**Time : 2 : 30 Hours**

**Total Marks : 300**

## Instructions :

1. *This Test Booklet contains **120** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.*
2. *You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.*
3. *All items carry equal marks.*
4. ***Penalty for wrong answers :***
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i) *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.*
   (ii) *If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii) *If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.*

1. If $\omega$ is a non-real cube root of 1, then what is the value of $\left|\dfrac{1-\omega}{\omega+\omega^2}\right|$ ?
   (a) $\sqrt{3}$
   (b) $\sqrt{2}$
   (c) 1
   (d) $\dfrac{4}{\sqrt{3}}$

2. What is the number of 6-digit numbers that can be formed only by using 0, 1, 2, 3, 4 and 5 (each once); and divisible by 6 ?
   (a) 96
   (b) 120
   (c) 192
   (d) 312

3. What is the binary number equivalent to decimal number 1011 ?
   (a) 1011
   (b) 111011
   (c) 11111001
   (d) 111110011

4. Let $A$ be a matrix of order $3 \times 3$ and $|A| = 4$. If $|2 \, \mathrm{adj}(3A)| = 2^{\alpha}3^{\beta}$ then what is the value of ($\alpha + \beta$) ?
   (a) 12
   (b) 13
   (c) 17
   (d) 24

5. If $\alpha$ and $\beta$ are the distinct roots of equation $x^2 - x + 1 = 0$, then what is the value of $\left|\dfrac{\alpha^{100}+\beta^{100}}{\alpha^{100}-\beta^{100}}\right|$ ?
   (a) $\sqrt{3}$
   (b) $\sqrt{2}$
   (c) 1
   (d) $\dfrac{1}{\sqrt{3}}$

6. Let A and B be symmetric matrices of same order, then which one of the following is correct regarding (AB – BA)?
   1. Its diagonal entries are equal but nonzero
   2. The sum of its non-diagonal entries is zero
   Select the correct answer using the code given below:
   (a) 1 only
   (b) 2 only
   (c) Both 1 and 2
   (d) Neither 1 nor 2

7. Consider the following statements in respect of square matrices A, B, C each of same order n :
   1. $AB = AC \Rightarrow B = C$ if A is non-singular
   2. If $BX = CX$ for every column matrix X having n rows then $B = C$
   Which of the statements given above is/are correct ?
   (a) 1 only
   (b) 2 only
   (c) Both 1 and 2
   (d) Neither 1 nor 2

8. The system of linear equations $x + 2y + z = 4$, $2x + 4y + 2z = 8$ and $3x + 6y + 3z = 10$ has
   (a) a unique solution
   (b) infinite many solutions
   (c) no solution
   (d) exactly three solutions

9. Let AX = B be a system of 3 linear equations with 3-unknowns. Let $X_1$ and $X_2$ be its two distinct solutions. If the combination $aX_1 + bX_2$ is a solution of AX = B; where $a, b$ are real numbers, then which one of the following is correct ?

**(a)** $a = b$                 **(b)** $a + b = 1$
**(c)** $a + b = 0$             **(d)** $a - b = 1$

**10.** What is the sum of the roots of the equation

$$\begin{vmatrix} 0 & x-a & x-b \\ 0 & 0 & x-c \\ x+b & x+c & 1 \end{vmatrix} = 0 ?$$

**(a)** $a + b + c$             **(b)** $a - b + c$
**(c)** $a + b - c$             **(d)** $a - b - c$

**11.** If $2 - i\sqrt{3}$ where $i = \sqrt{-1}$ is a root of the equation $x^2 + ax + b = 0$, then what is the value of $(a + b)$ ?

**(a)** $-11$                   **(b)** $-3$
**(c)** $0$                     **(d)** $3$

**12.** If $z = \dfrac{1 + i\sqrt{3}}{1 - i\sqrt{3}}$ where $i = \sqrt{-1}$, then what is the argument of $z$ ?

**(a)** $\dfrac{\pi}{3}$        **(b)** $\dfrac{2\pi}{3}$

**(c)** $\dfrac{4\pi}{3}$       **(d)** $\dfrac{5\pi}{6}$

**13.** If $a$, $b$, $c$ are in AP, then what is

$$\begin{vmatrix} x+1 & x+2 & x+3 \\ x+2 & x+3 & x+4 \\ x+a & x+b & x+3 \end{vmatrix} \text{ equal to ?}$$

**(a)** $-1$                    **(b)** $0$
**(c)** $1$                     **(d)** $2$

**14.** $\log_x a$, $a^x$ and $\log_b x$ are in GP, then what is $x$ equal to ?

**(a)** $\log_a(\log_b a)$      **(b)** $\log_b(\log_a b)$

**(c)** $\dfrac{\log_a(\log_b a)}{2}$   **(d)** $\dfrac{\log_b(\log_a b)}{2}$

**15.** If $2^{\frac{1}{c}}$, $2^{\frac{b}{ac}}$, $2^{\frac{1}{a}}$ are in GP, then which one of the following is correct?

