# NDA/NA
**National Defence Academy / Naval Academy**

## MATHEMATICS I

## QUESTION PAPER 2022

**Time : 2:30 Hour** **Total Marks : 300**

### Important Instructions :

1. *This test Booklet contains* **120** *items (questions). Each item is printed in* **English**. *Each item comprises four responses (answer's). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ONLY ONE response for each item.*
2. *You have to mark all your responses ONLY on the separate Answer Sheet provided.*
3. **All** *items carry equal marks.*
4. *Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions.*
5. *Penalty for wrong answers:*
   *THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.*
   **(i)** *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, one-third of the marks assigned to that question will be deducted as penalty.*
   **(ii)** *If a candidate gives more than one answer, it will be treated as a wrong answer even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   **(iii)** *If a question is left blank, i.e., no answer is given by the candidate, there will be no penalty for that question.*

**1.** If $\Delta_1 = \begin{vmatrix} 1 & p & q \\ 1 & q & r \\ 1 & r & p \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ q & r & p \\ r & p & q \end{vmatrix}$

where $p \neq q \neq r$, then $\Delta_1 + \Delta_2$ is

**(a)** 0
**(b)** always positive
**(c)** always negative
**(d)** positive if $p, q, r$ are positive else negative

**2.** *If* $(a - b)(b - c)(c - a) = 2$ and $abc = 6$, then what is the value of

$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$?

**(a)** 3 **(b)** 12
**(c)** 14 **(d)** 15

**3.** Under which of the following conditions does the determinant

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ vanish?

**(1)** $a + b + c = 0$
**(2)** $a^3 + b^3 + c^3 = 3abc$
**(3)** $a^2 + b^2 + c^2 - ab - be - ca = 0$

Select the correct answer using the code given below:

**(a)** 1 and 2 only **(b)** 2 and 3 only
**(c)** 1 and 3 only **(d)** 1, 2 and 3

**4.** Consider the following in respect of the matrices:

$A = [m\ n]$, $B = [-n\ -m]$ and $C = \begin{bmatrix} m \\ -m \end{bmatrix}$

**(1)** CA = CB
**(2)** AC = BC
**(3)** C(A + B) = CA + CB

Which of the above statements is/are correct?

**(a)** 1 only **(b)** 2 only
**(c)** 2 and 3 **(d)** 1 and 2

**5.** If $A = \begin{bmatrix} 2\sin\theta & \cos\theta & 0 \\ -2\cos\theta & \sin\theta & 0 \\ -1 & 1 & 1 \end{bmatrix}$, then

What is A(*adj*A) equal to?

**(a)** Null matrix **(b)** –I
**(c)** I **(d)** 2I

where I is the identity matrix.

**6.** For what value of $k$ is the matrix

$$\begin{bmatrix} 2\cos 2\theta & 2\cos 2\theta & 6 \\ 1-2\sin^2\theta & 2\cos^2\theta - 1 & 3 \\ k & 2k & 1 \end{bmatrix}$$

singular?

**(a)** 0 only **(b)** 1 only
**(c)** 2 only **(d)** Any real value

**7.** Let A be a non-singular matrix and B = *adj*A. Which of the following statements is/are correct?

**(1)** AB = BA
**(2)** AB is a scalar matrix
**(3)** AB can be a null matrix

Select the correct answer using the code given below:

**(a)** 1 only **(b)** 1 and 2 only
**(c)** 2 only **(d)** 1, 2 and 3

**8.** Consider the following statements in respect of square matrices A and B of same order:

**(l)** If AB is a null matrix, then a least one of A and B is a null matrix.
**(2)** If AB is an identity matrix, then BA = AB.

Which of the above statements is/are correct?

**(a)** 1 only **(b)** 2 only
**(c)** Both 1 and 2 **(d)** Neither 1 nor 2

**9.** If A is the identity matrix of order 3 and B is its transpose, then what is the value of the determinant of the matrix C = A + B?

**(a)** 1 **(b)** 2
**(c)** 4 **(d)** 8

**10.** Let A and B be non-singular matrices of the same order such that AB = A and BA = B. Which of the following statements is/are correct?

**(1)** $A^2 = A$
**(2)** $AB^2 = A^2B$

Select the correct answer using the code given below:

**(a)** 1 only **(b)** 2 only
**(c)** Both 1 and 2 **(d)** Neither 1 nor 2

**11.** How many terms are there in the expansion of

$$\left(1+\frac{2}{x}\right)^9\left(1-\frac{2}{x}\right)^9 ?$$

**(a)** 9 **(b)** 10
**(c)** 19 **(d)** 20

**12.** Consider the following statements in respect of the expansion of $(x + y)^{10}$:

**(1)** Among all the coefficients of the terms, the coefficient of the 6th term has the highest value
**(2)** The coefficient of the 3rd term is equal to coefficient of the 9th term

Which of the above statements is/are correct T

**(a)** 1 only **(b)** 2 only
**(c)** Both 1 and 2 **(d)** Neither 1 nor 2

**13.** If C$(3n, 2n)$ = C$(3n, 2n - 7)$, then what is the value of C$(n, n-5)$?

**(a)** 42 **(b)** 35
**(c)** 28 **(d)** 21

**14.** What is the value of

C(51, 21) – C(51, 22) + C(51, 23) – C(51, 24) + C(51, 25) – C(51, 26) + C(51, 27) – C(51, 28) + C(51, 29) – C(51, 30)?

**(a)** C(51, 25)
**(b)** C(51, 27)
**(c)** C(51, 51) – C(51, 0)
**(d)** C(51, 25) – C(51, 27)

**15.** How many odd numbers between 300 and 400 are there in which none of the digits is repeated?

**(a)** 32 **(b)** 36
**(c)** 40 **(d)** 45

**16.** How many permutations are there of the letters of the word 'TIGER' in which the vowels should not occupy the even positions?

**(a)** 72 **(b)** 36
**(c)** 18 **(d)** 12

**17.** Let $\alpha$ and $\beta$ be the roots of the equation $x^2 + px + q = 0$. If $\alpha^3$ and $\beta^3$ are the roots of the equation $x^2 + mx + n = 0$, then what is the value of $m + n$?

**(a)** $p^3 + q^3 + pq$ **(b)** $p^3 + q^3 - pq$
**(c)** $p^3 + q^3 + 3pq$ **(d)** $p^3 + q^3 - 3pq$

**18.** Let $\alpha$ and $\beta$ be the roots of the equation $x^2 - ax - bx + ab - c = 0$. What is the quadratic equation whose roots are $a$ and $b$?

**(a)** $x^2 - \alpha x - \beta x + \alpha\beta + c = 0$
**(b)** $x^2 - \alpha x - \beta x + \alpha\beta - c = 0$
**(c)** $x^2 + \alpha x + \beta x + \alpha\beta + c = 0$
**(d)** $x^2 + \alpha x + \beta x + \alpha\beta - c = 0$

**19.If the roots of the equation**

$x^2 - ax - bx - cx + bc + ca = 0$

are equal, then which one of the following is correct?

**(a)** $a+b+c=0$ **(b)** $a-b+c=0$
**(c)** $a+b-c=0$ **(d)** $-a+b+c=0$

**20.** Let $\alpha$ and $\beta$ $(\alpha > \beta)$ be the roots of the equation $x^2 - 8x + q = 0$. If $\alpha^2 - \beta^2 = 16$, then what is the value of $q$?
**(a)** $-15$ **(b)** $-10$
**(c)** 10 **(d)** 15

**21.** What is the maximum value of $n$ such that $5^n$ divides $(30! + 35!)$, where $n$ is a natural number?
**(a)** 4 **(b)** 6
**(c)** 7 **(d)** 8

**22.** What is the value of
$2(2 \times 1) + 3\,(3 \times 2 \times 1) + 4\,(4 \times 3 \times 2 \times 1) +$
$5(5 \times 4 \times 3 \times 2 \times 1) + \ldots\ldots\ \ldots\ldots\ \ldots\ldots +$
$9(9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 2$?
**(a)** 11! **(b)** 10!
**(c)** 10 + 10! **(d)** 11 + 10!

**23.** If A = {{1, 2, 3}}, then how many elements are there in the power set of A?
**(a)** 1 **(b)** 2
**(c)** 4 **(d)** 8

**24.** If $a, b, c$ are in GP where $a > 0, b > 0, c > 0$, then which of the following are correct?
**(1)** $a^2, b^2, c^2$ are in GP
**(2)** $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in GP
**(3)** $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in GP
Select the correct answer using the code given below:
**(a)** 1 and 2 only **(b)** 2 and 3 only
**(c)** 1 and 3 only **(d)** 1, 2 and 3

**25.** If $\frac{a+b}{2}, b, \frac{b+c}{2}$ are in HP,
then which one of the following is correct?
**(a)** $a, b,$ c are in AP
**(b)** $a, b, c$ are in GP
**(c)** $a+b, b+c, c+a$ are in GP
**(d)** $a+b, b+c, c+a$ are in AP

**26.** What is value of $\cot^2 15° + \tan^2 15°$?
**(a)** 12 **(b)** 14
**(c)** $8\sqrt{3}$ **(d)** 4

**27.** In a triangle ABC,
sin A – cos B – cos C = 0.
What is angle B equal to?
**(a)** $\frac{\pi}{6}$ **(b)** $\frac{\pi}{4}$
**(c)** $\frac{\pi}{3}$ **(d)** $\frac{\pi}{2}$

**28.** If $\alpha + \beta = \frac{\pi}{4}$ and $2\tan\alpha = 1$, then what is $\tan 2\beta$ equal to?
**(a)** $\frac{1}{3}$ **(b)** $\frac{2}{3}$
**(c)** $\frac{3}{4}$ **(d)** $\frac{3}{5}$

**29.** If $\tan(45° + \theta) = 1 + \sin 2\theta$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$, then what is the value of $\cos 2\theta$?
**(a)** 0 **(b)** $\frac{1}{2}$
**(c)** 1 **(d)** 2

**30.** Let $\sin 2\theta = \cos 3\theta$, where $\theta$ is acute angle. What is the value of $1 + 4\sin\theta$?
(given that $\sin 18° = \frac{\sqrt{5}-1}{4}$)
**(a)** $\sqrt{3}$ **(b)** 2
**(c)** $\sqrt{5}$ **(d)** 3

**31.** If $\tan\theta = -\frac{5}{12}$, then what can be the value of $\sin\theta$?
**(a)** $\frac{5}{13}$ but cannot be $-\frac{5}{13}$
**(b)** $-\frac{5}{13}$ but cannot be $\frac{5}{13}$
**(c)** $\frac{5}{13}$ or $-\frac{5}{13}$
**(b)** None of the above

**32.** What is the value of
$\cos^4\frac{7\pi}{8} + \cos^4\frac{5\pi}{8}$?
**(a)** $\frac{3}{2}$ **(b)** $\frac{3}{4}$
**(c)** $\frac{3}{8}$ **(d)** $\frac{3}{16}$

**33.** What is $\sin^2\left(\frac{\pi}{4}+\theta\right) - \sin^2\left(\frac{\pi}{4}-\theta\right)$ equal to?
**(a)** $\sin 2\theta$ **(b)** $\cos 2\theta$
**(c)** $2\sin\theta$ **(d)** $2\cos\theta$

**34.** A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height $h$. At a point on the plane the angles of elevation of the bottom and top of the flagstaff are $\theta$ and $2\theta$ respectively. What is the height of the tower?

(a) $h\cos\theta$ (b) $h\sin\theta$
(c) $h\cos 2\theta$ (d) $h\sin 2\theta$

**35.** The shadow of a tower becomes $x$ metre longer, when the angle of elevation of sun changes from 60° to θ. If the height of the tower is $\sqrt{3}$ $x$ metre, then which one of the following is correct?

(a) $0 < \theta < 30°$ (b) $30° < \theta < 45°$
(c) $45° < \theta < 60°$ (d) $60° < \theta < 90°$

**36.** If $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{x}{3}\right)=\frac{\pi}{4}$, where $0 < x < 6$, then what is $x$ equal to?

(a) 1 (b) 2
(c) 3 (d) 4

**37.** If $3\sin^{-1} x + \cos^{-1} x = \pi$, then what is $x$ equal to?

(a) 0 (b) $\frac{1}{2}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

**38.** If $\tan\alpha + \tan\beta = 1 - \tan\alpha \tan\beta$, where $\tan\alpha \tan\beta \neq 1$, then which of the following is one of the values of $(\alpha + \beta)$?

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$

**39.** If $(1 + \tan\theta)(1 + \tan 9\theta) = 2$, then what is the value of $\tan(10\theta)$?

(a) 0 (b) 1
(c) 2 (d) Infinite

**40.** What is the value of
$\sin 0° + \sin 10° + \sin 20° + \sin 30° + ... + \sin 360°$?

(a) –1 (b) 0
(c) 1 (d) 2

**41.** Consider all the subsets of the set A = {1, 2, 3, 4}. How many of them are supersets of the set {4}?

(a) 6 (b) 7
(c) 8 (d) 9

**42.** Consider the following statements in respect of two non-empty sets A and B:

(1) $x \notin (A \cup B) \Rightarrow x \notin A$ or $x \notin B$
(2) $x \notin (A \cap B) \Rightarrow x \notin A$ and $x \notin B$

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

**43.** Consider the following statements in respect of two non-empty sets A and B:

(1) $A \cup B = A \cup B$ if $A = B$
(2) $A \Delta B = \varphi$ if $A = B$

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

**44.** Consider the following statements in respect of the relation R in the set IN of natural numbers defined by $x$R$y$ if $x^2 - 5xy + 4y^2 = 0$:

(1) R is reflexive
(2) R is symmetric
(3) R is transitive

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) 1 and 2 only (d) 1, 2 and 3

**45.** Consider the following statements in respect of any relation R on a set A:

(1) R is reflexive, then $R^{-1}$ is also reflexive
(2) If R is symmetric, then $R^{-1}$ is also symmetric
(3) If R is transitive, then $R^{-1}$ is also transitive

Which of the above statements are correct?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

**46.** What is the principal argument of $\frac{1}{1+i}$ where $i = \sqrt{-1}$ ?

(a) $-\frac{3\pi}{4}$ (b) $-\frac{\pi}{4}$
(c) $\frac{\pi}{4}$ (d) $\frac{3\pi}{4}$

**47.** What is the modulus of $\left(\frac{\sqrt{-3}}{2}-\frac{1}{2}\right)^{200}$ ?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 1 (d) $2^{200}$

**48.** Consider the following statements:

(1) $\frac{n!}{3!}$ is divisible by 6, where $n > 3$
(2) $\frac{n!}{3!} + 3$ is divisible by 7, where $n > 3$

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

**49.** In how many ways can a team of 5 players be selected out of 9 players so as to exclude two particular players?

(a) 14 (b) 21
(c) 35 (d) 42

**50.** In the expansion of $\left(x+\frac{1}{x}\right)^{2n}$, what is the $(n+1)^{\text{th}}$ term from the end (when arranged in descending powers of $x$)?

**(a)** $C(2n, n)x$ **(b)** $C(2n, n-1)x$

**(c)** $C(2n, n)$ **(d)** $C(2n, n-1)$

**51.** If the sum of the first 9 terms of an AP is equal to sum of first 11 terms, then what is the sum of the first 20 terms?

**(a)** 20 **(b)** 10

**(c)** 2 **(d)** 0

**52.** If the, $5^{\text{th}}$ term of an AP is $\frac{1}{10}$ and its $10^{\text{th}}$ term is $\frac{1}{5}$ then what is the sum of first 50 terms?

**(a)** 25 **(b)** 25·5

**(c)** 26 **(d)** 26·5

**53.** What is $(1110011)_2 \div (10111)_2$ equal to?

**(a)** $(101)_2$ **(b)** $(1001)_2$

**(c)** $(111)_2$ **(d)** $(1011)_2$

**54.** If $x^3 + y^3 = (100010111)_2$ and $x + y = (11111)_2$, then what is $(x-y)^2 + xy$ equal to?

**(a)** $(1101)_2$ **(b)** $(1001)_2$

**(c)** $(1011)_2$ **(d)** $(1111)_2$

**55.** Consider the inequations

$5x - 4y + 12 < 0$, $x + y < 2$, $x < 0$ and $y > 0$. Which one of the following points lies in the common region?

**(a)** (0, 0) **(b)** (– 2, 4)

**(c)** (– 1, 4) **(d)** (– 1, 2)

**56.** Consider the following statements in respect of the function $y = [x]$, $x \in (-1, 1)$ where [.] is the greatest integer function:

**(1)** Its derivative is 0 at $x = 0·5$

**(2)** It is continuous at $x = 0$

Which of the above statements is/are correct?

**(a)** 1 only **(b)** 2 only

**(c)** Both 1 and 2 **(d)** Neither 1 nor 2

**57.** What is the degree of the differential equation

$$1 + \left(\frac{dy}{dx}\right)^2 = \left(\frac{d^2y}{dx^2}\right)^{\frac{4}{3}} ?$$

**(a)** $\frac{4}{3}$ **(b)** 2

**(c)** 3 **(d)** 4

**58.** A radioactive substance decays at a rate proportional to the amount of substance present. If half of the substance decays in 100 years, then what is the decay constant (proportionality constant)?

