NDA/NA
National Defence Academy / Naval Academy

MATHEMATICS II

QUESTION PAPER 2022

Time : 2:30 Hour **Total Marks : 300**

Important Instructions :

1. *This test Booklet contains* **120** *items (questions). Each item is printed in* **Mathematics**. *Each item comprises four responses (answer's). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ONLY ONE response for each item.*
2. *You have to mark all your responses ONLY on the separate Answer Sheet provided.*
3. **All** *items carry equal marks.*
4. *Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions.*
5. *Penalty for wrong answers :*
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i) *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, one third of the marks assigned to that question will be deducted as penalty.*
   (ii) *If a candidate gives more than one answer, it will be treated as a wrong answer even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii) *If a question is left blank, i.e., no answer is given by the candidate, there will be no penalty for that question.*

1. How many four-digit natural numbers are there such that all of the digits are odd?

(a) 625 **(b)** 400
(c) 196 **(d)** 120

2. What is $\sum_{r=0}^{n} 2^r\, C(n, r)$ equal to?

(a) 2^n **(b)** 3^n
(c) 2^{2n} **(d)** 3^{2n}

3. If different Permutations of the letters of the word 'MATHEMATICS' are listed as in a dictionary, how many words (with or without meaning) are there in the list before the first word that starts with *C* ?

(a) 302400 **(b)** 403600
(c) 907200 **(d)** 1814400

4. Consider the following statements :

1. If f is the subset of $Z \times Z$ defined by $f = \{(xy, x - y); x, y \in Z\}$, then f is a function from Z to Z.

2. If f is the subset of $N \times N$ defined by $f = \{(xy, x + y); x, y \in N\}$, then f is a function from N to N.

Which of the statements given above is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

5. Consider the determinant

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

If $a_{13} = yz$, $a_{23} = zx$, $a_{33} = xy$ and the minors of a_{13}, a_{23}, a_{33} are respectively $(z - y)$, $(z - x)$, $(y - x)$ then what is the value of Δ ?

(a) $(z - y)(z - x)(y - x)$
(b) $(x - y)(y - z)(x - z)$
(c) $(x - y)(z - x)(y - z)(x + y + z)$
(d) $(xy + yz + zx)(x + y + z)$

6. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & -\cos\theta \end{pmatrix}$, Then which of the following are correct ?

1. $A + adjA$ is a null matrix
2. $A^{-1} + adjA$ is a null matrix
3. $A - A^{-1}$ is a null matrix

Select the correct answer using the code given below:

(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3

7. If X is a matrix of order 3×3, Y is a matrix of order 2×3 and Z is a matrix of order 3×2, then which of the following are correct ?

1. $(ZY)X$ is a square matrix having 9 entries.

2. $Y(XZ)$is a square matrix having 4 entries.

3. $X(YZ)$is not defined.

Select the correct answer using the code given below:

(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

8. For how many quadratic equations, the sum of roots is equal to the product of roots ?

(a) 0 **(b)** 1
(c) 2 **(d)** Infinitely many

9. Consider the following statements:

1. The set of all irrational numbers between $\sqrt{2}$ and $\sqrt{5}$ is an infinite set.

2. The set of all odd integers less than 100 is a finite set.

Which of the statements given above is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

10. Consider the following statements :

1. $2+4+6+...+2n = n^2 + n$

2. The expression $n^2+ n + 41$ always gives a prime number for every natural number n

Which of the above statements is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

11. Let $p,q(p>q)$ be the roots of the quadratic equation $x^2+ bx + c = 0$ where $c > 0$. If $p^2 + q^2- 11pq = 0$, then what is $p - q$ equal to ?

(a) $3\sqrt{c}$ **(b)** 3c
(c) $9\sqrt{c}$ **(d)** $9c$

12. What is the diameter of a circle inscribed in a regular polygon of 12 sides, each of length 1 cm?

(a) $1 + \sqrt{2}$ cm **(b)** $2 + \sqrt{2}$ cm
(c) $2 + \sqrt{3}$ cm **(d)** $3 + \sqrt{3}$ cm

13. Let A = {7, 8, 9, 10, 11, 12, 13, 14, 15, 16} and let $f: A \to N$ be defined by $f(x)$ = the highest prime factor of x. How many elements are there in the range of f?

(a) 4 **(b)** 5
(c) 6 **(d)** 7

14. Let R be a relation from N to N defined by $R= \{(x,y):x,y \in N \text{ and } x^2=y^3\}$.Which of the following are ***not*** correct ?

1. $(x, x) \in R$ for all $x \in N$

2. $(x,y)\in R \Rightarrow (y,x)\in R$

3. $(x,y) \in R$ and$(y,z) \in R\Rightarrow(x, z)\in R$

Select the correct answer using the code given below:

(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

15. Consider the following :

1. $A \cap B = A \cap C \Rightarrow B = C$
2. $A \cup B = A \cup C \Rightarrow B = C$

Which of the above is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

Consider the following for the next **three (03)** items that follow :

$$\text{Let } z = \frac{1 + i \sin\theta}{1 - i \sin\theta} \text{ where } i = \sqrt{-1}$$

16. What is the modulus of z?

(a) 1 **(b)** $\sqrt{2}$
(c) $1 + \sin^2\theta$ **(d)** $\frac{1+\sin^2\theta}{1-\sin^2\theta}$

17. What is angle θ such that z is purely real?

(a) $\frac{n\pi}{2}$ **(b)** $\frac{(2n+1)\pi}{2}$
(c) $n\pi$ **(d)** $2n\pi$ only

where n is an integer

18. What is angle θ such that z is purely imaginary?

(a) $\frac{n\pi}{2}$ **(b)** $\frac{(2n+1)\pi}{2}$
(c) $n\pi$ **(d)** $2n\pi$

where n is an integer

Consider the following for the next **three (03)** items that follow :

Let P be the sum of first n positive terms of an increasing arithmetic progression A. Let Q be the sum of first n positive terms of another increasing arithmetic progression B. Let P: Q = $(5n + 4)$: $(9n + 6)$

19. What is the ratio of the first term of A to that of B?

(a) 1/3 **(b)** 2/5
(c) 3/4 **(c)** 3/5

20. What is the ratio of their 10th terms ?

(a) 11/29 **(b)** 22/49
(c) 33/59 **(d)** 44/69

21. If d is the common difference of A, and D is the common difference of B, then which one of the following is always correct ?

(a) $D > d$ **(b)** $D< d$
(c) $7D >12d$ **(d)** None of the above

Consider the following for the next **three (03)** items that follow :

Consider the Binomial Expansion of $(p + qx)^9$:

22. What is the value of q if the coefficients of x^3 and x^6 are equal ?

(a) p **(b)** $9p$

(c) $\frac{1}{p}$ **(d)** p^2

23. What is the ratio of the coefficients of middle terms in the expansion (when expanded in ascending powers of x)?

(a) pq **(b)** p/q

(c) $4p/5q$ **(d)** $1/(pq)$

24. Under what condition the coefficients of x^2 and x^4 are equal ?

(a) $p:q = 7:2$ **(b)** $p^2:q^2 = 7:2$

(c) $p : q = 2 : 7$ **(d)** $p^2: q^2 = 2 : 7$

Consider the following for the next **three (03)** items that follow :

Consider the word 'QUESTION':

25. How many 4-letter words each of two vowels and two consonants with or without meaning, can be formed ?

(a) 36 **(b)** 144

(c) 576 **(d)** 864

26. How many 8-letter words with or without meaning, can be formed such that consonants and vowels occupy alternate positions ?

(a) 288 **(b)** 576

(c) 1152 **(d)** 2304

27. How many 8-letter words with or without meaning, can be formed so that all consonants are together ?

(a) 5760 **(b)** 2880

(c) 1440 **(d)** 720

Consider the following for the next **three (03)** items that follow :

Let Δ be the determinant of a matrix A, where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ and C_{11}, C_{12}, C_{13} be the cofactors of a_{11}, a_{12}, a_{13} respectively.

28. What is the value of $a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$?

(a) 0 **(b)** 1

(c) Δ **(d)** –Δ

29. What is the value of $a_{21}C_{11} + a_{22}C_{12} + a_{23}C_{13}$?

(a) 0 **(b)** 1

(c) Δ **(d)** –Δ

30. What is the value of $\begin{vmatrix} a_{21} & a_{31} & a_{11} \\ a_{23} & a_{33} & a_{13} \\ a_{22} & a_{32} & a_{12} \end{vmatrix}$?

(a) 0 **(b)** 1

(c) Δ **(d)** –Δ

Consider the following for the next **three (03)** items that follow:

Let $f(x)$ be a function satisfying $f(x + y) = f(x)f(y)$ for all $x,y \in N$ such that $f(1) = 2$:

31. If $\sum_{x=2}^{n} f(x) = 2044$, then what is the value of n ?

(a) 8 **(b)** 9

(c) 10 **(d)** 11

32. What is $\sum_{x=1}^{5} f(2x-1)$ equal to?

(a) 341 **(b)** 682

(c) 1023 **(d)** 1364

33. What is $\sum_{x=1}^{6} 2^x f(x)$ equal to?

(a) 1365 **(b)** 2730

(c) 4024 **(d)** 5460

Consider the following for the next **three (03)** items that follow :

A university awarded medals in basket ball, football and volleyball. Only x students ($x < 6$) got medal in all the three sports and the medals went to a total of $15x$ students. It awarded $5x$ medals in basketball, $(4x+15)$ medals in football and $(x + 25)$ medals in volleyball.

34. How many received medals in exactly two of the three sports ?

(a) $30 - 4x$ **(b)** $35 - 7x$

(c) $40 - 7x$ **(d)** $45 - 5x$

35. How many received medals in at least two of three sports ?

(a) $30 - 6x$ **(b)** $35 - 6x$

(c) $40 - 5x$ **(d)** $40 - 6x$

36. How many received medals in exactly one of three sports ?

(a) $21x - 40$ **(b)** $21x - 35$

(c) $20x - 35$ **(d)** $20x - 25$

Consider the following for the next **three (03)** items that follow :

Let $A = \begin{pmatrix} 0 & \sin^2\theta & \cos^2\theta \\ \cos^2\theta & 0 & \sin^2\theta \\ \sin^2\theta & \cos^2\theta & 0 \end{pmatrix}$ and

$A = P + Q$ where P is symmetric matrix and Q is skew-symmetric matrix.

37. What is P equal to ?

(a) $\begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

(c) $\cos 2\theta \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

(d) $\cos 2\theta \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{pmatrix}$

38. What is Q equal to ?

(a) $\begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

(c) $\cos 2\theta \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$

(d) $\cos 2\theta \begin{pmatrix} 0 & -1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{pmatrix}$

39. What is the minimum value of determinant of A ?

(a) $\frac{1}{4}$ **(b)** $\frac{1}{2}$

(c) $\frac{3}{4}$ **(d)** 1

Consider the following for the next **three (03)** items that follow

ABC is a triangular plot with $AB = 16$ m, $BC = 10$ m and $CA = 10$ m. A lamp post is situated at the middle point of the side AB. The lamp post subtends an angle 45° at the vertex B.

40. What is the height of the lamp post ?

(a) 6 m **(b)** 7 m

(c) 8 m **(d)** 9 m

41. What is $\frac{AB}{\sin C}$ equal to ?

(a) 17 m **(b)** $\frac{50}{3}$ m

(c) $\frac{40}{3}$ m **(d)** 16

42. What is $\cos A + \cos B + \cos C$ equal to?

(a) 1 **(b)** $\frac{41}{25}$

(c) $\frac{37}{25}$ **(d)** $\frac{33}{25}$

Consider the following for the next **three (03)** items that follow:

There are two points P and Q due south of a leaning tower, which leans towards north. P is at a distance x and Q is at a distance y from the foot of the tower ($x>y$). The angles of elevation of the top of the tower from P and Q are 15° and 75° respectively.

43. At what height is the top of the tower above the ground level ?

(a) $\frac{x-y}{2\sqrt{3}}$ **(b)** $\frac{x-y}{4\sqrt{3}}$

(c) $\frac{x-y}{4}$ **(d)** $\frac{x-y}{2}$

44. If θ is the inclination of the tower to the horizontal, then what cotθ equal to?

(a) $2+\frac{\sqrt{3}(x-y)}{x+y}$ **(b)** $2-\frac{\sqrt{3}(x-y)}{x+y}$

(c) $2+\frac{\sqrt{3}(x+y)}{x-y}$ **(d)** $2-\frac{\sqrt{3}(x+y)}{x-y}$

45. What is the length of the tower ?

(a) $\frac{x-y}{2\sqrt{3}}\sqrt{1+\left\{2+\frac{\sqrt{3}(x+y)}{x-y}\right\}^2}$

(b) $\frac{x-y}{2\sqrt{3}}\sqrt{1+\left\{2-\frac{\sqrt{3}(x+y)}{x-y}\right\}^2}$

(c) $\frac{x-y}{4\sqrt{3}}\sqrt{1+\left\{2+\frac{\sqrt{3}(x+y)}{x-y}\right\}^2}$

(d) $\frac{x-y}{4\sqrt{3}}\sqrt{1+\left\{2-\frac{\sqrt{3}(x+y)}{x-y}\right\}^2}$

46. What is the value of $\operatorname{cosec}\left(-\frac{73\pi}{3}\right)$?

(a) $\frac{2}{\sqrt{3}}$ **(b)** $-\frac{2}{\sqrt{3}}$

(c) 2 **(d)** –2

47. What is the value of

$\cos\left(\frac{5\pi}{17}\right)+\cos\left(\frac{7\pi}{17}\right)+2\cos\left(\frac{11\pi}{17}\right)\cos\left(\frac{\pi}{17}\right)$?

(a) 0
(b) 1
(c) $4\cos\left(\frac{6\pi}{17}\right)\cos\left(\frac{\pi}{17}\right)$
(d) $4\cos\left(\frac{11\pi}{17}\right)\cos\left(\frac{\pi}{17}\right)$

48. What is the value of $\tan\left(\frac{3\pi}{8}\right)$?

(a) $\sqrt{2}-1$ **(b)** $\sqrt{2}+1$
(c) $1-\sqrt{2}$ **(d)** $-\left(\sqrt{2}+1\right)$

49. What is $\tan^{-1}\cot(\text{cosec}^{-1} 2)$ equal to ?

(a) $\frac{\pi}{8}$ **(b)** $\frac{\pi}{6}$
(c) $\frac{\pi}{4}$ **(d)** $\frac{\pi}{3}$

50. In a triangle *ABC*, $a = 4$, $b = 3$, $c = 2$. What is cos3C equal to ?

(a) $\frac{7}{128}$ **(b)** $\frac{11}{128}$
(c) $\frac{7}{64}$ **(d)** $\frac{11}{64}$

51. What is cos36° – cos72° equal to ?

(a) $\frac{\sqrt{5}}{2}$ **(b)** $-\frac{\sqrt{5}}{2}$
(c) $\frac{1}{2}$ **(d)** $-\frac{1}{2}$

52. If $\sec x = \frac{25}{24}$ and x lies in the fourth quadrant, then what is the value of $\tan x + \sin x$?

(a) $-\frac{625}{168}$ **(b)** $-\frac{343}{600}$
(c) $\frac{625}{168}$ **(d)** $\frac{343}{600}$

53. What is the value of $\tan^2 165° + \cot^2 165°$?

(a) 7 **(b)** 14
(c) $4\sqrt{3}$ **(d)** $8\sqrt{3}$

54. What is the value of

$\sin\left(2n\pi+\frac{5\pi}{6}\right)\sin\left(2n\pi-\frac{5\pi}{6}\right)$, where $n \in Z$?

(a) $-\frac{1}{4}$ **(b)** $-\frac{3}{4}$
(c) $\frac{1}{4}$ **(d)** $\frac{3}{4}$

55. If $1 + 2(\sin x + \cos x)(\sin x - \cos x) = 0$ where $0 < x < 360°$, then how many values does x take?

(a) Only one value **(b)** Only two values
(c) Only three values **(d)** Four values

56. Consider the following statements in respect of the line passing through origin and inclining at an angle of 75° with the positive direction of x-axis :

1. The line passes through the point $\left(1, \frac{1}{2\sqrt{3}}\right)$.
2. The line entirely lies in first and third quadrants.

Which of the statements given above is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

57. If $P(3, 4)$ is the mid-point of a line segment between the axes, then what is the equation of the line?

(a) $3x + 4y - 25 = 0$ **(b)** $4x + 3y - 24 = 0$
(c) $4x - 3y = 0$ **(d)** $3x - 4y + 7 = 0$

58. The base *AB* of an equilateral triangle *ABC* with side 8 cm lies along the y – axis such that the mid-point of *AB* is at the origin and *B* lies above the origin. What is the equation of line passing through (8,0) and parallel to the side *AC*?

(a) $x - \sqrt{3}y - 8 = 0$
(b) $x + \sqrt{3}y - 8 = 0$
(c) $\sqrt{3}x + y - 8\sqrt{3} = 0$
(d) $\sqrt{3}x - y - 8\sqrt{3} = 0$

59. The centre of the circle passing through origin and making positive intercepts 4 and 6 on the coordinate axes, lies on the line

(a) $2x - y + 1 = 0$
(b) $3x - 2y - 1 = 0$
(c) $3x - 4y + 6 = 0$
(d) $2x + 3y - 26 = 0$

60. The centre of an ellipse is at (0, 0), major axis is on the y-axis. If the ellipse passes through (3, 2) and (1, 6), then what is its eccentricity ?

(a) $\frac{\sqrt{3}}{2}$ **(b)** $\sqrt{3}$
(c) $\frac{\sqrt{5}}{2}$ **(d)** $\sqrt{5}$

61. An equilateral triangle is inscribed in a parabola $x^2 = \sqrt{3}y$ where one vertex of the triangle is at the vertex of the parabola. If p is the length of side of the triangle and q is the length of the latus rectum, then which one of the following is correct ?

(a) $p = q$ **(b)** $p = \sqrt{3}q$
(c) $p = 2\sqrt{3}q$ **(d)** $2\sqrt{3}p = q$

62. Consider the points $A(2, 4, 6)$, $B(-2, -4, -2)$, $C(4, 6, 4)$ and $D(8, 14, 12)$. Which of the following statements is/are correct?

1. The points are the vertices of a rectangle *ABCD.*

2. The mid-point of AC is the same as that of BD.

Select the correct answer using the code given below:

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

63. Consider the equation of a sphere $x^2 + y^2 + z^2 - 4x - 6y - 8z - 16 = 0$. Which of the following statements is/are correct ?