**(a)** $a, b, c$ are in AP     **(b)** $a, b, c$ are in GP
**(c)** $a, b, c$ are in HP     **(d)** $ab, be, ca$ are in AP

**16.** The first and the second terms of an AP are $\dfrac{5}{2}$ and $\dfrac{23}{12}$ respectively. If $n^{\text{th}}$ term is the largest negative term, what is the value of $n$ ?

**(a)** 5
**(b)** 6
**(c)** 7
**(d)** $n$ cannot be determined

**17.** For how many integral values of $k$, the equation $x^2 - 4x + k = 0$, where $k$ is an integer has real roots and both of them lie in the interval $(0, 5)$ ?

**(a)** 3                       **(b)** 4
**(c)** 5                       **(d)** 6

**18.** In an AP, the first term is $x$ and the sum of the first $n$ terms is zero. What is the sum of next $m$ terms ?

**(a)** $\dfrac{mx(m+n)}{n-1}$  **(b)** $\dfrac{mx(m+n)}{1-n}$

**(c)** $\dfrac{nx(m+n)}{m-1}$  **(d)** $\dfrac{nx(m+n)}{1-m}$

**19.** Consider the following statements :
1. $(25)! + 1$ is divisible by 26
2. $(6)! + 1$ is divisible by 7
Which of the above statements is/are correct ?
**(a)** 1 only                  **(b)** 2 only
**(c)** Both 1 and 2           **(d)** Neither 1 nor 2

**20.** If $z$ is a complex number such that $\dfrac{z-1}{z+1}$ is purely imaginary, then what is $|z|$ equal to ?

**(a)** $\dfrac{1}{2}$          **(b)** $\dfrac{2}{3}$

**(c)** $1$                     **(d)** $2$

**21.** How many real numbers satisfy the equation $|x - 4| + |x - 7| = 15$ ?
**(a)** Only one                **(b)** Only two
**(c)** Only three              **(d)** Infinitely many

**22.** A mapping $f : A \to B$ defined as $f(x) = \dfrac{2x+3}{3x+5}, x \in A$. If $f$ is to be onto, then what are A and B equal to ?

**(a)** $A = R \setminus \{-\dfrac{5}{3}\}$ and $B = R \setminus \{-\dfrac{2}{3}\}$

**(b)** $A = R$ and $B = R \setminus \{-\dfrac{5}{3}\}$

**(c)** $A = R \setminus \{-\dfrac{3}{2}\}$ and $B = R \setminus \{0\}$

**(d)** $A = R \setminus \{-\dfrac{5}{3}\}$ and $B = R \setminus \{\dfrac{2}{3}\}$

**23.** $\alpha$ and $\beta$ are distinct real roots of the quadratic equation $x^2 + ax + b = 0$. Which of the following statements is/are sufficient to find $\alpha$ ?
1. $\alpha + \beta = 0$, $\alpha^2 + \beta^2 = 2$
2. $\alpha\beta^2 = -1$, $a = 0$

Select the correct answer using the code given below:

(a) 1 only          (b) 2 only

(c) Both 1 and 2    (d) Neither 1 nor 2

**24.** If the sixth term in the binomial expansion of $\left( x^{-\frac{8}{3}} + x^2 log_{10}x \right)^8$ is 5600, then what is the value of $x$ ?

(a) 6          (b) 8

(c) 9          (d) 10

**25.** How many terms are there in the expansion of $(3x - y)^4 (x + 3y)^4$ ?

(a) 9          (b) 12

(c) 15         (d) 17

**26.** $p, q, r$ and $s$ are in AP such that $p + s = 8$ and $qr = 15$. What is the difference between largest and smallest numbers ?

(a) 6          (b) 5

(c) 4          (d) 3

**27.** Consider the following statements for a fixed natural number $n$ :

1.  $C(n, r)$ is greatest if $n = 2r$

2.  $C(n, r)$ is greatest if $n = 2r - 1$ and $n = 2r + 1$

Which of the statements given above is/are correct ?

(a) 1 only          (b) 2 only

(c) Both 1 and 2    (d) Neither 1 nor 2

**28.** $m$ parallel lines cut $n$ parallel lines giving rise to 60 parallelograms. What is the value of $(m + n)$?

(a) 6          (b) 7

(c) 8          (d) 9

**29.** Let $x$ be the number of permutations of the word 'PERMUTATIONS' and $y$ be the number of permutations of the word 'COMBINATIONS'. Which one of the following is correct ?

(a) $x = y$          (b) $y = 2x$

(c) $x = 4y$          (d) $y = 4x$

**30.** 5-digit numbers are formed using the digits 0, 1, 2, 4, 5 without repetition. What is the percentage of numbers which are greater than 50,000 ?