**(a)** $\frac{\ln 2}{100}$ **(b)** $\frac{\ln 5}{100}$

**(c)** $\frac{\ln 10}{100}$ **(d)** $\frac{2\ln 2}{100}$

**59.** What is the domain of the function

$f(x) = \sqrt{1-(x-1)^2}$ ?

**(a)** (0, 1) **(b)** [– 1, 1]

**(c)** (0, 2) **(d)** [0, 2]

**60.** The area of the region bounded by the parabola $y^2 = 4kx$, where $k > 0$ and its latus rectum is 24 square units. What is the value of $k$?

**(a)** 1 **(b)** 2

**(c)** 3 **(d)** 4

**61.** What is $\int_0^{\frac{\pi}{4}} \frac{dx}{(\sin x + \cos x)^2}$ equal to?

**(a)** $-\frac{1}{2}$ **(b)** $\frac{1}{2}$

**(c)** 1 **(d)** $\frac{3}{2}$

**62.** What is $\int (\sin x)^{-1/2} (\cos x)^{-3/2} dx$ equal to?

**(a)** $\sqrt{\tan x} + c$ **(b)** $2\sqrt{\tan x} + c$

**(c)** $\sqrt{\cot x} + c$ **(d)** $\sqrt{2\tan x} + c$

**63.** If $I_1 = \int \frac{e^x dx}{e^x + e^{-x}}$ and $I_2 = \int \frac{dx}{e^{2x}+1}$, then what is $I_1 + I_2$ equal to?

**(a)** $\frac{x}{2} + c$ **(b)** $x + c$

**(c)** $\ln(e^x + e^{-x}) + c$ **(d)** $\ln(e^x - e^{-x}) + c$

**64.** What is $\int_{-2}^{-1} \frac{x}{|x|} dx$ equal to?

**(a)** – 2 **(b)** – 1

**(c)** 1 **(d)** 2

**65.** How many extreme values does $\sin 4x + 2x$, where $0 < x < \frac{\pi}{2}$ have?

**(a)** 1 **(b)** 2

**(c)** 4 **(d)** 8

**66.** What is the maximum value of the function $f(x) = \frac{1}{\tan x + \cot x}$, where $0 < x < \frac{\pi}{2}$ ?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 1 (d) 2

**67.** If $4f(x)-f\left(\frac{1}{x}\right)=\left(2x+\frac{1}{x}\right)\left(2x-\frac{1}{x}\right)$, then what is $f(2)$ equal to?
(a) 0 (b) 1
(c) 2 (d) 4

**68.** If $f(x) = 4x + 3$, then what is $f_o f_o f(-1)$ equal to?
(a) –1 (b) 0
(c) 1 (d) 2

**69.** If $x^y y^x = 1$, then what is $\frac{dy}{dx}$ at (1, 1) equal to?
(a) –1 (b) 0
(c) 1 (d) 4

**70.** If $y = (x^x)^x$, then what is the value of $\frac{dy}{dx}$ at $x = 1$?
(a) $\frac{1}{2}$ (b) 1
(c) 2 (d) 4

**71.** Let $y = [x + 1]$, $-4 < x < -3$ where [.] is the greatest integer function. What is the derivative of $y$ with respect to $x$ at $x = -3{\cdot}5$?
(a) – 4 (b) – 3·5
(c) – 3 (d) 0

**72.** If $\frac{dy}{dx} = (\ln 5)y$ with $y(0) = \ln 5$, then what is $y(1)$ equal to?
(a) 0 (b) 5
(c) 2ln5 (d) 5ln5

**73.** Consider the following in respect of the function $f(x) = 10^x$:
(1) Its domain is $(-\infty, \infty)$
(2) It is a continuous function
(3) It is differentiable at $x = 0$
Which of the above statements are correct?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

**74.** What is $\lim_{x\to 0} x^3 (\operatorname{cosec} x)^2$ equal to?
(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) Limit does not exist

**75.** What is $\lim_{x\to 1}\frac{x^3-1}{\sqrt{x}-1}$ equal to?
(a) 0 (b) 3
(c) 6 (d) Limit does not exist

**76.** In which one of the following intervals is the function $f(x)=\frac{x^3}{3}-\frac{7x^2}{2}+6x+5$ decreasing?
(a) $(-\infty, 1)$ only (b) (1, 6)
(c) $(6, \infty)$ only (d) $(-\infty, 1) \cup (6, \infty)$

**77.** If the derivative of the function $f(x) = \frac{m}{x} + 2nx + 1$ vanishes at $x = 2$, then what is the value of $m + 8n$?
(a) – 2
(b) 0
(c) 2
(d) Cannot be determined due to insufficient data

**78.** What is the area included in the first quadrant between the curves $y = x$ and $y = x^3$?
(a) $\frac{1}{8}$ square unit (b) $\frac{1}{4}$ square unit
(c) $\frac{1}{2}$ square unit (d) 1 square unit

**79.** If $xy = 4225$ where $x$, $y$ are natural numbers, then what is the minimum value of $x + y$?
(a) 130 (b) 260
(c) 2113 (d) 4226

**80.** What does the equation $x\frac{dy}{dx}-2y=0$ represent?
(a) A family of straight lines
(b) A family of circles
(c) A family of parabolas
(d) A family of ellipses

**81.** If the points with coordinates $(-5, 0)$, $(5p^2, 10p)$ and $(5q^2, 10q)$ are collinear, then what is the value of $pq$ where $p \neq q$?
(a) – 2 (b) – 1
(c) 1 (d) 2

**82.** What is the equation of the straight line which passes through the point (1, – 2) and cuts off equal intercepts from the axes?
(a) $x + y - 1 = 0$ (b) $x - y - 1 = 0$
(c) $x + y + 1 = 0$ (d) $x - y - 2 = 0$

**83.** What is the equation of the circle which touches both the axes in the first quadrant and the line $y - 2 = 0$?
(a) $x^2 + y^2 - 2x - 2y - 1 = 0$
(b) $x^2 + y^2 + 2x + 2y + 1 = 0$
(c) $x^2 + y^2 - 2x - 2y + 1 = 0$
(d) $x^2 + y^2 - 4x - 4y + 4 = 0$

**84.** What is the equation of the parabola with focus (– 3, 0) and directrix $x - 3 = 0$?
**(a)** $y^2 = 3x$ **(b)** $x^2 = 12y$
**(c)** $y^2 = 12x$ **(d)** $y^2 = -12x$

**85.** What is the distance between the foci of the ellipse $x^2 + 2y^2 = 1$?
**(a)** 1 **(b)** $\sqrt{2}$
**(c)** 2 **(d)** $2\sqrt{2}$

**86.** Let $a, b, c$ be the lengths of sides BC, CA, AB respectively *of* a triangle ABC. If $p$ is the perimeter and $q$ is the area of the triangle, then what is $p(p - 2a) \tan\left(\frac{A}{2}\right)$ equal to?
**(a)** $q$ **(b)** $2q$
**(c)** $3q$ **(d)** $4q$

**87.** A straight line passes through the point of intersection of $x + 2y + 2 = 0$ and $2x - 3y - 3 = 0$. It cuts equal inter cepts in the fourth quadrant. What is the sum of the absolute values of the intercepts?
**(a)** 2 **(b)** 3
**(c)** 4 **(d)** 6

**88.** Under which one of the following conditions are the lines $ax + by + c = 0$ and $bx + ay + c = 0$ parallel $(a \neq 0, b \neq 0)$?
**(a)** $a - b = 0$ only **(b)** $a + b = 0$ only
**(c)** $a^2 - b^2 = 0$ **(d)** $ab + 1 = 0$

**89.** What is the equation of the locus of the mid-point of the line segment obtained by cutting the line $x + y = p$, (where $p$ is a real number) by the coordinate axes?
**(a)** $x - y = 0$ **(b)** $x + y = 0$
**(c)** $x - y = p$ **(d)** $x + y = p$

**90.** If the point $(x, y)$ is equidistant from the points $(2a, 0)$ and $(0, 3a)$ where $a > 0$, then which one of the following is correct?
**(a)** $2x - 3y = 0$ **(b)** $3x - 2y = 0$
**(c)** $4x - 6y + 5a = 0$ **(d)** $4x - 6y - 5a = 0$

Consider the following for the next **three** (03) items that follow:

The plane $6x + ky + 3z - 12 = 0$ where $k \neq 0$ meets the coordinate axes at A, B and C respectively. The equation of the sphere passing through the origin and A, B, C is $x^2 + y^2 + z^2 - 2x - 3y - 4z = 0$.

**91.** What is the value of $k$?
**(a)** 3 **(b)** 4
**(c)** 6 **(d)** 12

**92.** If $p$ is the perpendicular distance from the centre of the sphere to the plane, then which one of the following is correct?
**(a)** $0 < p < 0.5$ **(b)** $0.5 < p < 1$
**(c)** $1 < p < 1.5$ **(d)** $p > 1.5$

**93.** What is the equation of the line through the origin and the centre of the sphere?
**(a)** $x = y = z$ **(b)** $2x = 3y = 4z$
**(c)** $6x = 3y = 4z$ **(d)** $6x = 4y = 3z$

Consider the following for the next **two** (02) items that follow:

Let the plane $\frac{2x}{k} + \frac{2y}{3} + \frac{z}{3} = 2$ pass through the point (2, 3, – 6).

**94.** What are the direction ratios of a normal to the plane?
**(a)** $<3, 2, 1>$ **(b)** $<2, 3, 6>$
**(c)** $<6, 3, 2>$ **(d)** $<1, 2, 3>$

**95.** If $p, q$ and $r$ are the intercepts made by the plane on the coordinate axes respectively, then what is $(p + q + r)$ equal to?
**(a)** 10 **(b)** 11
**(c)** 12 **(d)** 13

**96.** If $4\hat{i} + \hat{j} - 3\hat{k}$ and $p\hat{i} + q\hat{j} - 2\hat{k}$ are collinear vectors, then what are the possible values of $p$ and $q$ respectively?
**(a)** 4, 1 **(b)** 1, 4
**(c)** $\frac{8}{3}, \frac{2}{3}$ **(d)** $\frac{2}{3}, \frac{8}{3}$

**97.** If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B, C respectively of a triangle ABC and G is the centroid of the triangle, then what is $\overrightarrow{AG}$ equal to?
**(a)** $\frac{\vec{a} + \vec{b} + \vec{c}}{3}$ **(b)** $\frac{2\vec{a} - \vec{b} - \vec{c}}{3}$
**(c)** $\frac{\vec{b} + \vec{c} - 2\vec{a}}{3}$ **(d)** $\frac{\vec{a} - 2\vec{b} - 2\vec{c}}{3}$

**98.** Consider the following statements:
**(1)** Dot product over vector addition is distributive
**(2)** Cross product over vector addition is distributive
**(3)** Cross product of vectors is associative

Which of the above statements is/ are correct?:
**(a)** 1 only **(b)** 2 only
**(c)** 1 and 2 only **(d)** 1, 2 and 3

**99.** Let $\vec{a}, \vec{b}, \vec{c}$ be three non–zero vectors such that $\vec{a} \times \vec{b} = \vec{c}$. Consider the following statements:

**(1)** $\vec{a}$ is unique if $\vec{b}$ and $\vec{c}$ are given

**(2)** $\vec{c}$ is unique if $\vec{a}$ and $\vec{b}$ are given

Which of the above statements is/are correct?

**(a)** 1 only **(b)** 2 only

**(c)** Both 1 and 2 **(d)** Neither 1 nor 2

**100.** Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $|\vec{a} - \vec{b}| < 2$. If $2\theta$ is the angle between $\vec{a}$ and $\vec{b}$, then which one of the following is correct?

**(a)** $0 < \sin\theta < 1$ only

**(b)** $-\frac{1}{2} < \sin\theta < \frac{1}{2}$ only

**(c)** $-1 < \sin\theta < 0$ only

**(d)** $-1 < \sin\theta < 1$

**101.** Two digits out of 1, 2, 3, 4, 5 are chosen at random and multiplied together. What is the probability that the last digit in the product appears as 0?

**(a)** $\frac{1}{10}$ **(b)** $\frac{1}{5}$

**(c)** $\frac{2}{5}$ **(d)** $\frac{4}{5}$

**102.** The frequency curve (assuming unimodal) corresponding to the data obtained in an experiment is skewed to the left. What conclusion can be drawn from the curve?

**(a)** Mean> Median> Mode

**(b)** Mean > Mode > Median

**(c)** Median > Mean > Mode

**(d)** Mode > Median > Mean

**103.** The variance of five positive observations is 3.6. If four of the observations are 2, 2, 4, 5 then what is the remaining observation?

**(a)** 4 **(b)** 5

**(c)** 7 **(d)** 9

**104.** What is the arithmetic mean of 50 terms of an AP with first term 4 and common difference 4?

**(a)** 50 **(b)** 51

**(c)** 100 **(d)** 102

**105.** What is the coefficient of mean deviation of 21, 34, 23, 39, 26, 37, 40, 20, 33, 27 (taken from mean)?

**(a)** 0·11 **(b)** 0·22

**(c)** 0·33 **(d)** 0·44

Consider the following for the next **three** (03) items that follow:

The algebraic sum of the deviations of a set of values $x_1, x_2, x_3, \ldots x_n$ measured from 100 is – 20 and the algebraic sum of the deviations of the same set of values measured from 92 is 140.

**106.** What is the mean of the values?

**(a)** 91 **(b)** 96

**(c)** 98 **(d)** 99

**107.** What is the algebraic sum of the deviations of the same set of values measured from 99?

**(a)** 0 **(b)** 10

**(c)** 20 **(d)** 40

**108.** If the algebraic sum of the deviations of the same set of values measured from $y$ is 180, then what is the value of $y$?

**(a)** 80 **(b)** 85

**(c)** 90 **(d)** 95

Consider the following data for the next **three** (03) items that follow:

The marks obtained by 51 students in a class are in AP with its first term 4 and common difference 3.

**109.** What is the mean of the marks?

**(a)** 67 **(b)** 71

**(c)** 75 **(d)** 79

**110.** What is the median of the marks?

**(a)** 79·5 **(b)** 79

**(c)** 78·5 **(d)** 77

**111.** What is the sum of the deviations measured from the median?

**(a)** – 1 **(b)** 0

**(c)** 1 **(d)** 2

Consider the following data for the next **three** (03) items that follow:

There are 90 applicants for a job. Some of them are graduates. Some of them have less than three years experience.

| | **Number of graduates** | **Number of non-graduates** |
|---|---|---|
| At least 3 years experience | 18 | 9 |
| Less than 3 years expenence | 36 | 27 |

Let G be the event that the first applicant interviewed is a graduate and T be the event that first applicant interviewed has at least 3 years experience.

**112.** What is $P(G \cap \overline{T})$ equal to?

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

**113.** What is $P(G | \overline{T})$ equal to?

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$

(c) $\frac{4}{7}$ (d) $\frac{5}{7}$

**114.** What is $P(\overline{T} | \overline{G})$ equal to?

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{3}{5}$ (d) $\frac{3}{4}$

Consider the following data for the next **three** (03) items at follow:

The incidence of suffering from a disease among workers in an industry has a chance of $33\frac{1}{3}\%$.

**115.** What is the probability that exactly 3 out of 6 workers suffer from a disease?

(a) $\frac{80}{729}$ (b) $\frac{10}{81}$

(c) $\frac{10}{243}$ (d) $\frac{160}{729}$

**116.** What is the probability that no one out of 6 workers suffers from a disease?

(a) $\frac{665}{729}$ (b) $\frac{64}{729}$

(c) $\frac{4}{243}$ (d) $\frac{1}{729}$

**117.** What is the probability that at least one out of 6 workers suffer from a disease?

(a) $\frac{728}{729}$ (b) $\frac{665}{729}$

(c) $\frac{653}{729}$ (d) $\frac{596}{729}$

Consider the following frequency distribution for the next **three** (03) items that follow:

| *Class* | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 |
|---|---|---|---|---|---|
| *Frequency* | 17 | $p+q$ | 32 | $p-3q$ | 19 |

The total frequency is 120. The mean is 50.

