, since third coordinate is irrational.

Now, we have three coordinates and one is irrational.

So, length of sides will be irrational.

Hence, area is also irrational.

Statement 2 is correct.

Hints:

(1) Find length of sides and put equal.

(2) By quadratic formula, get coordinates.

54. Option (d) is correct.

Explanation:

Third coordinate is $x = (1+\sqrt{3})$, $y = (1-\sqrt{3})$ from Q. 53

Difference of coordinates is:

$$x - y = (1+\sqrt{3}) - (1-\sqrt{3})$$

$$= 1 + \sqrt{3} - 1 + \sqrt{3}$$

$$= 2\sqrt{3}$$

Hint:

Take coordinates we find in Q. 53.

55. Option (c) is correct.

Explanation:

Given: ABCD is *a* parallelogram

A(1, 3) B(–1, 2) C(3, 5), D(x, y)

Since, oposite sides are parallel lines so slope equal.

Slope of AB = Slope of CD

$$\Rightarrow \quad \frac{3-2}{1-(-1)} = \frac{5-y}{3-x}$$

$$\left[\frac{y_2 - y_1}{x_2 - x_1} = \text{slope}\right]$$

$$\Rightarrow \quad \frac{1}{2} = \frac{5-y}{3-x}$$

$$\Rightarrow \quad 3 - x = 10 - 2y$$

$$\Rightarrow \quad x - 2y = -7 \quad \text{...(i)}$$

Slope of AD = Slope of BC

$\Rightarrow \quad \frac{5-2}{3+1} = \frac{y-3}{x-1}$

$\Rightarrow \quad \frac{3}{4} = \frac{y-3}{x-1}$

$\Rightarrow \quad 3x - 3 = 4y - 12$

$\Rightarrow \quad 3x - 4y = -9 \quad \ldots(ii)$

On solving (i), (ii), we get

$x = 5, y = 6$

So, coordinate D(5, 6)

Now, for equation of BD:

Let $\quad B(x_1, y_1) = (-1, 2)$

$D(x_2, y_2) = (5, 6)$

By point-slope equation:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$\Rightarrow \quad y - 2 = \frac{6-2}{5+1}(x+1)$

$\Rightarrow \quad y - 2 = — (x + 1)$

$\Rightarrow \quad 3y - 6 = 2x + 2$

$\Rightarrow \quad 2x - 3y + 8 = 0$

Hence, equation of BD is $2x - 3y + 8 = 0$

Hints:

(1) Slope of parallel lines are equal.

(2) Find by slope-point form.

56. Option (c) is correct.

Explanation:

We know, coordinates are A(1, 3), B(–1, 2), C(3, 5), D(5, 6).

Area of ΔABC:

$A(x_1, y_1) = (1, 3)$

$B(x_2, y_2) = (-1, 2)$

$C(x_3, y_3) = (3, 5)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad \ldots(i)$$

$$= \frac{1}{2}[x_1(y_2 - y_3) - y_1(x_2 - x_3) + x_2y_3 - x_3y_2]$$

$$= \frac{1}{2}[1(2-5) - 3(-1-3) + (-1)(5) - 3 \times 2]$$

$$= \frac{1}{2}[-3 - 3(-4) + (-5) - 6]$$

$$= \frac{1}{2}[-3 + 12 - 5 - 6]$$

$$= \frac{1}{2}[12 - 14] = \frac{1}{2} \times -2 = -1$$

Area of ΔABC = |–1| = +1

Now, Area of ΔACD

$A(x_1, y_1) = (1, 3)$

$C(x_2, y_2) = (3, 5)$

$D(x_3, y_3) = (5, 6)$

Again using (i)

$$\text{Area} = \frac{1}{2}[1(5-6) - 3(3-5) + (3)(6) - (5)(5)]$$

$$= \frac{1}{2}[-1 - 3(-2) + 18 - 25]$$

$$= \frac{1}{2}[-1 + 6 - 7]$$

$$= \frac{1}{2}[-2] = -1$$

Area of ΔACD = |–1| = +1

Hence,

Area of parallelogram = 1 + 1

= 2

Shortcut:

$$\text{Area of ABC} = \begin{vmatrix} 1 & 3 & 1 \\ -1 & 2 & 1 \\ 3 & 5 & 1 \end{vmatrix}$$

$= |(2 - 5) - 3(-1 - 3) + 1(-5 - 5)|$

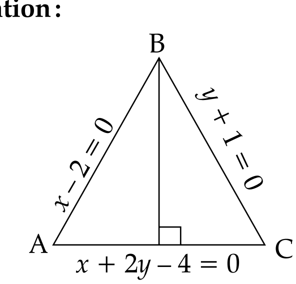
$= |-3 + 12 - 10| = |-1| = 1$

Area of parallelogram = 2×Arq ΔABC

= 2×1 = 2

57. Option (d) is correct.

Explanation:



First we find coordinate B:

AB is $x - 2 = 0$

BC is $y + 1 = 0$

So, $x = 2$, $y = -1$, we get:

Coordinate B(2, –1)

$$\text{Now, slope of line AC} = \frac{-\text{Coefficient of } x}{\text{Coefficient of } y}$$

$$= \frac{-1}{2}$$

Since, line AC and line BD are perpendicular

So, slope of line AC × Slope of line BD = –1

$$\Rightarrow \quad \frac{-1}{2} \times m = -1$$

$$\Rightarrow \quad m = 2$$

Therefore, for line BD, one point B(2, –1) and slope is 2.

By slope-point form of line:

$$\Rightarrow \quad y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - (-1) = 2(x - 2)$$

$$\Rightarrow \quad y + 1 = 2x - 4$$

$$\Rightarrow \quad 2x - y - 5 = 0$$

Hence, equation of altitude is $2x - y - 5 = 0$

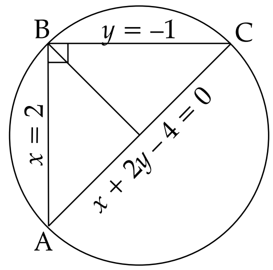
Hints:

(1) Get coordinate B.

(2) Use slope-line form.

58. Option (a) is correct.

Explanation:



$$x = 2 \quad \ldots\text{(i)}$$

$$y = -1 \quad \ldots\text{(ii)}$$

$$x + 2y - 4 = 0 \quad \ldots\text{(iii)}$$

Since, $x = 2$, $y = -1$ for AB and AC respectively, we get

ΔABC is right angle at B.

Now, Intersection of AB and AC is A and Intersection of BC and AC is C.

(1) Coordinate of A:

put $x = 2$ in (iii), we get

$2y - 2 = 0$

$y = 1$

$\Rightarrow$ Coordinate A is (2, 1).

(2) Coordinate of B:

Put $y = -1$ in (iii), we get:

$$\Rightarrow \quad x - 2 - 4 = 0$$

$$\Rightarrow \quad x = 6,$$

So, B = (6, –1)

Now,

Circumcentre of circle will be mid-points of line AC

A(2, 1), B(6, –1)

$$\text{Mid-point} = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

$$= \frac{2+6}{2}, \frac{1+(-1)}{2}$$

$$= (4, 0)$$

Hence, circumcentre is (4, 0).

Hints:

(1) Triangle ABC is a right angle triangle.

(2) Midpoint of hypotenuse (AC) is circumcenter of triangle ABC.

59. Option (b) is correct.

Explanation:

We have the end points of the latus rectum.

Focus lies on the midpoint of the segment joining the end points of the latus rectum.

So, the focus of the parabola is fixed.

We have the length of the latus rectum.

So, the distance between the focus and the directrix is fixed.

Directrix is parallel to the latus rectum.

So, the slope of directrix is fixed.

So, we can have two lines which are parallel to latus rectum and are at a fixed distance from the focus.

So, we have one focus and two possible directrices.

So, a maximum of two parabolas can be drawn.

Hints:

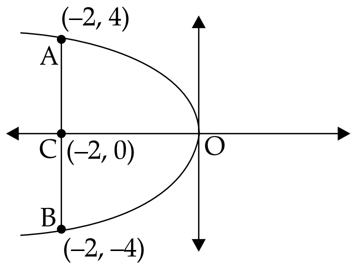
(1) Make parabola on given points.

(2) From both quadrant, two kind of parabola passes.

60. Option (c) is correct.

Explanation:

Statement 1:



Consider, parabola $y^2 = -4ax$

Point A and B on Latus Rectum.

So, $(+a, -2a) = (-2, 4)$

$(+a, +2a) = (-2, -4)$

On comparing: $a = -2$

Also, we get focus $(a, 0) = (-2, 0)$

Since, focus is at $(-2, 0)$

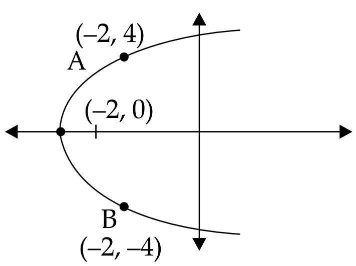
From figure and focus point:

End point of parabola O is (0, 0).

Hence, parabola passes through origin.

Statement 1 is correct.