(a) 20%          (b) 25%

(c) $\frac{100}{3}$%          (d) $\frac{110}{3}$%

*Consider the following for the next **two (02)** items that follow :*

Let $sin\beta$ be the GM of $sin\alpha$ and $cos\alpha$; $tan\gamma$ be the AM of $sin\alpha$ and $cos\alpha$.

**31.** What is $cos2\beta$ equal to ?

(a) $(cos\alpha - sin\alpha)^2$          (b) $(cos\alpha + sin\alpha)^2$

(c) $(cos\alpha - sin\alpha)^3$          (d) $\frac{(cos\alpha - sin\alpha)^2}{2}$

**32.** What is the value of $sec2\gamma$?

(a) $\frac{3 - sin2\alpha}{5 + 2sin2\alpha}$          (b) $\frac{5 + sin2\alpha}{3 - sin2\alpha}$

(c) $\frac{3 - 2sin2\alpha}{4 + sin2\alpha}$          (d) $\frac{3 - sin2\alpha}{4 + 3sin2\alpha}$

*Consider the following for the next **two (02)** items that follow :*

A flagstaff 20 m long standing on a pillar 10 m high subtends an angle $tan^{-1}(0.5)$ at a point $P$ on the ground. Let $\theta$ be the angle subtended by the pillar at this point $P$

**33.** If $x$ is the distance of P from bottom of the pillar, then consider the following statements:

1.  $x$ can take two values which are in the ratio $1 : 3$

2.  $x$ can be equal to height of the flagstaff

Which of the statements given above is/are correct ?

(a) 1 only          (b) 2 only

(c) Both 1 and 2    (d) Neither 1 nor 2

**34.** What is a possible value of $tan\theta$ ?

(a) $\frac{3}{4}$          (b) $\frac{2}{3}$

(c) $\frac{1}{3}$          (d) $\frac{1}{4}$

*Consider the following for the next **two (02)** items that follow :*

The perimeter of a triangle ABC is 6 times the AM of sine of angles of the triangle. Further $BC = \sqrt{3}$ and $CA = 1$.

**35.** What is the perimeter of the triangle ?

(a) $\sqrt{3} + 1$          (b) $\sqrt{3} + 2$

(c) $\sqrt{3} + 3$          (d) $2\sqrt{3} + 1$

**36.** Consider the following statements :

1.  ABC is right angled triangle

2.  The angles of the triangle are in AP

Which of the statements given above is/are correct ?

(a) 1 only          (b) 2 only

(c) Both 1 and 2    (d) Neither 1 nor 2

*Consider the following for the next **two (02)** items that follow :*

Let $x = \dfrac{\sin^2 A + \sin A + 1}{\sin A}$ where $0 < A \le \dfrac{\pi}{2}$

**37.** What is the minimum value of $x$ ?
(a) 1
(b) 2
(c) 3
(d) 4

**38.** At what value of $A$ does $x$ attain the minimum value ?
(a) $\dfrac{\pi}{6}$
(b) $\dfrac{\pi}{4}$
(c) $\dfrac{\pi}{3}$
(d) $\dfrac{\pi}{2}$

*Consider the following for the next **two (02)** items that follow :*

In the triangle ABC, $a^2 + b^2 + c^2 = ac + \sqrt{3}\,bc$

**39.** What is the nature of the triangle ?
(a) Equilateral
(b) Isosceles
(c) Right angled triangle
(d) Scalene but not right angled

**40.** If $c = 8$, what is the area of the triangle ?
(a) $4\sqrt{3}$
(b) $6\sqrt{3}$
(c) $8\sqrt{3}$
(d) $12\sqrt{3}$

*Consider the following for the next **two (02)** items that follow :*

Consider the function
$f(x) = |x - 2| + |3 - x| + |4 - x|$, where $x \in R$.

**41.** At what value of $x$ does the function attain minimum value ?
(a) 2
(b) 3
(c) 4
(d) 0

**42.** What is the minimum value of the function ?
(a) 2
(b) 3
(c) 4
(d) 0

*Consider the following for the next **two (02)** items that follow :*

Consider the sum
$S = 0! + 1! + 2! + 3! + 4! + \ldots + 100!$

**43.** If the sum $S$ is divided by 8, what is the remainder ?
(a) 0
(b) 1
(c) 2
(d) Cannot be determined

**44.** If the sum $S$ is divided by 60, what is the remainder ?

(a) 1
(b) 3
(c) 17
(d) 34

*Consider the following for the next **two (02)** items that follow :*

In a triangle PQR, P is the largest angle and $\cos P = \dfrac{1}{3}$. Further the in-circle of the triangle touches the sides PQ, QR and RP at N, L and M respectively such that the lengths PN, QL and RM are $n$, $n + 2$, $n + 4$ respectively where n is an integer.