**118.** What is the value of *p?*

(a) 25 (b) 26

(c) 27 (d) 28

**119.** What is the value of *q?*

(a) 1 (b) 2

(c) 3 (d) 4

**120.** If the frequency of each class is doubled, then what would be the mean?

(a) 25 (b) 50

(c) 75 (d) 100

## Answers

| Q. No. | Answer Key | Topic Name | Chapter Name |
|---|---|---|---|
| 1 | (c) | Properties of Determinants | Matrices and Determinants |
| 2 | (b) | Properties of Determinants | Matrices and Determinants |
| 3 | (d) | Properties of Determinants | Matrices and Determinants |
| 4 | (c) | Algebra of Matrices | Matrices and Determinants |
| 5 | (d) | Properties of Adjoint Matrices | Matrices and Determinants |
| 6 | (d) | Properties of Determinants | Matrices and Determinants |
| 7 | (b) | Properties of Adjoint Matrices | Matrices and Determinants |
| 8 | (b) | Algebra of Matrices | Matrices and Determinants |
| 9 | (d) | Properties of Determinants | Matrices and Determinants |
| 10 | (c) | Properties of Inverse | Matrices and Determinants |
| 11 | (b) | Number of Terms of Binomial Expansion | Algebra |
| 12 | (c) | Binomial Theorem | Algebra |
| 13 | (d) | Coefficients of Binomial Expansion for the Integral Index | Algebra |
| 14 | (c) | Coefficients of Binomial Expansion for the Integral Index | Algebra |
| 15 | (a) | Permutations and Combinations | Algebra |
| 16 | (b) | Permutations and Combinations | Algebra |
| 17 | (d) | Root and Coefficients | Algebra |
| 18 | (a) | Root and Coefficients | Algebra |
| 19 | (c) | Root and Coefficients | Algebra |
| 20 | (d) | Root and Coefficients | Algebra |
| 21 | (c) | Permutations and Combinations | Algebra |
| 22 | (b) | Permutations and Combinations | Algebra |
| 23 | (b) | Set Theory and Relations | Algebra |
| 24 | (d) | Geometric Progression | Algebra |
| 25 | (b) | Harmonic Progression | Algebra |
| 26 | (b) | Trigonometric Identities | Trigonometry |
| 27 | (d) | Trigonometric Equations | Trigonometry |
| 28 | (c) | Trigonometric Equations | Trigonometry |
| 29 | (c) | Trigonometric Equations | Trigonometry |
| 30 | (c) | Trigonometric Equations | Trigonometry |
| 31 | (c) | Trigonometric Equations | Trigonometry |

| Q. No. | Answer Key | Topic Name | Chapter Name |
|---|---|---|---|
| 32 | (b) | Trigonometric Identities | Trigonometry |
| 33 | (a) | Trigonometric Identities | Trigonometry |
| 34 | (c) | Heights and Distances | Trigonometry |
| 35 | (b) | Heights and Distances | Trigonometry |
| 36 | (a) | Inverse Trigonometric Identities | Trigonometry |
| 37 | (c) | Inverse Trigonometric Identities | Trigonometry |
| 38 | (b) | Trigonometric Identities | Trigonometry |
| 39 | (b) | Trigonometric Identities | Trigonometry |
| 40 | (b) | Trigonometric Identities | Trigonometry |
| 41 | (c) | Set Theory and Relations | Algebra |
| 42 | (d) | Set Theory and Relations | Algebra |
| 43 | (c) | Set Theory and Relations | Algebra |
| 44 | (a) | Set Theory and Relations | Algebra |
| 45 | (d) | Set Theory and Relations | Algebra |
| 46 | (b) | Arguments of Complex Number | Algebra |
| 47 | (c) | Modulus of Complex Number | Algebra |
| 48 | (d) | Permutations and Combinations | Algebra |
| 49 | (b) | Permutations and Combinations | Algebra |
| 50 | (c) | Binomial Theorem | Algebra |
| 51 | (d) | Arithmetic Progression | Algebra |
| 52 | (b) | Arithmetic Progression | Algebra |
| 53 | (a) | Binary System | Algebra |
| 54 | (b) | Binary System | Algebra |
| 55 | (d) | Inequalities | Algebra |
| 56 | (a) | Continuity and Differentiability | Differential Calculus |
| 57 | (d) | Differential Equations | Integral Calculus and Differential Equations |
| 58 | (a) | Differential Equations | Integral Calculus and Differential Equations |
| 59 | (d) | Function | Differential Calculus |
| 60 | (c) | Area under Curves | Integral Calculus & Differential Equations |
| 61 | (b) | Basics of Definite Integration | Integral Calculus & Differential Equations |
| 62 | (b) | Indefinite Integration | Integral Calculus & Differential Equations |

| Q. No. | Answer Key | Topic Name | Chapter Name |
|---|---|---|---|
| 63 | (b) | Basics of Integration | Integral Calculus & Differential Equations |
| 64 | (b) | Properties of Definite Integration | Integral Calculus & Differential Equations |
| 65 | (b) | Maxima and Minima | Differential Calculus |
| 66 | (b) | Maximum and Minimum Value | Trigonometry |
| 67 | (d) | Basics of Function | Differential Calculus |
| 68 | (a) | Composite Function | Differential Calculus |
| 69 | (a) | Logarithmic Differentiation | Integral Calculus & Differential Equations |
| 70 | (b) | Logarithmic Differentiation | Integral Calculus & Differential Equations |
| 71 | (d) | Basics of Differentiation | Integral Calculus & Differential Equations |
| 72 | (d) | Differential Equation | Integral Calculus & Differential Equations |
| 73 | (d) | Differentiability | Differential Calculus |
| 74 | (a) | Limit of Trigonometric Functions | Differential Calculus |
| 75 | (c) | Rationalization Method | Differential Calculus |
| 76 | (b) | Increasing and Decreasing Functions | Differential Calculus |
| 77 | (d) | Basic Differentiation | Integral Calculus & Differential Equations |
| 78 | (b) | Area under Curves | Integral Calculus & Differential Equations |
| 79 | (a) | Maxima and Minima | Differential Calculus |
| 80 | (c) | Differential Equations | Integral Calculus & Differential Equations |
| 81 | (c) | Area of Triangle | Matrices and Determinants |
| 82 | (c) | Equation of Straight Line | Analytical Geometry of 2 & 3 Dimensions |
| 83 | (c) | Equations of Circle | Analytical Geometry of 2 & 3 Dimensions |
| 84 | (d) | Basic of Parabola | Analytical Geometry of 2 & 3 Dimensions |
| 85 | (b) | Basic of Ellipse | Analytical Geometry of 2 & 3 Dimensions |
| 86 | (d) | Half Angle Formula and Area of Triangle | Analytical Geometry of 2 & 3 Dimensions |

| Q. No. | Answer Key | Topic Name | Chapter Name |
|---|---|---|---|
| **87** | **(a)** | Family of Straight Line | Analytical Geometry of 2 & 3 Dimensions |
| **88** | **(c)** | Parallel and Perpendicular Condition | Analytical Geometry of 2 & 3 Dimensions |
| **89** | **(a)** | Locus of Point | Analytical Geometry of 2 & 3 Dimensions |
| **90** | **(c)** | Distance Formula | Analytical Geometry of 2 & 3 Dimensions |
| **91** | **(b)** | Sphere | Analytical Geometry of 2 & 3 Dimensions |
| **92** | **(b)** | Perpendicular Distance from Point to Plane | Analytical Geometry of 2 & 3 Dimensions |
| **93** | **(d)** | Equation of Straight Line | Analytical Geometry of 2 & 3 Dimensions |
| **94** | **(a)** | Direction Ratio of Plane | Analytical Geometry of 2 & 3 Dimensions |
| **95** | **(b)** | Direction Ratio of Plane | Analytical Geometry of 2 & 3 Dimensions |
| **96** | **(c)** | Collinear Vectors | Vector Algebra |
| **97** | **(c)** | Centroide of Triangle | Vector Algebra |
| **98** | **(c)** | Properties of Dot and Cross Product of Vectors | Vector Algebra |
| **99** | **(b)** | Definition of Cross Product of Two Vectors | Vector Algebra |
| **100** | **(d)** | Properties of Dot Product of Vectors | Vector Algebra |
| **101** | **(c)** | Basics of Probability | Probability & Statistics |
| **102** | **(d)** | Normal Distribution and Skewed | Probability & Statistics |
| **103** | **(c)** | Variance | Probability & Statistics |
| **104** | **(d)** | Central Tendency | Probability & Statistics |
| **105** | **(b)** | Mean Deviation | Probability & Statistics |
| **106** | **(d)** | Deviation | Probability & Statistics |
| **107** | **(a)** | Deviation | Probability & Statistics |
| **108** | **(c)** | Deviation | Probability & Statistics |
| **109** | **(d)** | Central Tendency | Probability & Statistics |
| **110** | **(b)** | Central Tendency | Probability & Statistics |
| **111** | **(b)** | Central Tendency | Probability & Statistics |
| **112** | **(b)** | Conditional Probability | Probability & Statistics |
| **113** | **(c)** | Conditional Probability | Probability & Statistics |

| Q. No. | Answer Key | Topic Name | Chapter Name |
|---|---|---|---|
| **114** | **(d)** | Conditional Probability | Probability & Statistics |
| **115** | **(d)** | Binomial Distribution | Probability & Statistics |
| **116** | **(b)** | Binomial Distribution | Probability & Statistics |
| **117** | **(b)** | Binomial Distribution | Probability & Statistics |
| **118** | **(c)** | Central Tendency | Probability & Statistics |
| **119** | **(a)** | Central Tendency | Probability & Statistics |
| **120** | **(b)** | Central Tendency | Probability & Statistics |

## ANSWERS WITH EXPLANATION

**1. Option (c) is correct.**

**Explanation:**

$$\Delta_1 = \begin{vmatrix} 1 & p & q \\ 1 & q & r \\ 1 & r & p \end{vmatrix} = \overline{\begin{vmatrix} 1 & p & q \\ 1 & q & r \\ 1 & r & p \end{vmatrix}}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ q & r & p \end{vmatrix}$$

$$\begin{bmatrix} \text{Applying } C_1 \leftrightarrow C_2 \\ \text{and } \quad C_2 \leftrightarrow C_3 \end{bmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ q & r & p \\ r & p & q \end{vmatrix} = \Delta_2$$

Now, $\Delta_1 + \Delta_2 = 2\Delta_2$

$$= 2\begin{vmatrix} 1 & 1 & 1 \\ q & r & p \\ r & p & q \end{vmatrix}$$

$$\begin{bmatrix} \text{Applying } C_2 \to C_2 - C_1 \\ \text{and } \quad C_3 \to C_3 - C_1 \end{bmatrix}$$

$$= 2\begin{vmatrix} 1 & 0 & 0 \\ q & r-q & p-q \\ r & p-r & q-r \end{vmatrix}$$

$$= 2[(r-q)(q-r) - (p-q)(p-r)]$$
$$= -2[p^2 + q^2 + r^2 - pq - qr - rp]$$
$$= -[2p^2 + 2q^2 + 2r^2 - 2pq - 2qr - 2rp]$$
$$= -[(p-q)^2 + (q-r)^2 + (r-p)^2] < 0$$

Hence, value of $\Delta_1 + \Delta_2$ is always negative.

**Hints:**
- Use $|A^{-}| = |A|$
- Make sum of completing square of
$$2p^2 + 2q^2 + 2r^2 - 2pq - 2qr - 2rp$$
and use property sum of square of number

**2. Option (b) is correct.**

**Explanation:**

$$\text{Let } \Delta = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

(Taking common $a$, $b$, $c$ from $C_1$, $C_2$ and $C_3$ respectively)

[Applying $C_1 \to C_1 - C_2$ and $C_2 \to C_2 - C_3$]

$$= abc\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ (a-b)(a+b) & (b-c)(b+c) & c^3 \end{vmatrix}$$

$$= abc(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a+b & b+c & c^3 \end{vmatrix}$$

[Taking common $(a - b)$, $(b - c)$ from $C_1$ and $C_2$ respectively]

Expand through $R_1$

$$= abc(a-b)(b-c)(b+c-a-b)$$
$$= abc(a-b)(b-c)(c-a)$$
$$= 6 \times 2 = 12$$

**Hints:**
- Use properties of determinant and take common facter $abc$, $(a - b)$ $(b - c)$ $(c - a)$ from determinant.

**3. Option (d) is correct.**

**Explanation:**

$$\text{Let} \qquad \Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

[Applying $C_1 \to C_1 + C_2 + C_3$]

$$\Rightarrow \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix} = 0$$

$$\Rightarrow (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$$

[Take $a + b + c$ common from $C_1$]

[Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$$\Rightarrow (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0$$

[Expands through $C_1$] = 0

$\Rightarrow$ $(a + b + c)\ (-a^2 - b^2 - c^2 + ab + bc + ca) = 0$

$\Rightarrow$ $-(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$\Rightarrow$ $-(a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow a + b + c = 0$ *or* $a^2 + b^2 + c^2 - ab - bc - ca = 0$

or $a^3 + b^3 + c^3 = 3abc$

Hence, all statements 1, 2, 3 are correct.

**Hint:**

- Use properties of determinant and take common factor $(a + b + c)\ (a^2 - b^2 - c^2 + ab + bc + ca)$.
- Use algebraic identing $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)\ (a^2 - b^2 - c^2 - ab - bc - ca)$.

**4. Option (c) is correct.**

**Explanation:**

Statement 1:

$$CA = \begin{vmatrix} m \\ -m \end{vmatrix}\begin{bmatrix} m & n \end{bmatrix} = \begin{bmatrix} m^2 & mn \\ -m^2 & -mn \end{bmatrix}$$

$$CB = \begin{bmatrix} m \\ -m \end{bmatrix}\begin{bmatrix} -n - m \end{bmatrix} = \begin{bmatrix} -mn & -m^2 \\ mn & m^2 \end{bmatrix}$$

$CA \neq CB$

So, statement 1 is not true.

Statement 2.

$$AC = \begin{bmatrix} m & n \end{bmatrix}\begin{bmatrix} m \\ -m \end{bmatrix} = \begin{bmatrix} m^2 - mn \end{bmatrix}$$

$$BC = \begin{bmatrix} -n & -m \end{bmatrix}\begin{bmatrix} m \\ -m \end{bmatrix} = \begin{bmatrix} -mn + m^2 \end{bmatrix}$$

$\therefore$ $AC = BC$

So, statement 2 is true.

Statement 3:

$$C(A+B) = \begin{bmatrix} m \\ -m \end{bmatrix}\begin{bmatrix} m-n & n-m \end{bmatrix}$$

$$= \begin{bmatrix} m^2 - mn & mn - m^2 \\ -m^2 + mn & -mn + m^2 \end{bmatrix}$$

$$CA + CB = \begin{bmatrix} m^2 - mn & mn - m^2 \\ -m^2 + mn & -mn + m^2 \end{bmatrix}$$

$\therefore$ $C(A + B) = CA + CB$

So, statement 3 is true.

**Hint:**

- Use multiplication rule of matrices.

**5. Option (d) is correct.**

**Explanation:**

$$A \text{ dy } A = \begin{bmatrix} \sin\theta & 2\cos\theta & \sin\theta - 2\cos\theta \\ -\cos\theta & 2\sin\theta & -2\sin\theta - \cos\theta \\ 0 & 0 & 2 \end{bmatrix}^1$$

$$= \begin{bmatrix} \sin\theta & -\cos\theta & 0 \\ 2\cos\theta & 2\sin\theta & 0 \\ \sin\theta - 2\cos\theta & -2\sin\theta - \cos\theta & 2 \end{bmatrix}$$

$$A\ (\text{adj } A) = \begin{bmatrix} 2\sin\theta & \cos\theta & 0 \\ -2\cos\theta & \sin\theta & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} \sin\theta & -\cos\theta & 0 \\ 2\cos\theta & 2\sin\theta & 0 \\ \sin\theta - 2\cos\theta & -2\sin\theta - \cos\theta & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\sin^2\theta + 2\cos^2\theta & 0 & 0 \\ 0 & 2\cos^2\theta + 2\sin^2\theta & 0 \\ 0 & 0 & 2\sin^2\theta + 2\cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 2I$$

**Shortcut:**

We know that for a square matrix A of order $n$, A(adj A) = adj(A) A = |A|I

$$\therefore \quad |A| = \begin{bmatrix} 2\sin\theta & \cos\theta & 0 \\ -2\cos\theta & \sin\theta & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= 2\sin^2\theta + 2\cos^2\theta$$

[Expand through $C_3$]

$$= 2$$

$\therefore$ A(adj A) = |A| I = 2I

**Hints:**

- Use adj A = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

  Where $C_{ij}$ is cofactor of $a_{ij}$.

**6. Option (d) is correct.**

**Explanation:**

$$\text{Let} \quad \Delta = \begin{vmatrix} 2\cos 2\theta & 2\cos 2\theta & 6 \\ 1-2\sin^2\theta & 2\cos^2\theta - 1 & 3 \\ k & 2k & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 2\cos 2\theta & 2\cos 2\theta & 6 \\ \cos 2\theta & \cos 2\theta & 3 \\ k & 2k & 1 \end{vmatrix}$$

[$\because$ $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$]

$$= 0$$

[$\because$ $R_1$ and $R_2$ are identical]

Hence, for all real value of $k$, the values determinant of given matrix is zero. i.e., for any $k$ given matrix is singular.

**Hints:**

- Use $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
- Use property of determinant that if two rows or column of determinant are identical then its value is zero.
- If determinant of matrix is zero then matrix is called singular matrix.

**7. Option (b) is correct.**

**Explanation:**

We know that for a square matrix A of order n, A(adj A) = (adj A)A = |A|I given that B = adj A

$\therefore$ AB = BA

So, statement 1 is correct.

AB = |A|I is scalar matrix but not null matrix

So, statement 2 is correct and statement 3 is not correct.

**Hints:**

- Use A(adj A) = (adj A)A = |A|I
- If adj = constant for i = j then matrix A is called scalar matrix.

**8. Option (b) is correct.**

**Explanation:**

Let, $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ (Null matrix)}$$

But A and B are not null matrix.