1. z-axis is tangent to the sphere.
2. The centre of the sphere lies on the plane $x + y + z - 9 = 0$.

Select the correct answer using the code given below:

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

64. A plane cuts intercepts 2, 2, 1 on the coordinate axes. What are the direction cosines of the normal to the plane ?

(a) < 2/3, 2/3, 1/3 > **(b)** < 1/3, 2/3, 2/3 >
(c) $\left\langle \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right\rangle$ **(d)** $\left\langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$

65. Consider the following statements :

1. The direction ratios of y-axis can be < 0, 4, 0 >

2. The direction ratios of a line perpendicular to z-axis can be < 5, 6, 0 >

Which of the statements given above is /are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

66. $PQRS$ is a parallelogram. If $\overrightarrow{PR} = \vec{a}$ and $\overrightarrow{QS} = \vec{b}$, then what is $\overrightarrow{PQ}$ equal to?

(a) $\vec{a} + \vec{b}$ **(b)** $\vec{a} - \vec{b}$
(c) $\frac{\vec{a} + \vec{b}}{2}$ **(d)** $\frac{\vec{a} - \vec{b}}{2}$

67. Let $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular. What is the angle between $\vec{a}$ and $\vec{b}$?

(a) $\frac{\pi}{6}$ **(b)** $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ **(d)** $\frac{\pi}{2}$

68. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be unit vectors lying on the same plane. What is $\left\{\left(3\vec{a} + 2\vec{b}\right) \times \left(5\vec{a} - 4\vec{c}\right)\right\}.\left(\vec{b} + 2\vec{c}\right)$ equal to ?

(a) −8 **(b)** −32
(c) 8 **(d)** 0

69. What are the values of x for which the angle between the vectors $2x^2\hat{i} + 3x\hat{j} + \hat{k}$ and $\hat{i} - 2\hat{j} + x^2\hat{k}$ is obtuse ?

(a) $0 < x < 2$ **(b)** $x < 0$
(c) $x > 2$ **(d)** $0 \le x \le 2$

70. The position vectors of vertices A, B and C of triangle ABC are respectively $\hat{j} + \hat{k}$, $3\hat{i} + \hat{j} + 5\hat{k}$ and $3\hat{j} + 3\hat{k}$. What is angle C equal to ?

(a) $\frac{\pi}{6}$ **(b)** $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ **(d)** $\frac{\pi}{2}$

71. Let $z = [y]$ and $y = [x] - x$, where [.] is the greatest integer function. If x is not an integer but positive, then what is the value of z ?

(a) −1 **(b)** 0
(c) 1 **(d)** 2

72. If $f(x) = 4x + 1$ and $g(x) = kx + 2$ such that $fog(x) = gof(x)$, then what is the value of k ?

(a) 7 **(b)** 5
(c) 4 **(d)** 3

73. What is the minimum value of the function $f(x) = log_{10}(x^2 + 2x + 11)$?

(a) 0 **(b)** 1
(c) 2 **(d)** 10

74. What is $\int \left(x^x\right)^2 (1 + lnx)\, dx$ equal to ?

(a) $x^2x + c$ **(b)** $\frac{1}{2}x^2x + c$
(c) $2x^{2x} + c$ **(d)** $\frac{1}{2}xx + c$

75. What is $\int e^x \{1 + lnx + xlnx\}dx$ equal to?

(a) $xe^x lnx + c$ **(a)** $x^2e^x lnx + c$
(c) $x + e^x lnx + c$ **(d)** $xe^x + lnx + c$

76. What is $\int \frac{(\cos x)^{1.5} - (\sin x)^{1.5}}{\sqrt{\sin x . \cos x}} dx$ equal to?

(a) $\sqrt{\sin x} - \sqrt{\cos x} + c$
(b) $\sqrt{\sin x} + \sqrt{\cos x} + c$
(c) $2\sqrt{\sin x} + 2\sqrt{\cos x} + c$
(d) $\frac{1}{2}\sqrt{\sin x} + \frac{1}{2}\sqrt{\cos x} + c$

77. If $y = \frac{x\sqrt{x^2-16}}{2} - 8\ln\left|x+\sqrt{x^2-16}\right|$, then what is $\frac{dy}{dx}$ equal to ?

(a) $x\sqrt{x^2-16}$ (b) $x-\sqrt{x^2-16}$

(c) $\sqrt{x^2-16}$ (d) $4\sqrt{x^2-16}$

78. If $y = (x^x)^x$, then which one of the following is correct ?

(a) $\frac{dy}{dx} + xy(1+2lnx) = 0$

(b) $\frac{dy}{dx} - xy(1+2lnx) = 0$

(c) $\frac{dy}{dx} - 2xy(1+lnx) = 0$

(d) $\frac{dy}{dx} + 2xy(1+lnx) = 0$

79. What is the maximum value of 3 (sin x – cos x) + 4($\cos^3 x - \sin^3 x$) ?

(a) 1 (b) $\sqrt{2}$

(c) $\sqrt{3}$ (d) 2

80. What is the area of the region (in the first quadrant) bounded by $y=\sqrt{1-x^2}$ $y = x$ and $y = 0$?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{8}$ (d) $\frac{\pi}{12}$

81. What is the area of the region bounded by $x-|y|=0$ and $x-2=0$?

(a) 1 (b) 2

(c) 4 (d) 8

82. If $f(\alpha) = \sqrt{\sec 2\alpha - 1}$, then what is $\frac{f(\alpha)+f(\beta)}{1-f(\alpha)f(\beta)}$ equal to?

(a) $f(\alpha-\beta)$ (b) $f(\alpha+\beta)$

(c) $f(\alpha)(\beta)$ (d) $f(\alpha\beta)$

83. If $f(x) = ln(x+\sqrt{1+x^2})$, then which one of the following is correct ?

(a) $f(x)+f(-x)=0$ (b) $f(x)-f(-x)=0$

(c) $2f(x)=f(-x)$ (d) $f(x)=2f(-x)$

84. What is $\lim_{x\to 0}\frac{x}{\sqrt{1-\cos 4x}}$ equal to ?

(a) $\frac{1}{2\sqrt{2}}$

(b) $-\frac{1}{2\sqrt{2}}$

(c) $\sqrt{2}$

(d) Limit does not exist

85. What is $\lim_{x\to\frac{\pi}{2}}\frac{4x-2\pi}{\cos x}$ equal to ?

(a) –4 (b) –2

(c) 2 (d) 4

86. If $f(x) = \frac{x^2+x+|x|}{x}$, then what is $\lim_{x\to 0} f(x)$ equal to ?

(a) 0

(b) 1

(c) 2

(d) $\lim_{x\to 0} f(x)$ does not exist

87. What is $\lim_{h\to 0}\frac{\sin^2(x+h)-\sin^2 x}{h}$ equal to?

(a) $\sin^2 x$ (b) $\cos^2 x$

(c) $\sin 2x$ (d) $\cos 2x$

88. Let $f(x)$ be a function such that $f'(x)= g(x)$ and $f''(x) = -f(x)$. Let $h(x)= \{f(x)\}^2 + \{g(x)\}^2$. Then consider the following statements:

1. $h(3)= 0$

2. $h(l) = h(2)$

Which of the statements given above is/are correct ?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

89. If $y = \ln^2\left(\frac{x^2-x+1}{x^2+x+1}\right)$, then what is $\frac{dy}{dx}$ at $x = 0$ equalto?

(a) -2 (b) 0

(c) 1 (d) 2

90. If $\frac{d}{dx}\left(\frac{1+x^4+x^8}{1-x^2+x^4}\right) = ax+bx^3$, then which one of the following is correct ?

(a) $a = b$ (b) $a = 2b$

(c) $a + b = 0$ (d) $2a = b$

91. Under which one of the following conditions does the function$f(x) = (p \sec x)^2 + (q \operatorname{cosec} x)^2$ attains minimum value ?

(a) $\tan^2 x = \frac{q}{p}$ (b) $\cot^2 x = \frac{q}{p}$

(c) $\tan^2 x = pq$ (d) $\cot^2 x = pq$

92. Where does the function $f(x)=\sum_{j=1}^{7}(x-j)^2$ attains its minimum value ?

(a) $x = 3.5$ (b) $x = 4$

(c) $x = 4.5$ (d) $x = 5$

93. Consider the following statements in respect of the function $f(x)=\begin{cases}|x|+1, & 0<|x|\le 3\\ 1, & x=0\end{cases}$

1. The function attains maximum value only at $x = 3$
2. The function attains local minimum only at $x = 0$

Which of the statements given above is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

94. What is $\int_0^1 ln\left(\frac{1}{x}-1\right)dx$ equal to?

(a) -1 (b) 0
(c) 1 (d) $ln2$

95. If $\int_0^{\pi/2}\left(\sin^4 x+\cos^4 x\right)dx = k$, than what is the value of $\int_0^{20\pi}\left(\sin^4 x+\cos^4 x\right)dx$?

(a) k (b) $10k$
(c) $20k$ (d) $40k$

96. What is $\int_{-\pi/2}^{\pi/2}\left(e^{\cos x}\sin x+e^{\sin x}\cos x\right)dx$ equal to?

(a) $\frac{e^2-1}{e}$ (b) $\frac{e^2+1}{e}$
(c) $\frac{1-e^2}{e}$ (d) 0

97. What is the area of the region enclosed in the first quadrant by $x^2+y^2=\pi^2$, $y = \sin x$ and $x = 0$?

(a) $\frac{\pi^3}{4}-1$ (b) $\frac{\pi^3}{4}-2$
(c) $\frac{\pi^3}{2}-1$ (d) $\frac{\pi^2}{4}-2$

98. Consider the following statements :

1. The degree of the differential equation $\frac{dy}{dx}+\cos\left(\frac{dy}{dx}\right)=0$ is 1.
2. The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3+\cos\left(\frac{dy}{dx}\right)=0$ is 2.

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

99. What is the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis ?

(a) $x\frac{dy}{dx}+2y=0$ (b) $x\frac{dy}{dx}-2y=0$
(c) $y\frac{dy}{dx}+2x=0$ (d) $y\frac{dy}{dx}-2x=0$

100. What is the solution of the differential equation $(dy - dx) + \cos x(dy + dx) = 0$?

(a) $y = \tan\left(\frac{x}{2}\right)-x+c$
(b) $y = \frac{1}{2}\tan\left(\frac{x}{2}\right)-x+c$
(c) $y = 2\tan\left(\frac{x}{2}\right)-x+c$
(d) $y = \tan\left(\frac{x}{2}\right)-2x+c$

101. Let x be the mean of squares of first n natural numbers and y be the square of mean of first n natural numbers.If $\frac{x}{y}=\frac{55}{42}$, then what is the value of n?

(a) 24 (b) 25
(c) 27 (d) 30

102. What is the probability of getting a composite number in the list of natural numbers from 1 to 50?

(a) $\frac{7}{10}$ (b) $\frac{17}{25}$
(c) $\frac{18}{25}$ (d) $\frac{33}{50}$

103. If $n > 7$, then what is the probability that $C(n, 7)$ is a multiple of 7?

(a) 0 (b) $\frac{1}{7}$
(c) $\frac{1}{2}$ (d) 1

104. Two numbers x and y are chosen at random from a set of first 10 natural numbers. What is the probability that $(x + y)$ is divisible by 4 ?

(a) $\frac{1}{5}$ (b) $\frac{2}{9}$
(c) $\frac{8}{45}$ (d) $\frac{7}{45}$

105. A number x is chosen at random from first n natural numbers. What is the probability that the number chosen satisfies $x+\frac{1}{x}>2$?

(a) $\frac{1}{n}$ (b) $\frac{1}{(2n)}$
(c) $\frac{(n-1)}{n}$ (d) 1

106. Three fair dice are tossed once. What is the probability that they show different numbers that are in AP ?

(a) $\frac{1}{12}$ (b) $\frac{1}{18}$
(c) $\frac{1}{36}$ (d) $\frac{1}{72}$

107. If $P(A) = 0.5$, $P(B) = 0.7$ and $P(A \cap B) = 0.3$, than what is the value of $P(A' \cap B') + P(A' \cap B) + P(A \cap B')$?

(a) 0.6 **(b)** 0.7

(c) 0.8 **(d)** 0.9

108. Five coins are tossed once. What is the probability of getting at most four tails?

(a) $\frac{31}{32}$ **(b)** $\frac{15}{16}$

(c) $\frac{29}{32}$ **(d)** $\frac{7}{8}$

109. Three fair dice are thrown. What is the probability of getting a total greater than or equal to 15 ?

(a) $\frac{19}{216}$ **(b)** $\frac{1}{12}$

(c) $\frac{17}{216}$ **(d)** $\frac{5}{54}$

110. The probability that a person hits a target is 0.5. What is the probability of at least one hit in 4 shots ?

(a) $\frac{1}{8}$ **(b)** $\frac{1}{16}$

(c) $\frac{15}{16}$ **(d)** $\frac{7}{8}$

111. A box contains 2 white balls, 3 black balls and 4 red balls. What is the number of ways of drawing 3 balls from the box with at least one black ball ?

(a) 84 **(b)** 72

(c) 64 **(d)** 48

112. During war one ship out of 5 was sunk on an average in making a certain voyage. What is the probability that exactly 3 out of 5 ships would arrive safely ?

(a) $\frac{16}{625}$ **(b)** $\frac{32}{625}$

(c) $\frac{64}{625}$ **(d)** $\frac{128}{625}$

113. A card is drawn from a pack of 52 cards. A gambler bets that it is either a spade or an ace. The odds against his winning are

(a) 9 : 4 **(b)** 35 : 17

(c) 17 : 35 **(d)** 4 : 9

114. The coefficient of correlation between ages of husband and wife at the time of marriage for a given set of 100 couples was noted to be 0.7. Assume that all these couples survive to celebrate the silver jubilee of their marriage. The coefficient of correlation at that point of time will be

(a) 1 **(b)** 0.9

(c) 0.7 **(d)** 0.3

115. The completion of a construction job may be delayed due to strike. The probability of strike is 0.6. The probability that the construction job gets completed on time if there is no strike is 0.85 and the probability that the construction job gets completed on time if there is a strike is 0.35. What is the probability that the construction job will not be completed on time ?

(a) 0.35 **(b)** 0.45

(c) 0.55 **(d)** 0.65

Consider the following for the next two (02) items that follow:

The mean and standard deviation (SD) of marks obtained by 50 students of a class in 4 subjects are given below :

Subject	*Mathematics*	*Physics*	*Chemistry*	*Biology*
Mean Marks	40	28	38	36
SD	15	12	14	16

116. Which one of the following subjects shows highest variability of marks ?

(a) Mathematics

(b) Physics

(c) Chemistry

(d) Biology

117. What is the coefficient of variation of marks in Mathematics ?

(a) 37.5% **(b)** 38.0%

(c) 38.5% **(d)** 39.0%

Consider the following for the next **three (03)** items that follow :

Consider the following grouped frequency distribution :

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	1	2	4	6	4	3

118. What is the median of the distribution?

(a) 34 **(b)** 34.5

(c) 35 **(d)** 35.5

119. What is mean deviation about the median ?

(a) 11.4 **(b)** 11.1

(c) 10.8 **(d)** 10.5

120. What is the mean deviation about the mean ?

(a) 10.15 **(b)** 10.65

(c) 11.15 **(d)** 11.65

■■■

Answers

Mathematics

Q No	Answer Key	Topic Name	Chapter Name
1	**(a)**	Permutation	Permutation and combination
2	**(b)**	Binomial Theorem for Positive Integral Index	Binomial theorem and its applications
3	**(c)**	Permutation	Permutation and combination
4	**(d)**	Basics of function	Functions
5	**(a)**	Determinant of a Square Matrix	Matrices and determinants
6	**(d)**	Inverse of a Matrix	Matrices and determinants
7	**(d)**	Algebra of Matrices	Matrices and determinants
8	**(d)**	Relation between Roots and Coefficients	Quadratic equations
9	**(a)**	Basics of Sets	Set Theory
10	**(a)**	Series of Natural Numbers and other Miscellaneous Series	Sequences and Series
11	**(a)**	Relation between Roots and Coefficients	Quadratic Equations
12	**(c)**	Basics of circle	Circle
13	**(c)**	Basics of function	Functions
14	**(d)**	Algebra of Relations	Relations
15	**(d)**	Algebra of Sets	Set Theory
16	**(a)**	Modulus and Argument of Complex Numbers	Complex numbers
17	**(c)**	Basics of Complex Numbers	Complex numbers
18	**(b)**	Basics of Complex Numbers	Complex numbers
19	**(d)**	Arithmetic Progressions	Sequences and Series
20	**(c)**	Arithmetic Progressions	Sequences and Series
21	**(a)**	Arithmetic Progressions	Sequences and Series
22	**(a)**	Binomial Theorem for Positive Integral Index	Binomial theorem and its applications
23	**(b)**	Properties of Binomial Coefficients	Binomial theorem and its applications
24	**(b)**	Binomial Theorem for Positive Integral Index	Binomial theorem and its applications
25	**(d)**	Permutations	Permutation and combination
26	**(c)**	Permutations	Permutation and combination

Q No	Answer Key	Topic Name	Chapter Name
27	(b)	Permutations	Permutation and combination
28	(c)	Determinant of a Square Matrix	Matrices and determinants
29	(a)	Properties of Determinants	Matrices and determinants
30	(d)	Properties of Determinants	Matrices and determinants
31	(c)	Geometric Progressions	Sequences and Series
32	(b)	Geometric Progressions	Sequences and Series
33	(d)	Geometric Progressions	Sequences and Series
34	(c)	Algebra of Sets	Set Theory
35	(d)	Algebra of Sets	Set Theory
36	(a)	Algebra of Sets	Set Theory
37	(a)	Symmetric and Skew-Symmetric Matrices	Matrices and determinants
38	(d)	Symmetric and Skew-Symmetric Matrices	Matrices and determinants
39	(a)	Determinant of a Square Matrix	Matrices and determinants
40	(c)	Basics of Trigonometry	Trigonometric Ratios, Functions and Identities
41	(b)	Inradius, Exradii and Circumradius	Properties of Triangle
42	(d)	Relations Between Sides and Angles of a Triangle	Properties of Triangle
43	(a)	Heights and Distances	Heights and Distances
44	(d)	Heights and Distances	Heights and Distances
45	(b)	Heights and Distances	Heights and Distances
46	(b)	Trigonometric Functions and Properties	Trigonometric Ratios, Functions and Identities
47	(a)	Trigonometric Identities	Trigonometric Ratios, Functions and Identities
48	(b)	Trigonometric Identities	Trigonometric Ratios, Functions and Identities
49	(d)	Properties of Inverse Trigonometric Functions	Inverse Trigonometric Functions
50	(a)	Relations Between Sides and Angles of a Triangle	Properties of Triangle
51	(c)	Trigonometric Identities	Trigonometric Ratios, Functions and Identities
52	(b)	Basics of Trigonometry	Trigonometric Ratios, Functions and Identities
53	(b)	Trigonometric Functions and Properties	Trigonometric Ratios, Functions and Identities
54	(a)	Trigonometric Functions and Properties	Trigonometric Ratios, Functions and Identities