**45.** What is the value of n ?
(a) 4
(b) 6
(c) 8
(d) 10

**46.** What is the length of the smallest side ?
(a) 12
(b) 14
(c) 16
(d) 18

*Consider the following for the next **two (02)** items that follow :*

Given that
$\sin x + \cos x + \tan x + \cot x + \sec x + \operatorname{cosec} x = 7$

**47.** The given equation can be reduced to
(a) $\sin^2 2x - 44 \sin 2x + 36 = 0$
(b) $\sin^2 2x + 44 \sin 2x - 36 = 0$
(c) $\sin^2 2x - 22 \sin 2x + 18 = 0$
(d) $\sin^2 2x + 22 \sin 2x - 18 = 0$

**48.** If $\sin 2x = a - b\sqrt{c}$, where $a$ and $b$ are natural numbers and $c$ is prime number, then what is the value of $a - b + 2c$ ?
(a) 0
(b) 14
(c) 21
(d) 28

*Consider the following for the next **two (02)** items that follow :*

A quadratic equation is given by

$(3 + 2\sqrt{2})x^2 - (4 + 2\sqrt{3})x + (8 + 4\sqrt{3}) = 0$

**49.** What is the HM of the roots of the equation ?
(a) 2
(b) 4
(c) $2\sqrt{2}$
(d) $2\sqrt{3}$

**50.** What is the GM of the roots of the equation ?
(a) $\sqrt{2}\left(\sqrt{6} - \sqrt{3} + \sqrt{2} - 1\right)$
(b) $\sqrt{2}\left(\sqrt{6} + \sqrt{3} - \sqrt{2} - 1\right)$
(c) $\left(\sqrt{6} - \sqrt{3} + \sqrt{2} - 1\right)$
(d) $\left(\sqrt{6} + \sqrt{3} + \sqrt{2} - 1\right)$

*Consider the following for the next **two (02)** items that follow :*

$$\text{Let } \Delta(a, b, c, \alpha) = \begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix}$$

**51.** If $\Delta(a, b, c, \alpha) = 0$ for every $\alpha > 0$, then which one of the following is correct ?
(a) $a, b, c$ are in AP
(b) $a, b, c$ are in GP
(c) $a, 2b, c$ are in AP
(d) $a, 2b, c$ are in GP

**52.** If $\Delta(7, 4, 2, \alpha) = 0$, then $\alpha$ is a root of which one of the following equations ?
(a) $7x^2 + 4x + 2 = 0$
(b) $7x^2 - 4x + 2 = 0$
(c) $7x^2 + 8x + 2 = 0$
(d) $7x^2 - 8x + 2 = 0$

*Consider the following for the next **two (02)** items that follow :*

Given that $m(\theta) = \cot^2\theta + n^2\tan^2\theta + 2n$, where $n$ is a fixed positive real number.

**53.** What is the least value of $m(\theta)$?
(a) $n$
(b) $2n$
(c) $3n$
(d) $4n$

**54.** Under what condition does $m$ attain the least value ?
(a) $n = \tan^2\theta$
(b) $n = \cot^2\theta$
(c) $n = \sin^2\theta$
(d) $n = \cos^2\theta$

*Consider the following for the next **two (02)** items that follow :*

A quadrilateral is formed by the lines $x = 0$, $y = 0$, $x + y = 1$ and $6x + y = 3$.

**55.** What is the equation of diagonal through origin ?
(a) $3x + y = 0$
(b) $2x + 3y = 0$
(c) $3x - 2y = 0$
(d) $3x + 2y = 0$

**56.** What is the equation of other diagonal ?
(a) $x + 2y - 1 = 0$
(b) $x - 2y - 1 = 0$
(c) $2x + y + 1 = 0$
(d) $2x + y - 1 = 0$

*Consider the following for the next **two (02)** items that follow :*

P$(x, y)$ is any point on the ellipse $x^2 + 4y^2 = 1$. Let E, F be the foci of the ellipse.

**57.** What is PE + PF equal to ?
(a) 1
(b) 2
(c) 3
(d) 4

**58.** Consider the following points :

1. $\left( \dfrac{\sqrt{3}}{2}, 0 \right)$

2. $\left( \dfrac{\sqrt{3}}{2}, \dfrac{1}{4} \right)$
3. $\left( \dfrac{\sqrt{3}}{2}, -\dfrac{1}{4} \right)$

Which of the above points lie on latus rectum of ellipse ?
(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3

*Consider the following for the next **two (02)** items that follow :*

The line $y = x$ partitions the circle $(x - a)^2 + y^2 = a^2$ in two segments.