So, statement 1 is not correct.

Let, $AB = I \Rightarrow B = A^{-1}$

We, know that

$$AA^{-1} = I = A^{-1}A$$

$\Rightarrow$ AB = BA

So, statement 2 is correct.

**Hints:**

- If all element of matrix is zero then it is called null matrix.
- If AB = I then $B = A^{-1}$.

**9. Option (d) is correct.**

**Explanation:**

Given that $A = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

And $B = A' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

Now, C = A + B = 2I

$\therefore$ $|C| = |2I| = 2^3\,|I| = 8$

**Shortcut:**

Given, $A = I_3$ and $B = A' = I' = I$

$\therefore$ $C = 2I \Rightarrow |C| = 2^3\,|I| = 8$

**Hints:**

- Use $I' = I$
- Use $|kA| = k^n\ |A|$, where order of square matrix A is $n$.

**10. Option (c) is correct.**

**Explanation:**

Given that $|A| \neq 0$ and $|B| \neq 0$

$\because \quad AB = A \Rightarrow A^{-1}\,AB = A^{-1}\,A$

[Pre multiply by $A^{-1}$ both sides]

$\Rightarrow \quad IB = I \Rightarrow B = I \quad \ldots(i)$

and $\quad BA = B \Rightarrow B^{-1}\,BA = B^{-1}\,B$

[Pre multiply by $A^{-1}$ both sides]

$\Rightarrow \quad IA = I \Rightarrow A = I \quad \ldots(ii)$

Statement 1

$$A^2 = I^2 = I = A$$

(Statement 1 is correct.)

Statement 2

$$AB^2 = II^2 = I \text{ and } A^2B = I^2I = I$$

$[\because\ I^n = I]$

$\therefore \quad AB^2 = A^2B \quad$ (Statement 2 is correct.)

**Hints:**

- Premultiply by $A^{-1}$ and $B^{-1}$ in $AB = A$ and $BA = B$
- Use $I^n = I$.

**11. Option (b) is correct.**

**Explanation:**

$$\left(1+\frac{2}{x}\right)^9\left(1-\frac{2}{x}\right)^9 = \left(1-\frac{4}{x^2}\right)^9$$

$[\because\ (a+b)\,(a-b) = a^2 - b^2]$

We know that number of terms in expansion $(a+b)^n$ is $n+1$.

$\therefore$ Number of terms in expansion $\left(1-\frac{4}{x^2}\right)^9$ is $9 + 1 = 10$

**Hints:**

- Use $a^n b^n = (ab)^n$
- Use $(a+b)\,(a-b) = a^2 - b^2$
- Number of terms in expansion $(a + b)^n$ is $n + 1$

**12. Option (c) is correct.**

**Explanation:**

We know that coefficient of middle term of expansion $(x + y)^n$ has the highest value.

Middle term of $(x + y)^{10}$ is $\left(\frac{10+1+1}{2}\right)^{th}$ term = $6^{th}$ term

$\therefore$ The coefficient of the $6^{th}$ term has the highest value.

So, statement 1 is correct.

We know that in expansion $(x + y)^n$

coefficient of $k^{th}$ term = coefficient of $(n-k+2)^{th}$ term

$\therefore$ In expansion $(x + y)^{10}$

Coefficient of $3^{rd}$ term = coefficient of $(10-3+2)^{th}$ = coefficient of $9^{th}$ term.

So, statement 2 is correct.

**Hints:**

- Coefficient of middle term of expansion $(x + y)^n$ has the highest value.
- Coefficient of $k^{th}$ term = coefficient of $(n-k+2)^{th}$ term

**13. Option (d) is correct.**

**Explanation:**

Given that,

$$^{3n}C_{2n} = {}^{3n}C_{2n-7}$$

$[\because$ If $^{n}C_x = {}^{n}C_y$ then $x + y = n$ or $x = y]$

$\therefore\ 2n + 2n - 7 = 3n$

$\Rightarrow \quad 4n - 3n = 7$

$\Rightarrow \quad n = 7$

$$\therefore \quad ^{n}C_{n-5} = {}^{7}C_2 = \frac{7\times 6}{2\times 1} = 21$$

**Hints:**

- Use If $^{n}C_x = {}^{n}C_y$ then $x + y = n$ or $x = y$
- Use $^{n}C_r = \dfrac{n!}{(n-r)!\,r!}$

**14. Option (c) is correct.**

**Explanation:**

We, know that ${}^{n}C_{r} = {}^{n}C_{r-1}$

$$\therefore\ {}^{51}C_{21} - {}^{51}C_{22} + {}^{51}C_{23} - {}^{51}C_{24} + {}^{51}C_{25} - {}^{51}C_{26} + {}^{51}C_{27} - {}^{51}C_{28} + {}^{51}C_{29} - {}^{51}C_{30}$$

$$= {}^{51}C_{51-30} - {}^{51}C_{51-29} + {}^{51}C_{51-28} - {}^{51}C_{51-27} + {}^{51}C_{51-26} - {}^{51}C_{26} + {}^{51}C_{27} - {}^{51}C_{28} + {}^{51}C_{29} - {}^{51}C_{30}$$

$$= {}^{51}C_{30} - {}^{51}C_{29} + {}^{51}C_{28} - {}^{51}C_{27} + {}^{51}C_{26} - {}^{51}C_{26} + {}^{51}C_{27} - {}^{51}C_{28} + {}^{51}C_{29} - {}^{51}C_{30} = 0$$

Now, ${}^{51}C_{51} - {}^{51}C_{0} = 0$

$[\because\ {}^{n}C_{n} = {}^{n}C_{0}]$

**Hints:**

- Use ${}^{n}C_{r} = {}^{n}C_{r-1}$
- Use ${}^{n}C_{n} = {}^{n}C_{0} = 1$

**15. Option (a) is correct.**

**Explanation:**

For odd numbers between 300 and 400. When digits are not repeated.

| 3 | · | · |
|---|---|---|
| 1 | × 8 | × 4 |

Here one choice 3 for hundred place; 4 choice {1, 5, 7, 9} for unit place and remaining 8 choice for ten's place

$\therefore$ Total odd numbers between 300 and 400, when digits are not repeated = $1 \times 8 \times 4 = 32$.

**Hints:**

- Use for odd number unit place digits will be 1, 3, 5, 7, 9.
- 3 is fixed at hundred place.
- Use non repetition case.

**16. Option (b) is correct.**

**Explanation:**

Given, Word: TIGER

Vowels : I, E

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| □ | □ | □ | □ | □ |

Since, vowels not occupy the even positions.

So, there are 3 odd places are available of vowels.

$\therefore$ Number of words = ${}^{3}C_{2} \times 2! \times 3! = 36$

**Hints:**

- First select number of places for vowels.
- Than arrange number of vowels
- Also arrange number of consonant

**17. Option (d) is correct.**

**Explanation:**

Given that, $\alpha$ and $\beta$ are roots of the equation

$$x^2 + px + q = 0$$

$$\therefore\quad \alpha + \beta = -p \text{ and } \alpha\cdot\beta = q$$

Now,
$$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$$
$$= -p^3 - 3q(-p)$$
$$= 3pq - p^3$$
$$\alpha^3\cdot\beta^3 = (\alpha\beta)^3 = q^3$$

Since, $\alpha^3$ and $\beta^3$ are roots of the equation

$$x^2 + mx + n = 0$$

$$\therefore\quad \alpha^3 + \beta^3 = -m$$

$$\Rightarrow\quad m = -(\alpha^3 + \beta^3) = p^3 - 3pq$$

$$\alpha^3\cdot\beta^3 = n \Rightarrow n = q^3$$

So, $$m + n = p^3 + q^3 - 3pq$$

**Hints:**

- Use, If $\alpha$ and $\beta$ are roots of equation $ax^2 + bx = c = 0$ then, sum of roots $(\alpha + \beta) = -\frac{b}{a}$ and product of roots $(\alpha\cdot\beta) = \frac{c}{a}$
- Use algebraic identity $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

**18. Option (a) is correct.**

**Explanation:**

Given that, α and β are roots of the equation

$$x^2 - ax - bx + ab - c = 0$$

*i.e.,* $x^2 - (a + b)x + ab - c = 0$

$\therefore \quad \alpha + \beta = a + b$

$$\left[\because \text{ for } ax^2 + bx + c = 0;\ \alpha + \beta = -\frac{b}{a}, \alpha\cdot\beta = \frac{c}{a}\right]$$

$$\alpha\cdot\beta = ab - c$$

$\Rightarrow \quad ab = \alpha\cdot\beta + c$

Quadratic equation whose roots are $a$ and $b$ is

$$x^2 - (a + b)x + ab = 0$$

$\therefore \quad x^2 - (\alpha + \beta)x + \alpha\cdot\beta - c = 0$

$\Rightarrow \quad x^2 - \alpha x - \beta x + \alpha\beta - c = 0$

**Hints:**

- Use, If α and β are roots of the equation $ax^2 + bx + c = 0$ then $\alpha + \beta = \frac{-b}{a}$ and $\alpha\cdot\beta = \frac{c}{a}$
- Use, quadratic equation whose roots are α and β is $x^2 - (\alpha + \beta)x + \alpha\cdot\beta = 0$

**19. Option (c) is correct.**

**Explanation:**

Let, α be two equal roots of the equation

$$x^2 - ax - bx - cx + bc + ca = 0$$

*i.e.* $x^2 - (a + b + c)x + bc + ca = 0$

$\therefore \quad \alpha + \alpha = a + b + c$

$\Rightarrow \quad \alpha = \frac{a+b+c}{2} \quad \ldots\text{(i)}$

$$\alpha\cdot\alpha = bc + ca$$

$\Rightarrow \quad \alpha^2 = c(a + b) \quad \ldots\text{(ii)}$

From (i) and (ii)

$$4c(a + b) = [(a + b) + c]^2$$

$\Rightarrow \quad (a + b)^2 + c^2 - 2c(a + b) = 0$

$\Rightarrow \quad [(a + b) - c]^2 = 0$

$\Rightarrow \quad a + b - c = 0$

**Shortcut:**

For equal roots

$$(a + b + c)^2 - 4\times1\times(bc + ca) = 0$$

$\Rightarrow \quad [(a + b) - c]^2 = 0$

$$a + b - c = 0$$

**Hints:**

- Use, if quadratic equation $ax^2 + bx + c = 0$ has equal roots then $b^2 - 4ac = 0$
- Use $(a + b)^2 + c^2 - 2c(a + b) = [(a + b) - c]^2$

**20. Option (d) is correct.**

**Explanation:**

Given that, α and β are roots of the equation

$$x^2 - 8x + q = 0$$

$\therefore \quad \alpha + \beta = 8$ and $\alpha\cdot\beta = q$

Now, $\quad (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$$= 64 - 4q$$

$$\alpha - \beta = \sqrt{64 - 4q} \qquad (\because \alpha > \beta)$$

Now, $\quad \alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta) = 16$

$$8\left(\sqrt{64 - 4q}\right) = 16$$

$\Rightarrow \quad \sqrt{64 - 4q} = 2$

$\Rightarrow \quad 64 - 4q = 4$

$\Rightarrow \quad q = 15$

**Hints:**

- Use, If α and β are roots of the equation $ax^2 + bx + c = 0$ then $\alpha + \beta = \frac{-b}{a}$ and $\alpha\cdot\beta = \frac{c}{a}$
- Use $(a - b)^2 = (a + b)^2 - 4ab$ and $a^2 - b^2 = (a + b)(a - b)$

**21. Option (c) is correct.**

**Explanation:**

$$30! + 35! = 30! + 35\cdot34\cdot33\cdot32\cdot31\cdot30!$$

$$= 30!(1 + 35\cdot34\cdot33\cdot32\cdot31)$$

$\therefore$ $(1 + 35\cdot34\cdot33\cdot32\cdot31)$ not divisible by 5

$\therefore$ Only 30! Is divisible by $5^n$

So, maximum value of $n$ such that $5^n$ divides 30!

$$= \left[\frac{30}{5}\right] + \left[\frac{30}{5^2}\right] + \left[\frac{30}{5^3}\right]$$

$$= 6 + 1 + 0 = 7$$

Maximum value of $n = 7$

**Hints:**

- If $p$ is prime number then the highest power of $p$ in $n!$ is given by $\left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots.$
- $[x]$ is greatest integer less than equal to $x$ e.g. $[2.5] = [2.99] = 2$

**22. Option (b) is correct.**

**Explanation:**

$2(2\times1) + 3(3\times2\times1) + 4\cdot(4\times3\times2\times1)$

$+ \ldots\ldots + 9(9 \times 8 \times 7\ldots.. \times 1) + 2$

$= 2\cdot2! + 3\cdot3! + 4\cdot4! + \ldots\ldots. + 9.9! + 2$

$= 2\cdot2! + 2! + 3\cdot3! + 4\cdot4! + \ldots. 5\cdot5!$

$= 3\cdot2! + 3\cdot3! + 4\cdot4! + \ldots.9\cdot9!$

$= 3! + 3\cdot3! + 4\cdot4! + \ldots..9\cdot9!$

$= 4\cdot3! + 4\cdot4! + \ldots..9\cdot9!$

$= 4! + 4\cdot4! + \ldots..9\cdot9!$

Same way we solve, then we get

$= 9! + 9\cdot9! = 10\cdot9! = 10!$

**Hints:**

- Use, $n! = n(n-1)!$
- Use $n! + n\cdot n! = (n+1)n! = (n+1)!$

**23. Option (b) is correct.**

**Explanation:**

Given that, $A = \{\{1, 2, 3\}\}$

$\therefore$ $n(A) = 1$

Number of subset $= 2^1 = 2$

So, number of element in power set of A = 2

**Hints:**

- Use, if $n(A) = n$ then number of element in $p(A) = 2^n$. including with $\phi$

**24. Option (d) is correct.**

**Explanation:**

Given that, $a, b, c$ are in G.P

$\therefore$ $b^2 = ac$ …(i)

Squaring both sides, we get

$(b^2)^2 = a^2c^2$

$\Rightarrow a^2, b^2, c^2$ are also in G.P.

So, statement 1 is correct

From (i)

$$\frac{1}{b^2} = \frac{1}{ac} \Rightarrow \left(\frac{1}{b}\right)^2 = \frac{1}{a}\times\frac{1}{c}$$

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in G.P.

So, statement 2 is correct.

from (i)

$b^2 = ac \Rightarrow b = \sqrt{ac}$

$$\Rightarrow \left(\sqrt{b}\right)^2 = \sqrt{a}\sqrt{c}$$

$(\because a > 0, b > 0 > c > 0)$

$\Rightarrow \sqrt{a}, \sqrt{b}, \sqrt{c}$ are also in G.P.

So, statement 3 is correct.

**Hints:**

- Use, if $a, b, c$ are in G.P. then $b^2 = ac$

**25. Option (b) is correct.**

**Explanation:**

We know that if $a, b, c$ are in H.P, then $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

Given that, $\frac{a+b}{2}, b, \frac{b+c}{2}$ are in H.P.

$$\therefore \frac{2}{b} = \frac{2}{a+b} + \frac{2}{b+c} = \frac{2(a+2b+c)}{(a+b)(b+c)}$$

$\Rightarrow ab + ac + b^2 + bc = ab + 2b^2 + ca$

$ac = b^2$

$\therefore a, b, c$ are in G.P.

**Hints:**

- Use, if $a, b, c$ are in H.P then $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$
- Use, if $a, b, c$ are in G.P then $b^2 = ac$

**26. Option (b) is correct.**

**Explanation:**

$\cot^2 15° + \tan^2 15° = \operatorname{cosec}^2 15° - 1 + \sec^2 15° - 1$

$$= \frac{1}{\sin^2 15°} + \frac{1}{\cos^2 15°} - 2$$

$(\because \tan^2\theta = \sec^2\theta - 1, \cot^2\theta = \operatorname{cosec}^2\theta - 1$

$\sec\theta = \frac{1}{\cos\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta})$

$$= \frac{\sin^2 15° + \cos^2 15°}{\frac{1}{4}\left(4\sin^2 15°\cdot\cos^2 15°\right)} - 2$$

$$= \frac{4}{\sin^2 30°} - 2$$

$[\because \sin^2\theta + \cos^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cdot\cos\theta]$

$$= \frac{4}{\frac{1}{4}} - 2 = 16 - 2 = 14$$

**Hints:**

- Use trigonometric identities convert given expression in form of sin θ and cos θ and further solve.