Q No	Answer Key	Topic Name	Chapter Name
55	**(d)**	Trigonometric Equations	Trigonometric Equations
56	**(c)**	Straight Line and its Equations	Point and Straight Line
57	**(b)**	Straight Line and its Equations	Point and Straight Line
58	**(a)**	Straight Line and its Equations	Point and Straight Line
59	**(c)**	Interaction between Circle and a Line	Circle
60	**(a)**	Basics of Ellipse	Ellipse
61	**(c)**	Basics of Parabola	Parabola
62	**(b)**	Point in Cartesian Plane	Point and Straight Line
63	**(b)**	Sphere	Three Dimensional Geometry
64	**(c)**	Planes in 3D	Three Dimensional Geometry
65	**(c)**	Direction Cosines and Direction Ratios	Three Dimensional Geometry
66	**(d)**	Addition of Vectors	Vector Algebra
67	**(c)**	Scalar and Vector Products	Vector Algebra
68	**(d)**	Triple Products	Vector Algebra
69	**(a)**	Scalar and Vector Products	Vector Algebra
70	**(d)**	Scalar and Vector Products	Vector Algebra
71	**(a)**	Types of Functions	Functions
72	**(a)**	Composite Function	Functions
73	**(b)**	Maxima and Minima	Application of Derivatives
74	**(b)**	Integration by Substitution	Indefinite Integration
75	**(a)**	Integration by Parts	Indefinite Integration
76	**(c)**	Integration by Substitution	Indefinite Integration
77	**(c)**	Basics of Indefinite Integrals	Indefinite Integration
78	**(b)**	Logarithmic Differentiation	Differential Coefficient
79	**(b)**	Range of Trigonometric Expressions	Trigonometric Ratios, Functions and Identities
80	**(c)**	Area Bounded by Curves	Area under curves
81	**(c)**	Area Bounded by Curves	Area under curves
82	**(b)**	Basics of Functions	Functions
83	**(a)**	Even and Odd Functions	Functions
84	**(d)**	Basics of Limits	Limits
85	**(a)**	Methods of Evaluation of Limits	Limits
86	**(d)**	Basics of Limits	Limits
87	**(c)**	Methods of Evaluation of Limits	Limits
88	**(c)**	Rules of Differentiation	Differential Coefficient

Q No	Answer Key	Topic Name	Chapter Name
89	**(b)**	Rules of Differentiation	Differential Coefficient
90	**(d)**	Basics of Differentiation	Differential Coefficient
91	**(a)**	Maxima and Minima	Application of Derivatives
92	**(b)**	Maxima and Minima	Application of Derivatives
93	**(b)**	Maxima and Minima	Application of Derivatives
94	**(b)**	Basics of Definite Integrals	Definite Integration
95	**(d)**	Properties of Definite Integrals	Definite Integration
96	**(a)**	Properties of Definite Integrals	Definite Integration
97	**(b)**	Area Bounded by Curves	Area under curves
98	**(b)**	Basics of Differential Equations	Differential Equations
99	**(b)**	Formation of Differential Equations	Differential Equations
100	**(c)**	Variable Separable Form	Differential Equations
101	**(c)**	Series of Natural Numbers and other Miscellaneous Series	Sequences and Series
102	**(b)**	Basics of Probability	Probability
103	Bonus	Basics of Probability	Probability
104	**(b)**	Basics of Probability	Probability
105	**(c)**	Basics of Probability	Probability
106	**(b)**	Basics of Probability	Probability
107	**(b)**	Addition and Multiplication Theorems of Probability	Probability
108	**(a)**	Addition and Multiplication Theorems of Probability	Probability
109	**(d)**	Basics of Probability	Probability
110	**(c)**	Algebra of Probabilities	Probability
111	**(c)**	Combinations	Permutation and Combination
112	**(d)**	Bernoulli Trials and Binomial Distribution	Probability
113	**(a)**	Basics of Probability	Probability
114	**(c)**	Corelation	Statistics
115	**(b)**	Total Probability Theorem	Probability
116	**(d)**	Measures of Dispersion	Statistics
117	**(a)**	Measures of Dispersion	Statistics
118	**(c)**	Measures of Central Tendency	Statistics
119	**(d)**	Measures of Dispersion	Statistics
120	**(b)**	Measures of Dispersion	Statistics

ANSWERS WITH EXPLANATION

1. **Option (a) is correct.**

Solution:

We have to form 4-digit natural numbers using 1, 3, 5, 7, 9.

So for each digit we have 5 possible numbers.

$\Rightarrow$ Total number of 4-digit numbers possible

$= 5 \times 5 \times 5 \times 5$

$= 625$

Hint: Recall Fundamental principle of counting.

2. **Option (b) is correct.**

Solution:

As we know by binomial theorem, $(1+x)^n = \sum_{r=0}^{n} x^r c(n,r)$

Put $x = 2$ in above equation, we get

$(1+2)^n = \sum_{r=0}^{n} 2^r c(n,r)$

$\Rightarrow \sum_{r=0}^{n} 2^r c(n,r) = 3^n$

Hint: Use $(1+x)^n = \sum_{r=0}^{n} x^r \, {}^nC_r$

3. **Option (c) is correct.**

Solution:

Given alphabets are MATHEMATICS.

In the dictionary the words are arranged alphabetically.

The words in the dictionary before the first word that starts with C will start with A.

So we will fix letter A at first place and arrange 2M, A, 2T, H, E, I, C, S.

$\Rightarrow$ Number of required words $= \dfrac{10!}{2!2!}$

$= 907200$

Hint: The number of permutations of n things taken all at a time when 'p' of them are of same, 'q' of them are of same and rest are different is $\dfrac{n!}{p!q!}$

4. **Option (d) is correct.**

Solution:

Statement 1: $f = \{(xy, x - y), x, y \in z\}$

Let $a, b \in z$ and $x = a, y = b$

$\Rightarrow xy = ab$ and $x - y = a - b$

So image of ab is $a - b$ (1)

Now, as $a, b \in z$

$\Rightarrow -a, -b \in z$

Let $x = -a$ and $y = -b$

$\Rightarrow xy = (-a)(-b) = ab$ and $x - y = (-a) - (-b)$

$= -a + b$

So image of ab is $-a + b$ (2)

From (1) and (2), we get ab has two images $a - b$ and $-a + b$.

And as we know a function is a relationship between inputs and outputs where each input is related to exactly one output.

So f is not a function.

Similarly, Statement 2: $f = \{(xy; x + y), x, y \in N\}$

Let $a, b \in N$

$\Rightarrow 4a, 4b \in N$ and $x = 4a, y = 4b$.

$\Rightarrow xy = 16ab$ and $x + y = 4a + 4b$.

So image of $16ab$ is $4a + 4b$ (3)

Also $8a, 2b \in N$ and $x = 8a, y = 2b$

$\Rightarrow xy = 16\,ab$ and $x + y = 8a + 2b$

So image of $16\,ab$ is $8a + 2b$ (4)

From (3) and (4), f is also not a function.

So both of the statements are incorrect.

Hint: Use definition of function which states that function is a relationship between input and output such that each input is related to exactly one output.

5. Option (a) is correct.

Solution:

Given: $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$a_{13} = yz, a_{23} = zx, a_{33} = xy$

$M_{13} = z - y, M_{23} = z - x, M_{33} = y - x$

$\Rightarrow$ Cofactor of $a_{13} = C_{13} = (-1)^4 M_{13} = (z - y)$

$C_{23} = (-1)^5 M_{23} = -(z - x)$

$C_{33} = (-1)^6 M_{33} = (y - x)$

Now, expanding the determinant along column 3 we get

$\Delta = a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33}$

$= (yz)(z - y) + zx(x - z) + xy\,(y - x)$

$= z^2y - y^2z + x^2z - z^2x + xy^2 - x^2y$

$= z^2(y - x) - z\,(y^2 - x^2) + xy(y - x)$

$= (y - x)(z^2 - z(y + x) + xy)$

$= (y - x)(z^2 - zy - xz + xy)$

$= (y - x)(z\,(z - y) - x\,(z - y))$

$= (y - x)(z - y)(z - x)$

$= (z - y)(z - x)(y - x)$

Hint: Use $\Delta = a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33}$ where C denotes the cofactor.

6. Option (d) is correct.

Solution:

Given: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & -\cos\theta \end{bmatrix}$

$\text{adj}\,A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\theta & -\sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$

$\Rightarrow \text{adj}\,A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\theta & -\sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$

Now, $A + \text{adj}\,A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ = Null Matrix

Now, $A^{-1} = \dfrac{1}{|A|}(\text{adj}\,A)$

$|A| = 1(-\cos^2\theta - \sin^2\theta) - 0 + 0 = -1$

$\Rightarrow A^{-1} = (-1)\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\theta & -\sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & -\cos\theta \end{bmatrix}$

$A^{-1} + \text{adj}(A) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ = Null matrix

$A - A^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ = Null matrix

7. Option (d) is correct.

Solution:

Order of X is 3×3

Order of Y is 2×3

Order of Z is 3×2

Now, (ZY) X $= ([\ ]_{3\times2}\,[\ ]_{2\times3})\,[\ ]_{3\times3}$

$= ([\ ]_{3\times3})[\ ]_{3\times3}$

$= [\ ]_{3\times3}$

(ZY) X has 9 entries.

Similarly, Y(XZ) $= [\ ]_{2\times3}\,([\ ]_{3\times3}\,[\ ]_{3\times2})$

$= [\ ]_{2\times3}\,[\ ]_{3\times2}$

$= [\ ]_{2\times2}$

Y(XZ) has 4 entries

Now, X(YZ) $= [\ ]_{3\times3}\,([\ ]_{2\times3}\,[\ ]_{3\times2})$

$= [\ ]_{3\times3}\,[\ ]_{2\times2}$

X(YZ) is not defined as number of columns of the first matrix is not equal to the number of rows of the second matrix.

Hint: The multiplication of two matrices is possible if number of columns of the first matrix is equal to the number of rows of the second matrix.

8. Option (d) is correct

Solution:

Let the quadratic equation be $ax^2 + bx + c = 0$ and roots be α and β.

$\Rightarrow \alpha + \beta = \dfrac{-b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

Now, $\alpha + \beta = \alpha\beta$

$\Rightarrow \dfrac{-b}{a} = \dfrac{c}{a}$

$\Rightarrow b + c = 0$

$\therefore$ There are infinite possible values for which $b + c = 0$ so infinetly many quadratic equations possible.

Hint: For a quadratic equation $ax^2 + bx + c = 0$ sum of roots $= \frac{-b}{a}$ and product of roots $= \frac{c}{a}$.

9. Option (a) is correct.

Solution:

Statement 1: The set of all irrational numbers between $\sqrt{2}$ and $\sqrt{3}$ is an infinite set.

As we know, there are infinite irrational numbers between two irrational numbers.

So Statement 1 is true.

Statement 2: The set of odd integers less then 100 is a finite set.

Let the given set be X.

X $\in$ {..., –5, –3, –1, 1, 3, 5, ... 99}

So it is an infite set.

So statement 2 is false.

Hint: The number of irrational numbers between two irrational numbers are infinite.

10. Option (a) is correct

Solution:

$2 + 4 + 6 + ... + 2n = x$

$x = 2(1 + 2 + 3 + ... + n)$

$x = \frac{2[n(n+1)]}{2}$

$\Rightarrow x = n(n + 1)$

$\Rightarrow x = n^2 + n$

Let $y = n^2 + n + 41$

$\Rightarrow y = n(n + 1) + 41$

for $n = 40$

$y = 40(40 + 1) + 41$

$\Rightarrow y = 40(41) + 41$

$\Rightarrow y = 40(41) + 41$

y is not a prime number.

So only Statement 1 is correct.

Hint: Sum of first n natural numbers is $\frac{n(n+1)}{2}$

11. Option (a) is correct

Solution:

p, q are the roots of $x^2 + bx + c = 0$

$p + q = -b$ and $pq = c$

Now, $p^2 + q^2 - 11pq = 0$

$\Rightarrow p^2 + q^2 - 2pq - 9pq = 0$

$\Rightarrow (p - q)^2 - 9pq = 0$

$\Rightarrow (p - q)^2 = 9pq$

$\Rightarrow (p - q)^2 = 9c$

$\Rightarrow p - q = 3\sqrt{c}$

Hint: Use sum of roots of $ax^2 + bx + c = 0$ is $\frac{-b}{a}$ and product of roots is $\frac{c}{a}$.

12. Option (c) is correct.

Solution:

Given a regular polygon of 12 sides interior angle of a regular polygon of n sides

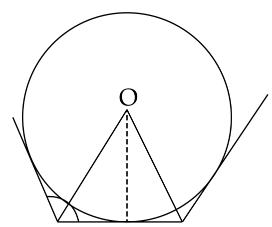
$= \frac{(n-2)\times 180^\circ}{n}$

So, interior angle of given regular polygon

$= \frac{(12-2)\times 180^\circ}{12} = 150^\circ$

The interior angle is bisected by the radius of the inscribed circle.

$\angle OAP = \frac{150^\circ}{2} = 75^\circ$



Let OP = r and AB = 1

$\Rightarrow AP = \frac{1}{2}$

$\therefore$ In ΔOAP,. tan 75° $= \frac{r}{1/2}$

$\Rightarrow 2 + \sqrt{3} = 2r$

Diameter = $(2 + \sqrt{3})$ cm

Hint: Interior angle of a regular polygon of n sides is $\frac{(n-2)}{n}\times 180^\circ$

13. Option (c) is correct.

Solution:

A = {7, 8, 9, 10, 11, 12, 13, 14, 15, 16} f: A $\to$ N,

$f(x)$ = The highest prime factor of x

$f(7) = 7$

$f(8) = 2$ $\{\because 8 = 2 \times 2 \times 2\}$

$f(9) = 3$ $\{\because 9 = 3 \times 3\}$

$f(10) = 5$ $\{\because 10 = 5 \times 2)$

$f(11) = 11$ {∵ $11 = 11 \times 1$}
$f(12) = 3$ {∵ $12 = 3 \times 2 \times 2$}
$f(13) = 13$ {∵ $13 = 13 \times 1$}
$f(14) = 7$ {∵ $14 = 7 \times 2$}
$f(15) = 5$ {∵ $15 = 5 \times 3$}
$f(16) = 2$ {∵ $16 = 2 \times 2 \times 2 \times 2$}
Range of f = {2, 3, 5, 7, 11, 13}
Number of elements = 6

Hint: Find highest prime factor of each element of A.

14. Option (d) is correct.

Solution:

$R = \{(x, y) : x, y \in N \text{ and } x^2 = y^3\}$

For $x = y$, $x^2 \neq x^3$

$\Rightarrow (x, x) \notin R$

Now, $(x, y) \in R$

$\Rightarrow x^2 = y^3$

$\Rightarrow x^3 = (x^2)(x)$

$\Rightarrow x^3 = y^3 x$

$\Rightarrow x^3 \neq y^2$

$\Rightarrow (y, x) \notin R$

Now, $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow x^2 = y^3$ and $y^2 = z^3$

$\Rightarrow x^2 = y(z^3)$

$\Rightarrow x^2 \neq z^2$

$\Rightarrow (x; z) \notin R$

Hint: Let $x = 2 \Rightarrow 2^2 \neq 2^3$
$\Rightarrow (x, x) \notin R$
Also, $(8, 4) \in R$ as $8^2 = 4^3$
but $4^2 \neq 8^3$
$\Rightarrow (4, 8) \notin R$

15. Option (d) is correct.

Solution:

Let A = {2, 4, 6, 8, 10}
B = {2, 4, 12}
C = {2, 4, 14}

$\Rightarrow A \cap B = \{2, 4\}$ and $A \cap C = \{2, 4\}$

$\Rightarrow A \cap B = A \cap C$

But $B \neq C$

So statement 1 is incorrect.

Let A = {2, 4, 6, 8, 10}
B = {2, 4, 6}
C = {2, 4, 8}

$\Rightarrow A \cup B = A$ and $A \cup C = A$

$\Rightarrow A \cup B = A \cup C$

But $B \neq C$

So statement 2 is also incorrect.

Hints: (1) Intersection of two sets contain elements present in both sets.
(2) Union of two sets contain all elements present in both sets.

16. Option (a) is correct.

Solution:

$$z = \frac{1 + i\sin\theta}{1 - i\sin\theta}$$

Now, $|z| = \left|\frac{1 + i\sin\theta}{1 - i\sin\theta}\right|$

$$\Rightarrow |z| = \frac{|1 + i\sin\theta|}{|1 - i\sin\theta|}$$

$$\Rightarrow |z| = \frac{\sqrt{1 + \sin^2\theta}}{\sqrt{1 + \sin^2\theta}}$$

$\Rightarrow |z| = 1$

17. Option (c) is correct.

Solution:

$$z = \frac{1 + i\sin\theta}{1 - i\sin\theta} \times \frac{(1 + i\sin\theta)}{(1 + i\sin\theta)}$$

$$\Rightarrow z = \frac{(1 + i\sin\theta)^2}{(1 - i^2\sin^2\theta)}$$

$$\Rightarrow z = \frac{(1 - \sin^2\theta) + (2\sin\theta)i}{1 + \sin^2\theta}$$

z is purely real if img $(z) = 0$

$$\Rightarrow \frac{2\sin\theta}{1 + \sin^2\theta} = 0$$

$\Rightarrow \sin\theta = 0$

$\Rightarrow \theta = n\pi$

Hint: Simplify z and put img $(z) = 0$.

18. Option (b) is correct.

Solution:

$$z = \frac{1 + i\sin\theta}{1 - i\sin\theta} \times \frac{1 + i\sin\theta}{1 + i\sin\theta}$$

$$\Rightarrow z = \frac{(1 - \sin^2\theta) + 2i\sin\theta}{1 + \sin^2\theta}$$

z is purely imaginary, if Re(z) = 0.

$\Rightarrow 1 - \sin^2\theta = 0$

$\Rightarrow \sin\theta = \pm 1$

$\Rightarrow \theta = \dfrac{(2n+1)\pi}{2}$

Hint: Simplify z and put Re(z) = 0.