**59.** What is the area of minor segment ?
(a) $\dfrac{(\pi - 2)a^2}{4}$
(b) $\dfrac{(\pi - 1)a^2}{4}$
(c) $\dfrac{(\pi - 2)a^2}{2}$
(d) $\dfrac{(\pi - 1)a^2}{2}$

**60.** What is the area of major segment ?
(a) $\dfrac{(3\pi - 2)a^2}{4}$
(b) $\dfrac{(3\pi + 2)a^2}{4}$
(c) $\dfrac{(3\pi - 2)a^2}{2}$
(d) $\dfrac{(3\pi + 2)a^2}{2}$

*Consider the following for the next **two (02)** items that follow :*

Let A$(1, -1, 2)$ and B$(2, 1, -1)$ be the end points of the diameter of the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$.

**61.** What is $u + v + w$ equal to ?
(a) $-2$
(b) $-1$
(c) 1
(d) 2

**62.** If P$(x, y, z)$ is any point on the sphere, then what is PA$^2$ + PB$^2$ equal to ?
(a) 15
(b) 14
(c) 13
(d) 6·5

*Consider the following for the next **two (02)** items that follow :*

Consider two lines whose direction ratios are $(2, -1, 2)$ and $(k, 3, 5)$. They are inclined at an angle $\dfrac{\pi}{4}$.

**63.** What is the value of $k$ ?
(a) 4
(b) 2
(c) 1
(d) $-1$

**64.** What are the direction ratios of a line which is perpendicular to both the lines ?
(a) $(1, 2, 10)$
(b) $(-1, -2, 10)$
(c) $(11, 12, -10)$
(d) $(11, 2, -10)$

*Consider the following for the next **two (02)** items that follow :*

Let $\vec{a} = 3\hat{i} + 3\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{j} - \hat{k}$. Let $\vec{b}$ be such that $\vec{a} \cdot \vec{b} = 27$ and $\vec{a} \times \vec{b} = 9\vec{c}$

**65.** What is $\vec{b}$ equal to ?

(a) $3\hat{i} + 4\hat{j} + 2\hat{k}$    (b) $5\hat{i} + 2\hat{j} + 2\hat{k}$
(c) $5\hat{i} - 2\hat{j} + 6\hat{k}$    (d) $3\hat{i} + 3\hat{j} + 4\hat{k}$

**66.** What is the angle between $(\vec{a} + \vec{b})$ and $\vec{c}$ ?

(a) $\dfrac{\pi}{2}$        (b) $\dfrac{\pi}{3}$

(c) $\dfrac{\pi}{4}$        (d) $\dfrac{\pi}{6}$

*Consider the following for the next **two (02)** items that follow :*

Let a vector $\vec{a} = 4\hat{i} - 8\hat{j} + \hat{k}$ make angles $\alpha$, $\beta$, $\gamma$ with the positive directions of $x$, $y$, $z$ axes respectively.

**67.** What is $\cos\alpha$ equal to ?

(a) $\dfrac{1}{3}$        (b) $\dfrac{4}{9}$

(c) $\dfrac{5}{9}$        (d) $\dfrac{2}{3}$

**68.** What is $\cos2\beta + \cos2\gamma$ equal to ?

(a) $-\dfrac{32}{81}$      (b) $-\dfrac{16}{81}$

(c) $\dfrac{16}{81}$       (d) $\dfrac{32}{81}$

*Consider the following for the next **two (02)** items that follow :*

The position vectors of two points $A$ and $B$ are $\hat{i} - \hat{j}$ and $\hat{j} + \hat{k}$ respectively.

**69.** Consider the following points :
1. $(-1, -3, 1)$
2. $(-1, 3, 2)$
3. $(-2, 5, 3)$
Which of the above points lie on the line joining A and B ?

(a) 1 and 2 only     (b) 2 and 3 only
(c) 1 and 3 only     (d) 1, 2 and 3

**70.** What is the magnitude of $\overrightarrow{AB}$ ?

(a) 2          (b) 3
(c) $\sqrt{6}$        (d) $\sqrt{3}$

*Consider the following for the next **three (03)** items that follow :*

Let $f(x) = Pe^x + Qe^{2x} + Re^{3x}$, where $P$, $Q$, $R$ are real numbers. Further $f(0) = 6$, $f'(\ln 3) = 282$ and
$$\int_0^{\ln 2} f(x)\,dx = 11$$

**71.** What is the value of $Q$ ?
(a) 1              (b) 2
(c) 3              (d) 4

**72.** What is the value of $R$ ?
(a) 1              (b) 2
(c) 3              (d) 4

**73.** What is $f'(0)$ equal to ?
(a) 18           (b) 16
(c) 15           (d) 14

*Consider the following for the next **two (02)** items that follow :*

Suppose $E$ is the differential equation representing family of curves $y^2 = 2cx + 2c\sqrt{c}$ where $c$ is a positive parameter.