Statement 2:



We know, points of Latus Rectum:

$(+a, -2a) = (-2, 4)$

$(a, +2a) = (-2, -4)$

On comparing, $a = -2$

So, focus is $(a, 0) = (-2, 0)$

Hence, statement 2 is correct.

Hints:

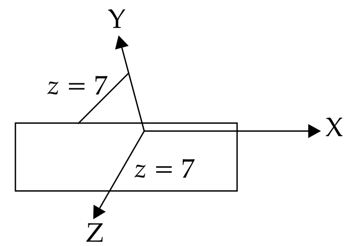
(1) Make figure of both parabola.

(2) Use formula of Latus Rectum.

61. Option (d) is correct.

Explanation:

Given, point $z = 7$



It is clear if point moving on Z-axis means it is parallel to XY-plane.

Hint:

Make diagram for plane.

62. Option (c) is correct.

Explanation:

Statement 1: It is properties of line that in space it can have infinitely many direction ratios. Since, in space line is free to move in any direction and can revolve in any direction x, y, z.

Hence, direction ratios are infinitely.

It is correct.

Statement 2: Incorrect, because sum of squares of direction cosines is always 1.

Example,

(1) $\cos\alpha = \dfrac{v_x}{\sqrt{v_x^2 + v_y^2 + v_z^2}}$

(2) $\cos\beta = \dfrac{v_y}{\sqrt{v_x^2 + v_y^2 + v_z^2}}$

(3) $\cos\gamma = \dfrac{v_z}{\sqrt{v_x^2 + v_y^2 + v_z^2}}$

Square and add both sides.

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Hint:

(1) Properties of direction cosines.

63. Option (c) is correct.

Explanation:

Given: xy plane divides line segment.

Consider, two points:

$(x_1, y_1, z_1) = (-1, 3, 4)$

$(x_2, y_2, z_2) = (2, -5, 6)$

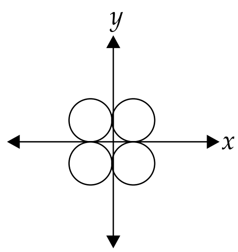
Now, xy plane divides by:

$$\text{Ratio} = \frac{-z_1}{z_2} = \frac{-4}{6} = \frac{-2}{3}$$

Since, division in negative.
So, externally division.
Hence, in ratio 2 : 3 externally.

Hint:
(1) Division by XY-plane. So, only Z-coordinates.

64. Option (c) is correct.
Explanation:
Consider, xy axis.



Circles touching coordinate axes in xy-axis are 4. Now, for sphere we have to consider 3D plane means x, y, z axis.
There are 8 quadrants in 3D, so for a given radius there will be 8 possible spheres which will touch all 3 axis.

Hints:
(1) Make circle an xy-axis.
(2) Use xz, yz axis and make sphere by each circle.

65. Option (b) is correct.
Explanation:
Consider, angles given between lines.
Direction cosines of lines by formula, we get:

$\Rightarrow$ $AB = (12 - 0, 0 - 0, 0 - 0)$
$\Rightarrow$ $AB = (12, 0, 0)$ ...(i)
$\Rightarrow$ $AG = (12 - 0, 6 - 0, 4 - 0)$
$\Rightarrow$ $AG = (12, 6, 4)$ ...(ii)
$\Rightarrow$ $AC = (12 - 0, 6 - 0, 0 - 0)$
$\Rightarrow$ $AC = (12, 6, 0)$ ...(iii)

Now, for angle between AB and AG:
Direction cosines of AB = $(a_1, b_1, c_1) = (12, 0, 0)$
Direction cosines of AG = $(a_2, b_2, c_2) = (12, 6, 4)$

$$\cos\alpha = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\times\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\alpha = \frac{144}{12\times\sqrt{12^2+6^2+4^2}}$$

(By (i), (ii))

$$\cos\alpha = \frac{12}{14} = \frac{6}{7} \quad \text{...(iv)}$$

Now, similarly for AC and AG:

$$\cos\beta = \frac{12\times12+6\times6+4\times0}{\sqrt{12^2+6^2+4^2}\times\sqrt{12^2+6^2+0^2}}$$

(By (ii) and (iii))

$$\cos\beta = \frac{144+36}{\sqrt{144+36+24}\times\sqrt{144+36}}$$

$$\cos\beta = \frac{180}{\sqrt{196}\times\sqrt{180}}$$

$$\cos\beta = \frac{\sqrt{180}}{14} \quad \text{...(v)}$$

Now, for find let $I = \cos 2\alpha + \cos 2\beta$, we get

$$I = 2\cos^2\alpha - 1 + 2\cos^2\beta - 1$$

$$= 2\left(\frac{6}{7}\right)^2 + 2\left(\frac{\sqrt{180}}{14}\right)^2 - 2$$

[By (iv), (v)]

$$= 2\times\frac{36}{49} + 2\times\frac{180}{196} - 2$$

$$= \frac{72}{49} + \frac{90}{49} - 2$$

$$= \frac{64}{49}$$

Hints:
(1) Find direction cosines.
(2) Put in formula of angle.

66. Option (c) is correct.
Explanation:
Given: $\vec{a}, \vec{b}, \vec{c}$ are unit vectors

So, $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

Now, also given $\vec{a}\times\vec{b}$ perpendicular to $\vec{c}$ ...(i)

Statement 1: Take LHS

$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$$

$$= 1\times1\ \sin\theta\,\hat{n}$$

$$= \sin\theta\,\vec{c}$$

[$\vec{c}$ is perpendicular to $\vec{a}\times\vec{b}$ and of unit modulus]

Hence, $\vec{a}\times\vec{b} = \sin\theta\,\vec{c}$

Statement 2: Take LHS

$$\vec{a}\cdot\left(\vec{b}\times\vec{c}\right) = [\vec{a}\vec{b}\,\vec{c}]$$
$$= -[\vec{a}\,\vec{c}\,\vec{b}]$$
$$= [\vec{c}\,\vec{a}\,\vec{b}]$$
$$= \vec{c}\cdot[\vec{a}\times\vec{b}]$$
$$= 0$$

$\because$ Given, $\vec{a}\perp\vec{b}\times\vec{c}$

Hint:

(1) Use angle given between them.

67. Option (d) is correct.

Explanation:

Consider, given vectors:

$$\vec{a}+3\vec{b} = 3\hat{i}-\hat{j} \quad \ldots(i)$$
$$2\vec{a}+\vec{b} = \hat{i}-2\hat{j} \quad \ldots(ii)$$

Multiply (i) by 2

$$2\vec{a}+6\vec{b} = 6\hat{i}-2\hat{j} \quad \ldots(iii)$$

Subtract (ii) from (iii)

$$\left(2\vec{a}+6\hat{b}\right)-\left(2\vec{a}+\hat{b}\right) = \left(6\hat{i}-2\hat{j}\right)-\left(\hat{i}-2\hat{j}\right)$$
$$\Rightarrow \quad 5\hat{b} = 5\hat{i}$$
$$\Rightarrow \quad \vec{b} = \hat{i}$$

Put $\vec{b} = \hat{i}$ in (ii), we get

$$\Rightarrow \quad 2\vec{a}+\hat{i} = \hat{i}-2\hat{j}$$
$$\Rightarrow \quad \vec{a} = -\hat{j}$$

So, $\vec{a} = -\hat{j}$

$\vec{b} = \vec{i}$

Angle between $\hat{i}$ (x-axis) and $\hat{j}$ (y-axis) is 90° *i.e.*, $\frac{\pi}{2}$.

Hints:

(1) Multiply (i) by 2.

(2) Subtract and find $\vec{a}$, $\vec{b}$.

68. Option (a) is correct.

Explanation:

Given: $\left(\vec{a}+\vec{b}\right)$ perpendicular to $\vec{a}$

So,

$$\Rightarrow \quad \left(\vec{a}+\vec{b}\right)\cdot\vec{a} = 0$$
$$\Rightarrow \quad \vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{a} = 0$$
$$\left|\vec{a}\right|^2+\vec{a}\cdot\vec{b} = 0 \quad \left[\because \vec{a}\cdot\vec{b} = -\vec{b}\cdot\vec{a}\right]$$
$$\Rightarrow \quad \vec{a}\cdot\vec{b} = \left|\vec{a}\right|^2 \quad \ldots(i)$$

Now, given $\left|\vec{b}\right| = 2\left|\vec{a}\right| \quad \ldots(ii)$

To find let $\mathrm{I} = \left(4\vec{a}+\vec{b}\right)\cdot\vec{b}$

$$\mathrm{I} = 4\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}$$
$$= 4\vec{a}\cdot\vec{b}+\left|\vec{b}\right|^2$$
$$= -\ 4\left|\vec{a}\right|^2+\left[2\left|\vec{a}\right|\right]^2$$

[By (i), (ii)]

$$= 4\left|\vec{a}\right|^2-4\left|\vec{a}\right|^2$$
$$= 0$$

Hints:

(1) Use given conditions.

(2) Put in formula.

69. Option (c) is correct.

Explanation:

Given: $\vec{a},\vec{b},\vec{c}$ are coplanar.