19. Option (d) is correct.

Solution:

Given: P = sum of first n positive terms of arithmetic progression A.

Q = Sum of first n positive terms of another arithmetic progression B.

Let first term of A be a and first term of Q be b.

For n = 1, P = a and Q = b.

$\because \dfrac{P}{Q} = \dfrac{5n+4}{9n+6}$

For n = 1, $\dfrac{P}{Q} = \dfrac{a}{b} = \dfrac{9}{15}$

$\Rightarrow \dfrac{a}{b} = \dfrac{3}{5}$

Hint: For n = 1, P = First term of A and Q = First term of B.

20. Option (c) is correct.

Solution:

Let first term of arithmetic progression A be a and common difference be d_1.

And first term of arithmetic progressoin B be b and common difference be d_2.

So, $P = \dfrac{n}{2}[2a+(n-1)d_1]$

And, $Q = \dfrac{n}{2}[2b+(n-1)d_2]$

$\because \dfrac{P}{Q} = \dfrac{5n+4}{9n+6}$

$\Rightarrow \dfrac{2a+(n-1)d_1}{2b+(n-1)d_2} = \dfrac{5n+4}{9n+6}$ ...(i)

Now, 10^{th} term of A, $(T_{10})_A = a + 9d_1$.

10^{th} term of B, $(T_{10})_B = b + 9d_2$.

Now, $\dfrac{(T_{10})_A}{(T_{10})_B} = \dfrac{a+9d_1}{b+9d_2}$

$\dfrac{(T_{10})_A}{(T_{10})_B} = \dfrac{2a+18d_1}{2b+18d_2}$

Put n = 19 in equation (i), we get

$\dfrac{2a+18d_1}{2b+18d_2} = \dfrac{5(19)+4}{9(19)+6} = \dfrac{99}{177} = \dfrac{33}{59}$

$\Rightarrow \dfrac{(T_{10})_A}{(T_{10})_B} = \dfrac{33}{59}$

Hints:

1. n^{th} term of AP is $T_n = a + (n-1)d$, where a = first term and d = common difference.

2. Sum of nth term of A.P. is $S_n = \dfrac{n}{2}[2a+(n-1)d]$ where a = first term and d = common difference.

21. Option (a) is correct.

Solution:

Let first term of A be a and first term of B be b.

Given: d = Common difference of A

D = Common difference of B.

$\because \dfrac{P}{Q} = \dfrac{5n+4}{9n+6}$

For n = 1, $\dfrac{a}{b} = \dfrac{9}{15} = \dfrac{3}{5}$ ...(i)

$\Rightarrow a < b$

For n = 2, $\dfrac{2a+d}{2b+D} = \dfrac{14}{24} = \dfrac{7}{12}$

$\Rightarrow 24a + 12d = 14b + 7D$

$\Rightarrow 24a - 14b + 12d = 7D$

$\Rightarrow 7D = 24a - 14\left(\dfrac{5}{3}a\right) + 12d$

$\Rightarrow 7D = \dfrac{2a}{3} + 12d$

$\Rightarrow D = \dfrac{2a}{21} + \dfrac{12}{7}d$

$\Rightarrow D > d$

Hint: Put n = 1 and n = 2 in given relation and solved further.

22. Option (a) is correct.

Solution:

Given: The binomial expansion of $(p + qx)^9$ general term $T_{r+1} = {}^9C_r(P)^{9-r}(qx)^r$

$= {}^9C_r(P)^{9-r}(q)^r x^r$

Now, coefficient of x^3 = coefficient of x^6

$\Rightarrow {}^9C_3(P)^{9-3}(q)^3 = {}^9C_6(P)^{9-6}(q)^6$

$\Rightarrow {}^9C_3p^6q^3 = {}^9C_3p^3q^6$

$\Rightarrow p^3 = q^3$

$\Rightarrow p = q$

Hint: Find the general term of given expansion and then find the coefficient of x^3 and x^6.

23. Option (b) is correct.

Solution:

Given expansion of $(p + qx)^9$

The middle term of given expansion are $\frac{9+1}{2}$ and $\frac{9+3}{2}$

$\Rightarrow 5^{th}$ and 6^{th} term.

$\because$ General term $T_{r+1} = {}^9C_r(P)^{9-r}(qx)^r$

$\Rightarrow T_5 = {}^9C_4(P)^5q^4x^4$ and $T_6 = {}^9C_5(P)^4q^5x^5$

$$\Rightarrow \frac{\text{Coefficient of } T_5}{\text{Coefficient of } T_6} = \frac{{}^9C_4P^5q^4}{{}^9C_5P^4q^5}$$

$$= \frac{{}^9C_4p}{{}^9C_4q}$$

$$= \frac{p}{q}$$

Hint: The middle terms in the expansion of $(a + bx)^n$ for n is odd are $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ terms.

24. Option (b) is correct.

Solution:

$T_{r+1} = {}^9C_r(p)^{9-r}(qx)^r$

Coefficient of x^2 = coefficient of x^4

${}^9C_2p^7q^2 = {}^9C_4p^5q^4$

$$\Rightarrow \frac{p^2}{q^2} = \frac{{}^9C_4}{{}^9C_2} = \frac{9\times8\times7\times6}{4\times3\times2}\times\frac{2}{9\times8}$$

$$\Rightarrow \frac{p^2}{q^2} = \frac{7}{2}$$

Hint: The general term of expansion of $(a + bx)^n$ is $T_{r+1} = {}^nC_r(a)^{n-r}(bx)^r$

25. Option (d) is correct.

Solution:

Given word is QUESTION

Vowels are UEIO and consonants are QSTN.

We have to form 4 letter word using 2 vowels and 2 consonants.

So firstly we will choose 2 vowels and 2 consonants and then arrange them

So required words = $4_{C_2} \times 4_{C_2} \times 4!$

$$= \frac{4\times3}{2}\times\frac{4\times3}{2}\times4\times3\times2$$

= 864

Hint: Choose 2 vowels and 2 consonents and then arrange them.

26. Option (c) is correct.

Solution:

There are 4 vowels UEIO and 4 consonants QSTN in QUESTION

For alternate positions of vowels and consonants there are 2 possibilities.

VCVCVCVC or CVCVCVCV

So we will arrange 4 vowels and 4 consonants seperately.

So required words are 2 × 4! × 4!

= 2 × 24 × 24

= 1152

Hint: For alternate positions of vowels and consonents 2 possibilities are VCVCVCVC or CVCVCVCV.

27. Option (b) is correct.

Solution:

There are 4 vowels UEIO and 4 consonants QSTN.

Let us consider 4 consonants as one alphabet. We will arrange these 5 alphabets and then we will arrange 4 consonants.

So required words = 5! × 4!

= 120 × 24

= 2880

Hint: Consider 4 consonants as one alphabet and then arrange them.

28. Option (c) is correct.

Solution:

$$\text{Given } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

c_{11}, c_{12} and c_{13} are cofactors of a_{11}, a_{12}, a_{13}.

As we know,

$|A| = a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13}$

$\Delta = a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13}$

Hint: Use formula to find the value of a determinant.

29. Option (a) is correct.

Solution:

$$\text{Given } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$c_{11} = (-1)^2 (a_{22}a_{33} - a_{23}a_{32})$

$c_{12} = (-1)^3 (a_{21}a_{33} - a_{31}a_{23})$

$c_{13} = (-1)^4 (a_{21}a_{32} - a_{22}a_{31})$

$\Rightarrow a_{21}c_{11} + a_{22}c_{12} + a_{23}c_{13}$

$= a_{21}(a_{22}a_{33} - a_{23}a_{32}) - a_{22}(a_{21}a_{33} - a_{31}a_{23}) + a_{23}(a_{21}a_{32} - a_{22}a_{31})$

$\Rightarrow a_{21}c_{11} + a_{22}c_{12} + a_{23}c_{13} = 0$

Hint: Use $C_{ij} = (-1)^{i+j} \det (M_{ij})$.
where C_{ij} is cofactor and M_{ij} is minor.

30. Option (d) is correct.

Solution:

$$\text{Given } \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Transposing the elements we get,

$$\Delta = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

Applying $c_1 \leftrightarrow c_2$, we get

$$\Rightarrow \Delta = (-1)\begin{vmatrix} a_{21} & a_{11} & a_{31} \\ a_{22} & a_{12} & a_{32} \\ a_{23} & a_{13} & a_{33} \end{vmatrix}$$

Again applying $c_2 \leftrightarrow c_3$, we get

$$\Rightarrow \Delta = \begin{vmatrix} a_{21} & a_{31} & a_{11} \\ a_{22} & a_{32} & a_{12} \\ a_{23} & a_{33} & a_{13} \end{vmatrix}$$

Applying $R_2 \leftrightarrow R_3$, we get

$$\Rightarrow \Delta = (-1)\begin{vmatrix} a_{21} & a_{31} & a_{11} \\ a_{23} & a_{33} & a_{13} \\ a_{22} & a_{32} & a_{12} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} a_{21} & a_{31} & a_{11} \\ a_{23} & a_{33} & a_{13} \\ a_{22} & a_{32} & a_{12} \end{vmatrix} = -\Delta$$

Hint: Apply row and column transformations to get the desired result.

31. Option (c) is correct.

Solution:

$f(x + y) = f(x) f(y)$ and $f(1) = 2$

$\Rightarrow f(2) = f(1)\ f(1) = 2^2$, $f(3) = f(2)\ f(1) = 2^3$, $f(4) = 2^4$

$\Rightarrow f(n) = 2^n$.

Now, $\sum_{x=2}^{n} f(x) = 2044$

$\Rightarrow 2^2 + 2^3 + ... + 2^n = 2044.$

$\Rightarrow \dfrac{2^2(2^{n-1} - 1)}{2-1} = 2044 \quad \left\{\text{Sum of GP} = \dfrac{a(r^n - 1)}{r-1}\right\}$

$\Rightarrow 2^{n-1} = 511 - 1 = 510$

$\Rightarrow n - 1 = 9$

$\Rightarrow n = 10$

Hint: Use sum of a GP with a as first term, r as common ratio and n number of terms is $\dfrac{a(r^n - 1)}{r - 1}$

32. Option (b) is correct.

Solution:

$f(x + y) = f(x) f(y)$ and $f(1) = 2$

$\Rightarrow f(2) = f(1) f(1) = 2^2, f(3) = f(2) f(1) = 2^3, f(4) = 2^4$

$\Rightarrow f(n) = 2^n$.

$\therefore \sum_{x=1}^{5} f(2x-1) = f(1) + f(3) + f(5) + f(7) + f(9)$

$= 2 + 2^3 + 2^5 + 2^7 + 2^9$

$= \dfrac{2(2^2)^5 - 1}{2^2 - 1} \quad \left\{\text{Sum of GP} = \dfrac{a(r^n - 1)}{r-1}\right\}$

$= \dfrac{2(1024 - 1)}{3}$

$= \frac{2 \times 1023}{3}$

$= 682$

Hint: Use sum of GP $= \frac{a(r^n - 1)}{r - 1}$, where a = first term and r = common ratio.

33. Option (d) is correct.

Solution:

$f(x + y) = f(x) f(y)$ and $f(1) = 2$

$\Rightarrow f(2) = f(1) f(1) = 2^2, f(3) = f(2) f(1) = 2^3, f(4) = f(3) f(1) = 2^4$

$\Rightarrow f(n) = 2^n$.

$$\therefore \sum_{x=1}^{6} 2^x f(x) = 2f(1) + 2^2 f(2) + 2^3 f(3) + 2^4 f(4) + 2^5 f(5) + 2^6 f(6)$$

$= 2(2) + 2^2(2^2) + 2^3(2^3) + 2^4(2^4) + 2^5(2^5) + 2^6(2^6)$

$= 2^2 + 2^4 + 2^6 + 2^8 + 2^{10} + 2^{12}$

$= \frac{2^2(2^{12} - 1)}{2^2 - 1}$ $\left\{\text{Sum of GP} = \frac{a(r^n - 1)}{r - 1}\right\}$

$= \frac{4 \times 4095}{3}$

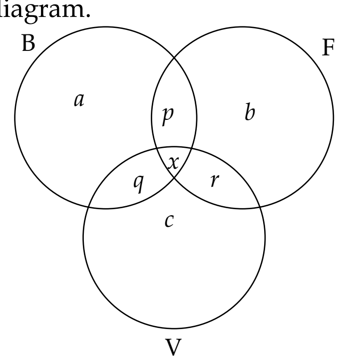
$= 5460$

Hint: Use sum of GP $= \frac{a(r^n - 1)}{r - 1}$, where a = first term and r = common ratio.

34. Option (c) is correct.

Solution:

Let B, F, V represents Basketball, football and volleyball and a, b, c, p, q, r, x represented in venn diagram.



$a + b + c + p + q + r + x = 15x$

$\Rightarrow a + b + c + p + q + r = 14x$ ....(1)

& $a + p + q + x = 5x$

& $a + p + q = 4x$ ....(2)

& $b + p + r + x = 4x + 15$

& $b + p + r = 3x + 15$ ....(3)

& $c + q + r + x = x + 25$

& $c + q + r = 25.$ ....(4)

Adding above (2)+(3)+(4) equations, we get

$a + b + c + 2(p + q + r) = 7x + 40$ (5)

Eq. (5) – (1), we get

$p + q + r = 7x + 40 - 14x = 40 - 7x$

Hint: Use venn diagram to solve it.

35. Option (d) is correct.

Solution:

Using final answer of question No. 34

Medals in at least two sports $= p + q + r + x$

$= 40 - 7x + x$

$= 40 - 6x$

Hint: Use venn diagram to solve it.

36. Option (a) is correct.

Solution:

Medals in exactly one of the sport $= a + b + c$.

Putting $p + q + r = 40 - 7x$ in equation (1) of question No. 34. we get.

$a + b + c = 14x - 40 + 7x$

$= 21x - 40$

Hint: Use venn diagram to solve it.

37. Option (a) is correct.

Solution:

$$A = \begin{bmatrix} 0 & \sin^2\theta & \cos^2\theta \\ \cos^2\theta & 0 & \sin^2\theta \\ \sin^2\theta & \cos^2\theta & 0 \end{bmatrix} \text{ and } A = P + Q$$

$\because A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

where $\frac{1}{2}(A + A^T)$ is a symmetric matrix.

$\Rightarrow P = \frac{1}{2}(A + A^T)$

$$\frac{1}{2}\left\{\begin{bmatrix} 0 & \sin^2\theta & \cos^2\theta \\ \cos^2\theta & 0 & \sin^2\theta \\ \sin^2\theta & \cos^2\theta & 0 \end{bmatrix} + \begin{bmatrix} 0 & \cos^2\theta & \sin^2\theta \\ \sin^2\theta & 0 & \cos^2\theta \\ \cos^2\theta & \sin^2\theta & 0 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & \sin^2\theta+\cos^2\theta & \cos^2\theta+\sin^2\theta \\ \cos^2\theta+\sin^2\theta & 0 & \sin^2\theta+\cos^2\theta \\ \sin^2\theta+\cos^2\theta & \cos^2\theta+\sin^2\theta & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Hint: Use $A = \frac{1}{2}(A - A^T) + \frac{1}{2}(A - A^T)$ where $\frac{1}{2}(A + A^T)$ is a symmetric matrix.

38. Option (d) is correct.

Solution:

$$A = \begin{bmatrix} 0 & \sin^2\theta & \cos^2\theta \\ \cos^2\theta & 0 & \sin^2\theta \\ \sin^2\theta & \cos^2\theta & 0 \end{bmatrix} \text{ and } A = P + Q$$

$\because A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

where $\frac{1}{2}(A + A^T)$ is a skew symmetric matrix.

$\Rightarrow Q = \frac{1}{2}(A - A^T)$

$$\frac{1}{2}\left\{\begin{bmatrix} 0 & \sin^2\theta & \cos^2\theta \\ \cos^2\theta & 0 & \sin^2\theta \\ \sin^2\theta & \cos^2\theta & 0 \end{bmatrix} - \begin{bmatrix} 0 & \cos^2\theta & \sin^2\theta \\ \sin^2\theta & 0 & \cos^2\theta \\ \cos^2\theta & \sin^2\theta & 0 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & \sin^2\theta-\cos^2\theta & \cos^2\theta-\sin^2\theta \\ \cos^2\theta-\sin^2\theta & 0 & \sin^2\theta-\cos^2\theta \\ \sin^2\theta-\cos^2\theta & \cos^2\theta-\sin^2\theta & 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -\cos 2\theta & \cos 2\theta \\ \cos 2\theta & 0 & -\cos 2\theta \\ -\cos 2\theta & \cos 2\theta & 0 \end{bmatrix}$$

$$= \cos 2\theta \begin{bmatrix} 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

Hint: Use $A = \frac{1}{2}(A - A^T) + \frac{1}{2}(A - A^T)$ where $\frac{1}{2}(A + A^T)$ is a skew symmetric matrix.

39. Option (a) is correct.

Solution:

$$A = \begin{bmatrix} 0 & \sin^2\theta & \cos^2\theta \\ \cos^2\theta & 0 & \sin^2\theta \\ \sin^2\theta & \cos^2\theta & 0 \end{bmatrix}$$

$|A| = 0 - \sin^2\theta(0 - \sin^4\theta) + \cos^2\theta(\cos^4\theta - 0)$

$\Rightarrow |A| = \sin^6\theta + \cos^6\theta$

$\Rightarrow |A| = (\sin^2\theta + \cos^2\theta)$
$(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

$\Rightarrow |A| = (1)\,[\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta]$

$\Rightarrow |A| = 1 - 3(\sin\theta\cos\theta)^2$

$\Rightarrow |A| = 1 - \frac{3}{4}\sin^2 2\theta$

For $|A|$ to be minimum, $\sin^2 2\theta = 1$

$\Rightarrow |A| = 1 - \frac{3}{4} = \frac{1}{4}$

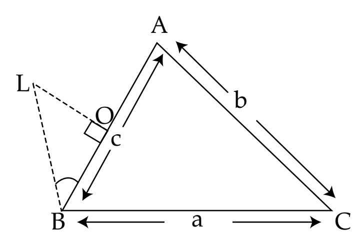
Hint: Use trigonometric and algebraic identities to solve it.

40. Option (c) is correct.

Solution:

ABC is a plot. Let OL be the lamp post.

$\angle OBL = 45°$ and $\angle BOL = 90°$



In Δ BOL, $\tan 45° = \frac{OL}{OB} = \frac{OL}{8}$

$\Rightarrow OL = 8$.