**74.** What is the order of the differential equation ?
(a) 1              (b) 2
(c) 3              (d) 4

**75.** What is the degree of the differential equation ?
(a) 2
(b) 3
(c) 4
(d) Degree does not exist

*Consider the following for the next **three (03)** items that follow :*

Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

**76.** What is $f(0)$ equal to ?
(a) $-1$         (b) 0
(c) 1           (d) 2

**77.** What is $\lim\limits_{x \to 0} \dfrac{f(x)}{x}$ equal to ?

(a) $-1$         (b) 0
(c) 1           (d) 2

**78.** What is $\lim\limits_{x \to 0} \dfrac{f(x)}{x^2}$ equal to ?

(a) $-1$         (b) 0
(c) 1           (d) 2

*Consider the following for the next **two (02)** items that follow :*

Let $f(x) = \sin[\pi^2]x + \cos[-\pi^2]x$ where [.] is a greatest integer function

**79.** What is $f\left(\dfrac{\pi}{2}\right)$ equal to ?

(a) $-1$       (b) $0$
(c) $1$       (d) $2$

**80.** What is $f\left(\dfrac{\pi}{4}\right)$ equal to ?

(a) $-\dfrac{1}{\sqrt{2}}$       (b) $-1$

(c) $1$       (d) $\dfrac{1}{\sqrt{2}}$

*Consider the following for the next **three (03)** items that follow :*

Let $I_1 = \displaystyle\int_0^\pi \dfrac{x}{1+\cos^2 x}\,dx$ and $I_2 = \displaystyle\int_0^\pi \dfrac{x}{1+\sin^2 x}\,dx$

**81.** What is the value of $\dfrac{I_1 + I_2}{I_1 - I_2}$ ?

(a) $1$       (b) $\pi$
(c) $\pi^2/2$       (d) $\dfrac{\pi+1}{\pi-1}$

**82.** What is the value of $8I_1^2$ ?

(a) $\pi$       (b) $\pi^2$
(c) $\pi^3$       (d) $\pi^4$

**83.** What is the value of $I_2$ ?

(a) $\dfrac{\pi}{\sqrt{2}}$       (b) $\dfrac{\pi^2}{2\sqrt{2}}$

(c) $\dfrac{3\pi}{2\sqrt{2}}$       (d) $\dfrac{\pi}{4\sqrt{2}}$

*Consider the following for the next **two (02)** items that follow :*

Let $l = \displaystyle\int_a^b \dfrac{|x|}{x}\,dx,\ a < b$

**84.** What is $l$ equal to when $a < 0 < b$ ?
(a) $a + b$       (b) $a - b$

(c) $b - a$       (d) $\dfrac{(a+b)}{2}$

**85.** What is $l$ equal to when $a < b < 0$ ?
(a) $a + b$       (b) $a - b$

(c) $b - a$       (d) $\dfrac{(a+b)}{2}$

*Consider the following for the next **three (03)** items that follow :*

Let $f(x) = |lnx|,\ x \neq 1$

**86.** What is the derivative of $f(x)$ at $x = 0.5$ ?
(a) $-2$       (b) $-1$
(c) $1$       (d) $2$

**87.** What is the derivative of $f(x)$ at $x = 2$ ?

(a) $-\dfrac{1}{2}$       (b) $-1$

(c) $\dfrac{1}{2}$       (d) $2$

**88.** What is the derivative of $f \circ f(x)$, where $1 < x < 2$ ?

(a) $\dfrac{1}{lnx}$       (b) $\dfrac{1}{xlnx}$

(c) $-\dfrac{1}{lnx}$       (d) $-\dfrac{1}{xlnx}$

*Consider the following for the next **two (02)** items that follow :*

Let $f(x) = \begin{cases} x+6, & x \leq 1 \\ px+q, & 1 < x < 2 \\ 5x, & x \geq 2 \end{cases}$

and $f(x)$ is continuous

**89.** What is the value of $p$ ?
(a) $2$       (b) $3$
(c) $4$       (d) $5$

**90.** What is the value of $q$ ?
(a) $2$       (b) $3$
(c) $4$       (d) $5$

**91.** Consider the following statements :
1. $f(x) = lnx$ is increasing in $(0, \infty)$
2. $g(x) = e^x + e^x$ is decreasing in $(0, \infty)$
Which of the statements given above is/are correct ?
(a) 1 only       (b) 2 only
(c) Both 1 and 2       (d) Neither 1 nor 2

**92.** What is the derivative of $\sin^2 x$ with respect to $\cos^2 x$ ?
(a) $-1$       (b) $1$
(c) $\sin 2x$       (d) $\cos 2x$