Since, they are co-planar so scalar triple product is zero, we get:

$$\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] = 0$$
$$\vec{a}\cdot\left(\vec{b}\times\vec{c}\right) = 0 \quad \ldots(i)$$

Now,

Statement 1: Consider,

$$\mathrm{I} = \left(\vec{a}\times\vec{b}\right)\times\vec{c}$$
$$= (\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$$

Now, $\left\{(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}\right\}\cdot\left(\vec{a}\times\vec{b}\right)$

$$= (\vec{a}\cdot\vec{c})\left[\vec{b}\,\vec{a}\,\vec{b}\right]-\left(\vec{b}\cdot\vec{c}\right)\left[\vec{a}\,\vec{a}\,\vec{b}\right]$$
$$= 0 - 0 = 0 \quad \ldots(ii)$$
$$\left(\because \left[\vec{a}\,\vec{a}\vec{b}\right] = \left[\vec{b}\,\vec{a}\,\vec{b}\right] = 0\right)$$

So, $\left(\vec{a}\times\vec{b}\right)\times\vec{c}$ is co planar with $\vec{a}$ and $\vec{b}$.

Statement 2 is correct.

Statement 2: Consider,

$$I = \left[\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right]\cdot\left[\vec{a}\times\vec{b}\right]$$

$$I = 0$$

[From (ii)]

Hint:

(1) Use triple product rule.

70. Option (c) is correct.

Explanation:

Given: $\vec{A} = \left(\sqrt{2}-1\right)\hat{i} - \hat{j}$

$\vec{B} = \hat{i} + \left(\sqrt{2}+1\right)\hat{j}$

$\overrightarrow{AB} = \left[1-\left(\sqrt{2}-1\right)\right]\hat{i} + \left[\left(\sqrt{2}+1\right)-(-1)\right]\hat{j}$

$= \left[2-\sqrt{2}\right]\hat{i} + \left[\sqrt{2}+2\right]\hat{j}$

$\left|\overrightarrow{AB}\right| = \sqrt{\left(2-\sqrt{2}\right)^2 + \left(2+\sqrt{2}\right)^2}$

[Magnitude of vector]

$= \sqrt{4+2-4\sqrt{2}+4+2+4\sqrt{2}}$

$\left|\overrightarrow{AB}\right| = \sqrt{12}$

$\left|\overrightarrow{AB}\right| = 2\sqrt{3}$

Hints:

(1) Make $\overrightarrow{AB}$.

(2) Find magnitude.

71. Option (b) is correct.

Explanation:

Product rule of differentiation says:

$$\frac{d}{dx}(a\cdot b) = (a)\frac{d}{dx}(b) + (b)\frac{d}{dx}(a)$$

Now,

$y = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})$

Differentiate with respect to x

$\frac{dy}{dx} = (1+x)(1+x^2)(1+x^4)(1+x^8)\frac{d}{dx}(1+x^{16})$

$+ (1+x)(1+x^2)(1+x^4)(1+x^{16})\frac{d}{dx}(1+x^8)$

$+ (1+x)(1+x^2)(1+x^8)(1+x^{16})\frac{d}{dx}(1+x^4)$

$+ (1+x)(1+x^4)(1+x^8)(1+x^{16})\frac{d}{dx}(1+x^2)$

$+ (1+x^2)(1+x^4)(1+x^8)(1+x^{16})\frac{d}{dx}(1+x)$

$\frac{dy}{dx}(x=0) = 0+0+0+0+(1\times1\times1\times1\times1)$

$= 0+1$

$= 1$ $\left[\frac{d}{dx}\left(x^n\right) = nx^{n-1}\right]$

Hint:

(1) Use differentiation Rule of product.

72. Option (a) is correct.

Explanation:

Given:

$y = \cos x\cdot\cos 4x\cdot\cos 8x$ ...(i)

We find differentiation by product rule.

$\frac{d}{dx}(a\cdot b\cdot c) = a\cdot b\frac{d}{dx}(c) + b\cdot c\frac{d}{dx}(a) + a\cdot c\frac{d}{dx}(b)$

Similarly,

$\frac{d}{dx}(\cos x\cdot\cos 4x\cdot\cos 8x) = \cos x\cdot\cos 4x\frac{d}{dx}(\cos 8x)$

$+ \cos 8x\cdot\cos 4x\frac{d}{dx}(\cos x) + \cos x\cos 8x\frac{d}{dx}(\cos 4x)$

$\Rightarrow \cos x\cos 4x \times -\sin 8x \times 8 + \cos 8x\cos 4x$

$\times -\sin x + \cos x\cos 8x \times -\sin 4x \times 4$

$\Rightarrow -8\cos x\cos 4x\sin 8x - \sin x\cos 4x\cos 8x$

$- 4\sin 4x\cos x\cos 8x$

We get:

$\frac{dy}{dx} = -8\cos x\cos 4x\sin 8x - \sin x\cos 4x\cos 8x$

$- 4\sin 4x\cos x\cos 8x$ ....(ii)

Now, to find $I = \frac{1}{y}\frac{dy}{dx}$

Use value from (i) and (ii),

$$I = \frac{1}{\cos x\cos 4x\cos 8x}$$

$[-8\cos x\cos 4x\sin 8x - \sin x\cos 4x\cos 8x$

$- 4\sin 4x\cos x\cos 8x]$

$\Rightarrow \frac{-8\sin 8x}{\cos 8x} - \frac{\sin x}{\cos x} - \frac{4\sin 4x}{\cos 4x}$

$\Rightarrow -8\tan 8x - \tan x - 4\tan 4x$

$$\left[\because \frac{\sin\theta}{\cos\theta} = \tan\theta\right]$$

Put $x = \frac{\pi}{4}$

$\Rightarrow$ $-8\tan 2\pi - \tan\frac{\pi}{4} - 4\tan\pi$ [$\because$ $\tan\pi = 0$ and $\tan 2\pi = 0$]

$\Rightarrow$ $0 - \tan\frac{\pi}{4} - 0$

$\Rightarrow$ $-\tan\frac{\pi}{4} = -1$

$$\left[\because \tan\frac{\pi}{4} = 1\right]$$

Hints:

(1) Find $\frac{dy}{dx}$ using product rule.

(2) Put value of $x = \frac{\pi}{4}$.

Shortcut:

log y = log cos x +log cos $4x$ +log cos $8x$

$\frac{1}{y}\frac{dy}{dx} = \tan x - 4\tan 4x - 8\sin 8x$

Put x=II

=–1 –4.0 – 8.0=–1

73. Option (c) is correct.

Explanation:

Given: $fof(x) = x^4$

$f[f(x)] = x^4$

[composition of function]

$f[f(x)] = (x^2)^2$

On comparing, we get

$f(x) = x^2$

Differentiate with respect x

$f'(x) = 2x$

Now, $f'(x)$ at $x = 1$

$f'(1) = 2 \times 1 = 2$

Hint:

Use composition of function.

74. Option (c) is correct.

Explanation:

Given: $\lim_{n\to\infty} \frac{a^n + b^n}{a^n - b^n}$

Taking a^n common

$$\Rightarrow \lim_{n\to\infty} \frac{a^n\left[1+\frac{b^n}{a^n}\right]}{a^n\left[1-\frac{b^n}{a^n}\right]}$$

$$\Rightarrow \lim_{n\to\infty} \frac{a^n\left[1+\left(\frac{b}{a}\right)^n\right]}{a^n\left[1-\left(\frac{b}{a}\right)^n\right]}$$

$$\Rightarrow \lim_{n\to\infty} \frac{1+\left(\frac{b}{a}\right)^n}{1-\left(\frac{b}{a}\right)^n}$$

Put limit, $n \to \infty$, we get

$$= \frac{1+\left(\frac{b}{a}\right)^\infty}{1-\left(\frac{b}{a}\right)^\infty}$$

$$= \frac{1+0}{1-0} \quad \left\{b < a, \text{So } \frac{b}{a} < 1\right\}$$

$$= 1$$

Hints:

(1) Take common a^n.

(2) Use $\left(\frac{b}{a}\right)^\infty = 0$

75. Option (d) is correct.

Explanation:

Given:

$$f(x) = \begin{cases} 1+\frac{x}{2k}, 0 < x < 2 \\ kx, 2 \le x < 4 \end{cases}$$

$\lim_{x\to 2} f(x)$ exist mean at $x = 2$, we get:

$\Rightarrow$ Left hand limit = Right hand limit

Now,

Case I: Right hand limit

for $x = 2 + h$

$f(x)$ taken $= kx$

$f(2+h) = \lim_{h\to 0} k(2+h)$

[put limit]

$= 2k$ ...(i)

Case II: Left hand limit

for $x = 2 - h$

$$f(x) \text{ taken as } 1 + \frac{x}{2k}$$

$$f(2-h) \Rightarrow \lim_{h \to 0} 1 + \frac{2-h}{2k}$$

[put limit]

$$\Rightarrow \quad 1 + \frac{2}{2k}$$

$$\Rightarrow \quad 1 + \frac{1}{k} \quad \ldots\text{(ii)}$$

Using limit exist condition and (i), (ii), we get

$$\Rightarrow \quad 2k = 1 + \frac{1}{k}$$

$$\Rightarrow \quad 2k - \frac{1}{k} = 1$$

$$\Rightarrow \quad 2k^2 - 1 = k$$

$$\Rightarrow \quad 2k^2 - k - 1 = 0$$

$$\Rightarrow \quad 2k^2 - 2k + k - 1 = 0$$

(middle term split)

$$\Rightarrow 2k(k-1) + 1(k-1) = 0$$

$$\Rightarrow \quad (k-1)(2k+1) = 0$$

$$\Rightarrow \quad k = 1, \frac{-1}{2}$$

Option given (d), $k = 1$

Hints:

(1) Solve limits by LHL, RHL.