Hint: Use $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

41. Option (b) is correct.

Solution:

From the given question no. 40,

As we know, by sin rule.

$$\frac{AB}{\sin C} = \frac{BC}{\sin A} = \frac{AC}{\sin B} = 2R$$

Where R is a circumradius and $R = \frac{abc}{4\Delta}$ where

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ and $s = \frac{AB+CB+CA}{2}$

Here $s = \frac{16+10+10}{2} = 18$

$\Delta = \sqrt{18(18-16)(18-10)(18-10)}$

$\Rightarrow \Delta = \sqrt{18\times 2\times 8\times 8}$

$\Rightarrow \Delta = 48$

$\Rightarrow$ **R =** $\frac{(AB)(BC)(CA)}{4\Delta} = \frac{16\times 10\times 10}{4\times 48} = \frac{25}{3}$

$\Rightarrow \frac{AB}{\sin c} = 2R = 2\times\frac{25}{3} = \frac{50}{3}$

Hint:
Use sin rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin c} = 2\left(\frac{abc}{4\Delta}\right)$

where $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ and

$s = \frac{a+b+c}{2}$

42. Option (d) is correct.

Solution:

As we know, by cosine rule

$$\cos A = \frac{b^2+c^2-a^2}{2bc}, \cos B = \frac{a^2+c^2-b^2}{2ac},$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

From the given question no. 40,

$\Rightarrow$ cos A + cos B + cos C

$= \frac{(10)^2+(16)^2-(10)^2}{2(10)(16)} + \frac{(10)^2+(16)^2-(10)^2}{2(10)(16)} + \frac{(10)^2+(10)^2-(16)^2}{2(10)(10)}$

$= \frac{100+256-100}{320} + \frac{100+256-100}{320} + \frac{100+100-256}{200}$

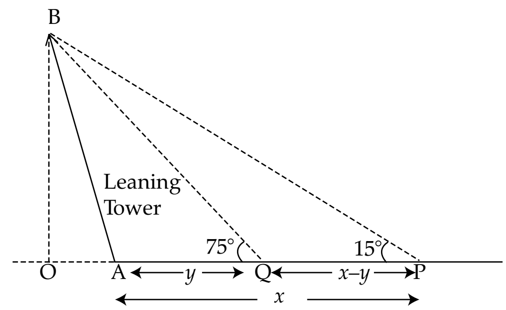
$= \frac{4}{5}+\frac{4}{5}-\frac{7}{25}$

$= \frac{20+20-7}{25} = \frac{33}{25}$

Hint: Use cosine rule, cos A = $\frac{b^2+c^2-a^2}{2bc}$

43. Option (a) is correct.

Solution:



Let AB be learning tower which leans towards north. And OB = height of top of the tower above ground level.

In ΔOBQ, tan 75° = $\frac{OB}{OA+y}$

$\Rightarrow$ OA + y = OB cot 75° ...(i)

In ΔOBP, tan 15° = $\frac{OB}{OA+x}$

$\Rightarrow$ OA + x = OB cot 15° ...(ii)

Equation (ii) – Equation (i), we get

$x - y$ = OB (cot 15° – cot 75°)

$\Rightarrow OB = \frac{x-y}{(2+\sqrt{3})-(2-\sqrt{3})}$

$\Rightarrow OB = \frac{x-y}{2\sqrt{3}}$

Hint:
Find OB using the concept of height and distance.

44. Option (d) is correct.

Solution:

From the given question No.43.

$h = \frac{x-y}{2\sqrt{3}}$

In ΔOBP, tan 15° = $\frac{h}{OA+x}$

$\Rightarrow$ OA + x = h cot 15° ...(ii)

Put value of h in equation (ii), we get

$\Rightarrow OA + x = \frac{x-y}{2\sqrt{3}}\cdot(2+\sqrt{3})$

$\Rightarrow 2\sqrt{3}OA = (2+\sqrt{3})x - y(2+\sqrt{3}) - 2\sqrt{3}x$

$$\Rightarrow 2\sqrt{3}OA = x(2-\sqrt{3}) - y(2+\sqrt{3})$$

$$\Rightarrow OA = \frac{1}{2\sqrt{3}}\left(x(2-\sqrt{3}) - y(2+\sqrt{3})\right) \quad ...(iii)$$

Now, $\cot\theta = \frac{OA}{n}$

$$\Rightarrow \cot\theta = \frac{\frac{1}{2\sqrt{3}}\left(2(x-y) - \sqrt{3}(x+y)\right)}{\frac{1}{2\sqrt{3}}(x-y)}$$

$$\Rightarrow \cot\theta = 2 - \sqrt{3}\frac{(x+y)}{x-y}$$

45. Option (b) is correct.

Solution:

From the given question no. 43.

Now, $\cot\theta = \frac{OA}{h}$

Using equation No (iii) of question no. 43.

$$\Rightarrow \cot\theta = \frac{\frac{1}{2\sqrt{3}}\left(2(x-y) - \sqrt{3}(x+y)\right)}{\frac{1}{2\sqrt{3}}(x-y)}$$

$$\Rightarrow \cot\theta = 2 - \sqrt{3}\frac{(x+y)}{x-y}$$

$\Rightarrow$ In ΔOAB, $\sin\theta = \frac{OB}{AB}$

$\Rightarrow AB = OB \operatorname{cosec}\theta$

$\Rightarrow AB = \frac{x-y}{2\sqrt{3}} \operatorname{cosec}\theta \quad ...(iii)$

As we know $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$$\Rightarrow \operatorname{cosec}^2\theta = 1 + \left(2 - \sqrt{2}\frac{(x+y)}{x-y}\right)^2$$

$$\Rightarrow \operatorname{cosec}\theta = \sqrt{1 + \left\{2 - \sqrt{3}\frac{(x+y)}{x-y}\right\}^2}$$

Put value of cosecθ in equation (iii), we get

$$AB = \frac{x-y}{2\sqrt{3}}\sqrt{1 + \left\{2 - \frac{\sqrt{3}(x+y)}{x-y}\right\}^2}$$

46. Option (b) is correct.

Solution:

Let $A = \operatorname{cosec}\left(-\frac{73\pi}{3}\right)$

$$\Rightarrow A = \operatorname{cosec}\left(\frac{73\pi}{3}\right) \quad \{\because \operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta\}$$

$$\Rightarrow A = -\operatorname{cosec}\left(24\pi + \frac{\pi}{3}\right)$$

{∵ cosec(2π + θ) = −cosec θ in Ist quadrant.}

$$\Rightarrow A = -\operatorname{cosec}\frac{\pi}{3}$$

$$\Rightarrow A = -\left(\frac{2}{\sqrt{3}}\right)$$

Hint: Use cosec (−θ) = −cosec θ and cosec (2π + θ) = cosec θ in Ist quadrant.

47. Option (a) is correct.

Solution:

Let

$$A = \cos\left(\frac{5\pi}{17}\right) + \cos\left(\frac{7\pi}{17}\right) + 2\cos\left(\frac{11\pi}{17}\right)\cos\left(\frac{\pi}{17}\right)$$

$$\Rightarrow A = 2\cos\left(\frac{\frac{5\pi}{17} + \frac{7\pi}{17}}{2}\right)\cos\left(\frac{\frac{7\pi}{17} - \frac{5\pi}{17}}{2}\right) + 2\cos\left(\frac{11\pi}{17}\right)\cos\left(\frac{\pi}{17}\right)$$

$$\Rightarrow A = 2\cos\left(\frac{6\pi}{17}\right)\cos\left(\frac{\pi}{17}\right) + 2\cos\left(\frac{11\pi}{17}\right)\cos\left(\frac{\pi}{17}\right)$$

$$\left[\because \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$$

$$\Rightarrow A = 2\cos\left(\frac{\pi}{17}\right)\left\{\cos\left(\frac{6\pi}{17}\right) + \cos\left(\frac{11\pi}{17}\right)\right\}$$

$$\Rightarrow A = 2\cos\left(\frac{\pi}{17}\right)2\cos\left(\frac{\pi}{2}\right)\cos\left(\frac{51\pi}{34}\right)$$

$$\Rightarrow A = 0 \quad \left\{\because \cos\frac{\pi}{2} = 0\right\}$$

Hint: Use

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

48. Option (b) is correct

Solution:

Let $A = \tan\left(\frac{3\pi}{8}\right)$

$$\Rightarrow A = \tan\left(\frac{\pi}{2} - \frac{\pi}{8}\right)$$

$$\Rightarrow A = \cot\left(\frac{\pi}{8}\right) \quad \left\{\because \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta\right\}$$

$$\Rightarrow A = \frac{\cos(\pi/8)}{\sin(\pi/8)}$$

$$\Rightarrow A = \frac{\sqrt{1/2(1+\cos\pi/4)}}{\sqrt{1/2(1-\cos\pi/4)}}$$

$$\left\{\because \begin{array}{l} 1+\cos 2\theta = 2\cos^2\theta \\ 1-\cos 2\theta = 2\sin^2\theta \end{array}\right\}$$

$$\Rightarrow A = \sqrt{\frac{1+\cos\pi/4}{1-\cos\pi/4}}$$

$$\Rightarrow A = \sqrt{\frac{1+1/\sqrt{2}}{1-1/\sqrt{2}}}$$

$$\Rightarrow A = \sqrt{\frac{(\sqrt{2}+1)(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}}$$

$$\Rightarrow A = \sqrt{2}+1$$

Hint: Use $2\cos^2\theta = 1 + \cos^2\theta$ and $2\sin^2\theta = 1 - \cos^2\theta$.

49. Option (d) is correct.

Solution:

Let A = $\tan^{-1}\cot(\text{cosec}^{-1}2)$ & $\left\{\because \cos 36° = \frac{\sqrt{5}+1}{4}\right.$

$\Rightarrow$ A = $\tan^{-1}[\cot(\cot^{-1}\sqrt{3})]$ $\left.\sin 18° = \frac{\sqrt{5}-1}{4}\right\}$

$$\left\{\because \text{cosec}\,\theta = \frac{h}{p}, \cot\theta = \frac{b}{p}\right\}$$

$\Rightarrow$ A = $\tan^{-1}(\sqrt{3})]$ $\{\because \cot(\cot^{-1}\alpha) = \alpha\}$

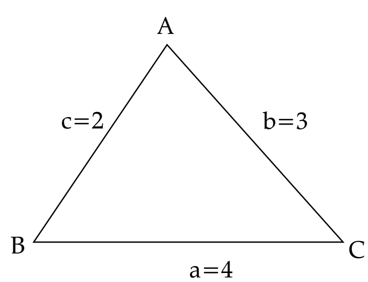
$\Rightarrow$ A = $\pi/3$

Hint: Use $\text{cosec}\,\theta = \frac{\text{Hyp.}}{\text{perp.}}$ and $\cot\theta = \frac{\text{Base}}{\text{perp.}}$

50. Option (a) is correct.

Solution:

As we know, $\cos 3C = 4\cos^3 C - 3\cos C$



Also by cosine rule, $\cos C = \frac{a^2+b^2-c^2}{2ab}$

$$\Rightarrow \cos C = \frac{(4)^2+(3)^2-(2)^2}{2(4)(3)}$$

$$\Rightarrow \cos C = \frac{16+9-4}{24}$$

$$\Rightarrow \cos C = \frac{7}{8}$$

$$\because \cos 3C = 4\left(\frac{7}{8}\right)^3 - 3\left(\frac{7}{8}\right)$$

$$\Rightarrow \cos 3C = \frac{343-336}{128}$$

$$\Rightarrow \cos 3C = \frac{7}{128}$$

Hint: Use $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ and cosine rule.

51. Option (c) is correct.

Solution:

Let A = cos 36° − cos 72°

$\Rightarrow$ A = cos 36° – 18° $\{\because \cos(90° - \theta) = \sin\theta\}$

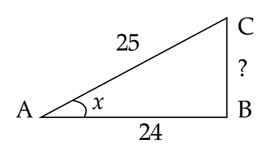
$$\Rightarrow A = \left(\frac{\sqrt{5}+1}{4}\right) - \left(\frac{\sqrt{5}-1}{4}\right)$$

$$\Rightarrow A = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Hint: use $\cos 36° = \frac{\sqrt{5}+1}{4}$ $\sin 18° = \frac{\sqrt{5}-1}{4}$

52. Option (b) is correct

Solution:



Given: $\sec x = \frac{25}{24}$

$\Rightarrow$ Perpendicular = $\sqrt{(25)^2-(24)^2}$

= 7

$$\Rightarrow \tan x = \frac{7}{24} \text{ and } \sin x = \frac{7}{25}$$

$\because$ x lies in the fourth quadrant where sin θ and tan θ are negative

$$\Rightarrow \tan x = -\frac{7}{24} \text{ and } \sin x = \frac{-7}{25}$$

$$\Rightarrow \tan x + \sin x = \frac{-7}{24} - \frac{7}{25} = \frac{-343}{600}$$

Hint: tan θ and sin θ are negative when θ lies in fourth quadrant.

53. Option (b) is correct.

Solution:

Let $A = \tan^2 165° + \cot^2 165°$

$\Rightarrow A = \tan^2(180° - 15°) + \cot^2(180° - 15°)$

$\Rightarrow A = \tan^2 15° + \cot^2 15°$

$\Rightarrow A = \left(2-\sqrt{3}\right)^2 + \left(2+\sqrt{3}\right)^2$

$\Rightarrow A = 4+3-4\sqrt{3}+4+3+4\sqrt{3}$

$\Rightarrow A = 14$

Hint: use $\tan 15° = 2-\sqrt{3}$ and $\cot 15° = 2+\sqrt{3}$

54. Option (a) is correct.

Solution:

Let $A = \sin\left(2n\pi + \frac{5\pi}{6}\right)\sin\left(2n\pi + \frac{5\pi}{6}\right)$

As we know, sin θ is −ve in fourth quadrant and + ve in first quadrant.

$\Rightarrow A = \left(\sin\frac{5\pi}{6}\right)\left(-\sin\frac{5\pi}{6}\right)$

$\Rightarrow A = -\left\{\sin\left(\pi - \frac{\pi}{6}\right)\right\}^2$

$\Rightarrow A = -\sin^2\frac{\pi}{6}$

$\Rightarrow A = -\frac{1}{4}$

Hint: use the sign convention of trigonometric ratios in four quadrants.

55. Option (d) is correct.

Solution:

$1 + 2(\sin x + \cos x)(\sin x - \cos x) = 0$

$\Rightarrow 2(\sin^2 x - \cos x) = -1$

$\Rightarrow \cos 2x = \frac{1}{2}$

$\Rightarrow \cos 2x = \cos\frac{\pi}{3}$

$\Rightarrow 2x = 2n\pi + -\frac{\pi}{3}$

$\Rightarrow x = n\pi \pm \pi/6$

For $n = 0$, $x = \frac{\pi}{6}$ $\{\because x \in (0°, 360°)\}$

For $n = 1$, $x = \frac{5\pi}{6}, \frac{7\pi}{6}$

For $n = 2$, $x = \frac{11\pi}{6}$

Hints: (1) use $\cos 2\theta = \cos^2\theta - \sin^2\theta$.

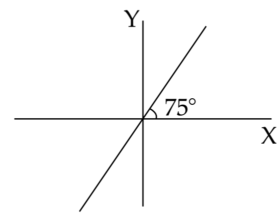
(2) General solution of $\cos x = \cos\theta$ is $x = 2n\pi \pm \theta$.

56. Option (c) is correct.

Solution:

Given: Line passes through origin

Line makes 75° with positive x axis



Slope = $m = \tan 75°$

$\Rightarrow m = 2+\sqrt{3}$

∵ line passes through origin so equation of the line is $y = mx$.

$\Rightarrow y = \left(2+\sqrt{3}\right)x$

Statement 1: At $x = 1$,

$y = \left(2+\sqrt{3}\right)(1) = 2+\sqrt{3} = \frac{1}{2-\sqrt{3}}$

$\Rightarrow$ The line passes through $\left(1, \frac{1}{2-\sqrt{3}}\right)$

So statement 1 is correct

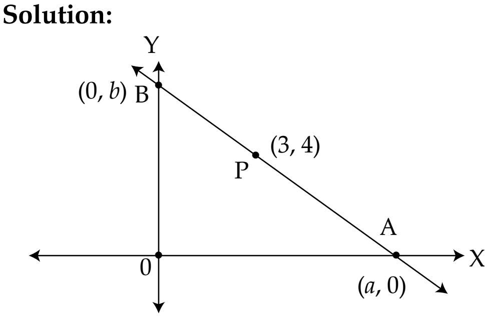
Statement 2: $y = \left(2+\sqrt{3}\right)x$

If x is +ve then y is also +ve always and if x is –ve then y is also –ve always so the line entirely lies is 1st and 3rd quadrant.

Hint: Slope of a line is tan θ where θ is the angle made by the line with positive x-axis.

57. Option (b) is correct.

Solution:



Let the line be AB with A ≡ $(a, 0)$ and B ≡ $(0, b)$ and p is mid point of AB.

By mid point formula,

$3 = \frac{0+a}{2} \Rightarrow a = 6$

and $4 = \frac{b+0}{2} \Rightarrow b = 8$

By intercept form of line, equation of AB is $\frac{x}{a}+\frac{y}{b}=1$

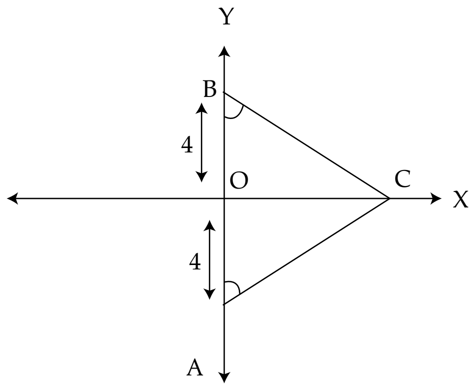
$\Rightarrow \frac{x}{6}+\frac{y}{8}=1$

$\Rightarrow 4x + 3y - 24 = 0$

Hint: use mid point formula and intercept form of equation of line.

58. Option (a) is correct.

Solution:



AB = BC = 8

OB = OA = 4

$\Rightarrow$ B ≡ (0, 4) & A ≡ (0, −4)

∠OCB = 30°

$\therefore$ In Δ OCB, $\tan 30 = \frac{4}{OC}$

$\Rightarrow OC = 4\sqrt{3}$

$\therefore C \equiv \left(4\sqrt{3},0\right)$

Now, slope of $AC = \frac{0+4}{4\sqrt{3}-0} = \frac{1}{\sqrt{3}}$

The required line has slope $\frac{1}{\sqrt{3}}$ and passes through (8, 0)

$\Rightarrow$ The equation of line is $y - 0 = \frac{1}{\sqrt{3}}(x-8)$

$\Rightarrow \sqrt{3}y = x - 8$

$\Rightarrow x - \sqrt{3}y - 8 = 0$

Hint: The equation of line having slope m and passing through (x_1, y_1) is $(y - y_1) = m(x - x_1)$

59. Option (c) is correct

Solution:

Let the equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

$\because$ circle passes through origin.