**27. Option (d) is correct.**

**Explanation:**

Given that,

sin A – cos B – cos C = 0

$$\sin A = \cos B + \cos C$$

$$\Rightarrow \quad \sin A = 2\cos\frac{B+C}{2}\cdot\cos\left(\frac{B-C}{2}\right)$$

$$\left[\text{In } \Delta ABC\ A+B+C = \frac{\pi}{2}\right]$$

$$\Rightarrow \quad \frac{1}{2}\sin\frac{A}{2}\cdot\cos\frac{A}{2} = 2\cos\left(\frac{\pi}{2}-\frac{A}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)$$

$$[\because \sin 2\theta = 2\sin\theta\cdot\cos\theta]$$

$$\Rightarrow \quad \frac{1}{2}\sin\frac{A}{2}\cdot\cos\frac{A}{2} = 2\sin\frac{A}{2}\cdot\cos\frac{B-C}{2}$$

$$\Rightarrow \quad \cos\frac{A}{2} = \cos\frac{B-C}{2}$$

$$\Rightarrow \quad \frac{A}{2} = \frac{B}{2}-\frac{C}{2}$$

$$\Rightarrow \quad \frac{A+C}{2} = \frac{B}{2}$$

$$\Rightarrow \quad \frac{\pi}{2}-\frac{B}{2} = \frac{B}{2}$$

$$\Rightarrow \quad B = \frac{\pi}{2}$$

**Hints:**

- Use, $\cos x + \cos y = 2\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2}$
- Use $\sin\theta = 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}$
- Use in ΔABC, $A + B + C = \frac{\pi}{2}$

**28. Option (c) is correct.**

**Explanation:**

Given that, 2 tan α = 1

$$\Rightarrow \quad \tan\alpha = \frac{1}{2}$$

$$\text{and} \quad \alpha+\beta = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan(\alpha+\beta) = \tan\frac{\pi}{4}$$

$$\Rightarrow \quad \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta} = 1$$

$$\left[\because \tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\cdot\tan B}\right]$$

$$\Rightarrow \quad \tan\alpha+\tan\beta = 1-\tan\alpha\cdot\tan\beta$$

$$\Rightarrow \quad \frac{1}{2}+\tan\beta = 1-\frac{\tan\beta}{2}$$

$$\Rightarrow \quad 1+2\tan\beta = 2-\tan\beta$$

$$\Rightarrow \quad \tan\beta = \frac{1}{3}$$

$$\text{Now,} \quad \tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta}$$

$$= \frac{\frac{2}{3}}{1-\frac{1}{9}} = \frac{2}{3}\times\frac{9}{8} = \frac{3}{4}$$

**Hints:**

- Use $\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\cdot\tan B}$
- Use $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

**29. Option (c) is correct.**

**Explanation:**

Given that, tan(45° + θ) = 1 + sin 2θ

$$\Rightarrow \quad \frac{\tan 45°+\tan\theta}{1-\tan 45°\cdot\tan\theta}-1 = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\left[\begin{array}{l}\because \tan(A+B) = \dfrac{\tan A+\tan B}{1-\tan A\cdot\tan B}\\ \sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}\end{array}\right]$$

$$\Rightarrow \quad \frac{1+\tan\theta}{1-\tan\theta}-1 = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\Rightarrow \quad \frac{2\tan\theta}{1-\tan\theta} = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\Rightarrow \quad \tan\theta(1+\tan^2\theta) = \tan\theta(1-\tan\theta)$$

$$\Rightarrow \quad \tan^3\theta+\tan^2\theta = 0$$

$$\Rightarrow \quad \tan^2\theta(\tan\theta+1) = 0$$

$$\Rightarrow \quad \tan\theta = 0 \text{ or } \tan\theta = -1 \text{ (Not possible)}$$

$$\left[\because -\frac{\pi}{4} < \theta < \frac{\pi}{4}\right]$$

$$\Rightarrow \quad \theta = 0°$$

$$\text{Now,} \quad \cos 2\theta = \cos 0° = 1$$

**Hints:**

- Use $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$
- Use $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$
- Simplify and solve the trigonometric equation

**30. Option (c) is correct.**

**Explanation:**

Given that $\sin 2\theta = \cos 3\theta$

$\Rightarrow \quad \sin 2\theta = \sin(90° - 3\theta)$

$\Rightarrow \quad 2\theta = 90° - 3\theta$

$\Rightarrow \quad \theta = 18°$

Now, $1 + 4\sin\theta = 1 + 4\sin 18°$

$$= 1 + 4\cdot\frac{\sqrt{5}-1}{4} \quad \left[\because \sin 18° = \frac{\sqrt{5}-1}{4}\right]$$

$$= 1 + \sqrt{5} - 1$$

$$= \sqrt{5}$$

**Hints:**

- Use $\cos\theta = \sin(90° - \theta)$
- Solve angle $\theta$
- Use $\sin 18° = \dfrac{\sqrt{5}-1}{4}$

**31. Option (c) is correct.**

**Explanation:**

Given that,

$$\tan\theta = -\frac{5}{12} = \frac{p}{b}$$

Let, $p = 5k$ and $b = 12k$

$$H = \sqrt{p^2 + b^2} = \sqrt{25k^2 + 144k^2} = 13k$$

Since value of $\tan\theta$ is –ve that represent $\theta$ lies in $2^{nd}$ and $4^{th}$ quadrant.

$$\therefore \quad \sin\theta = \pm\frac{p}{h} = \pm\frac{5}{13}$$

**Hints:**

- Use $\tan\theta = \dfrac{p}{b}$ and $\sin\theta = \dfrac{p}{h}$
- Use $h^2 = p^2 + b^2$
- Value of $\tan\theta$ in $2^{nd}$ and $4^{th}$ quadrant is negative.

**32. Option (b) is correct.**

**Explanation:**

$$\cos^4\frac{7\pi}{8} + \cos^4\frac{5\pi}{8} = \left(\cos^2\frac{7\pi}{8}\right)^2 + \left(\cos^2\frac{5\pi}{8}\right)^2$$

$$= \left(\cos^2\frac{7\pi}{8} - \cos^2\frac{5\pi}{8}\right)^2 + 2\cos^2\frac{7\pi}{8}\cdot\cos^2\frac{5\pi}{8}$$

$$= \left[-\sin\left(\frac{7\pi}{8}+\frac{5\pi}{8}\right)\cdot\sin\left(\frac{7\pi}{8}-\frac{5\pi}{8}\right)\right]^2 + \frac{1}{2}\left[2\cos\frac{7\pi}{8}\cdot\cos\frac{5\pi}{8}\right]^2$$

$$= \left[-\sin\frac{3\pi}{2}\cdot\sin\frac{\pi}{4}\right]^2 + \frac{1}{2}\left[\cos\left(\frac{7\pi}{8}+\frac{5\pi}{8}\right)+\cos\left(\frac{7\pi}{8}-\frac{5\pi}{8}\right)\right]^2$$

$$= \left[-(-1)\cdot\frac{1}{\sqrt{2}}\right]^2 + \frac{1}{2}\left[\cos\frac{3\pi}{2}+\cos\frac{\pi}{4}\right]^2$$

$$= \frac{1}{2} + \frac{1}{2}\left[0 + \frac{1}{\sqrt{2}}\right]^2$$

$$= \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

**Hints:**

- Use $\cos^2 x - \cos^2 y = (-1)\sin(x+y)\cdot\sin(x-y)$
- Use $2\cos x\cdot\cos y = \cos(x+y) + \cos(x-y)$
- Use $a^2 + b^2 = (a-b)^2 + 2ab$

**33. Option (a) is correct.**

**Explanation:**

$$\sin^2\left(\frac{\pi}{4}+\theta\right) - \sin^2\left(\frac{\pi}{4}-\theta\right)$$

$$= \left[\sin\left(\frac{\pi}{4}+\theta\right)+\sin\left(\frac{\pi}{4}-\theta\right)\right]\left[\sin\left(\frac{\pi}{4}+\theta\right)-\sin\left(\frac{\pi}{4}-\theta\right)\right]$$

$$= 2\sin\frac{\left(\frac{\pi}{4}+\theta+\frac{\pi}{4}-\theta\right)}{2}\cdot\cos\frac{\left(\frac{\pi}{4}+\theta-\frac{\pi}{4}+\theta\right)}{2}$$

$$\cdot 2\cos\frac{\left(\frac{\pi}{4}+\theta+\frac{\pi}{4}-\theta\right)}{2}\times\sin\frac{\left(\frac{\pi}{4}+\theta-\frac{\pi}{4}+\theta\right)}{2}$$

$$= 2\sin\frac{\pi}{4}\cdot\cos\theta\cdot 2\cos\frac{\pi}{4}\cdot\sin\theta$$

$$= 2\sin\frac{\pi}{4}\cdot\cos\frac{\pi}{4}\cdot 2\sin\theta\cdot\cos\theta$$

$$= \sin\frac{\pi}{2}\cdot\sin 2\theta = \sin 2\theta$$

**Shortcut:**

We, know that

$\sin^2 x - \sin^2 y = \sin(x+y)\cdot\sin(x-y)$

$$\therefore\ \sin^2\left(\frac{\pi}{4}+\theta\right)-\sin^2\left(\frac{\pi}{4}-\theta\right)$$

$$= \sin\left[\frac{\pi}{4}+\theta+\frac{\pi}{4}-\theta\right]\cdot\sin\left[\frac{\pi}{4}+\theta-\frac{\pi}{4}+\theta\right]$$

$$= \sin\frac{\pi}{2}\cdot\sin 2\theta = \sin 2\theta$$

**Hints:**

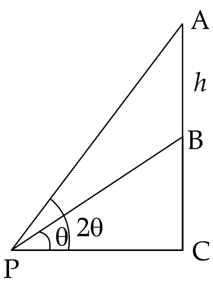
- Use $a^2 - b^2 = (a+b)(a+b)$
- Use $\sin A + \sin B = 2\sin\frac{A+B}{2}\cdot\cos\frac{A-B}{2}$

  and $\sin A - \sin B = 2\cos\frac{A+B}{2}\cdot\sin\frac{A-B}{2}$
- Use $\sin 2\theta = 2\sin\theta\cdot\cos\theta$

**34. Option (c) is correct.**

**Explanation:**

Let, BC is vertical tower and AB is flag staff of height $h$.




In $\Delta$PBC

$$\tan\theta = \frac{BC}{PC}$$

$$\Rightarrow\quad PC = BC\cot\theta$$

In $\Delta$APC

$$\tan 2\theta = \frac{BC+h}{PC}$$

$$\Rightarrow\quad \tan 2\theta\cdot PC = BC + h$$

$$\Rightarrow\quad \frac{2\tan\theta}{1-\tan^2\theta}\cdot\cot\theta\, BC = BC + h$$

$$\Rightarrow\quad \frac{2}{1-\tan^2\theta}\,BC - BC = h$$

$$[\because\ \tan\theta\cdot\cot\theta = 1]$$

$$\Rightarrow\quad \frac{1+\tan^2\theta}{1-\tan^2\theta}\,BC = h$$

$$\Rightarrow\quad BC = \frac{1-\tan^2\theta}{1+\tan^2\theta}\cdot h$$

$$\Rightarrow\quad BC = h\cos 2\theta$$

$$\left[\because \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}\right]$$

**Hints:**

- Draw diagram according to given condition
- Use $\tan\theta\cdot\cot\theta = N$
- Use $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

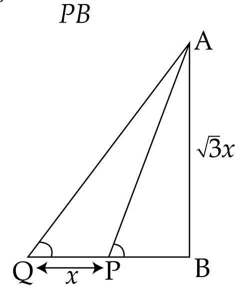
**35. Option (b) is correct.**

**Explanation:**

Let, AB be $a$ tower of height $\sqrt{3}\,x$ metre.

In $\Delta$ABP

$$\tan 60^\circ = \frac{\sqrt{3}\,x}{PB}$$




$$\Rightarrow\quad \sqrt{3} = \frac{\sqrt{3}\,x}{PB}$$

$\Rightarrow \quad PB = x$

In $\Delta ABQ$

$$\tan\theta = \frac{AB}{BQ} = \frac{\sqrt{3}\,x}{x+x} = \frac{\sqrt{3}\,x}{2x} = \frac{\sqrt{3}}{2}$$

$\because \frac{1}{\sqrt{3}} < \frac{\sqrt{3}}{2} < 1$

$\therefore 30° < \theta < 45°$

**Hints:**

- Draw diagram according to given condition.
- Use $\tan\theta = \frac{P}{B}$

**36. Option (a) is correct.**

**Explanation:**

Given that,

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{x}{3}\right) = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{x}{3}\right) = \frac{\pi}{4} - \tan^{-1}\frac{1}{2}$$

$$= \tan^{-1}1 - \tan^{-1}\frac{1}{2}$$

$$\left[\because \tan^{-1}1 = \frac{\pi}{4}\right]$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{x}{3}\right) = \tan^{-1}\left(\frac{1-\frac{1}{2}}{1+1-\frac{1}{2}}\right)$$

$$\Rightarrow \quad \frac{x}{3} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$$

$$\Rightarrow \quad x = 1$$

**Hints:**

- Write $\tan^{-1}1$ in place of $\frac{\pi}{4}$
- Use $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

**37. Option (c) is correct.**

**Explanation:**

Given that,

$$3\sin^{-1}x + \cos^{-1}x = \pi$$

$$\Rightarrow 2\sin^{-1}x + \sin^{-1}x + \cos^{-1}x = \pi$$

$$\Rightarrow \quad 2\sin^{-1}x = \pi - \frac{\pi}{2}$$

$$\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

$$\Rightarrow \quad 2\sin^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow \quad \sin^{-1}x = \frac{\pi}{4}$$

$$\Rightarrow \quad x = \sin — = \frac{1}{\sqrt{2}}$$

**Hints:**

- Use $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$
- And solve $x$

**38. Option (b) is correct.**

**Explanation:**

Given that,

$$\tan\alpha + \tan\beta = 1 - \tan\alpha\cdot\tan\beta$$

$$\Rightarrow \quad \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\cdot\tan\beta} = 1$$

$$[\because \tan\alpha\cdot\tan\beta \neq 1]$$

$$\Rightarrow \quad \tan(\alpha+\beta) = 1$$

$$\left[\because \tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta}\right]$$

$$\Rightarrow \quad \alpha+\beta = \tan^{-1}1 = \frac{\pi}{4}$$

**Hints:**

- Use $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta}$
- Use $\tan^{-1}1 = \frac{\pi}{4}$

**39. Option (b) is correct.**

**Explanation:**

Given that,

$$(1+\tan\theta)(1+\tan 9\theta) = 2$$

$$\Rightarrow \quad 1 + \tan 9\theta + \tan\theta + \tan\theta\cdot\tan 9\theta = 2$$

$$\Rightarrow \quad \tan 9\theta + \tan\theta = 1 - \tan\theta\cdot\tan 9\theta$$

$$\Rightarrow \quad \frac{\tan 9\theta + \tan\theta}{1-\tan\theta\cdot\tan 9\theta} = 1$$

$$\Rightarrow \quad \tan(9\theta+\theta) = 1$$

$$\Rightarrow \quad \tan 10\theta = 1$$

**Hints:**

- Expand the given equation and change to form $\frac{\tan 9\theta + \tan\theta}{1 - \tan\theta \cdot \tan 9\theta}$
- Use $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

**40. Option (b) is correct.**

**Explanation:**

$\sin 0° + \sin 10° + \sin 20° + \sin 30° + \ldots + \sin 360°$

$= (\sin 0° + \sin 360°) + (\sin(10° + \sin 350°)$
$+ (\sin 20° + \sin 340°)) + \ldots\ldots + \sin 180°$
$= 2\sin 180° \cdot \cos 180° + 2\sin 180° \cdot \cos 170° +$
$2\sin 180° \cdot \cos 160° + \ldots + \sin 180°$
$= 0 + 0 + 0 + \ldots + 0 = 0$

$[\because \sin 180° = 0]$

**Hints:**

- Make pair whose sum of angle is 360°
- Use $\sin A + \sin B = 2\sin\frac{A+B}{2} \cdot \cos\frac{A-B}{2}$

**41. Option (c) is correct.**

**Explanation:**

Given set A = {1, 2, 3, 4}

Super sets of the set {4} are {4}, {4,3}, {4,2}, {4,1}, {4, 3, 2}, {4, 3, 1}, {4, 2, 1}, {4, 3, 2,1}

Total number of supersets = 8

**Shortcut:**

No. of element except 4 {1, 2, 3} is 3.

$\therefore$ Number of super sets of {4} = $2^3 = 8$

**Hints:**

- Number of subsets of set contain $n$ element is $2^n$.
- If each element of set A belongs to set B then B is called super set of set A.

**42. Option (d) is correct.**

**Explanation:**

If $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \notin A$ and $x \notin B$

So, statement 1 is not correct.

Let, A = {1, 2} and B = {2, 3}

$\therefore$ $A \cap B = \{2\}$

$\because$ $1 \notin A \cap B$ but $1 \in A$

$3 \notin A \cap B$ but $3 \in B$

So, statement 2 is not correct.

**Hint:**

- Use properties of union and intersection

**43. Option (c) is correct.**

**Explanation:**

Let $x \in A$

$\Rightarrow x \in A \cup B$

$\Rightarrow x \in A \cap B$

$[\because A \cup B = A \cap B]$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow A \subseteq B$ …(i)

Let, $y \in B$

$\Rightarrow y \in A \cup B$

$\Rightarrow y \in A \cap B$

$[\because A \cup B = A \cap B]$

$\Rightarrow y \in A$ and $y \in B$

$\Rightarrow B \subseteq A$ …(ii)

From, (i) and (ii) A = B

So, statement 1 is correct.