(2) Satisfy on given condition.

76. Option (a) is correct.

Explanation:

Let $g(x) = |x|$

Statement 1:

$$g(x) = \begin{cases} +x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

(1) Left hand limit ($x < 0$)

$$\lim_{h \to 0} -(x-h)$$

$$\lim_{h \to 0} -x + h$$

$$= 0$$

(2) Right hand limit ($x > 0$)

$$\lim_{h \to 0} +(x+h)$$

$$\lim_{h \to 0} +x + h$$

$$= 0$$

(3) $g(x)$ at $x = 0$

$$g(0) = +0 = 0$$

Hence,

Left hand limit = Right hand limit = $g(0)$

So, $g(x)$ continuous at $x = 0$

Hence, $f(x) = g(x) - 1$

$\Rightarrow f(x) = |x| - 1$ is continuous at $x = 1$

Statement 1 is correct.

Statement 2:

Again,

$$g(x) = \begin{cases} +x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

(1) Left hand derivative

$$\lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$$

$$\lim_{h \to 0} \frac{-(0-h) - 0}{-h}$$

$$\lim_{h \to 0} \frac{+h}{-h}$$

$$= -1$$

(2) Right hand derivative

$$\lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$\lim_{h \to 0} \frac{(0+h) - 0}{+h}$$

$$= +1$$

Hence,

Left hand derivative ≠ Right hand derivative

Hence, $f(x) = g(x) - 1$ also not differentiable at $x = 0$

Statement 2 is incorrect.

Hints:

(1) Find continuity and differentiability at $x = 0$.

(2) Use continuity and differentiability rule.

77. Option (c) is correct.

Explanation:

Consider, $f(x) = \dfrac{[x]}{|x|}$

Here, $[x]$ is greatest integer function.

Now, to find right hand limit:

$$\lim_{h\to 0}\frac{[1+h]}{|1+h|}$$

[In greatest Integer function, gives greatest possible integer less than or equal to the number. Example, [1 · 3] = 1

$$\lim_{h\to 0}\frac{1}{|1+h|}$$

Put limit, we get

$$\frac{1}{1+0}=1$$

Hints:

(1) Greatest integer function give greatest possible integer less than or equal to the number.

(2) Solve limits.

78. Option (b) is correct.

Explanation:

Consider, $f(x) = \sin\left(\frac{1}{x^2}\right)$

Statement 1: Given $f(0) = 0$

To check continuity to $x = 0$

(1) Right hand limit ($x > 0$)

$$\lim_{h\to 0}\sin\left(\frac{1}{(0+h)^2}\right)$$

$$\lim_{h\to 0}\sin\left(\frac{1}{h^2}\right)$$

[Put limit, $h = 0$]

$$= \sin\left(\frac{1}{0}\right)$$

$$= \sin\infty$$

We know, sine function value lies between –1 and +1.

Hence, $-1 \le \sin x \le +1$

So, $-1 \le \sin x\ \infty \le 1$

Right hand limit $\ne f(0)$

Hence, $f(x)$ not continuous at $x = 0$.

Statement 2:

To check continuity at $x = \frac{2}{\sqrt{\pi}}$

(1) At $x = \frac{2}{\sqrt{\pi}}$

$$f(x) = \sin\left(\frac{1}{x^2}\right)$$

$$f\left(\frac{2}{\sqrt{\pi}}\right) = \sin\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{\sqrt{2}}$$

(2) Right hand limit

$$\lim_{h\to 0}\sin\left(\frac{1}{\left(\frac{2}{\sqrt{\pi}}+h\right)^2}\right)$$

$$\lim_{h\to 0}\sin\left(\frac{1}{\frac{4}{\pi}+h^2+\frac{4h}{\sqrt{\pi}}}\right)$$

[Put limit, $h = 0$]

$$= \sin\left(\frac{1}{\frac{4}{\pi}+0}\right)$$

$$= \sin\frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}}$$

(3) Left hand limit

$$\lim_{h\to 0}\sin\left(\frac{1}{\left(\frac{2}{\sqrt{\pi}}-h\right)^2}\right)$$

$$\lim_{h\to 0}\sin\left(\frac{1}{\frac{4}{\pi}+h^2-\frac{4h}{\sqrt{\pi}}}\right)$$

[Put limit, $h = 0$]

$$= \sin\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{\sqrt{2}}$$

So,

Left hand limit = Right hand limit = $f\left(\frac{2}{\sqrt{\pi}}\right)$

Hence, $f(x)$ is continuous at $x = \frac{2}{\sqrt{\pi}}$.

Statement 2 is correct.

Hints:

(1) Check continuity at $x = 0$.

(2) Since value of sin x lies between –1 and +1.

79. Option (b) is correct.

Explanation:

To find Range of $f(x) = 1 - \sin x$

We know, value of sine function lies between –1 and +1, we get:

$\Rightarrow \quad -1 \le \sin x \le +1$

[Multiply by –1]

$\Rightarrow \quad -1 \le -\sin x \le +1$

[Add 1]

$\Rightarrow 1 - 1 \le 1 - \sin x \le 1 + 1$

$\Rightarrow \quad 0 \le 1 - \sin x \le 2$

$\Rightarrow \quad 0 \le f(x) \le 2$

Hence, $f(x) = 1 - \sin x$ lies between 0 and 2, we get:

Range: [0, 2]

Hint:

(1) Consider Range of sin x.

80. Option (a) is correct.

Explanation:

$$y = \cos^{-1} \cos (x)$$

$\Rightarrow \quad \cos y = \cos x$

$\Rightarrow \quad \frac{d}{dx}(\cos y) = \frac{d}{dx}(\cos x)$

$\Rightarrow \quad -\sin y \frac{dy}{dx} = -\sin x$

$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin x}{\sin y}$

Now, at $x = -\frac{\pi}{4}$

$$y = \cos^{-1}\left[\cos\left(-\frac{\pi}{4}\right)\right]$$

$$= \cos^{-1}\left(\cos\frac{\pi}{4}\right)$$

[cos(– θ) = cos θ]

$$= \frac{\pi}{4}$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{x=-\frac{\pi}{4}} = \frac{\sin\left(-\frac{\pi}{4}\right)}{\sin\left(\frac{\pi}{4}\right)}$$

[sin (–θ) = –sin θ]

$$= \frac{-\sin\frac{\pi}{4}}{\sin\frac{\pi}{4}}$$

$$= -1$$

Hence, slope of tangent at $\left(x = -\frac{\pi}{4}\right) = -1$

Hints:

(1) At $x = -\frac{\pi}{4}$, $y = -\cos^{-1}(\cos x)$.

(2) Differentiation, $\frac{dy}{dx} = -1$.

81. Option (d) is correct.

Explanation:

Given: $f(x) = 1 + x^2 + x^4$

We have to find w.r.t x^2, we get:

$$I = \int\left(\ + x^2 + x^4\right) d\left(x^2\right)$$

Let $y = x^2$, we get

$$I = \int\left(1 + y + y^2\right) dy$$

$$I = y + \frac{y^2}{2} + \frac{y^3}{3} + c$$

$\left[\int x^n\, dx = \frac{x^{n+1}}{n+1}\right]$

[put $y = x^2$]

$$I = x^2 + \frac{\left(x^2\right)^2}{2} + \frac{\left(x^2\right)^3}{3} + c$$

$$I = x^2 + \frac{x^4}{2} + \frac{x^6}{3} + c$$

Hints:

(1) Put $y = x^2$ and Modify function.

(2) Integrate according to function.

82. Option (b) is correct.

Explanation:

Given: $f(x) = x^2 + 1$, interval = (1, 2)

Differentiate with Repeat to x

$$f'(x) = 2x$$

put $f'(x) = 0$ for critical values

$\Rightarrow$ $2x = 0$

$\Rightarrow$ $x = 0$

$$f''(x) = 2 < 0$$

$\therefore$ $f(1)$ is maximum at $x = 0$

$\therefore$ minimum value = 2

In the interval (1, 2)

Statement 1 incorrect.

Statement 2 correct.

Hints:

If $f'(a) = 0$ and $f''(a) > 0$ then $f'(x)$ is minimum at $x = a$.

83. Option (b) is correct.

Explanation:

Given: $f(1) = f(4)$ ...(i)

Mean value of $f(x)$ at $x = 1, 4$ is equal

Now, given $I = \int_1^4 f'(x)\,dx$

$$I = \left[f(x)\right]_1^4$$

[Integrate of differentials function gives function itself.]

Putting limits

$$I = f(4) - f(1)$$

$I = 0$ [using (i)]

Hints:

(1) Integration of differentiate function.

(2) Using given values.