$\therefore c = 0$.

$\Rightarrow$ The equation of circle becomes

$x^2 + y^2 + 2gx + 2fy = 0$

Now, intercept on x axis = 4

$\Rightarrow 2\sqrt{g^2 - c} = 4$

$\Rightarrow g^2 = 4$

$\Rightarrow g = \pm 2$

and intercept on y axis = 6

$\Rightarrow 2\sqrt{f^2 - c} = 6$

$\Rightarrow f^2 = 9$

$\Rightarrow f = \pm 3$

Now, centre of the circle is $(-g, -f)$

$\Rightarrow$ centre ≡ (± 2, ± 3)

$\because$ Intercepts are made with positive axes so center lies in I quadrant.

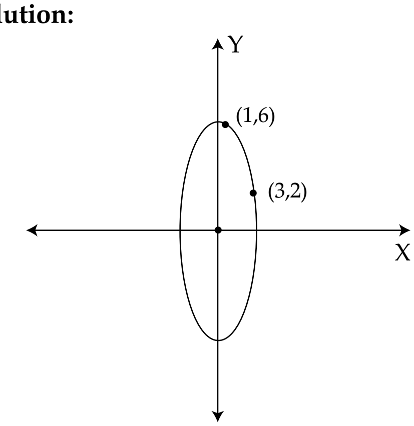
$\Rightarrow$ centre ≡ (2, 3)

Among all the options (2, 3) lies on $3x - 4y + 6 = 0$

Hint: If the general equation of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ the intercept on x axis $= 2\sqrt{g^2 - c}$ and intercept on y axis $= 2\sqrt{f^2 - c}$.

60. Option (a) is correct.

Solution:



$\because$ center ≡ (0, 0) and major axis is on y axis.

$\Rightarrow$ Equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (a<b)$

$\because$ Ellipse passes through (3, 2) and (1, 6)

$\Rightarrow \frac{9}{a^2}+\frac{4}{b^2}=1$ and $\frac{1}{a^2}+\frac{36}{b^2}=1$

$$\Rightarrow \frac{9}{a^2}+\frac{4}{b^2}=\frac{1}{a^2}+\frac{36}{b^2}$$

$$\Rightarrow \frac{8}{a^2}=\frac{32}{b^2}$$

$$\Rightarrow \frac{a^2}{b^2}=\frac{1}{4}$$

Now, eccentricity $e=\sqrt{1-\frac{a^2}{b^2}}$

$$e=\sqrt{1-\frac{1}{4}}$$

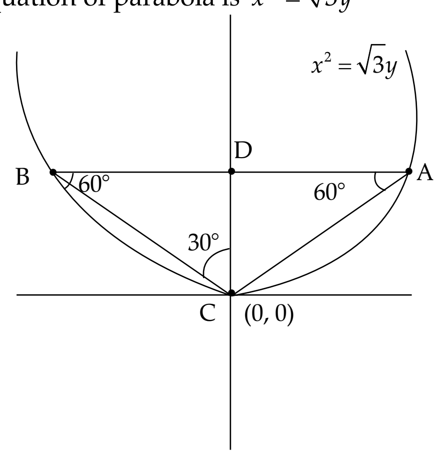
$$e=\frac{\sqrt{3}}{2}$$

Hint: Use eccentricity $e=\sqrt{\frac{1-a^2}{b^2}}$, where equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; $b > a$.

61. Option (c) is correct.

Solution:

Equation of parabola is $x^2=\sqrt{3}y$



ΔABC is equilateral.

$\angle$ACD = 30° = $\angle$BCD

Let coordinates of A be (x, y)

In ΔACD, sin 60° = $\frac{CD}{AC}$

$$\Rightarrow \frac{\sqrt{3}}{2}=\frac{y}{p}$$

$$\Rightarrow y=\frac{\sqrt{3}}{2}p$$

In ACD, cos60° = $\frac{AD}{AC}$

$$\Rightarrow \frac{1}{2}=\frac{x}{p}$$

$$\Rightarrow x=\frac{p}{2}$$

So, $A=\left(\frac{p}{2},\frac{\sqrt{3}p}{2}\right)$

$\because$ Point A lies on parabola $x^2=\sqrt{3}y$

$$\because \left(\frac{p}{2}\right)^2=\sqrt{3}\frac{\sqrt{3}}{2}p$$

$$\Rightarrow \frac{p^2}{4}=\frac{3}{2}p$$

$\Rightarrow p = 6$

Now length of latus rectum, $q=\sqrt{3}$

$\therefore p=2\sqrt{3}q$

62. Option (b) is correct.

Solution:

Given points A(2, 4, 6), B(–2, –4, –2), C(4, 6, 4) and D(8, 14, 12)

Now, $AB=\sqrt{(-4)^2+(-8)^2(-8)^2}=\sqrt{144}=12$

$$BC=\sqrt{(6)^2+(10)^2+(6)^2}=\sqrt{172}$$

$$CD=\sqrt{(4)^2+(8)^2+(8)^2}=\sqrt{144}=12$$

$$DA=\sqrt{(-6)^2+(-10)^2+(-6)^2}=\sqrt{172}$$

$$AC=\sqrt{(2)^2+(2)^2+(-2)^2}=2\sqrt{3}$$

$$BD=\sqrt{(10)^2+(18)^2+(14)^2}=\sqrt{620}$$

$\because$ AB = CD and BC = DA and AC $\neq$ BD.

$\because$ Given points are the vertices of parallelogram ABCD. So, statement 1 is incorrect.

Now, midpoint of

$$AC=\left(\frac{2+4}{2},\frac{4+6}{2},\frac{6+4}{2}\right)=(3,5,5)$$

midpoint of

$$BD=\left(\frac{8-2}{2},\frac{14-4}{2},\frac{12-2}{2}\right)=(3,5,5)$$

So statement 2 is correct.

Hints:

1. For rectangle AB = CD, BC = DA and AC = BD.
2. Midpoint of two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$

63. Option (b) is correct.

Solution:

Equation of sphere is $x^2 + y^2 + z^2 - 4x - 6y - 8z - 16 = 0$. As we know plane is the tangent of the sphere. So, z-axis cannot be tangent of sphere.

So, statement 1 is incorrect.

Now, coordinates of the centre of sphere

$= (2, 3, 4)$

Equation of plane is $x + y + z - 9 = 0$.

$\because 2 + 3 + 4 - 9 = 0$

$\therefore$ Point (2, 3, 4) lies on the plane $x + y + z - 9 = 0$

So, statement 2 is correct.

Hints:

1. Plane is the tangent of the sphere.
2. Coordinates of the centre of the sphere $x^2 + y^2 + z^2 + 2gx + 2fy + 2kz + c$ are $(-g, -f, -k)$.

64. Option (c) is correct.

Given: Plane cuts intercepts 2, 2, 1 on the coordinate axes.

$\therefore$ Equation of plane is $\frac{x}{2}+\frac{y}{2}+\frac{z}{1}=1$

$\Rightarrow x + y + 2z = 2$

So, direction ratios of the normal to plane

$= \{1, 1, 2\}$

Direction cosines of the normal to plane

$= \left\{\frac{1}{k}, \frac{1}{k}, \frac{2}{k}\right\}$

where $k = \sqrt{1^2+1^2+2^2} = \sqrt{6}$

So, direction cosines of normal to plane

$= \left\{\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right\}$

Hints:

1. If plane cuts intercepts a, b, c on the coordinate axes, then equation of plane in $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$
2. Direction ratios of the normal to plane $Ax + By + Cz + d = 0$ are A, B, C.

65. Option (c) is correct.

Solution:

As we As we know that cosines of y-axis

$= \{\cos 90°, \cos 0°, \cos 90°\}$

$= \{0, 1, 0\}$

$\therefore$ Direction ratios of y-axis $= \{0, k, 0\}$; $k \in R$

So, statement 1 is correct.

Direction cosines of z-axis

$= \{\cos 90°, \cos 90°, \cos 0°\}$

$= \{0, 0, 1\}$

$\therefore$ Direction ratios of z-axis $= \{0, 0, \lambda\}$; $\lambda \in R$

Now, $5(0) + 6(0) + 0(\lambda) = 0$

$\therefore$ line whose direction ratios are {5, 6, 0} is perpendicular to z-axis.

So, statement 2 is also correct.

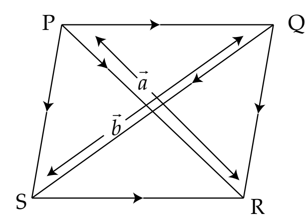
Hints:

1. Direction cosines of y-axis = {0, 1, 0}
2. Direction cosines of z-axis = {0, 0, 1}
3. If $a_1a_2 + b_1b_2 + c_1c_2 = 0$, then lines whose direction ratios are $\{a_1, b_1, c_1\}$ and $\{a_2, b_2, c_2\}$, are perpendicular.

66. Option (d) is correct.

Solution:

Given: $\overrightarrow{PR} = \vec{a}$ and $\overrightarrow{QS} = \vec{b}$



Let $\overrightarrow{PQ} = \vec{x}$ and $\overrightarrow{QR} = \vec{y}$

Then $\overrightarrow{SR} = \vec{x}$ and $\overrightarrow{PS} = \vec{y}$

By triangle law of vector addition.

$\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

$\vec{x} + \vec{y} = \vec{a}$ ...(i)

Applying triangle law of vector addition in ΔQRS

$\overrightarrow{QR} + \overrightarrow{RS} = \overrightarrow{QS}$

$\vec{y} - \vec{x} = \vec{b}$ ...(ii)

Equation (i)–Equation (ii), we get

$2\vec{x} = \vec{a} - \vec{b}$

$\Rightarrow \vec{x} = \frac{\vec{a}-\vec{b}}{2}$

$\Rightarrow \overrightarrow{PQ} = \frac{\vec{a}-\vec{b}}{2}$

Hint:

Apply triangle law of addition in ΔPQR and ΔQRS and solve further.

67. Option (c) is correct.

Solution:

Given: $|\vec{a}| = |\vec{b}| = 1$

And vectors $(\vec{a}+2\vec{b})$ and $(5\vec{a}-4\vec{b})$ are perpendicular.

$\Rightarrow (\vec{a}+2\vec{b})\cdot(5\vec{a}-4\vec{b}) = 0$

$\Rightarrow 5|\vec{a}|^2 - 4\vec{a}\cdot\vec{b} + 10\vec{b}\cdot\vec{a} - 8|\vec{b}|^2 = 0$

$\Rightarrow 5(1) - 4\vec{a}\cdot\vec{b} + 10\vec{a}\cdot\vec{b} - 8(1) = 0$

$\Rightarrow 6\vec{a}\cdot\vec{b} = 3$

$\Rightarrow \vec{a}\cdot\vec{b} = \dfrac{1}{2}$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = \dfrac{1}{2}$, where θ = Angle between $\vec{a}$ and $\vec{b}$

$\Rightarrow \cos\theta = \dfrac{1}{2}$

$\Rightarrow \theta = \dfrac{\pi}{3}$

Hint:
Dot product of perpendicular vectors is zero.

68. Option (d) is correct.

Solution:

Given: $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

Let $A = \{(3\vec{a}+2\vec{b})\times(5\vec{a}-4\vec{c})\}\cdot(\vec{b}+2\vec{c})$

$\Rightarrow A = \{15\vec{a}\times\vec{a} - 12(\vec{a}\times\vec{c}) + 10(\vec{b}\times\vec{a}) - 8(\vec{b}\times\vec{c})\}\cdot(\vec{b}+2\vec{c})$

$\Rightarrow A = \{0 - 12(\vec{a}\times\vec{c}) + 10(\vec{b}\times\vec{a}) - 8(\vec{b}\times\vec{c})\}\cdot(\vec{b}+2\vec{c})$

$\Rightarrow A = -12[(\vec{a}\times\vec{c})\cdot\vec{b}] + 10(\vec{b}\times\vec{a})\cdot\vec{b} - 8(\vec{b}\times\vec{c})\cdot\vec{b}$
$\quad -24(\vec{a}\times\vec{c})\cdot\vec{c} + 20(\vec{b}\times\vec{a})\cdot\vec{c} - 1(\vec{b}\times\vec{c})\cdot\vec{c}$

$\Rightarrow A = -12[\vec{a}\,\vec{c}\,\vec{b}] + 10(0) - 8(0) - 24(0)$
$\quad +20[\vec{a}\,\vec{c}\,\vec{b}] - 16(0)$

$\Rightarrow A = +12[\vec{a}\,\vec{b}\,\vec{c}] - 20[\vec{a}\,\vec{b}\,\vec{c}]$

$\Rightarrow A = -8[\vec{a}\,\vec{b}\,\vec{c}]$

$\Rightarrow$ A = –8(0) $\{\because \vec{a}, \vec{b}$ and $\vec{c}$ are coplanar$\}$

$\Rightarrow$ A = 0

Hint:
Scalar triple product of three coplanar vector is 0.

69. Option (a) is correct.

Solution:

Let $\vec{A} = 2x^2\hat{i} + 3x\hat{j} + \hat{k}$

$\vec{B} = \hat{i} - 2\hat{j} + x^2\hat{k}$

Let angle between $\vec{A}$ and $\vec{B}$ be θ

$\therefore \cos\theta = \dfrac{\vec{A},\vec{B}}{|\vec{A}||\vec{B}|}$

$\Rightarrow \cos\theta = \dfrac{2x^2 - 6x + x^2}{\sqrt{4x^4+9x^2+1}\cdot\sqrt{1+4+x^4}}$

$\Rightarrow \cos\theta = \dfrac{3x^2 - 6x}{\sqrt{4x^4+9x^2+1}\cdot\sqrt{x^4+5}}$

For obtuse angle, cosθ < 0

$\therefore 3x^2 - 6x < 0$

$\Rightarrow x(x-2) < 0$

$\Rightarrow 0 < x < 2$

Hint:
For obtuse angle,
$(2x^2\hat{i} + 3x\hat{j} + \hat{k})\cdot(\hat{i} - 2\hat{j} + x^2\hat{k}) < 0$

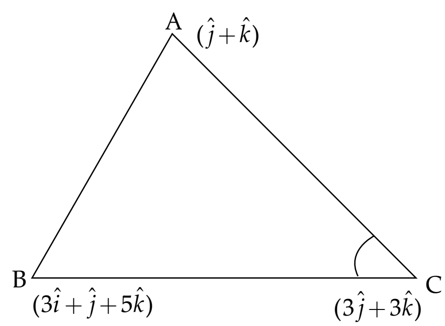
70. Option (d) is correct.

Solution:

Given: Position vector of A, $\overrightarrow{OA} = \hat{j} + \hat{k}$

Position vector of B, $\overrightarrow{OB} = 3\hat{i} + \hat{j} + 5\hat{k}$

Position vector of C, $\overrightarrow{OC} = 3\hat{j} + 3\hat{k}$



Now, $\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC} = (\hat{j}+\hat{k}) - (3\hat{j}+3\hat{k})$

$= -2\hat{j} - 2\hat{k}$

$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} = (3\hat{i}+\hat{j}+5\hat{k}) - (3\hat{j}+3\hat{k})$

$= 3\hat{i} - 2\hat{j} + 2\hat{k}$

Now, $\overrightarrow{CA}\cdot\overrightarrow{CB} = |\overrightarrow{CA}||\overrightarrow{CB}|\cos c$

$\Rightarrow 0 + 4 - 4 = |\overrightarrow{CA}||\overrightarrow{CB}|\cos c$

$\Rightarrow c = \frac{\pi}{2}$

Hint:
Find $\overrightarrow{CA}$ and $\overrightarrow{CB}$ and use
$\overrightarrow{CA} \cdot \overrightarrow{CB} = |\overrightarrow{CA}||\overrightarrow{CB}|\cos c$

71. Option (a) is correct.
Solution:
Given: $z = [y]$ and $y = [x] - x$
As we know $x = [x] + \{x\}$
$\Rightarrow [x] - x = -\{x\}$
$\therefore z = [-\{x\}]$
As we know $0 < \{x\} < 1$
$\Rightarrow z = -1$

Hint: Use $x = [x] + \{x\}$ and $0 < \{x\} < 1$.

72. Option (a) is correct.
Solution:
Given: $f(x) = 4x + 1$ and $g(x) = kx + 2$
And $fog\,(x) = gof(x)$
$\Rightarrow f[kx + 2] = g[4x + 1]$
$\Rightarrow 4(kx + 2) + 1 = k(4x + 1) + 2$
$\Rightarrow 4\,kx + 8 + 1 = 4\,kx + k + 2$
$\Rightarrow k = 7$

Hint:
Use $fog\,(x) = f[g(x)]$

73. Option (b) is correct.
Solution:
Given: $f(x) = \log_{10}(x^2 + 2x + 11)$

$$\Rightarrow f(x) = \frac{\log_e(x^2+2x+11)}{\log_e 10}$$

$$\Rightarrow f'(x) = \frac{1}{\log_e 10}\left[\frac{1}{x^2+2x+11}\frac{d}{dx}(x^2+2x+11)\right]$$

$$\Rightarrow f'(x) = \frac{1}{\log_e 10}\left[\frac{2(x+1)}{x^2+2x+11}\right]$$

For critical points, $f'(x) = 0$

$$\Rightarrow \frac{2(x+1)}{\log_e 10(x^2+2x+11)} = 0$$

$\Rightarrow x = -1$
So, $f(x)$ has minimum value at $x = -1$
$\therefore [f(x)]_{min} = \log 10\,(1 - 2 + 11) = \log_{10}10 = 1$

74. Option (b) is correct.
Solution:
Let I = $\int (x^x)^2(1+\ln x)dx$
Let x^x = u
ln x^x = ln u
$x \ln x = \ln u$

$$\Rightarrow x\left(\frac{1}{x}\right) + lnx = \frac{1}{u}\frac{\partial u}{\partial x}$$

$$\Rightarrow (1+\ln x)x^x = \frac{\partial u}{\partial x}$$

$$\Rightarrow \partial u = x^x(1+\ln x)\partial x$$

$$\Rightarrow I = \int u\,du$$

$$\Rightarrow I = \frac{u^2}{2} + c$$

$$\Rightarrow I = \frac{1}{2}(x^2)^2 + c$$

$$\Rightarrow I = \frac{1}{2}x^{2x} + c$$

Hint: Assume x^x = 4u and solve further.