Now, $A \Delta B = \phi = (A - B) \cup (B - A)$

$\Rightarrow A - B = \phi$ and $B - A = \phi$

$\Rightarrow A = B$ Converse is also true.

So, statement 2 is correct.

**Hints:**

- Use $x \in A \cup B \Rightarrow x \in A$ or $x \in B$ and $x \in A \cap B \Rightarrow x \in A$ and $x \in B$
- Use $A \Delta B = (A - B) \cup (B - A)$

**44. Option (a) is correct.**

**Explanation:**

For reflexive

Put, $y = x$

$x^2 - 5x^2 + 4x^2 = 0$

$\therefore$ $x R x$

$\therefore$ R is reflexive.

So, statement 1 is correct.

For symmetric

Let, $x R y$

$$\Rightarrow \quad x^2 - 4xy + 4y^2 = 0$$

$$\Rightarrow \quad x = y \text{ or } x = 4y$$

$$2f\ x \neq y \text{ then } y \neq 4x$$

$$\Rightarrow \quad (y, x) \notin R$$

$\therefore$ R is not symmetric.

So, statement 2 is not correct.

For transitive

Let, $\quad x\,R\,y \Rightarrow x = y$ or $x = 4y$ ...(i)

and $\quad y\,R\,z \Rightarrow y = z$ or $y = 4z$ ...(ii)

from, (i) and (ii)

If $\quad x = 4y$ and $y = 42$

Then, $\quad x = 162$

$\therefore \quad (x, 2) \notin R$

$\therefore$ R is not transitive.

So, statement 3 is not correct.

**Hints:**

- If $x\,R\,x$. then R is reflexive.
- If $x\,R\,y$ then $y\,R\,x$, so R is symmetric.
- If $x\,R\,y$ and $y\,R\,z$ then $x\,R\,z$, so R is transitive.

**45. Option (d) is correct.**

**Explanation:**

We, know that If $(x, y) \in R \Leftrightarrow (y, x) \in R^{-1}$

Since, R is reflexive.

$\therefore (x, x) \in R \Rightarrow (x, x) \in R^{-1}$

So, $R^{-1}$ is reflexive.

Since, R is symmetric.

So, If $(x, y) \in R \Rightarrow (y, x) \in R$

$\Rightarrow$ If $(y, x) \in R^{-1}$ then $(x, y) \in R^{-1}$

So, $R^{-1}$ is symmetric.

Since, R is transitive.

If $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$

$\Rightarrow (y, x) \in R^{-1}, (z, y) \in R^{-1}$ and $(z, x) \in R^{-1}$

So, $R^{-1}$ is transitive.

**Hints:**

- If $(x, y) \in R \Leftrightarrow (y, x) \in R^{-1}$
- If $(x, x) \in$R then R is reflexive.
- If $(x, y) \in R$ and $(y, x) \in R$, then R is symmetric.
- If $(x, y) \in R$, $(y, z) \in R$ and $(x, z) \in R$, then R is transitive.

**46. Option (b) is correct.**

**Explanation:**

$$\frac{1}{1+i} \times \frac{1-i}{1-i} = \frac{1-i}{1-i^2}$$

$$= \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i$$

$$\tan\theta = \left|\frac{\frac{1}{2}}{-\frac{1}{2}}\right| = 1 = \tan\frac{\pi}{4}$$

$$\theta = \frac{\pi}{4}$$

Since, $x$ is positive and $y$ is negative.

So, principal argument $= -\dfrac{\pi}{4}$

**Hints:**

- Ratinalization denominator
- If $z$ lies in 3rd quadrant then principal argument is $-\theta$.

**47. Option (c) is correct.**

**Explanation:**

$$\left(\frac{\sqrt{-3}}{2} - \frac{1}{2}\right)^{200} = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{200}$$

$$\left[\because \sqrt{-1} = i\right]$$

$$\left|\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{200}\right| = \left|-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right|^{200} = \left(\sqrt{\frac{1}{4} + \frac{3}{4}}\right)^{200}$$

$$= 1^{200} = 1$$

**Hints:**

- If $z = x + iy$ then $|z| = \sqrt{x^2 + y^2}$
- Use $|z^n| = |z|^n$

**48. Option (d) is correct.**

**Explanation:**

Statement 1

Put $n = 4$

$$\therefore \quad \frac{n!}{3!} = \frac{4!}{3!} = 4 \text{ is not divisible by } 6$$

So, statement 1 is not correct.

Statement 2

Put $n = 5$

$\frac{n!}{3!} + 3 = \frac{5!}{3!} + 3 = 5.4 + 3 = 23$ is not divisibile by

So, statement 2 is not correct.

**Hints:**

- Put $n = 4, 5 \ldots.$ and check validity
- Use $n! = n(n-1)!$

**49. Option (b) is correct.**

**Explanation:**

Number of way of selecting 5 players out of 9 players as to exclude two particular players

$$= {}^{7}C_5 = \frac{7!}{5!2!} = 21$$

**Hints:**

- Number of ways of selecting $r$ things out of $n$ things such that $k$ things excluded $(r > k) = {}^{n-k}C_r$

**50. Option (c) is correct.**

**Explanation:**

$(n+1)^{th}$ term of $\left(x + \frac{1}{x}\right)^{2n}$ from end = $(n+1)^{th}$ term of $\left(\frac{1}{x} + x\right)^{2n}$ from beginning

$\therefore$ $(n+1)^{th}$ term of $\left(\frac{1}{x} + x\right)^{2n}$

$$T_{n+1} = {}^{2n}C_n \left(\frac{1}{x}\right)^n x^n = {}^{2n}C_n$$

**Hints:**

- $k^{th}$ term of $(a+b)^n$ from end = $k^{th}$ term of $(b+a)^n$ from beginning
- Use $T_{r+1} = {}^{n}C_r\ a^r\ b^{n-r}$

**51. Option (d) is correct.**

**Explanation:**

Given that,

$$S_9 = S_{11}$$

$$\Rightarrow \quad \frac{9}{2}(2a + 8d) = \frac{11}{2}(2a + 10d)$$

$$\Rightarrow \quad 18a + 72d = 22a + 110d$$

$$\Rightarrow \quad 4a + 38d = 0$$

$$\Rightarrow \quad 2a + 19d = 0 \quad \ldots(i)$$

$$\therefore \quad S_{20} = \frac{20}{2}(2a + 19d) = 0$$

**Hint:**

- Use $S_n = \frac{n}{2}[2a + (n+1)d]$

**52. Option (b) is correct.**

**Explanation:**

According to question

$$a_5 = \frac{1}{10}$$

$$\Rightarrow \quad a + 4d = \frac{1}{10} \quad \ldots(i)$$

$$a_{10} = \frac{1}{5}$$

$$\Rightarrow \quad a + 9d = \frac{1}{5} \quad \ldots(ii)$$

from (i) and (ii)

$$a + 9d = \frac{1}{5}$$

$$a + 4d = \frac{1}{10}$$

$$-\quad -\qquad -$$

$$5d = \frac{2-1}{10} = \frac{1}{10}$$

$$d = \frac{1}{50}$$

$$\therefore \quad a = \frac{1}{10} - \frac{4}{50} = \frac{5-4}{50} = \frac{1}{50}$$

$$\therefore \quad S_{50} = \frac{50}{2}\left[\frac{2}{50} + \frac{49}{50}\right]$$

$$= \frac{50}{2} \times \frac{51}{50} = 25.5$$

**53. Option (a) is correct.**

**Explanation:**

$$(1110011)_2 = 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 2^0$$

$$= 64 + 32 + 16 + 2 + 1 = 115$$

$$(10111)_2 = 1 \times 2^4 + 0 + 2^3 + 1 \times 2^2 + 1 \times 2^1 + 2^0$$

$$= 16 + 4 + 2 + 1 = 23$$

$\therefore (1110011)_2 \div (10111)_2 = 115 \div 23 = 5$

Now, $(101)_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$$= 4 + 1 = 5$$

**Hints:**

- Convert $13_{10}$ to binary

| Division by 2 | Quotient | Remainder |
|---|---|---|
| $\frac{13}{2}$ | 6 | 1 |
| $\frac{6}{2}$ | 3 | 0 |
| $\frac{3}{2}$ | 1 | 1 |
| $\frac{1}{2}$ | 0 | 1 |

So, $13_{10} = (1101)$

**54. Option (b) is correct.**

**Explanation:**

$$x^3 + y^3 = (100010111)_2$$
$$= 1 \times 2^8 + 1 \times 2^4 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$
$$= 256 + 16 + 4 + 2 + 1 = 279$$
$$\Rightarrow x^3 + y^3 = 279$$
$$x + y = (11111)_2 = 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$
$$= 16 + 8 + 4 + 2 + 1 = 31$$
$$\text{Now,} (x - y)^2 + xy = x^2 + y^2 - 2xy + xy$$
$$= x^2 + y^2 - xy$$
$$= \frac{(x+y)(x^2 - xy + y^2)}{x+y} = \frac{x^3 + y^3}{x+y}$$
$$= \frac{279}{31} = 9$$

| Division by 2 | Quotient | Remainder |
|---|---|---|
| $\frac{9}{2}$ | 4 | 1 |
| $\frac{4}{2}$ | 2 | 0 |
| $\frac{2}{2}$ | 1 | 0 |
| $\frac{1}{2}$ | 0 | 1 |

$\therefore \quad 9_2 = 1001$

So, $(x - y)^2 + xy = (1001)_2$

**Hint:**

- Use $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- Convert binary to decimal and answer convert decimal to binary

**55. Option (d) is correct.**

**Explanation:**

Since $x < 0$ and $y > 0$

**(a)** $\therefore$ (0, 0) not lies in the common region

**(b)** $-2 + 4 = 2$, So, $(-2, 4)$ not lies in the common region

**(c)** $-1 + 4 > 2$, So, $(-1, 4)$ not lies in the common region

**(d)** $-1 + 2 < 2$, and $5(-1) - 4(2) = -13 < 0$

So, $(-1, 2)$ lies in the common region

**Hints:**

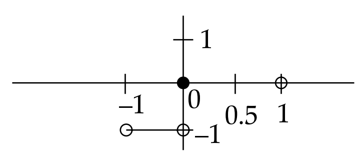
- Substitute all given option in all given in equations and check validity

**56. Option (a) is correct.**

**Explanation:**

$$y = \{x\}, x \in (-1, 1)$$

$$\therefore \quad f(x) = \begin{cases} -1 & x \in (-1, 1) \\ 0 & x \in [0, 1) \end{cases}$$




It is clear from figure that $f(x)$ is differentiable at $x = 0.5$ and $f'(x) = 0$

But discontinuous at $x = 0$

**Hints:**

- Use $[x] = -1, x \in (-1, 0)$ and $[x] = 0, x \in [0, 1)$
- If L.H.L = R.H.L = $f(a)$ then $f(x)$ is continuous at $n = a$
- If L.H.D = R.H.D then $f'(x)$ is exists.

**57. Option (d) is correct.**

**Explanation:**

$$1+\left(\frac{dy}{dx}\right)^2 = \left(\frac{d^2y}{dx^2}\right)^{\frac{4}{3}}$$

$$\Rightarrow \quad \left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^4$$

$\therefore$ Degree = 4

**Hints:**

- Degree of differential equation is the power of highest order derivative when differential equation in the form of polynomial.

**58. Option (a) is correct.**

**Explanation:**

Given that

$$\frac{dp}{dt}\alpha - p$$

$$\Rightarrow \quad \frac{dp}{dt} = -kp$$

(where $p$ is radio active substance initially)

$$\Rightarrow \quad \int \frac{dp}{p} = -k\int dt$$

$$\Rightarrow \quad \ln p = -kt + c$$

When, $t = 0$ $c = \ln p$

Put, $t = 100$ and substance $\frac{p}{2}$

$$\therefore \quad \ln\left(\frac{p}{2}\right) = -100\,k + \ln p$$

$$\Rightarrow \ln p - \ln 2 = -100k + \ln p$$

$$\Rightarrow \quad k = \frac{\ln 2}{100}$$

**Hints:**

- Use $\frac{dp}{dt}$ = rate of change of radio active substance
- Take negative sign for decays
- Use variable separable to solve it

**59. Option (d) is correct.**

**Explanation:**

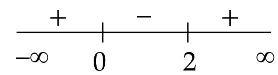
$$f(x) = \sqrt{1-(x-1)^2}$$

$$\therefore \quad 1-(x-1)^2 \geq 0$$

$$\Rightarrow \quad 0 - x^2 + 2x \geq 0$$

$$\Rightarrow \quad x^2 - 2x \leq 0$$

$$\Rightarrow \quad x(x-2) \leq 0$$




$\therefore$ Domain = [0, 2]

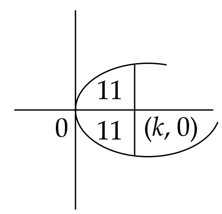
**Hints:**

- Use $y = \sqrt{f(x)}$ is define when $f(x) \geq 0$
- To solve in equation use wavy curve method

**60. Option (c) is correct.**

**Explanation:**

$$\text{Required area} = 2.2\sqrt{k}\int_0^k \sqrt{x}\,dx = 24$$




$$\Rightarrow \quad \sqrt{k}\cdot\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^k = 6$$

$$\Rightarrow \quad k^{\frac{1}{2}}\cdot k^{\frac{3}{2}} = 6\times\frac{3}{2} = 9$$

$$\Rightarrow \quad k^2 = 9$$

$$\Rightarrow \quad k = 3 \qquad [\because k > 0]$$

**Hints:**

- Focus of parabola $y^2 = 4kx$ is $(k, 0)$
- Equation of latus rectum is $x = k$

**61. Option (b) is correct.**

**Explanation:**

$$\int_0^{\frac{\pi}{4}} \frac{dx}{(\sin x+\cos x)^2} = \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{dx}{\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)^2}$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2\left(\frac{\pi}{4}-x\right)}$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \sec^2\left(\frac{\pi}{4} - x\right) dx$$

$$= -\frac{1}{2}\left[\tan\left(\frac{\pi}{4} - x\right)\right]_0^{\frac{\pi}{4}}$$

$$= -\frac{1}{2}[0 - 1] = \frac{1}{2}$$

**Hints:**

- Use

$$a\sin x + b\cos x = \sqrt{a^2 + b^2}\left[\frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x\right]$$

**62. Option (b) is correct.**

**Explanation:**

$$\mathrm{I} = \int (\sin x)^{-\frac{1}{2}} (\cos x)^{\frac{-3}{2}}\, dx$$

$$= \int \frac{1}{\sqrt{\sin x \cdot \cos x} \cdot \cos x} dx$$

$$= \int \frac{1}{\cos x \sqrt{\frac{\sin x}{\cos^2 x} \cdot \cos x \cdot \cos x}} dx$$

$$= \int \frac{\sec^2 x}{\sqrt{\tan x}} dx$$

Let, $\sqrt{\tan x} = t$

$$\Rightarrow \frac{1}{2\sqrt{\tan x}} \cdot \sec^2 x\, dx = dt$$

$$\Rightarrow \frac{\sec^2 x}{\sqrt{\tan x}} dx = 2dt$$

$$\mathrm{I} = 2\int dt = 2t + c = 2\sqrt{\tan x} + c$$

**Hints:**

- Make positive power
- Convert given function in the form of $\tan x$ and $\sec x$.
- By substitution solve it

**63. Option (b) is correct.**

**Explanation:**

$$\mathrm{I}_1 = \int \frac{e^x}{e^x + e^{-x}} dx$$

$$= \int \frac{e^x}{e^x + \frac{1}{e^x}} dx = \int \frac{e^{2x}}{e^{2x}+1} dx$$

$$\text{Now, } \mathrm{I}_1 + \mathrm{I}_2 = \int \left(\frac{e^{2x}}{e^{2x}+1} + \frac{1}{e^{2x}+1}\right) dx$$

$$= \int \frac{e^{2x}+1}{e^{2x}+1} dx$$

$$= \int 1\, dx = x + c$$

**Hints:**

- Simplify function in $\mathrm{I}_1$ and add $\mathrm{I}_1 + \mathrm{I}_2$.