84. Option (c) is correct.

Explanation:

Consider, $I = \int_0^{\frac{\pi}{2}} e^{\ln(\cos x)}\,dx$ ...(i)

We know,

$e^{\ln x} = x$ ...(ii)

Using (ii) in (i), we get

$$I = \int_0^{\frac{\pi}{2}} \cos x\,dx$$

$$I = [\sin x]_0^{\frac{\pi}{2}}$$

[Integration of cos x is sin x]

$I = \sin\frac{\pi}{2} - \sin 0$ [put limits]

$I = 1$

Hints:

(1) $e^{\ln x} = x$ property.

(2) Definite integration.

85. Option (b) is correct.

Explanation:

Given:

$I = \int \sqrt{1 - \sin 2x}\,dx = A\sin x + B\cos x + C$ ...(i)

Consider,

$$I = \int \sqrt{1 - \sin 2x}\,dx$$

[Putting values, $\sin^2 x + \cos^2 x = 1$
$\sin 2x = 2\sin x\cos x$]

$$I = \int \sqrt{\sin^2 x + \cos^2 x - 2\sin x\cos x}\,dx$$

[using $(A + B)^2 = A^2 + B^2 - 2AB$]

$$I = \int \sqrt{(\cos x - \sin x)^2}\,dx$$

$I = \int (\cos x - \sin x)\,dx$ $\left[\because 0 < x < \frac{\pi}{4}\right]$

$I = \sin x + \cos x = 1$

Comparing using (i)

$\cos x + \sin x + c = A\sin x + B\cos x + c$

We get: $A = 1$

$B = 1$

So,

$\Rightarrow$ $A + B = 1 + 1 = 2$

$\Rightarrow$ $A + B = 2$

$\Rightarrow$ $A + B - 2 = 0$

Hints:

(1) Integrate using formula.

(2) Compare with given equation to find value.

86. Option (b) is correct.

Explanation:

Equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(i)

Consider, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Differentiate with respect to x

$$\Rightarrow \quad \frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$$

$$\Rightarrow \quad \frac{x}{a^2}+\frac{y}{b^2}\frac{dy}{dx}=0 \quad \ldots\text{(ii)}$$

Again differentiate with respect to x

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}\left[y\frac{d^2y}{dx^2}+\frac{dy}{dx}\times\frac{dy}{dx}\right]=0$$

$$\Rightarrow \quad \frac{1}{b^2}\left[y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right]=-\frac{1}{a^2}$$

$$\Rightarrow \quad -\left[y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right]=\frac{b^2}{a^2}$$

$$\Rightarrow \quad -y\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^2=\frac{b^2}{a^2}$$

$$\Rightarrow \quad -y\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^2=\frac{-y}{x}\frac{dy}{dx}$$

[By (ii)]

$$\Rightarrow \quad y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=\frac{y}{x}\frac{dy}{dx}$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2-\frac{y}{x}\frac{dy}{dx}=0$$

$$\Rightarrow \quad xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$$

Order of differential equation is the highest derivative of differential equation.

Hence, order = 2

Hints:

(1) Differentiation two times.

(2) Remove constant using previous equation.

Shortcut:

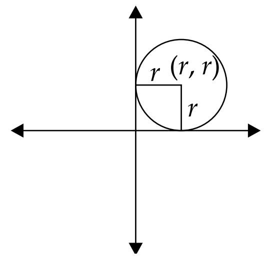
Equation of ellipse is

$$\frac{x^2}{a^2}+\frac{x^2}{b^2}=1$$

Since, it has two arbitrary constant So, order = 2

87. Option (b) is correct.

Explanation:



Let circle of Radius = r

Centre (r, r)

Equation of circle of radius r:

$\Rightarrow \quad (x-r)^2+(y-r)^2=r^2$

In above equation number of aribitrary constant = 1

Hence, the order of differential equation = 1 and degree =1

Hints:

(1) Do differentiation.

(2) Make differential equation using value of constant.

88. Option (b) is correct.

Explanation:

$$y=\mathrm{A}-\frac{\mathrm{B}}{x}$$

To find differential equation,

Differentiate both side with respect to x. We get:

Now, $$y=\mathrm{A}-\frac{\mathrm{B}}{x}$$

$$\Rightarrow \quad \frac{dy}{dx}=0-\mathrm{B}\left(-\frac{1}{x^2}\right)$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{\mathrm{B}}{x^2}$$

$$\left[\text{A \& B = constant Differentiation of } \frac{1}{x} \text{ is } -\frac{1}{x^2}\right]$$

$$\Rightarrow \quad y_1=\frac{\mathrm{B}}{x^2} \quad \ldots\text{(i)}$$

Differentiate again with respect to x.

$$\Rightarrow \quad y_2=\mathrm{B}\times\left(\frac{-2}{x^3}\right)$$

$$\Rightarrow \quad y_2=\frac{-2\mathrm{B}}{x^3}$$

$$\Rightarrow \quad y_2 = \frac{-2}{x} \times \frac{B}{x^2}$$

$$\Rightarrow \quad y_2 = \frac{-2}{x} \times y_1 \qquad \text{[using (i)]}$$

$$\Rightarrow \quad xy_2 = -2y_1$$

$$\Rightarrow \quad xy_2 + 2y_1 = 0$$

Hints:

(1) Differentiate two times.

(2) Put value from y_1 in y_2.

89. Option (a) is correct.

Explanation:

Consider,

$$I = \int_0^{\pi} \log \tan \frac{x}{2}\, dx \qquad \ldots(i)$$

$$I = \int_0^{\pi} \log \tan \left(\frac{\pi}{2} - \frac{x}{2}\right) dx$$

$$\left[\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$I = \int_0^{\pi} \log \cot \frac{x}{2}\, dx \qquad \ldots(ii)$$

Add (i) and (ii)

$$I + I = \int_0^{\pi} \log \tan \frac{x}{2}\, dx + \int_0^{\pi} \log \cot \frac{x}{2}\, dx$$

$$2I = 0$$

$$I = 0$$

Hint:

Use $\left[\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$

90. Option (b) is correct.

Explanation:

Tangent to the curve at (0, 1)

Consider,

$$y = e^x$$

Differentiate with respect to x

$$\frac{dy}{dx} = e^x$$

Slope of tangent at (0, 1)

We know, $\frac{dy}{dx}(0, 1)$ gives slope of tangent, we get

$$\text{Slope} = \frac{dy}{dx}(0, 1) = e^{\circ} = 1$$

Now, by slope point form

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - 1 = 1(x - 0)$$

$$\Rightarrow \quad y - 1 = x \qquad \ldots(i)$$

Given that line touches x-axis, so $y = 0$ in (i)

$$\Rightarrow \quad 0 - 1 = x$$

$$\Rightarrow \quad x = -1$$

Hence, it meets curve at (–1, 0).

Hint:

(1) Find slope $\frac{dy}{dx}$ and equation.

91. Option (a) is correct.

Explanation:

Consider,

$$f(x) = x + \frac{1}{x}$$

Differentiate with respect to x

$$f'(x) = 1 - \frac{1}{x^2}$$

$$\left[\frac{d}{dx}(x^n) = nx^{n-1}\right]$$

Find critical point by $f'(x) = 0$, we get

$$\Rightarrow \quad 1 - \frac{1}{x^2} = 0$$

$$\Rightarrow \quad x^2 = 1$$

$$\Rightarrow \quad x = \pm 1$$

Now, critical points are +1, –1

Consider, $f'(x) = 1 - \frac{1}{x^2}$

Again differentiate with respect to x

$$f''(x) = \frac{2}{x^3}$$

Check at $x = +1, -1$

Now, At $x = +1$, $f''(x) =$ is (+)ve,

$x = -1$, $f''(x)$ is negative.

By rule of second differention, we get

At $x = -1$, $f(x)$ is maximum and

at $x = +1$, $f(x)$ is minimum

Hence,

at $x = -1$, $f(x) = -2$ and at $x = +1$, $f(x) = +2$

So, maximum value is –2 and minimum value is +2.

Statement 1: It is correct.

Statement 2: It is incorrect.

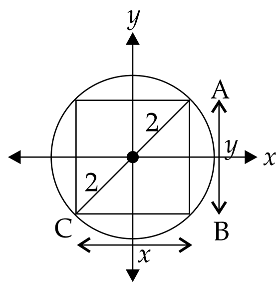
Hints:

(1) Find critical point.

(2) By rule of second differentiation.