75. Option (a) is correct.
Solution:
Let $A = \int e^x(1+\ln x + x\ln x)\partial x$
Let $f(x) = x \ln x$

$$\Rightarrow f'(x) = x\left(\frac{1}{x}\right) + \ln x$$

$\Rightarrow f'(x) = 1 + \ln x$

$$\therefore A = \int e^x(f(x) + f'(x))dx$$

$\Rightarrow$ A = $e^x f(x) + c$
$\Rightarrow$ A = $e^x x \ln x + c$
$\Rightarrow$ A = $x e^x \ln x + c$

Hint: Use $\int e^x[f(x)+f'(x)]dx = e^x f(x) + c$

76. Option (c) is correct.
Solution:

$$\text{Let } A = \int \frac{(\cos x)^{1.5} - (\sin x)^{1.5}}{\sqrt{\sin x \cdot \cos x}}$$

$$\Rightarrow A = \int \frac{\cos x\sqrt{\cos x} - \sin x\sqrt{\sin x}}{\sqrt{\sin x \cdot \cos x}}dx$$

$$\Rightarrow A = \int \frac{\cos x}{\sqrt{\sin x}}dx - \int \frac{\sin x}{\sqrt{\cos x}}dz$$

Let sin $x = t$ in first integral and cos x = 4 in second integral.

$\therefore\ A = \int \frac{dt}{\sqrt{t}} + \int \frac{du}{\sqrt{u}}$

$\Rightarrow A = 2\sqrt{t} + 2\sqrt{u} + c$

$\Rightarrow A = 2\sqrt{\sin x} + 2\sqrt{\cos x} + c$

Hints:

1. Simplify given integral and solve further using method of substitution.
2. $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x}$

77. Option (c) is correct.

Solution:

Given: $y = \frac{x\sqrt{x^2-16}}{2} - 8\ln\left|x+\sqrt{x^2-16}\right|$

As we know

$\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right|$

$\therefore \int \sqrt{x^2-16}dx = \frac{x}{2}\sqrt{x^2-16} - 8\ln|x\sqrt{x^2-16}|$

$\Rightarrow \sqrt{x^2-16} = \frac{d}{dx}\left(\frac{x}{2}\sqrt{x^2-16} - 8\ln|x+\sqrt{x^2-16}|\right)$

$\Rightarrow \frac{dy}{dx} = \sqrt{x^2-16}$

Hint: Use

$\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln|x+\sqrt{x^2-a^2}|$

78. Option (b) is correct.

Solution:

Given: $y = (x^x)^x$

$\ln y = x \ln x^x$

$\ln y = x^2 \ln x$

Differentiating the above equation w.r.t. x, we get

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 2x\ln x + x^2\left(\frac{1}{x}\right)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 2x\cdot\ln x + x$

$\Rightarrow \frac{dy}{dx} = 2xy\ln x + xy$

$\Rightarrow \frac{dy}{dx} = xy(2\ln x + 1)$

$\Rightarrow \frac{dy}{dx} - xy(1+2\ln x) = 0$

Hint: Take logarithm both the sides and solve further using the concept of differentiation of implicit function.

79. Option (b) is correct.

Solution:

Let $f(x) = 3(\sin x - \cos x) + 4(\cos^3 x - \sin x)$

$f(x) = 3\sin x - 4\sin^3 x + 4\cos^3 x - 3\cos x$

$f(x) = \sin 3x + \cos 3x$

As we know

$-\sqrt{a^2+b^2} \le a\sin kx + b\cos kx \le \sqrt{a^2+b^2}$

$\therefore [f(x)]_{max} = \sqrt{1^2+1^2} = \sqrt{2}$

Hints:

1. Use $\sin 3x = 3\sin x - 4\sin^3 x$ and $\cos^3 x = 4\cos^3 x - 3\cos x$
2. Use $-\sqrt{a^2+b^2} \le a\sin kx + b\cos kx \le \sqrt{a^2+b^2}$

80. Option (c) is correct.

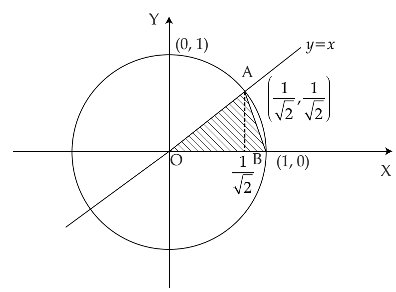
Solution:

Given curves $y = \sqrt{1-x^2}$, $y = x$

$\therefore\ y = \sqrt{1-x^2}$

$\Rightarrow y^2 = 1 - x^2$

$\Rightarrow x^2 + y^2 = 1$



Required area $= \frac{1}{2}$ Area of $\Delta OAB + \int_{\frac{1}{\sqrt{2}}}^{1} \sqrt{1-x^2}dx$

$= \frac{1}{2}\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}} + \left[\frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_{\frac{1}{\sqrt{2}}}^{1}$

$= \frac{1}{4} + \left[\frac{1}{2}(0) + \frac{1}{2}\sin^{-1}(1) - \frac{1}{2}\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}} - \frac{1}{2}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$

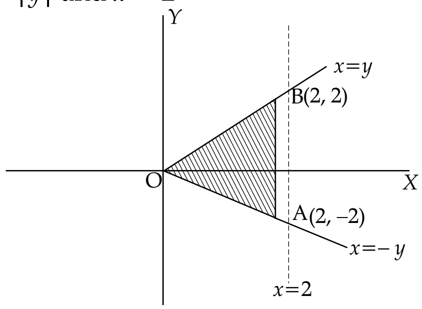
$= \frac{1}{4} + \frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{1}{4} - \frac{1}{2}\left(\frac{\pi}{4}\right)$

$= \frac{\pi}{4} - \frac{\pi}{8}$

$= \frac{\pi}{8}$

Hint:
Required area = Area of ΔOAB + $\int_{1/\sqrt{2}}^{1} \sqrt{1-x^2}\,dx$

81. Option (c) is correct.
Solution:
Given curves $x - |y| = 0$ and $x - 2 = 0$
$\Rightarrow x = |y|$ and $x = 2$



Required area = Area of ΔOAB

$= \frac{1}{2} \times AB \times OA$

$= \frac{1}{2} \times 4 \times 2$

= 4 square units.

Hint:
Required area = Area of ΔOAB.

82. Option (b) is correct.
Solution:

Given: $f(\alpha) = \sqrt{\sec^2\alpha - 1}$

$\Rightarrow f(\alpha) = \sqrt{\tan^2\alpha}$ $\{\because 1 + \tan^2\alpha = \sec^2\alpha\}$

$\Rightarrow f(\alpha) = \tan\alpha$

Now, $\frac{f(\alpha) + f(\beta)}{1 - f(\alpha)\cdot f(\beta)} = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$

$= \tan(\alpha + \beta)$
$= f(\alpha + \beta)$

Hints:

1. Use $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$
2. Use $1 + \tan^2\theta = \sec^2\theta$

83. Option (a) is correct.
Solution:

Given: $f(x) = \ln\left(x + \sqrt{1+x^2}\right)$

$\Rightarrow f(-x) = \ln(-x + \sqrt{1+x^2})$

$\Rightarrow f(-x) = \ln\left[\frac{-x + \sqrt{1-x^2}}{\sqrt{1+x^2} + x} \cdot \left(\sqrt{1+x^2} + x\right)\right]$

$\Rightarrow f(-x) = \ln\left[\frac{1 + x^2 - x^2}{x + \sqrt{1+x^2}}\right]$

$\{\because (a+b)(a-b) = a^2 - b^2\}$

$\Rightarrow f(-x) = \ln\left[\frac{1}{x + \sqrt{1+x^2}}\right]$

$\Rightarrow f(-x) = \ln(1) - \ln(x + \sqrt{1+x^2})$

$\Rightarrow f(-x) = 0 - \ln(x + \sqrt{1+x^2})$

$f(-x) = -f(x)$

$f(x) + f(-x) = 0$

Hints:

1. Find $f(-x)$ and solve further using the concept of rationalisation.
2. Use $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

84. Option (d) is correct.
Solution:

Let $A = \lim_{x \to 0} \frac{x}{\sqrt{1 - \cos 4x}}$

$\Rightarrow A = \lim_{x \to 0} \frac{x}{\sqrt{2}\,|\sin 2x|}$ $\{\because 1 - \cos^2 x = 2\sin^2 x\}$

Now, $\lim_{x \to 0^-} \frac{x}{\sqrt{2}\,|\sin 2x|} = \lim_{x \to 0^-} \frac{x}{\sqrt{2}(-\sin 2x)}$

$= \lim_{x \to 0^-} -\left(\frac{2x}{2\sqrt{2}\sin 2x}\right)$

$= -\frac{1}{2\sqrt{2}}$

And $\lim_{x \to 0^+} \frac{x}{\sqrt{2}\,|\sin 2x|} = \lim_{x \to 0^+} \frac{x}{\sqrt{2}\sin 2x}$

$= \lim_{x \to 0^+} \frac{1}{2\sqrt{2}}\left(\frac{2x}{\sin 2x}\right)$

$= \frac{1}{2\sqrt{2}}$

$\because \lim_{x \to 0^-} \frac{x}{\sqrt{2}\,|\sin 2x|} \neq \lim_{x \to 0^+} \frac{x}{\sqrt{2}\,|\sin 2x|}$

$\therefore \lim_{x \to 0} \frac{x}{\sqrt{1 - \cos 4x}}$ does not exist.

Hints:

1. Find L.H.L and R.H.L at $x = 0$ and analyse further.
2. If $\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$, then $\lim_{x \to a} f(x)$ does not exist.

85. Option (a) is correct.

Solution:

Let $A = \lim_{x\to\frac{\pi}{2}} \frac{4x-2\pi}{\cos x}$

Above limit has $\frac{0}{0}$ indeterminate form. So applying L'hospital rule.

$$\therefore\ A = \lim_{x\to\frac{\pi}{2}} \frac{\frac{d}{dx}(4x-2\pi)}{\frac{d}{dx}(\cos x)}$$

$$\Rightarrow A = \lim_{x\to\frac{\pi}{2}} \frac{4}{-\sin x}$$

$\Rightarrow$ A = –4

Hint:

If $\lim_{x\to a}\frac{f(x)}{g(x)}$ has indeterminate form of $\frac{0}{0}$ or $\frac{\infty}{\infty}$ then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

86. Option (d) is correct.

Solution:

Given: $f(x) = \frac{x^2+x+|x|}{x}$

Now, L.H.L at x = 0, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\frac{x^2+x-x}{x}$

$\{\because |x| = -x;\ x < 0\}$

$$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x = 0$$

Now, R.H.L at x = 0, $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\frac{x^2+x+x}{x}$

$\{\because |x| = x;\ x > 0\}$

$$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(x+2) = 2$$

$$\therefore \lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$$

$\therefore \lim_{x\to 0^-} f(x)$ does not exist.

Hints:

1. Find L.H.L and R.H.L at x = 0 and analyse further.

2. Use $|x| = \begin{cases} x;\ x>0 \\ -x;\ x<0 \end{cases}$

87. Option (c) is correct.

Solution:

Let $A = \lim_{h\to 0}\frac{\sin^2(x+h)-\sin^2 x}{h}$

$$A = \lim_{h\to 0}\frac{\sin(x+h+x)-\sin(x+h-x)}{h}$$

$\{\because \sin^2A - \sin^2B = \sin(A+B)\cdot\sin(A-B)\}$

$$A = \lim_{h\to 0}\sin(2x+h)\cdot\frac{\sin h}{h}$$

A = sin2x · (1) = sin2x

Hint:

Simplify given limit using $\sin^2A - \sin^2B = \sin(A+B)\cdot\sin(A-B)$ and solve further.

88. Option (c) is correct.

Solution:

Given: $f'(x) = g(x)$ ...(i)

And $f''(x) = -f(x)$ ...(ii)

And $h(x) = \{f(x)\}^2 + \{g(x)\}^2$

$\because f''(x) = -f(x)$

$\Rightarrow g'(x) = -f(x)$ $\{\because f'(x) = g(x)\}$

Now, $h'(x) = 2f(x)f'(x) + 2g(x)g'(x)$

$\Rightarrow h'(x) = 2f(x)g(x) + 2g(x)[-f(x)]$

$\Rightarrow h'(x) = 0$

$\Rightarrow h'(3) = 0$

So, statement 1 is correct.

$\because h'(x) = 0$

$\Rightarrow h(x) = k$, where k is a constant.

$\Rightarrow h(2) = h(1) = k$

So, statement 2 is also correct.

89. Option (b) is correct.

Solution:

Given: $y = \ln^2\left(\frac{x^2-x+1}{x^2+x+1}\right)$

$$\Rightarrow \frac{dy}{dx} = 2\ln\left(\frac{x^2-x+1}{x^2+x+1}\right)\frac{d}{dx}\left[\ln\left(\frac{x^2-x+1}{x^2+x+1}\right)\right]$$

$$\Rightarrow \frac{dy}{dx} = 2\ln\left(\frac{x^2-x+1}{x^2+x+1}\right)\frac{d}{dx}\left[\ln(x^2-x+1)-\ln(x^2+x+1)\right]$$

$$\Rightarrow \frac{dy}{dx} = 2\ln\left(\frac{x^2-x+1}{x^2+x+1}\right)\left[\frac{2x-1}{x^2-x+1}-\frac{2x+1}{x^2+x+1}\right]$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 2\ln(1)[-1-1]$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 0 \qquad \{\because \ln 1 = 0\}$$

Hint:

Use $\frac{d}{dx}[f(g(x))] = f'(g(x))\cdot g'(x)$

90. Option (d) is correct.

Solution:

Given: $\frac{d}{dx}\left(\frac{1+x^4+x^8}{1-x^2+x^4}\right) = ax+bx^3$

$\Rightarrow \frac{d}{dx}\left(\frac{(x^4+x^2+1)(x^4+1-x^2)}{1-x^2+x^4}\right) = ax+bx^3$

$\Rightarrow \frac{d}{dx}(x^4+x^2+1) = ax+bx^3$

$\Rightarrow 4x^3 + 2x = bx^3 + ax$

$\Rightarrow b = 4$ and $a = 2$

$\Rightarrow 2a = b$

Hint: Use $1 + x^4 + x^8 = (x^4 + x^2 + 1)(x^4 + 1 - x^2)$ and solve further.

91. Option (a) is correct.

Solution:

Given: $f(x) = (p \sec x)^2 + (q \operatorname{cosec} x)^2$

$\Rightarrow f'(x) = 2p^2 \sec x (\sec x \tan x) + 2q^2 \operatorname{cosec} x (-\operatorname{cosec} x \cot x)$

$\Rightarrow f'(x) = 2(p^2\sec^2 x \cdot \tan x - q^2\operatorname{cosec}^2 x \cdot \cot x)$

For critical points, $f'(x) = 0$

$\Rightarrow p^2\sec^2 x \tan x = q^2\operatorname{cosec}^2 x \cdot \cot x$

$\Rightarrow p^2 \frac{\sin x}{\cos^3 x} = q^2 \frac{\cos x}{\sin^3 x}$

$\Rightarrow \frac{\sin^4 x}{\cos^4 x} = \frac{q^2}{p^2}$

$\Rightarrow \tan^4 x = \frac{q^2}{p^2}$

$\Rightarrow \tan^2 x = \frac{q}{p}$

$\Rightarrow \cot^2 x = \frac{p}{q}$

Hint: $f(x)$ attains minimum value where $f'(x) = 0$.

92. Option (b) is correct.

Solution:

Given: $f(x) = \sum_{j=1}^{7}(x-j)^2$

$\Rightarrow f(x) = (x-1)^2 + (x-2)^2 + (x-3)^2 + (x-4)^2 + (x-5)^2 + (x-6)^2 + (x-7)^2$

$\Rightarrow f'(x) = 2(x-1) + 2(x-2) + 2(x-3) + 2(x-4) + 2(x-5) + 2(x-6) + 2(x-7)$

$\Rightarrow f'(x) = 2\,[7x - 28]$

For critical points $f'(x) = 0$

$\Rightarrow 7x - 28 = 0$

$\Rightarrow x = \frac{28}{7} = 4$

Now, $f''(x) = 2(7 - 0) = 14 > 0$

$\therefore f(x)$ has minimum value at $x = 4$.

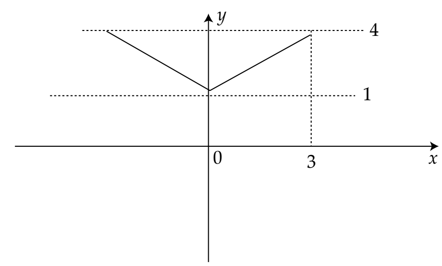
Hint: Find critical points using $f'(x) = 0$ and analyse further.

93. Option (b) is correct.

Solution:

Given: $f(x) = \begin{cases} |x|+1, & 0 < |x| \le 3 \\ 1, & x = 0 \end{cases}$

$\Rightarrow f(x) = \begin{cases} -x+1 & ; \ -3 \le x < 0 \\ x+1 & ; \ 0 < x \le 3 \\ 1 & ; \ x = 0 \end{cases}$



It is clear from the graph function $f(x)$ has maximum value at $x = +-3$ and attains local minimum at $x = 0$. So only statement 2 is correct.

Hint:

Generalise the $f(x)$ using the definition of modulus function and draw the curve and analyse further.

94. Option (b) is correct.

Solution:

Let $A = \int_0^1 \ln\left(\frac{1}{x}-1\right) dx$

$\Rightarrow A = \int_0^1 \ln\left(\frac{1-x}{x}\right) dx$

$\Rightarrow A = \int_0^1 \ln(1-x)\, dx - \int_0^1 \ln x\, dx$

Let $B = \int_0^1 \ln(1-x)\, dx$

Let $1 - x = u \Rightarrow dx = -du$

$\Rightarrow B = \int_1^0 -\ln u\, du$

$\Rightarrow B = \int_0^1 \ln u\, du$

$\Rightarrow B = \int_0^1 \ln x\, du$

$\therefore$ A = 0

Hint: Simplify given integral using $\ln\left(\frac{m}{n}\right) = \ln m - \ln n$ and solve further using properties of definite integral.