**64. Option (b) is correct.**

**Explanation:**

$$\because \quad |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\therefore \quad \mathrm{I} = \int_{-2}^{-1} \frac{x}{|x|} dx = \int_{-2}^{-1} \frac{x}{-x} dx$$

$$= \int_{-2}^{-1} (-1)\, dx$$

$$= (-1)[x]_{-2}^{-1}$$

$$= (-1)\,[-1 + 2] = -1$$

**Hints:**

- $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

**65. Option (b) is correct.**

**Explanation:**

Let $f(x) = \sin 4x + 2x$

$f'(x) = 4\cos 4x + 2 = 0$

$$\cos 4x = -\frac{1}{2}$$

$$\left[\begin{array}{l} \because 0 < x < \frac{\pi}{2} \\ \Rightarrow 0 < 4x < 2\pi \end{array}\right]$$

$$\therefore \quad 4x = \pi - \frac{\pi}{3} \text{ or } 4x = \pi + \frac{\pi}{3}$$

$$x = \frac{\pi}{6} \text{ or } \frac{\pi}{3}$$

**Hints:**

- For extreme value $f'(x) = 0$

**66. Option (b) is correct.**

**Explanation:**

$$f(x) = \frac{1}{\tan x + \cot x} = \frac{1}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$$

$$= \frac{\sin x \cos x}{\sin^2 x + \cos^2 x} = \sin x \cdot \cos x$$

$$= \frac{1}{2} \sin 2x$$

We know that

$$0 \le \sin 2x \le 1$$

$$\Rightarrow \quad \le \frac{1}{2} \sin 2x \le \frac{1}{2}$$

$$\left[\because 0 < x < \frac{\pi}{2}\right]$$

$$\Rightarrow \quad 0 \le f(x) \le \frac{1}{2}$$

$\therefore$ Maximum value $= \frac{1}{2}$

**Hints:**

- Use $-1 \le \sin \theta \le 1$

**67. Option (d) is correct.**

**Explanation:**

Given that,

$$4f(x) - f\left(\frac{1}{x}\right) = \left(2x + \frac{1}{x}\right)\left(2x - \frac{1}{x}\right)$$

$$= 4x^2 - \frac{1}{x^2} \qquad \ldots\text{(i)}$$

Replace $x$ by $\frac{1}{x}$

$$4f\left(\frac{1}{x}\right) - f(x) = \frac{4}{x^2} - x^2 \qquad \ldots\text{(ii)}$$

from 4(i) + (ii)

$$16f(x) - 4f\left(\frac{1}{x}\right) = 16x^2 - \frac{4}{x^2}$$

$$4f\left(\frac{1}{x}\right) - f(x) = \frac{4}{x^2} - x^2$$

$$\overline{15f(x) = 15x^2}$$

$$\Rightarrow \quad f(x) = x^2$$

$$f(2) = (2)^2 = 4$$

**Hints:**

- Replace $x$ by $\frac{1}{x}$ and further solve it
- Find $f(x)$

**68. Option (a) is correct.**

**Explanation:**

Given that, $f(x) = 4x + 3$

$$\therefore \quad fofof(-1) = f(f(f(-1)))$$

$$= f(f(4(-1) + 3)) = f(f(-1))$$

$$= f(-4 + 3) = f(-1) = -4 + 3 = -1$$

**Hints:**

- Use $fofof(x) = f(f(f(x)))$

**69. Option (a) is correct.**

**Explanation:**

Given that, $\quad x^y y^x = 1$

Taking log both side

$$\ln(x^y y^x) = \ln 1$$

$$\ln x^y + \ln y^x = 0$$

$$y \ln x + x \ln y = 0$$

Differentiate w.r.t or $x$

$$\frac{dy}{dx} \ln x + \frac{y}{x} + 1.\ln y + \frac{x}{y}\frac{dy}{dx} = 0$$

Put, $x = 1$ and $y = 1$, we get

$$\frac{dy}{dx} \ln 1 + 1 + \ln 1 + \frac{dy}{dx} = 0$$

$$0 + 1 + 0 + \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -1$$

**Hints:**

- Taking log both sides
- Use logrithumic properties

$$\log(m \cdot n) = \log m + \log n$$

$$\log m^n = n \log m$$

**70. Option (b) is correct.**

**Explanation:**

$$y = \left(x^x\right)^x = x^{x^2}$$

Taking log both side

$$\ln y = x^2 \ln x$$

Differentiate w.r.t or $x$

$$\frac{1}{y}\frac{dy}{dx} = 2x \ln x + x^2 \cdot \frac{1}{x} \qquad \ldots(i)$$

When $x = 1, y = 1$

$$\therefore \quad 1\frac{dy}{dx} = 2\ln 1 + 1$$

$$\frac{dy}{dx} = 0 + 1 = 1$$

**Hints:**

- Use $(x^m)^n = x^{mn}$
- Taking log both sides
- Use properties of logritham
  $\log m^n = n\log m$

**71. Option (d) is correct.**

**Explanation:**

$\because \quad -4 < x < -3$

$\Rightarrow \quad -3 < x + 1 < -2$

So, $\quad y = [x + 1] = -3$

$$\frac{dy}{dx} = 0$$

**Hints:**

- If $-4 < x < -3$ then $-3 < x + 1 < -2$

**72. Option (d) is correct.**

**Explanation:**

$$\frac{dy}{dx} = (\ln 5)y$$

$$\int \frac{1}{y}dy = (\ln 5)\int dx$$

$$\ln y = (\ln 5)x + c$$

put $x = 0, \; y = \ln 5$

$\ln(\ln 5) = c$

$\therefore \quad \ln y = (\ln 5)x + \ln(\ln 5)$

Put $x = 1$

$$\ln y = \ln 5 + \ln(\ln 5)$$

$$\ln y = \ln[5 \cdot \ln 5]$$

$$y = 5\ln 5$$

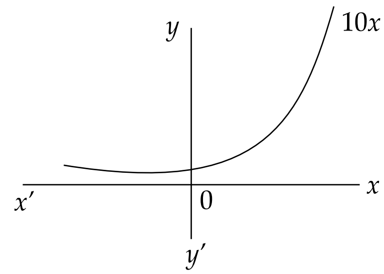
**Hints:**

- Solve differential equation by variable separable method

**73. Option (d) is correct.**

**Explanation:**

It is clear from graph that domain of $f(x)$ is $(-\infty, \infty)$.




It is continuous function unique tangent can be drawn at $x = 0$.

**Hints:**

- Draw the graph of $10^x$ and check statement.

**74. Option (a) is correct.**

**Explanation:**

$$\lim_{n \to 0} x^3 (\text{cosec } x)^2$$

$$= \lim_{n \to 0} x \cdot \frac{x^2}{\sin^2 x} = 0.1 = 0$$

$$\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 = 1$$

**Hints:**

- Use $\because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = \lim_{\theta \to 0} \frac{\theta}{\sin \theta} = 1$

**75. Option (c) is correct.**

**Explanation:**

$$\lim_{x \to 1} \frac{x^3 - 1}{\sqrt{x} - 1} = \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{\sqrt{x}-1} \times \frac{\sqrt{x}+1}{\sqrt{x}+1}$$

$$= \lim_{x \to 1} \frac{(x-1)(x^2+x+1)(\sqrt{x}+1)}{x-1}$$

$$= (1 + 1 + 1)\left(\sqrt{1} + 1\right) = 6$$

**Hints:**

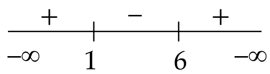
- Rationalize the denominator
- Factorize numerator by using formula
  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

**76. Option (b) is correct.**

**Explanation:**

$$f(x) = \frac{x^3}{3} - \frac{7x^2}{2} + 6x + 5$$

$$f'(x) = x^2 - 7x + 6 = 0$$
$$x^2 - 6x - x + 6 = 0$$
$$x(x-6) - 1(x-6) = 0$$
$$(x-6)(x-1) = 0$$
$$x = 1 \text{ or } x = 6$$




$\therefore f(x)$ is decreasing on (1, 6).

**Hints:**

- Equate $f'(x) = 0$ and find critical point
- If $f'(x) > 0$ in interval $(a, b)$ then increasing
- If $f'(x) < 0$ in interval $(a, b)$ then decreasing.

**77. Option (d) is correct.**

**Explanation:**

Given that, $f'(2) = 0$

$$f(x) = \frac{m}{x} + 2nx + 1$$
$$f'(x) = -\frac{m}{x^2} + 2n$$
$$f'(2) = -\frac{m}{4} + 2n = 0$$
$$8n - m = 0$$

Cannot be determined value of $m + 8n$ due to insufficient data.

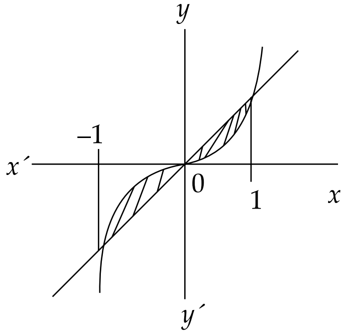
**Hints:**

- $f(x)$ is vanish at $x = 2$ i.e., $f'(2) = 0$

**78. Option (b) is correct.**

**Explanation:**

$$y = x \quad \text{...(i)}$$
$$y = x^3 \quad \text{...(ii)}$$




Solving equation (i) and (ii)

We get $x = 0, -1, 1$

Required area

$$= \left[\int_0^1 x\,dx - \int_0^1 x^3\,dx\right]$$
$$= \left[\left(\frac{x^2}{2}\right)_0^1 - \left[\frac{x^4}{4}\right]_0^1\right]$$
$$= \left[\frac{1}{2} - \frac{1}{4}\right] = \frac{1}{4} \text{ square unit}$$

**Hints:**

- Draw the graph $y = x$ and $y = x^3$
- Find inter section point of given curves

**79. Option (a) is correct.**

**Explanation:**

$$xy = 4225$$
$$\Rightarrow \quad y = \frac{4225}{x}$$

Let, $S = x + y = x + \dfrac{4225}{x}$

$$\frac{dS}{dx} = 1 - \frac{4225}{x^2} = 0$$
$$\Rightarrow \quad x^2 = 4225$$
$$\Rightarrow \quad x = 65 \qquad (\because x, y \in \text{N})$$
$$\frac{d^2S}{dt^2} = \frac{2 \times 4225}{x^3} = \frac{2 \times 4225}{(65)^3} > 0$$

$\therefore x + y$ is minimum at $x = 65$

Minimum value $= 65 + \dfrac{4225}{65} = 130$

**Hints:**

- Equate $f'(x) = 0$ and find critical point
- If $f''(x) > 0$ at critical point then $f(x)$ is minimum.

**80. Option (c) is correct.**

**Explanation:**

$$x\frac{dy}{dx} - 2y = 0$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{2y}{x}$$
$$\Rightarrow \quad \int \frac{1}{y}\,dy = 2\int \frac{1}{x}\,dx$$
$$\ln y = \ln x^2 + \ln c = \ln cx^2$$

$$y = cx^2$$

Represent *a* family of parabolas

**Hints:**

- Solve differential equation by variable separable method.

**81. Option (c) is correct.**

**Explanation:**

Given that, (–5, 0), $(5p^2, 10p)$ and $(5q^2, 10q)$ are collinear.

$$\therefore \quad \begin{vmatrix} -5 & 0 & 1 \\ 5p^2 & 10p & 1 \\ 5q^2 & 10q & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad 50\begin{vmatrix} -1 & 0 & 1 \\ p^2 & p & 1 \\ q^2 & q & 1 \end{vmatrix} = 0$$

Applying $C_1 \to C_1 + C_3$

$$\Rightarrow \quad \begin{vmatrix} 0 & 0 & 1 \\ p^2+1 & p & 1 \\ q^2+1 & q & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad q(p^2 + 1) - p(q^2 + 1) = 0$$

$$\Rightarrow \quad p^2q + q - pq^2 - p = 0$$

$$\Rightarrow \quad p^2q - pq^2 + q - p = 0$$

$$\Rightarrow \quad pq(p - q) - (p - q) = 0$$

$$\Rightarrow \quad (p - q)(pq - 1) = 0$$

$$\Rightarrow \quad pq - 1 = 0$$

$$\Rightarrow \quad pq = 1 \qquad (\because p \neq q)$$

**Hints:**

- $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$ is collinear if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

**82. Option (c) is correct.**

**Explanation:**

Since, straight line cuts off equal intercepts from the axes.

$$\therefore \quad \frac{x}{a} + \frac{y}{a} = 1$$

$$\Rightarrow \quad x + y = a$$

Since it is passes through (1, –2)

$$\therefore \quad 1 - 2 = a$$

$$\Rightarrow \quad a = -1$$

$\therefore$ Required equation is

$$x + y + 1 = 0$$

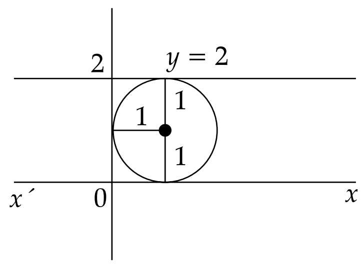
**Hints:**

- Equation of straight line cuts off equal intercept is $\frac{x}{a} + \frac{y}{a} = 1$

**83. Option (c) is correct.**

**Explanation:**

Since, circle which touches both the axes in the first quadrant and the line $y - 2 = 0$.




Then centre of circle is (1, 1) and radius is 1 unit.

$\therefore$ Equation of circle is

$$(x - 1)^2 + (y - 1)^2 = 1$$

$$\Rightarrow \quad x^2 + y^2 - 2x - 2y + 1 = 0$$

**Hints:**

- When circle touches $x$-axis and $y = 2$ then diameter is 2 units
- Equation of circle with centre $(h, k)$ and radius $r$ is $(x - h)^2 + (y - k)^2 = r^2$

**84. Option (d) is correct.**

**Explanation:**

Given that, focus (–3, 0) and directrix

$x - 3 = 0$ of parabola

$\therefore \quad a = -3$ and axis is $x$-axis

$\therefore$ Equation of parabola is

$$y^2 = 4(-3)x$$

$$\Rightarrow \quad y^2 = -12x$$

**Hints:**

- Focus and direction of parabola $y^2 = 4ax$ is $(a, 0)$ and $x - a = 0$ respectively.

**85. Option (b) is correct.**

**Explanation:**

Given, equation ellipse is

$$x^2 + 2y^2 = 1$$

$$\Rightarrow \quad \frac{x^2}{1} + \frac{y^2}{\left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

$$\therefore \quad a = 1 \text{ and } b = \frac{1}{\sqrt{2}}$$

We know that

$$b^2 = a^2 - c^2$$

$$\Rightarrow \quad \frac{1}{2} = 1 - c^2$$

$$\Rightarrow \quad c^2 = \frac{1}{2}$$

$$\Rightarrow \quad c = \pm\frac{1}{\sqrt{2}}$$

Distance between the *foci* = $|2c|$

$$= 2\cdot\frac{1}{\sqrt{2}} = \sqrt{2}$$

**86. Option (d) is correct.**

**Explanation:**

Given that, $a$, $b$, $c$ are sides of triangle ABC, perimeter $= a + b + c = p$ and $ar\ \Delta ABC = \Delta = q$

We know that

$$\tan\frac{A}{2} = \frac{\Delta}{s(s-a)}$$

(where $s$ = semiperimeter)

$$\Rightarrow \quad s(s-a)\ \tan\frac{A}{2} = \Delta$$

$$\Rightarrow \quad \frac{p}{2}\left(\frac{p}{2} - a\right)\tan\frac{A}{2} = q$$

$$\Rightarrow \quad p(p-2a)\frac{A}{2} = 4q$$

**Hints:**

- Use formula $\tan\frac{A}{2} = \frac{\Delta}{s(s-a)}$

**87. Option (a) is correct.**

**Explanation:**

Equation of straight line passes through point of intersection of two given lines is

$$x + 2y + 2 + \lambda(2x - 3y - 3) = 0$$

$$\Rightarrow \quad (1 + 2\lambda)x + (2 - 3\lambda)y + (2 - 3\lambda) = 0$$

Since, it cuts equal intercepts in the fourth quadrant

$$x\text{-intercept} = \frac{-(2-3\lambda)}{1+2\lambda}$$

$$y\text{-intercept} = \frac{-(2-3\lambda)}{(2-3\lambda)} = -1$$

$\therefore$ $x$-intercept $= 1$

Sum of absolute values of the intercepts = $|-1| + |1| = 2$

**Hints:**

- Family of straight lines passes through the point of intersection of two given lines is $L_1 + \lambda L_2 = 0$

**88. Option (c) is correct.**

**Explanation:**

Since, lines $ax + by + c = 0$ and $bx + ay + c = 0$ are parallel.

$$\therefore \quad \frac{a}{b} = \frac{b}{a}$$

$$\Rightarrow \quad a^2 = b^2$$

$$\Rightarrow \quad a^2 - b^2 = 0$$

$(\because a \neq 0 \text{ and } b \neq 0)$

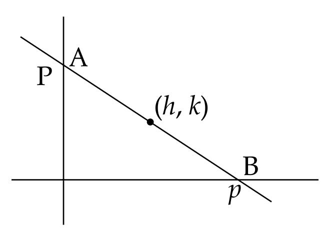
**Hints:**

- Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$.

**89. Option (a) is correct.**

**Explanation:**

Given, lines is $x + y = p$



$\Rightarrow \quad \frac{x}{p}+\frac{y}{p}=1$

Let $(h, k)$ he the coordinate of line segment AB.

$\therefore \quad \left(\frac{p}{2}, \frac{p}{2}\right)=(h, k)$

$\Rightarrow \quad h=k$

$\therefore$ equation of locus is $y=x$

$\Rightarrow \quad x-y=0$

**Hints:**

- Use intercept form of line $\frac{x}{a}+\frac{y}{b}=1$
- Mid-point of $(x_1, y_1)$ and $(x_2, y_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

**90. Option (c) is correct.**

**Explanation:**

Let, $P(x, y)$, $A(2a, 0)$ and $B(0, 3a)$

According to question

$$AP = BP \Rightarrow AP^2 = BP^2$$

$$(x-2a)^2+(y-0)^2=(x-0)^2+(y-3a)^2$$

$\Rightarrow \quad x^2-4ax+4a^2+y^2=x^2+y^2-6ay+9a^2$

$\Rightarrow \quad 4ax-6ay+5a^2=0$

$\Rightarrow \quad 4x-6y+5a=0$

**Hints:**

- Distance between $(x_1, y_1)$ and $(x_2, y_2)$ is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

For questions 91 to 93.