92. Option (c) is correct.

Explanation:



Let Radius $(r) = 2$ units of circle and for Rectangle, Let x = length and y = breadth

Now, In ΔABC, By Pythagoras theorem

$$\Rightarrow \quad x^2 + y^2 = 4^2$$

$$\Rightarrow \quad x^2 + y^2 = 16$$

$$\Rightarrow \quad y^2 = 16 - x^2$$

$$\Rightarrow \quad y = \sqrt{16 - x^2} \quad \ldots(i)$$

Now, Area of rectangle = length × breadth

$$A = xy$$

$$A = x\sqrt{16 - x^2}$$

Differentiate with respect to x

$$\frac{dA}{dx} = \left[x \times \frac{1}{2\sqrt{16 - x^2}} \times (0 - 2x)\right] + \left[\sqrt{16 - x^2} \times 1\right]$$

$$\frac{dA}{dx} = \frac{-2x^2}{2\sqrt{16 - x^2}} + \sqrt{16 - x^2}$$

$$\frac{dA}{dx} = \frac{-x^2 + (16 - x^2)}{\sqrt{16 - x^2}}$$

$$\frac{dA}{dx} = \frac{16 - 2x^2}{\sqrt{16 - x^2}}$$

$$\frac{dA}{dx} = \frac{2(8 - x^2)}{\sqrt{16 - x^2}}$$

Put $\frac{dy}{dx} = 0$, we get

$$2(8 - x^2) = 0$$

$$x = 2\sqrt{2}$$

Put $x = 2\sqrt{2}$ in (i),

$$\Rightarrow \quad y = \sqrt{16 - 8}$$

$$\Rightarrow \quad y = \sqrt{8}$$

$$\Rightarrow \quad y = 2\sqrt{2}$$

Hence, we get $x = 2\sqrt{2}$, $y = 2\sqrt{2}$

Now, $\quad \frac{dA}{dx} = \frac{2(8 - x^2)}{\sqrt{16 - x^2}}$

Differentiate with repeat to x

$$\frac{d^2A}{dx^2} = 2 \cdot \frac{\sqrt{16 - x^2} \times (-2x) - (8 - x^2) \times \frac{1}{\sqrt{16 - x^2}} \times -x}{\left(\sqrt{16 - x^2}\right)^2}$$

Put $x = 2\sqrt{2}$ in $\frac{d^2A}{dx^2}$, we get

$\frac{d^2A}{dx^2}$ is negative.

Hence, by second differential rule if second derivative is negative, then function maximum.

Hence, maximum area of rectangle:

$$A = 2\sqrt{2} \times 2\sqrt{2}$$

$$A = 8 \text{ square units}$$

Hints:

(1) Make figure and area of rectangle.

(2) Differentiate and get maximum area of rectangle.

93. Option (a) is correct.

Explanation:

Consider,

$$I = \int \frac{dx}{x(x^2 + 1)}$$

[Multiply and Divide by x, we get]

$$I = \int \frac{x}{x^2(x^2 + 1)} dx$$

[Multiply and Divide by 2, we get]

$$I = \frac{1}{2}\int \frac{2x}{x^2(x^2+1)}dx \quad ....(i)$$

Put $t = x^2$

Differentiate both sides

$$dt = 2x\,dx$$

Put values in (i), we get

$$I = \frac{1}{2}\int \frac{dt}{t(t+1)}$$

[Add and Subtract t in Numerator, we get]

$$I = \frac{1}{2}\int \frac{(t+1)-t}{t(t+1)}dt$$

$$I = \frac{1}{2}\int \left[\frac{(t+1)}{t(t+1)} - \frac{t}{t(t+1)}\right]dt$$

$$I = \frac{1}{2}\int \left[\frac{1}{t} - \frac{1}{t+1}\right]dt$$

$$I = \frac{1}{2}[\log|t| - \log|t+1| + c$$

$$I = \frac{1}{2}\log\left|\frac{t}{t+1}\right| + c$$

[put $t = x^2$]

$$I = \frac{1}{2}\log\left[\frac{x^2}{x^2+1}\right] + c$$

Hints:

(1) Multiply and Divide by x and 2.

(2) Put $t = x^2$.

94. Option (a) is correct.

Explanation:

Consider, $u = e^{e^x}, v = e^x$ …(i)

Now, $u = e^{e^x} = e^v$ [using (i)]

Differentiate with respect to x

[Differentiation of e^x is e^x itself and use change rule]

$$\Rightarrow \quad \frac{du}{dx} = e^v \times \frac{dv}{dx}$$

$$\Rightarrow \quad \frac{du}{dx} = e^v\frac{dv}{dx} \quad …(ii)$$

Now, $v = e^x$

$$\Rightarrow \quad \frac{dv}{dx} = e^x \quad …(iii)$$

Divide (ii) by (iii), we get

$$\Rightarrow \quad \frac{du}{dv} = \frac{e^v\frac{dv}{dx}}{e^x}$$

$$\Rightarrow \quad \frac{du}{dv} = \frac{e^v \times e^x}{e^x}$$

[using (iii)]

$$\Rightarrow \quad \frac{du}{dv} = e^v$$

$$\Rightarrow \quad \frac{du}{d(e^x)} = e^{e^x} \quad \text{[using (i)]}$$

Hints:

(1) Differentiate both separately.

(2) Divide both result.

95. Option (b) is correct.

Explanation:

Consider,

$$f(x) = x^3 + x^2 + kx$$

Differentiate with respect to x

$$f'(x) = 3x^2 + 2x + k$$

Now, given that $f(x)$ has no local extremum, we get

$$f'(x) \neq 0$$

$$\Rightarrow \quad 3x^2 + 2x + k \neq 0$$

$$\Rightarrow \quad 3x^2 + 2x + k > 0 \text{ or } 3x^2 + 2x + k < 0$$

{For quadratic equation if $ax^2 + bx + c \neq 0$ then, discriminant < 0}

Now, By using above condition, we get

$$\Rightarrow \quad D < 0$$

$$\Rightarrow \quad b^2 - 4ac < 0$$

$$\Rightarrow \quad (2)^2 - 4 \times 3 \times k < 0$$

$$\Rightarrow \quad 4 - 12k < 0$$

$$\Rightarrow \quad \frac{4}{12} < k$$

$$\Rightarrow \quad \frac{1}{3} < k$$

$$\Rightarrow \quad 1 < 3k$$

Hints:

(1) No local extremum $f'(x) \neq 0$.

(2) Condition D < 0.

96. Option (d) is correct.

Explanation:

Given: $f(x) = 2^x$

Differentiate with respect to x

$[\because f(x) = a^x \text{ gives } f'(x) = a^x \log a]$

$\Rightarrow \quad f'(x) = 2^x \log 2 \quad \ldots(i)$

Now, we have to find

$$I = \int_2^{10} \frac{f'(x)}{f(x)} dx$$

$$I = \int_2^{10} \frac{2^x \log 2}{2^x} dx \quad \text{[using (i)]}$$

$$I = \int_2^{10} \frac{2^x \log 2}{2^x} dx$$

$$I = \int_2^{10} \log 2 \, dx$$

[cut 2^x in numerator & denominator]

$$I = \log 2 \int_2^{10} 1 \, dx$$

[log 2 is constant]

$$I = \log 2 \times [x]_2^{10}$$

[Integration of 1 in x]

$I = \log 2 \times [10 - 2]$

$I = \log 2 \times 8$

$I = 8 \log 2$

Hints:

(1) Differentiation of a^x is $a^x \log a$.

(2) Put value and cut common.

97. Option (d) is correct.

Explanation:

Consider,

$$\int_{-2}^{0} f(x) dx = k$$

Let $f(x) = x$, we get

$$\Rightarrow \quad \int_{-2}^{0} x \, dx = k$$

$$\Rightarrow \quad \left[\frac{x^2}{2}\right]_{-2}^{0} = k$$

$$\Rightarrow \quad \frac{1}{2}\left[(0)^2 - (-2)^2\right] = k$$

$$\Rightarrow \quad \frac{1}{2}[-4] = k$$

$\Rightarrow \quad -2 = k$

Hence, $k = -2 \quad \ldots(i)$

Now, consider $\quad I = \int_{-2}^{0} |f(x)| dx$

$$I = \int_{-2}^{0} |x| dx$$

$$I = -\left[\frac{x^2}{2}\right]_{-2}^{0}$$

[Since, limits is negative, So, I is negative]

$$I = -\frac{1}{2}\left[x^2\right]_{-2}^{0}$$

$$I = -\frac{1}{2}\left[0^2 - (-2)^2\right]$$

$$I = -\frac{1}{2}[-4]$$

$I = 2$

If, Let $f(x) = 1$ than $\int_{-2}^{0} |f(x)| dn = K$

Hence, I greater than or equal to k,

Depending upon limit.

Hints:

(1) Put $f(x) = x$.

(2) Find k and $\int_{-2}^{0} |f(x)|$.

98. Option (a) is correct.

Explanation:

Given: $\quad f(x) = x^2 - kx$

$f(x)$ is increasing on values $(1, \infty)$

Now, consider

$$f(x) = x^2 - kx$$

Differentiate with respect to x

$$f'(x) = 2x - k$$

Since, function increasing. So $f'(x) > 0$

$\Rightarrow \quad 2x - k > 0$

$\Rightarrow \quad -k > -2x$

$\Rightarrow \quad k < 2x$

Using given interval $(1, \infty)$

first, put $x = 1$

$\Rightarrow \quad k < 2 \times 1$

$\Rightarrow \quad k < 2$

second, put $x = \infty$

$\Rightarrow \quad k < 2 \times \infty$

$\Rightarrow \quad k < \infty$

Common value, we get $x < 2$.

Hints:

(1) Differentiate and keep $f'(x) > 0$

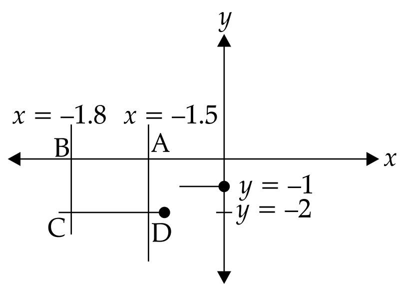
(2) Put values of x and get common value.