**93.** For what value of $m$ with $m < 0$, is the area bounded by the lines $y = x$, $y = mx$ and $x = 2$ equal to 3 ?

(a) $-\dfrac{1}{2}$       (b) $-1$

(c) $-\dfrac{3}{2}$       (d) $-2$

**94.** What is the derivative of $\csc(x°)$ ?

(a) $-\csc(x°)\cot(x°)$

(b) $-\dfrac{\pi}{180}\csc(x°)\cot(x°)$

(c) $\dfrac{\pi}{180}\csc(x°)\cot(x°)$

(d) $-\dfrac{\pi}{180}\csc(x)\cot(x)$

**95.** A solution of the differential equation $\left(\dfrac{dy}{dx}\right)^2 - x\dfrac{dy}{dx} = 0$ is

(a) $y = x^2/2 + c$

(b) $y = 2x + 4$

(c) $y = x^2 + 1$

(d) $y = \dfrac{(x^2 - x)}{2}$

**96.** If $f(x) = x^2 + 2$ and $g(x) = 2x - 3$, then what is $(fg)(1)$ equal to ?

(a) 3

(b) 1

(c) –2

(d) –3

**97.** What is the range of the function $f(x) = x + |x|$ if the domain is the set of real numbers ?

(a) $(0, \infty)$

(b) $[0, \infty)$

(c) $(-\infty, \infty)$

(d) $[1, \infty)$

**98.** If $f(x) = x(4x^2 - 3)$, then what is $f(\sin\theta)$ equal to ?

(a) $-\sin3\theta$

(b) $-\cos3\theta$

(c) $\sin3\theta$

(d) $-\sin4\theta$

**99.** What is $\lim\limits_{x\to5}\dfrac{5-x}{|x-5|}$ equal to ?

(a) – 1

(b) 0

(c) 1

(d) Limit does not exist

**100.** What is $\lim\limits_{x\to1}\dfrac{x^9-1}{x^3-1}$ equal to ?

(a) – 1

(b) – 3

(c) 3

(d) Limit does not exist

**101.** The mean and variance of five obser-vations are 14 and 13.2 respectively. Three of the five observations are 11, 16 and 20. What are the other two observations ?

(a) 8 and 15

(b) 9 and 14

(c) 10 and 13

(d) 11 and 12

**102.** Let A and B be two independent events such that $P(A') = 0.7$, $P(B') = k$, $P(A \cup B) = 0.8$. What is the value of $k$ ?

(a) $\dfrac{5}{7}$

(b) $\dfrac{4}{7}$

(c) $\dfrac{2}{7}$

(d) $\dfrac{1}{7}$

**103.** A biased coin with the probability of getting head equal to $\dfrac{1}{4}$ is tossed five times. What is the probability of getting tail in all the first four tosses followed by head ?

(a) $\dfrac{81}{512}$

(b) $\dfrac{81}{1024}$

(c) $\dfrac{81}{256}$

(d) $\dfrac{27}{1024}$

**104.** A coin is biased so that heads comes up thrice as likely as tails. In four independent tosses of the coin, what is probability of getting exactly three heads ?

(a) $\dfrac{81}{256}$

(b) $\dfrac{27}{64}$

(c) $\dfrac{27}{256}$

(d) $\dfrac{9}{256}$

**105.** Let X and Y be two random variables such that $X + Y = 100$. If $X$ follows Binomial distribution with parameters $n = 100$ and $p = \dfrac{4}{5}$, what is the variance of Y?

(a) 1

(b) $\dfrac{1}{2}$

(c) 16

(d) $\dfrac{1}{16}$

**106.** If two lines of regression are $x + 4y + 1 = 0$ and $4x + 9y + 7 = 0$, then what is the value of $x$ when $y = -3$?

(a) −13

(b) −5

(c) 5

(d) 7

**107.** The central angles $p$, $q$, $r$ and $s$ (in degrees) of four sectors in a Pie Chart satisfy the relation $9p = 3q = 2r = 6s$. What is the value of $4p - q$ ?

(a) 12

(b) 24

(c) 30

(d) 36

**108.** The observations 4, 1, 4, 3, 6, 2, 1, 3, 4, 5, 1, 6 are outputs of 12 dices thrown simultaneously. If $m$ and $M$ are means of lowest 8 observations and highest 4 observations respectively, then what is $(2m + M)$ equal to ?

(a) 10

(b) 12

(c) 17

(d) 21

**109.** A bivariate data set contains only two points (–1, 1) and (3, 2). What will be the line of regression of y on $x$ ?

(a) $x - 4y + 5 = 0$

(b) $3x + 2y - 1 = 0$

(c) $x + 4y + 1 = 0$

(d) $5x - 4y + 1 = 0$

**110.** A die is thrown 10 times and obtained the following outputs:

1, 2, 1, 1, 2, 1, 4, 6, 5, 4

What will be the mode of data so obtained ?

(a) 6                              (b) 4
(c) 2                              (d) 1

**111.** Consider the following frequency distribution:

| $x$ | 1 | 2 | 3 | 5 |
|-----|---|---|---|---|
| $f$ | 4 | 6 | 9 | 7 |

What is the value of median of the distribution?