Given equation of plane is

$$6x+ky+3z-12=0$$

$\Rightarrow \quad \frac{x}{2}+\frac{y}{\frac{12}{k}}+\frac{z}{4}=1$

$\therefore$ A(2, 0, 0), B$(0, \frac{12}{k}, 0)$ and C(0, 0, 4)

We know that

Centre of sphere

$x^2+y^2+z^2+2gx+2fy+2hz+c=0$ is $(-g, -f, -h)$.

$\therefore$ Centre of $x^2+y^2+z^2-2x-3y-4z=0$

is $\left(1, \frac{3}{2}\right)$.

**91. Option (b) is correct.**

**Explanation:**

Since, $B\left(0, \frac{12}{k}, 0\right)$ lies on sphere

$\therefore \quad 0+\left(\frac{12}{k}\right)^2+0-0-3\left(\frac{12}{k}\right)-0=0$

$$\left(\frac{12}{k}\right)\left[\frac{12}{k}-3\right]=0$$

$\Rightarrow \quad k=4$

**92. Option (b) is correct.**

**Explanation:**

Distance of $\left(1, \frac{3}{2}, 2\right)$ to the plane

$$=\frac{6(1)+4\left(\frac{3}{2}\right)+3(2)-12}{\sqrt{36+16+9}}$$

$$=\frac{6}{\sqrt{65}}=0.74$$

**Hints:**

- A perpendicular distance from point $(x_1, y_1, z_1)$ to the plane $ax+by+cz+d=0$
$$=\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}$$

**93. Option (d) is correct.**

**Explanation:**

Equation of line passes through (0, 0, 0) and $\left(1, \frac{3}{2}, 2\right)$

$$\frac{x-0}{1}=\frac{y-0}{\frac{3}{2}}=\frac{z-0}{2}$$

$$6x=4y=3z$$

**Hints:**

- Equation of line passes through two point is $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$

[For Q. 94 to Q. 95]

Equation of plane is $\frac{2x}{k}+\frac{2y}{3}+\frac{z}{3}=2$

$$\Rightarrow \quad \frac{x}{\frac{k}{2}}+\frac{y}{\frac{3}{2}}+\frac{z}{3}=2$$

is passes through (2, 3, –6) is

$$\frac{4}{k}+\frac{3\times 2}{3}+\frac{-6}{3}=q$$

$$\Rightarrow \quad \frac{4}{k}=2$$

$$k=2$$

$$\therefore \quad x+\frac{2y}{3}+\frac{z}{3}=2$$

$$3x+2y+z=6$$

**94. Option (a) is correct.**

**Explanation:**

Direction ratios of plane is 3, 2, 1

**95. Option (b) is correct.**

**Explanation:**

$$3x+2y+z=6$$

$$\frac{x}{2}+\frac{y}{3}+\frac{z}{6}=1$$

$$\therefore \quad p=2, q=3, r=6$$

Now, $\quad p+q+r=2+3+6=11$

**96. Option (c) is correct.**

**Explanation:**

Given that, vectors $4\hat{i}+\hat{j}-3\hat{k}$ and $p\hat{i}+q\hat{j}-2\hat{k}$ are collinear

$$\therefore \quad \frac{4}{p}=\frac{1}{q}=\frac{3}{2}$$

$$\Rightarrow \quad \frac{4}{p}=\frac{3}{2} \text{ and } \frac{1}{p}=\frac{3}{2}$$

$$\Rightarrow \quad p=\frac{8}{3} \text{ and } q=\frac{2}{3}$$

**Hints:**

- If $a_1\hat{i}+b_1\hat{y}+c_1\hat{k}$ and $a_2\hat{i}+b_2\hat{y}+c_2\hat{k}$ are collinear then $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$.

**97. Option (c) is correct.**

**Explanation:**

Given that $\vec{a}$, $\vec{b}$ and $\vec{i}$ are the position vectors of the vertices. A, B, C respectively of triangle ABC

$\therefore$ Centroide (G) of triangle ABC

$$=\frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

Now, $\quad \overrightarrow{AG}=\frac{\vec{a}+\vec{b}+\vec{c}}{3}-\vec{a}$

$$=\frac{\vec{b}+\vec{c}-2\vec{a}}{3}$$

**98. Option (c) is correct.**
**Explanation:**

Statement 1

$$\because \quad \vec{a}\cdot\left(\vec{b}+\vec{c}\right)=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$$

So, but product over vector addition is distribute

$\therefore$ Statement 1 is correct.

Statement 2

$$\vec{a}\times\left(\vec{b}+\vec{c}\right)=\vec{a}\times\vec{b}+\vec{a}\times\vec{c} \text{ (By properties)}$$

So, cross product over vector addition is distributive.

$\therefore$ Statement 2 is correct.

Statement 3

$$\vec{a}\times\left(\vec{b}\times\vec{c}\right)=(\vec{a}\cdot\vec{c})\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{c}$$

$$\left(\vec{a}\times\vec{b}\right)\times\vec{c}=(\vec{a}\cdot\vec{c})\vec{b}-\left(\vec{b}\cdot\vec{c}\right)\vec{a}$$

$$\therefore \quad \vec{a}\times\left(\vec{b}\times\vec{c}\right)\neq\left(\vec{a}\times\vec{b}\right)\times\vec{c}$$

$\therefore$ Statement 3 is correct.

**99. Option (b) is correct.**

**Explanation:**

Given that, $\vec{a}\neq 0$, $\vec{b}\neq 0$, $\vec{c}\neq 0$ and $\vec{a}\times\vec{b}=\vec{c}$

$\Rightarrow \vec{a}\perp\vec{c}$ and $\vec{b}\perp\vec{c}$

If vector $\vec{b}$ and $\vec{c}$ are given and vector $\vec{c}$ is perpendicular to vector $\vec{b}$ then $\vec{c}$ is also perpendicular to all vector which is coplanar with vector $\vec{b}$, then $\vec{a}$ is not unique.

So, statement 1 is correct.

If vectors $\vec{a}$ and $\vec{b}$ are given then cross product two vector is unique vector.

So, statement 2 is correct.

**100. Option (d) is correct.**

**Explanation:**

Given that, $|\vec{a}| = |\vec{b}| = 1$

and $|\vec{a}-\vec{b}| < 2$

$\Rightarrow \quad |\vec{a}-\vec{b}|^2 < 4$

$\Rightarrow \quad (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b}) < 4$

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}| \cos 2\theta < 4$

$\Rightarrow \quad 1 + 1 - 2\cdot1\cdot1\cdot\cos 2\theta < 4$

$\Rightarrow \quad -\cos 2\theta < 1$

$\Rightarrow \quad 2\sin^2\theta - 1 < 1$

$\Rightarrow \quad \sin^2\theta < 1$

$\Rightarrow \quad -1 < \sin\theta < 1$

**Hints:**

- Use $|\vec{a}|^2 = \vec{a}\cdot\vec{b}$

  $(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$

**101. Option (c) is correct.**

**Explanation:**

Two digits taken from 1, 2, 3, 4, 5 in ${}^5C_2$ ways

$\therefore \quad n(S) = {}^5C_2$

Product of two digits whose last digit is zero i.e. (2, 5), (4, 5), (5, 2), (5, 4)

$n(E) = 4$

$\therefore \quad p(E) = \dfrac{4}{{}^5C_2} = \dfrac{4\times2!}{5.4} = \dfrac{2}{5}$

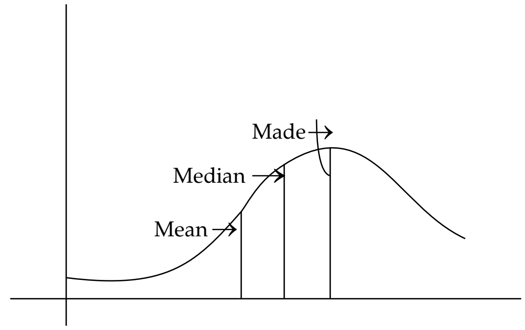
**Hints:**

- When $r$ things is taken out from $n$ things is ${}^nC_r$.

**102. Option (d) is correct.**

**Explanation:**

If the distribution of data is skewed to the left, then the mean is less than the median and median is less than mode.




Left (Negative0 skewed frequency distribution.
Mean < Median < Mode

**103. Option (c) is correct.**

**Explanation:**

Let $x$ be the $5^{th}$ observation

$$\therefore \text{ Variance} = \frac{2^2+2^2+4^2+5^2+x^2}{5} - \left(\frac{2+2+4+5+x}{5}\right)^2$$

$$3.6 = \frac{49+x^2}{5} - \frac{(13+x)^2}{25}$$

$\Rightarrow \quad 3.6 \times 25 = 5(49 + x^2) - (13 + x)^2$

$\Rightarrow \quad 90 = 245 + 5x^2 - 169 - x^2 - 26x$

$\Rightarrow \quad 2x^2 - 13x - 7 = 0$

$\Rightarrow \quad x = 7,\ x = -\dfrac{1}{2}$

($\because$ observation is positive)

$\therefore \quad x = 7$

**Hints:**

- Mean $= \dfrac{\sum x_i}{N}$
- Variance $= \dfrac{\sum x_i^2}{N} - \left(\dfrac{\sum x_i}{N}\right)^2$

**104. Option (d) is correct.**

**Explanation:**

Given that, $a = 4$ and $d = 4$

$\therefore \quad S_{50} = \dfrac{50}{2}[8 + 49(4)] = \dfrac{50}{2} \times 204$

$= 50 \times 102$

Mean of 50 terms $= \dfrac{50\times102}{50} = 102$

**Hints:**

- Use sum of $n$ term of A.P
- $S_n = \frac{n}{2}(2a + (n-1)d)$
- Mean $= \frac{\text{Sum of observation}}{\text{Number of observation}}$

**105. Option (b) is correct.**

**Explanation:**

$$\text{Mean} = \frac{21+34+23+39+26 \\ +37+40+20+33+27}{10}$$

$$= \frac{300}{10} = 30$$

Mean deviation from mean

$$= |21-30| + |34-30| + |23-30| + |39-30| + |26-30| + |37-30| + |40-30| + |20-30| + |33-30| + |27-30|$$

$$= \frac{9+4+7+9+4 +7+10+10+3+3}{10}$$

$$= \frac{66}{10} = 6.6$$

Coefficient of mean deviation

$$= \frac{\text{Mean deviation}}{\text{Mean}} = \frac{6.6}{30} = 0.22$$

**Hints:**

- Mean $(\bar{x}) = \frac{\sum x_i}{N}$
- Mean deviation from mean $= \frac{\sum |x_i - \bar{x}|}{N}$
- Coefficient of mean deviation $= \frac{\text{Mean deviation}}{\text{Mean}}$

**[For Q. 106 to Q. 108]**

According to question

$(x_1 - 100) + (x_2 - 100) + \ldots.. + (x_n - 100) = -20$

$\Rightarrow \quad (x_1 + x_2 + x_3 + \ldots.. + x_n) - n \times 100 = -20$

$\Rightarrow \quad x_1 + x_2 + x_3 + \ldots.. + x_n = 100n - 20 \quad \ldots.(i)$

Similarly

$\Rightarrow \quad x_1 + x_2 + x_3 + \ldots.. + x_n = 92n + 140 \quad \ldots(ii)$

From, (i) and (ii)

$$100x - 20 = 92_n + 140$$
$$8n = 160$$
$$n = 20$$

$\therefore x_1 + x_2 + x_3 \ldots\ldots\ldots\ldots + x_n = 2000 - 20 = 1980$

**106. Option (d) is correct.**

**Explanation:**

$$\text{Mean} = \frac{x_1 + x_2 + x_3 \ldots.. + x_n}{n}$$

$$= \frac{1980}{20} = 99$$

**107. Option (a) is correct.**

**Explanation:**

Sum of the deviations from 99

$$= 1980 - 20 \times 99 = 0$$

**108. Option (c) is correct.**

**Explanation:**

Sum of the deviations from $y$

$$= 1980 - 20y = 180$$

$\Rightarrow \quad 20y = 1800 = 90$

**Hints:**

- Sum of deviation of $n$ observations from '$a$' is

$$= \sum x_i - \text{an}$$

**[For Q. 109 to Q. 111]**

Given that, $a = 4$ and $d = 3$

$$\therefore \quad \sum x_i = S_{51} = \frac{51}{2}[8 + 50 \times 3]$$

$$= \frac{51}{2} \times 158 = 51 \times 79$$

**109. Option (d) is correct.**

**Explanation:**

$$\text{Mean marks} = \frac{\sum x_i}{n}$$

$$= \frac{51 \times 79}{51} = 79$$

**110. Option (b) is correct.**

**Explanation:**

Median of the marks = middle term

$$= \frac{(51+1)^{\text{th}}}{2} \text{ term} = 26^{\text{th}} \text{ term}$$

$\therefore$ Median $= T_{26} = 4 + 25 \times 3 = 79$

**111. Option (b) is correct.**

**Explanation:**

Sum of deviation from median

$$= \sum x_i - \text{Median} \times n$$
$$= 51 \times 79 - 51 \times 79 = 0$$

**Hint:**

- Use sum of deviation of $x$ observation from '$a$' $= \sum x_i - n \times a$

**112. Option (b) is correct.**

**Explanation:**

$$n(S) = 90$$

G $\rightarrow$ Graduate

T $\rightarrow$ At least 3 years experience

$$\therefore \quad n(G \cap \overline{T}) = 36$$

$$\therefore \quad p(G \cap \overline{T}) = \frac{36}{90} = \frac{2}{5}$$

**113. Option (c) is correct.**

**Explanation:**

$$n(G \cap \overline{T}) = 36, \; n(\overline{T}) = 36 + 27 = 63$$

$$\therefore \quad p\left(\frac{G}{\overline{T}}\right) = \frac{p(G \cap \overline{T})}{p(\overline{T})} = \frac{36}{63} = \frac{4}{7}$$

**114. Option (d) is correct.**

**Explanation:**

$$n(\overline{T} \cap \overline{G}) = 27, \; n(\overline{G}) = 9 + 27 = 36$$

$$\therefore \quad p\left(\frac{\overline{T}}{\overline{G}}\right) = \frac{p(\overline{T} \cap \overline{G})}{p(\overline{G})} = \frac{27}{36} = \frac{3}{4}$$

**Hint:**

- $$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)}$$

**[For Q. 115 to Q. 117]**

P(suffering from a disease) $= P = 33\frac{1}{3}\% = \frac{1}{3}$

P(not suffering from a disease) $= q = 1 - \frac{1}{3} = \frac{2}{3}$

**115. Option (d) is correct.**

**Explanation:**

Given that, number of trial $n = 6$

$$\therefore P(x = 3) = {}^6C_3 \cdot \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3 = \frac{160}{729}$$

**116. Option (b) is correct.**

**Explanation:**

$$P(x = 0) = {}^6C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^6$$

$$= \frac{64}{729}$$

**117. Option (b) is correct.**

**Explanation:**

$$P(x \geq 1) = 1 - P(x = 0)$$

$$= 1 - \frac{64}{729} = \frac{665}{729}$$

**Hints:**

P(r) = nCrpr qn-r, r= 0, 1, 2, 3

- Binomial distribution

  $n \rightarrow$ no. of trial

  $p \rightarrow$ probability of success

**[For Q. 118 to Q. 120]**

| C.I | $f_i$ | $x_i$ | $f_i x_i$ |
|---|---|---|---|
| 0-20 | 17 | 10 | 170 |
| 20-40 | $p + q$ | 30 | $30p + 30q$ |
| 40-60 | 32 | 50 | 1600 |
| 60-80 | $p - 3q$ | 70 | $70p - 210q$ |
| 80-100 | 19 | 90 | 1710 |
| Total | $68 + 2p - 2q$ | | $3480 + 100p - 180q$ |

According to question

$$\sum f_i = 68 + 2p - 2q = 120$$

$$\Rightarrow \quad p - q = 26 \qquad \ldots(i)$$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$50 = \frac{3480 + 100p - 180q}{120}$$

$\Rightarrow \quad 6000 = 3480 + 100p - 180q$

$\Rightarrow \quad 5p - 9q = 126 \quad \ldots\text{(ii)}$

Solving equation (i) and (ii) we get

$$p = 27, \text{ and } q = 1$$

**118. Option (c) is correct.**

**119. Option (a) is correct.**

**120. Option (b) is correct.**

**Explanation:**

If the frequency of each class is doubled then

$$\text{Mean} = \frac{\sum 2 f_i x_i}{\sum 2 f_i} = \frac{2 \sum f_i x_i}{2 \sum f_i} = \frac{\sum f_i x_i}{\sum f_i}$$

$= 50$ (Mean remains same)

**Hints:**

- $\text{Mean} = \dfrac{\sum f_i x_i}{\sum f_i}$
- If the frequency of each class multiply by same constant then mean remains same.