95. Option (d) is correct.

Solution:

Given: $\int_0^{\pi/2} (\sin^4 x + \cos^4 x)dx = k$

Let $f(x) = \sin^4 x + \cos^4 x$

$\Rightarrow f\left(\frac{\pi}{2} + x\right) = \cos^4 x + \sin^4 x = f(x)$

So, period of $f(x)$ is $\frac{\pi}{2}$

Let $I = \int_0^{20\pi} (\sin^4 x + \cos^4 x)dx$

$\Rightarrow I = \int_0^{40\left(\frac{\pi}{2}\right)} (\sin^4 x + \cos^4 x)dx$

As we know $\int_0^{nT} f(x)dx = n\int_0^{T} f(x)\,dx;$ where T is the period of function $f(x)$.

$\Rightarrow I = 40\int_0^{\frac{\pi}{2}} (\sin^4 x + \cos^4 x)dx$

$\Rightarrow$ I = 40 k

Hint:

Use $\int_0^{nT} f(x)dx = n\int_0^{T} f(x)dx$ where T = period of $f(x)$.

96. Option (a) is correct.

Solution:

Let A = $A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (e^{\cos x} \sin x + e^{\sin x} \cos x)dx$

$\Rightarrow A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\cos x} \sin x\, dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\sin x} \cos x dx$

$\Rightarrow A = 0 + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\sin x} \cos x\, dx$

$\left\{\because \int_{-a}^{a} f(x)dx = 0; f(-x) = -f(x)\right\}$

Let $\sin x = u \Rightarrow \cos x\, dx = du$

$\therefore A = \int_{-1}^{1} e^u du$

$\Rightarrow A = [e^u]_{-1}^{1}$

$\Rightarrow A = e - e^{-1}$

$\Rightarrow A = e - \frac{1}{e}$

$\Rightarrow A = \frac{e^2 - 1}{e}$

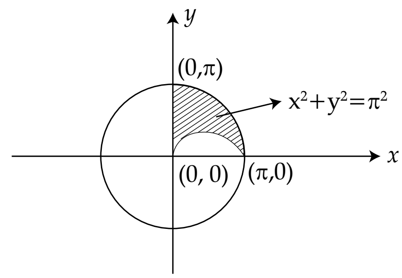
Hints:

1. Use $\int_{-a}^{a} f(x)dx = 0; f(-x) = -f(x)$

2. $\int_a^b f(x)dx = F(b) - F(a); \int f(x)dx = F(x)$

97. Option (b) is correct.

Solution:



Required area

$= \frac{1}{4}(\text{Area of circle } x^2 + y^2 = \pi^2) - \int_0^{\pi} \sin x\, dx$

$= \frac{1}{4}(\pi \cdot \pi^2) - [-\cos x]_0^{\pi}$ $\quad \therefore \{9 = \pi\}$

$= \frac{\pi^3}{4} - [-\cos\pi + \cos 0]$

$= \frac{\pi^3}{4} - [-(-1) + 1]$

$= \frac{\pi^3}{4} - 2$

Hint:

Required area

$= \frac{1}{4}(\text{Area of circle } x^2 + y^2 = \pi^2) - \int_0^{\pi} \sin x\, dx$

98. Option (b) is correct.

Solution:

Given differential equation is $\frac{dy}{dx} + \cos\left(\frac{dy}{dx}\right) = 0$

The given differential equation is not a polynomial equation in derivatives. Hence, the degree of the equation is not defined.

So, statement 1 is incorrect.

Given differential equation is

$\left(\frac{d^2y}{dx^2}\right)^3 + \cos\left(\frac{dy}{dx}\right) = 0$

As we know the order of differential equation is the order of the highest derivative present in the equation.

$\therefore$ Order of the given differential equation is 2.

So, statement 2 is correct.

Hint:

1. The degree of the differential equation is represented by the power of the highest order derivative in the given differential equation. The differential equation must be a polynomial in derivatives for the degree to be defined.
2. The order of differential equation is the order of the highest derivative present in the equation.

99. Option (b) is correct.

Solution:

The equation of the family of parabolas having vertex at the origin and axis along positive y-axis is $x^2 = 4ay$ ....(i), where a is a parameter.

Differentiation the above equation w.r.t. x, we get $2x = 4a\frac{dy}{dx}$

$$\Rightarrow a = \frac{x}{2\frac{dy}{dx}}$$

Substitute the value of a in equation (i), we get

$$x^2 = 4\frac{x}{2\frac{dy}{dx}}y$$

$$\Rightarrow x\frac{dy}{dx} = 2y$$

$$\Rightarrow x\frac{dy}{dx} - 2y = 0$$, which is the required differential equation.

Hint:

Equation of family of parabolas having vertex at the origin and axis along positive y-axis is $x^2 = 4ay$. Differentiate the equation and eliminate 'a' and solve further.

100. Option (c) is correct.

Solution:

Given: $(dy - dx) + \cos x\,(dy + dx) = 0$

$$\Rightarrow \frac{dy}{dx} - 1 + \cos x\frac{dy}{dx} + \cos x = 0$$

$$\Rightarrow \frac{dy}{dx}(1 + \cos x) = 1 - \cos x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \tan^2\frac{x}{2}$$

$$\Rightarrow \frac{dy}{dx} = \sec^2\frac{x}{2} - 1$$

$$\Rightarrow \int dy = \int \sec^2\frac{x}{2}dx - \int dx$$

$$\Rightarrow y = 2\tan\frac{x}{2} - x + c$$

Hint:

Divide the given equation by dx and solve further using variable separable method.

101. Option (c) is correct.

Solution:

Given: x = mean of squares of first n natural number

$$\Rightarrow x = \frac{1^2 + 2^2 + 3^2 + \cdots + n^2}{n} = \frac{n(n+1)(2n+1)}{6n}$$

And y = squares of mean of first n natural numbers.

$$\Rightarrow y = \left(\frac{1 + 2 + 3 + \cdots + n}{n}\right)^2$$

$$\Rightarrow y = \left(\frac{n(n+1)}{2n}\right)^2$$

$$\Rightarrow y = \frac{(n+1)^2}{4}$$

$$\therefore \frac{x}{y} = \frac{55}{42}$$

$$\Rightarrow \frac{(n+1)(2n+1)4}{6(n+1)^2} = \frac{55}{42}$$

$7(8n + 4) = 55(n + 1)$

$56n + 28 = 55n + 55$

$n = 27$

Hints:

1. Mean of data $x_1, x_2, x_3, ..., x_n$ is given by $\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$
2. $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$
3. $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

102. Option (b) is correct.

Solution:

Composite numbers are those numbers that have factors other than 1 and itself i.e., the numbers that are not prime numbers. There are 15 prime numbers from 1 to 50 so composite numbers from 1 to 50 are 50 – 15 – 1 = 34 numbers

{∵ 1 is neither prime nor composite number}.

Required probability $= \frac{34}{50}$

$= \frac{17}{25}$

> **Hint:**
> 1. Composite numbers are those numbers that have factors other than 1 and itself.
> 2. 1 is neither prime nor composite number.

103. (BONUS)

Solution:

$C(n, 7) = {}^nC_7$

$= \frac{n!}{(n-7)!7!}$

$= \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)}{7\times6\times5\times4\times3\times2}$

For $n > 7$, the numerator of $C(n, 7)$ will have 2 factors of 7 for $n \in [49, 50]$ as 49 will be present and on solving it, this will be factor of 7.

So for $n > 7$ there are many values for which ${}^nC_7$ will be multiple of 7.

So, question is incorrect.

104. Option (b) is correct.

Solution:

x and y are chosen from first 10 natural numbers

$\Rightarrow (x + y)$ can be maximum 19 and minimum 3

$(x + y)$ is divisible by 4 when:

$x + y = 4 \Rightarrow (x, y) = (1, 3)$

$x + y = 8 \Rightarrow (x, y) = (1, 7), (2, 6), (3, 5)$

$x + y = 12 \Rightarrow (x, y) = (10, 2), (9, 3), (8, 4), (7, 5)$

$x + y = 16 \Rightarrow (x, y) = (10, 6), (9, 7)$

So possible cases are 10 and total number of case are ${}^{10}C_2$

Required probability $= \frac{10}{{}^{10}C_2}$

$= \frac{10\times2}{10\times9}$

$= \frac{2}{9}$

> **Hint:** Write possible cases for $x + y$ is multiple of 4 and then use classical definition of probability.

105. Option (c) is correct.

Solution:

x is chosen from first n natural numbers let 2 numbers be x and $\frac{1}{x}$

AM $= \frac{x+\frac{1}{x}}{2}$ and G.M. $= \sqrt{x\left(\frac{1}{x}\right)} = 1$

as we know, AM ≥ GM

$\Rightarrow x+\frac{1}{x} \geq 2$

But $x+\frac{1}{x} = 2$ only when $x = 1$.

For rest of the natural number $x+\frac{1}{x} > 2$

So, number of cases $= n - 1$.

So, required probability $= \frac{n-1}{n}$

> **Hint:** Use AM ≥ GM.

106. Option (b) is correct.

Solution:

3 dice are rolled.

For AP, $d = 1$, possible cases are (1, 2, 3), (2, 3, 4), (3, 4, 5), (4, 5, 6)

$d = 2$, possible cases are (1, 3, 5), (2, 4, 6)

Now all these APs can be possible in descending order also.

So total cases $= 6 \times 2$

Required probability $= \frac{6\times2}{6\times6\times6}$

$= \frac{1}{18}$

> **Hint:** Write possible cases for common difference $d = 1$ and 2.

107. Option (b) is correct.

Solution:

$P(A) = 0.5, P(B) = 0.7, P(A \cap B) = 0.3$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

{By addition theorem of probability}

$\Rightarrow P(A \cup B) = 0.5 + 0.7 - 0.3 = 0.9$

By De Morgan's law, $P(A' \cap B') = P(A \cup B)'$

$\Rightarrow P(A' \cap B') = 1 - P(A \cup B)$

{By complement rule}

$\Rightarrow P(A' \cap B') = 1 - 0.9 = 0.1$

Also, $P(A' \cap B) = P(B) - P(A \cap B)$

$\Rightarrow P(A' \cap B) = 0.7 - 0.3 = 0.4$

and $P(A \cap B') = P(A) - P(A \cap B)$

$\Rightarrow P(A \cap B') = 0.5 - 0.3 = 0.2$

$\therefore P(A' \cap B') + P(A' \cap B) + P(A \cap B')$

$= 0.1 + 0.4 + 0.2 = 0.7$

Hint: Use addition theorem of probability i.e., $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

108. Option (a) is correct.

Solution:

P(getting head) = P(getting tail) = $\frac{1}{2}$

P(at most 4 tails in 5 toss) = 1 – P(5 tails in 5 toss)

$$= 1-\left\{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right\}$$

$$= 1-\frac{1}{32}$$

$$= \frac{31}{32}$$

Hint: Use P(getting at most 4 tails in 5 toss) = 1 – P (getting 5 tails in 5 toss).

109. Option (d) is correct.

Solution:

P(getting total ≥ 15) = P(getting total of 15, 16, 17, 18).

For 15: (5, 5, 5) i.e., 1 case and (6, 4, 5) i.e., 3! cases or 6 cases. (6, 6, 3) i.e., 3 cases

For 16: (6, 6, 4) i.e., $\frac{3!}{2!} = 3$ cases and (5, 5, 6) i.e., $\frac{3!}{2!} = 3$ cases.

For 17: (6, 6, 5) i.e., $\frac{3!}{2!} = 3$ cases.

For 18: (6, 6, 6) i.e., 1 case.

Total possible cases = 3 + 1 + 6 + 3 + 3 + 1 = 20 cases.

Required probability = $\frac{20}{6\times6\times6} = \frac{5}{54}$

Hint: P(getting total ≥ 15) = P(getting total of 15, 16, 17, 18).

110. Option (c) is correct.

Solution:

P(hitting a target) = 0.5

P(not hitting a target) = 1 – 0.5 = 0.5

P(at least one hit in 4 shots) = 1 – P(no hits in 4 shots)

= 1 – {0.5 × 0.5 × 0.5 × 0.5}

$$= 1-\frac{1}{16}$$

$$= \frac{15}{16}$$

Hint: P(at least one hit in 4 shots) = 1 – P (no hits in 4 shots).

111. Option (c) is correct.

Solution:

We have 2 white, 3 black and 4 red balls.

Required ways = Total ways – no. of ways of drawing no black ball.

$$= 9_{C_3} - 6_{C_3}$$

$$= \frac{9\times8\times7}{3\times2} - \frac{6\times5\times4}{3\times2}$$

= 64 ways.

Hint: Required ways = Total ways – no. of ways of drawing no black balls.

112. Option (d) is correct.

Solution:

p = P(ship sinks) = $\frac{1}{5}$

q = P (ship arrives) = $\frac{4}{5}$

$$P(x=3) = 5_{C_3}\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)^2$$

{Using Binomial probability theorem}

$$= \frac{5\times4}{2}\times\frac{64}{5\times625}$$

$$= \frac{128}{625}$$

Hint: Use binomial probability theorem and solve it.

113. Option (a) is correct.

Solution:

There are 13 spades (including ace of spade) and rest 3 aces.

P(getting a spade or an ace) = $\frac{13+3}{52}$

$$= \frac{16}{52} = \frac{4}{13}$$

Odds against winning

$$= \frac{\text{Number of unfavorable choices}}{\text{Number of favorable choices}}$$

$$= \frac{13-4}{4} = \frac{9}{4}$$

Hint: Odds against an event is the ratio of number of unfavorable choices to the number of favorable choices.

114. Option (c) is correct.

Solution:

Given: coefficient of correlation between ages of husband and wife at the time of marriage, P = 0.7.

As we know the coefficient of correlation is independent of change of scale.

$\therefore$ Coefficient of correlation at silver jubilee will be 0.7.

Hint: Coefficient of correlation is independent of change of scale.

115. Option (b) is correct.

Solution:

Let A: Event that job will be completed on time

B: Event that strike happens.

P(B) = 0.6 $\quad P(\bar{B}) = 1 - 0.6 = 0.4$

$P\left(\frac{A}{\bar{B}}\right) = 0.85 \quad P\left(\frac{A}{B}\right) = 0.35$

$$P(A) = P(B)P\left(\frac{A}{B}\right) + P(\bar{B})P\left(\frac{A}{\bar{B}}\right)$$

= (0.6) (0.35) + (0.4)(0.85)

= 0.21 + 0.34

= 0.55

$P(\bar{A})$ = 1 – 0.55 = 0.45

Hint: Use total probability theorem.

116. Option (d) is correct.

Solution:

Subject	Mathe-matics	Physics	Chemistry	Biology
Mean marks	40	28	38	36
S.D	15	12	14	16

Coefficient of variance of Mathematics data

C.V. (M) = $\frac{15}{40} \times 100$ = 37.5%

Coefficient of variance of physics data,

C.V (P) = $\frac{12}{28} \times 100$ = 42.8%

Coefficient of variance of chemistry data,

C.V. (C) = $\frac{14}{38} \times 100$ = 36.8%

Coefficient of variance of biology data,

C.V. (B) = $\frac{16}{36} \times 100$ = 44.4%

$\therefore$ C.V (B) > C.V (P) > C.V (M) > C.V (C)

$\therefore$ Biology shows the heighest variability in marks.

Hints:

1. Find coefficient of variance for each subject data and use high coefficient of variance shows high variability.
2. Coefficient of variance = $\frac{S.D.}{Mean} \times 100$

117. Option (a) is correct.

Solution:

Subject	Mathe-matics	Physics	Chemistry	Biology
Mean marks	40	28	38	36
S.D.	15	12	14	16

Coefficient of variation of marks in mathematics.

C.V. (M) = $\frac{S.D.}{Mean} \times 100$

= $\frac{15}{40} \times 100$

= 37.5%

Hint:

Coefficient of variation of data,

C.V. = $\frac{S.D.}{Mean} \times 100$

118. Option (c) is correct.

Solution:

Class interval	Frequency	Cumulative frequency
0–10	1	1
10–20	2	3
20–30	4	7
30–40	6	13
40–50	4	17
50–60	3	20

Here n = 20

$\frac{n}{2} = \frac{20}{2} = 10$

Thus the observations lies between the class interval 30–40, which is called the median class.

Therefore,

Lower class limit l = 30

Class size h = 10

Frequency of the median class f = 6

Cumulative frequency of the class preceding the median class cf = 7

As we know, median = $l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$= 30 + \left(\frac{10-7}{6}\right) \times 10$

$= 30 + 5$

$= 35$

Hint: Find cumulative frequencies of all classes and $\frac{n}{2}$. After that, locate the class whose cumulative frequence is greater than (nearest to) $\frac{n}{2}$. The class is called the median class.

After finding median class use the below formula median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

where,

119. Option (d) is correct.

Solution:

Class interval	Frequency	*c.f.*	Midpoint (x_i)	$\lvert x_i - m \rvert$	$f_i \lvert x_i - m \rvert$
0–10	1	1	5	30	30
10–20	2	3	15	20	40
20–30	4	7	25	10	40
30–40	6	13	35	0	0
40–50	4	17	45	10	40
50–60	3	20	55	20	60
		N = 20			$\sum f_i \lvert x_i - m \rvert = 210$

As we know, M.D $= \frac{1}{N}\sum_{i=1}^{n} f_i \lvert x_i - m \rvert$

Where,
N = Number of observations
M = Median
x_i = Midpoint of class interval
f_i = frequency of x_i

$\therefore$ M.D $= \frac{1}{20}(2.0) = 10.5$

Hint:
Mean deviation about median M.D $= \frac{1}{N}\sum_{i=1}^{n} f_i \lvert x_i - m \rvert$

120. Option (b) is correct.

Solution:

Class interval	Frequency	Midpoint (x_i)	$x_i f_i$
0–10	1	5	5
10–20	2	15	30
20–30	4	25	100
30–40	6	35	210
40–50	4	45	180
50–60	3	55	165
	$\sum f = 20$		$\sum f_i x_i = 690$

So, Mean $(\bar{x}) = \frac{1}{\sum f} \times \sum f_i x_i$

$\bar{x} = \frac{690}{20} = 34.5$

Class interval	Frequency	Midpoint (x_i)	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
0–10	1	5	29.5	29.5
10–20	2	15	19.5	39.0
20–30	4	25	9.5	38.0
30–40	6	35	0.5	3.00
40–50	4	45	10.5	42.0
50–60	3	55	20.5	61.5
	N = $\sum f$ = 20			$\sum f_i \lvert x_i - \bar{x} \rvert = 213$

So, mean deviation about median

$= \frac{1}{N}\sum f_i \lvert x_i - \bar{x} \rvert$

$= \frac{1}{20}(213)$

$= 10.65$

Hint:
Mean deviation about mean $= \frac{1}{N}\sum f_i \lvert x_i - \bar{x} \rvert$