99. Option (c) is correct.

Explanation:

Consider, $y = [x]$ is greatest integer function.

First, we make graph for this function and $x = -1.5$, $x = -1.8$, then find common area.



ABCD is required area.

Now, $\quad AB = (-1.8) - (-1.5)$

$AB = -0.3$

{But we have to find area, take length only}

$AB = 0.3$ units

$AD = 2$ units

{on y-axis length}

Area of ABCD $= AB \times AD$

$= 0.3 \times 2$

$= 0.6$ square units

Hints:

(1) Make diagram.

(2) Find length and area.

100. Option (d) is correct.

Explanation:

Given: Curve is $x^2 = y$

Point (1, 1)

Now, consider curve equation

$$x^2 = y$$

Differentiate with respect to x

$$2x = \frac{dy}{dx}$$

$\frac{dy}{dx}$ gives slope of tangent at any point (x, y)

Put $(x, y) = (1, 1)$

We get, $\frac{dy}{dx}(1, 1) = 2 \times 1 = 2$

So, slope of tangent $= 2$

Slope of tangent is defined as tangent of angle made with positive direction of x-axis.

So, $\quad \tan\theta = 2$

Now, we know

$$\tan\frac{\pi}{3} = \sqrt{3} = 1.73 \quad \text{(approx.)}$$

$$\tan\frac{\pi}{2} = \infty$$

Therefore, $\tan\frac{\pi}{3} < \tan\theta < \tan\frac{\pi}{2}$

$\Rightarrow \quad \frac{\pi}{3} < \theta < \frac{\pi}{2}$

Hints:

(1) Differentiate given curve and find $\frac{dy}{dx}$.

(2) Put given value (1, 1) and get slope.

(3) Slope is $\tan\theta$.

101. Option (c) is correct.

Explanation:

Consider,

Statement 1: $P(E \cap F) \geq P(E) + P(F) - 1$

Now, we can write

$\Rightarrow \quad 1 \geq P(E) + P(F) - P(E \cap F)$

$\Rightarrow \quad 1 \geq P(E \cup F)$

[By sets union rule]

$\Rightarrow$ This is possible because probability lies between 0 and 1.

Hence, statement 1 correct.

Now, we know for any two sets

$P(E \cup F) = P(E) + P(F) - P(E \cap F) \quad \ldots(i)$

So, statement 2 is incorrect.

Now, using (i)

$\Rightarrow \quad P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$\Rightarrow P(E \cup F) + P(E \cap F) = P(E) + P(F)$

$\Rightarrow \quad P(E \cup F) \le P(E) + P(F)$

$[\because P(E \cap F) \ge 0]$

Hence, statement 3 is correct.

Hint:
(1) Use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

102. Option (a) is correct.

Explanation:

Consider, $P\left(\frac{A}{B}\right) < P(A)$

$\Rightarrow \quad \frac{P(A \cap B)}{P(B)} < P(A)$

$\Rightarrow \quad P(A \cap B) < P(A) \times P(B)$

{cross multiply}

$\Rightarrow \quad \frac{P(A \cap B)}{P(A)} < P(B)$

$\Rightarrow \quad P\left(\frac{B}{A}\right) < P(B)$

{By probability formula}

Hint:
(1) Use direct formula $P\left(\frac{A}{B}\right) < P(A)$ gives

$P\left(\frac{A \cap B}{P(B)}\right) < P(A)$

103. Option (c) is correct.

Explanation:

Standard deviation is measure of amount of variation or dispersion net of values.

A low standard deviation indicates that the values tend to be close to the mean of the set, while high deviation means in central tendency, more deviation and not exact result.

Hence, In measure of central tendency since standard deviation measured using variance. We get exact Result.

Hint:
(1) Properties of standard deviation.

104. Option (d) is correct.

Explanation:

Consider,

Probability of A to solve question $= P(A) = \frac{1}{2}$

Probability of A to not solve question $= P(\bar{A})$

$= 1 - \frac{1}{2} = \frac{1}{2}$ ...(i)

Similarly, probability B and C to not solve question:

$P(\bar{B}) = 1 - \frac{3}{4} = \frac{1}{4}$ ...(ii)

$P(\bar{C}) = 1 - P$ ...(iii)

Now, Given probability to solve question $= \frac{29}{32}$

$\Rightarrow 1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C}) = \frac{29}{32}$

[By (i), (ii), (iii)]

$\Rightarrow 1 - \frac{1}{2} \times \frac{1}{4} \times (1 - P) = \frac{29}{32}$

$\Rightarrow \quad \frac{1}{2} \times \frac{1}{4} \times (1 - P) = \frac{3}{32}$

$\Rightarrow \quad (1 - P) = \frac{3}{4}$

$\Rightarrow \quad 4 - 4P = 3$

$\Rightarrow \quad P = \frac{1}{4}$

Hint:
(1) Find probability to solve question.

105. Option (d) is correct.

Explanation:

Given: Out of 60 Bolls, Hit sixes 8.

Now, number of Balls on which he does not hit six = 60 – 8 = 52

Probability he not hit six $= \frac{\text{Favourable case}}{\text{Total case}}$

$\Rightarrow \quad \bar{P} = \frac{52}{60}$

$\Rightarrow \quad \bar{P} = \frac{13}{15}$

Hint:
(1) Find balls, he not hit sixes.

106. Option (c) is correct.

Explanation:

Consider,

Statement 1:

From first regression line given:

$$3x - 4y + 8 = 0$$

$$\Rightarrow \quad -4y = -3x - 8$$

$$\Rightarrow \quad -y = -\left[\frac{3x+8}{4}\right]$$

$$\Rightarrow \quad y = \frac{3x}{4} + \frac{8}{4}$$

$$\Rightarrow \quad y = \frac{3x}{4} + 2$$

Hence, Regression line of y on x is

$$y = \frac{3x}{4} + 2$$

Statement 1 is correct.

Now,

Statement 2:

From second regression line given:

$$4x - 3y - 1 = 0$$

$$\Rightarrow \quad 4x = 3y + 1$$

$$\Rightarrow \quad x = \frac{3y+1}{4}$$

$$\Rightarrow \quad x = \frac{3y}{4} + \frac{1}{4}$$

Hence, Regression line of x on y is

$$x = \frac{3y}{4} + \frac{1}{4}$$

Statement 2 is correct.

Hint:

(1) Find x and y from two regression lines given.

107. Option (a) is correct.

Explanation:

Regression of y on x is $y = \frac{3}{4}x + 2$ ...(i)

$\therefore b_{yx} = \frac{3}{4}$

Regression of x on y is $y = \frac{3}{4}y + \frac{1}{4}$ ...(ii)

$\therefore b_{xy} = \frac{3}{4}$

Correlation of coefficient $= \sqrt{b_{yx}\, b_{xy}} = \sqrt{\frac{3}{4} \times \frac{3}{4}}$

$= \frac{3}{4}$

Statement 1 is correct.

Now, from given lines:

$$3\bar{x} - 4\bar{y} + 8 = 0 \quad \text{...(iii)}$$

$$4\bar{x} - 3\bar{y} - 1 = 0 \quad \text{...(iv)}$$

On solving (iii), (iv) by elimination method

$$\Rightarrow \quad \bar{y} = 5 \text{ and } \bar{x} = 4$$

Hence, statement 2 is incorrect.

Hints:

(1) Find coefficient and product.

(2) Find x, y by solving both equation.

108. Option (c) is correct.

Explanation:

Marks	Number Students	C.F.
15–20	4	4
20–25	5	9
25–30	11	20
30–35	6	26
35–40	5	31
40–45	8	39
45–50	9	48
50–55	6	54
55–60	4	58
60–65	2	60

$$\left[\frac{N}{2} = \frac{60}{2} = 30 \text{ i.e., Interval is } 35-40\right]$$

$$\text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$= 35 + \frac{\frac{60}{2} - 26}{5} \times 5$$

$$= 35 + \frac{30-26}{5} \times 5$$

$$= 35 + 4$$

$$= 39$$

Hint:

(1) Find C.F. and $\frac{N}{2}$, then select interval and apply formula of median.

109. Option (b) is correct.

Explanation:

To find mode, consider maximum frequency interval, we get:

$$f_1 = 11 \text{ of } 25 - 30 \text{ interval}$$

Now, consider preceding and successive frequency of 25 – 30 interval, we get:

So, $f_0 = 5, f_1 = 11, f_2 = 6$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 25 + \frac{11-5}{2\times 11-5-6} \times 5$$

[l = lower limit of 25–30]

$$= 25 + \frac{6\times 5}{22-11}$$

$$= 25 + \frac{30}{11}$$

$$= 25 + 2.73$$

$$= 27.73$$

Hints:

(1) Select interval maximum frequency then one prederive and successive frequency.

(2) Apply formula.

110. Option (c) is correct.

Explanation:

$$\text{Sum of } n \text{ natural number} = \frac{n(n+1)}{2}$$

Now,

$$\text{Sum of 14 natural number} = \frac{14(14+1)}{2} = 105$$

$$\text{Sum of 64 natural number} = \frac{64(64+1)}{2} = 2080$$

So, sum of terms in interval [15, 64]:

= Sum of 64 number – Sum of 14 number

= 2080 – 105

= 1975

$$\text{Mean of terms in interval} = \frac{1975}{50} = 39.5$$

Hint:

(1) Sum of n natural number.

111. Option (a) is correct.

Explanation:

Consider, given terms

$x, x, x + 2, x + 3, x + 10$

$$\text{Mean} = \frac{\text{Sum of terms}}{5}$$

$$= \frac{x+x+x+2+x+3+x+10}{5}$$

$$= \frac{5x+15}{5}$$

$$= x + 3$$

Mode is defined as that frequency which occur maximum time, we get

Mode = x

Median is defined as middle term odd number of series median is $\frac{(n+1)^{th}}{2}$ term where n is add in ordered series.

$$\text{Median} = \frac{(n+1)^{th}}{2} \text{ term}$$

$= 3^{rd}$ term

$= x + 2$

Hence, Mean $= x + 3$

Mode $= x$

Median $= x + 2$

Statement 1: Mean > Mode is correct.

Statement 2: Median > Mean is incorrect.

Hint:

(1) Use definition of mean, mode, median term.

112. Option (b) is correct.

Explanation:

Mean of 10 observations = 5.5

$$\Rightarrow \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}}{10} = 5.5$$

$$\Rightarrow \frac{\sum_1^{10} x_i}{10} = 5.5 \quad \text{....(i)}$$

Now, each term multiplied by 4 and subtracted from 44.

$$\text{New Mean} = 44 - \frac{4\sum_1^{10} x_i}{10}$$

$= 44 - 4 \times 5.5$ [By (i)]

$= 44 - 22$

$= 22$

Hints:

(1) Mean get multiply by 4.

(2) New mean according to given situation.

113. Option (c) is correct.

Explanation:

Geometric mean = g(given)

Now, g of given numbers is given by:

$g = (\text{product})^{\frac{1}{n}}$, where n is number of observations or terms

$$g = \left(2\times4\times8\times16\times32\times64 \times128\times256\times512\times1024\right)^{\frac{1}{10}}$$

{Power is $\frac{1}{10}$ because 10 digits}

$$g = \left(2^1\times2^2\times2^3\times2^4\times2^5 \times2^6\times2^7\times2^8\times2^9\times2^{10}\right)^{\frac{1}{10}}$$

$$g = \left(2^{1+2+3+4+5+6+7+8+9+10}\right)^{\frac{1}{10}} \quad \ldots(i)$$

Now, series = 1 + 2 + 3 ………… + 10

Sum of series given by:

$$S = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{10}{2}[2\times1 + (10-1)\times1]$$

$$= 5[2 + 9]$$

$$= 5 \times 11$$

$$= 55$$

Put S = 55 in (i), we get

$$g = \left(2^{55}\right)^{\frac{1}{10}}$$

$$g = 2^{\frac{11}{2}}$$

So, $2^5 < 2^{\frac{11}{2}} < 2^6$

$32 < g < 64$

Hint:

Geometric mean is $(\text{product})^{\frac{1}{n}}$, where n is number of terms.

114. Option (c) is correct.

Explanation:

Harmonic mean of two number a and b is given by $\frac{2ab}{a+b}$.

Now, Harmonic mean given is 48.

Let $a = 60$, $b = x$

So, $\frac{2\times60\times x}{60+x} = 48$

$\Rightarrow \frac{120x}{60+x} = 48$

$\Rightarrow 120x = 48(60 + x)$

$\Rightarrow 120x = 2880 + 48x$

$\Rightarrow 72x = 2880$

$\Rightarrow x = 40$

Hint:

(1) Use direct formula of harmonic mean.

115. Option (a) is correct.

Explanation:

First 10 even natural number are:

x_i = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

$$\text{Mean } (\bar{x}) = \frac{2+4+6+8+10+12+14+16+18+20}{10}$$

$$= \frac{110}{10} = 11$$

$$\text{Mean deviation} = \frac{\sum|x_i - \bar{x}|}{N}$$

$$= \frac{|2-11|+|4-11|+|6-11|+|8-11|+|10-11|+|12-11|+|14-11|+|16-11|+|18-11|+|20-11|}{10}$$

$$= \frac{9+7+5+3+1+1+3+5+7+9}{10}$$

$$= \frac{50}{10}$$

$$= 5$$

Hint:

(1) Use formula $\frac{\sum|x_i - \bar{x}|}{N}$ for mean deviation.

116. Option (b) is correct.

Explanation:

Given: $\sum_{i=1}^{10} x_i = 110,$

Number of terms = 10

$$\sum_{i=1}^{10} x_i^2 = 1540$$

$$\text{Variance} = \frac{\sum_{i=1}^{10} x_i^2}{N} - \left(\frac{\sum_{i=1}^{10} x_i}{N}\right)^2$$

$$= \frac{1540}{10} - \left(\frac{110}{10}\right)^2$$

$$= 154 - (11)^2$$

$$= 154 - 121$$

$$= 33$$

Hint:

(1) Use variance formula directly.

117. Option (a) is correct.

Explanation:

We have to make 3 digit number by 1, 3, 7 that is divisible by 3.

Divisibility rule of 3 says that sum of digits must be divisible by 3.

Since, repetition not allowed.

So, sum of digits = 1 + 3 + 7 = 11

Since, 11 not divisible by 3.

Hence, any number by 1, 3, 7 that is divisible by 3 is not possible.

Hence, probability = 0

Hint:

(1) Use divisibility rule of 3.

118. Option (a) is correct.

Explanation:

Given equation:

$$x^2 + x + n = 0 \quad \ldots\text{(i)}$$

Also, given $n \in N$, $n < 4$

So, possible values of $n = 1, 2, 3$

We get equations By (i):

$\Rightarrow \quad x^2 + x + 1 = 0$

$\Rightarrow \quad x^2 + x + 2 = 0$

$\Rightarrow \quad x^2 + x + 3 = 0$

Now, we know if the discriminant,

$D = b^2 - 4ac$ is negative then no real roots exist.

Case 1: $x^2 + x + 1 = 0$

$$D = b^2 - 4ac$$
$$= (1)^2 - 4 \times 1 \times 1$$
$$= 1 - 4 = -3$$

So, no real root exist for

$$x^2 + x + 1 = 0$$

Case 2: $x^2 + x + 2 = 0$

$$D = b^2 - 4ac$$
$$= (1)^2 - 4 \times 1 \times 2$$
$$= -7$$

So, no real root.

Case 3: $x^2 + x + 3 = 0$

$$D = b^2 - 4ac$$
$$= 1^2 - 4 \times 1 \times 3$$
$$= -11$$

So, no real root.

Hence, no real root exist for any given condition

So, probability = 0

Hints:

(1) If $D < 0$ then no real root.

(2) $n = 1, 2, 3$ condition taken.

119. Option (d) is correct.

Explanation:

For two events, A and B

$$P(\text{not } A) = P(\bar{A}) = \frac{7}{10} \quad \text{(given)}$$

So, $$P(A) = 1 - \frac{7}{10}$$

$\Rightarrow$ $$P(A) = \frac{3}{10} \quad \ldots\text{(i)}$$

Again,

$$P(\text{Not } B) = P(\bar{B}) = \frac{3}{10} \quad \text{(given)}$$

So, $$P(B) = 1 - \frac{3}{10}$$

$\Rightarrow$ $$P(B) = \frac{7}{10} \quad \ldots\text{(ii)}$$

Now, given $P\left(\frac{A}{B}\right) = \frac{3}{14}$

$$\Rightarrow \quad \frac{P(A \cap B)}{P(B)} = \frac{3}{14}$$

$$\Rightarrow \quad P(A \cap B) = \frac{3}{14} \times P(B)$$

$$\Rightarrow \quad P(A \cap B) = \frac{3}{14} \times \frac{7}{10} \qquad \text{[By (ii)]}$$

$$\Rightarrow \quad P(A \cap B) = \frac{3}{20} \qquad \ldots\text{(iii)}$$

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$$

$$= \frac{\frac{3}{20}}{\frac{3}{10}} \qquad \text{[By (i), (iii)]}$$

$$= \frac{1}{2}$$

Hints:

(1) Use P(A), P(B)

(2) Use $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

120. Option (a) is correct.

Explanation:

Total balls to arrange = 7 + 3 = 10

Ways to arrange 10 balls = 10!

Ways to arrange 7 white balls =7!

Ways to arrange 3 Black balls such that no two balls should be adjacent = $^8P_3$

Favourable events = 7! × $^8P_3$

Total events (sample space) = 10!

$$\text{Probability} = \frac{7! \times {}^8P_3}{10!}$$

$$P = \frac{7! \times \frac{8!}{5!}}{10!}$$

$$\left\{ {}^nP_r = \frac{n!}{(n-r)!} \right\}$$

$$P = \frac{8 \times 7 \times 6}{10 \times 9 \times 8}$$

$$P = \frac{7}{15}$$

Hints:

(1) Total events and sample space.

(2) Black balls arrange ways is $^8P_3$.