(a) 1                              (b) 2
(c) 3                              (d) 3-5

**112.** For data –1, 1, 4, 3, 8, 12, 17, 19, 9, 11; if M is the median of first 5 observations and N is the median of last five observations, then what is the value of 4M – N ?

(a) 7                              (b) 4
(c) 1                              (d) 0

**113.** Let P, Q, R represent mean, median and mode. If for some distribution $5P = 4Q = \dfrac{R}{2}$, then

what is $\dfrac{P+Q}{2P+0.7R}$ equal to ?

(a) $\dfrac{1}{12}$              (b) $\dfrac{1}{7}$

(c) $\dfrac{2}{9}$               (d) $\dfrac{1}{4}$

**114.** If $G$ is the geometric mean of numbers 1, 2, $2^2$, $2^3$,. . ., $2^{n-1}$, then what is the value of $1 + 2\log_2 G$?

(a) 1                              (b) 4
(c) $n$ - 1                        (d) $n$

**115.** If H is the harmonic mean of numbers 1, 2, $2^2$, $2^3$, . . . , $2^{n-1}$, then what is $n/H$ equal to ?

(a) $2 - \dfrac{1}{2^{n+1}}$      (b) $2 - \dfrac{1}{2^{n-1}}$

(c) $2 + \dfrac{1}{2^{n-1}}$      (d) $2 - \dfrac{1}{2^{n}}$

**116.** Let P be the median, Q be the mean and R be the mode of observations $x_1, x_2, x_3, \dots x_n$. Let $S = \sum_{i=1}^{n}(2x_i - a)^2$ S takes minimum value, when $a$ is equal to

(a) P                              (b) $\dfrac{Q}{2}$

(c) 2Q                             (d) R

**117.** One bag contains 3 white and 2 black balls, another bag contains 2 white and 3 black balls. Two balls are drawn from the first bag and put it into the second bag and then a ball is drawn from the second bag. What is the probability that it is white ?

(a) $\dfrac{6}{7}$               (b) $\dfrac{33}{70}$

(c) $\dfrac{3}{10}$              (d) $\dfrac{1}{70}$

**118.** Three dice are thrown. What is the probability that each face shows only multiples of 3 ?

(a) $\dfrac{1}{9}$               (b) $\dfrac{1}{18}$

(c) $\dfrac{1}{27}$              (d) $\dfrac{1}{3}$

**119.** What is the probability that the month of December has 5 Sundays ?

(a) 1                              (b) $\dfrac{1}{4}$

(c) $\dfrac{3}{7}$               (d) $\dfrac{2}{7}$

**120.** A natural number $n$ is chosen from the first 50 natural numbers. What is the probability that

$n + \dfrac{50}{n} < 50$ ?

(a) $\dfrac{23}{25}$             (b) $\dfrac{47}{50}$

(c) $\dfrac{24}{25}$             (d) $\dfrac{49}{50}$

## ANSWER KEY

| Q No | Answer Key | Topic | Chapter |
|------|-----------|-------|---------|
| 1 | a | Cube root of unity | Complex Numbers |
| 2 | d | Number of ways | Permutations and Combinations |
| 3 | **Bonus** | Binary operation | Sets |
| 4 | b | Adjoint of a matrix | Matrices |
| 5 | d | Cube root of unity | Complex Numbers |
| 6 | b | Properties of matrices | Matrices |
| 7 | d | Properties of determinants | Determinants |
| 8 | b | System of equations | Determinants |
| 9 | b | Properties of determinants | Determinants |
| 10 | b | Expansion of determinant | Determinants |
| 11 | d | Roots of Equations | Complex Numbers |
| 12 | b | Argument | Complex Numbers |
| 13 | b | Expansion of determinant | Determinants |
| 14 | c | Geometric Progression | Sequence and Series |
| 15 | a | Geometric Progression | Sequence and Series |
| 16 | b | Arithmetic Progression | Sequence and Series |
| 17 | a | Nature of roots | Quadratic Equations |
| 18 | b | Suum of $n$ terms | Sequence and Series |
| 19 | b | Factorial | Permutations and Combinations |
| 20 | c | Modulus | Complex Numbers |
| 21 | b | Roots of Equations | Equations |
| 22 | d | Onto Functions | Relations and Functions |
| 23 | a | Roots of Equations | Quadratic Equations |
| 24 | d | $N^{th}$ term | Binomial Theorem |
| 25 | c | Binomial Expansion | Binomial Theorem |
| 26 | a | Arithmetic Progression | Sequence and Series |
| 27 | c | Combinations | Permutations and Combinations |
| 28 | d | Combinations | Permutations and Combinations |
| 29 | c | Number of permutations | Permutations and Combinations |
| 30 | b | Number of ways | Permutations and Combinations |
| 31 | a | Trigonometric Identities | Trigonometry |
| 32 | b | Trigonometric Identities | Trigonometry |
| 33 | a | Height and Distance | Trigonometry |
| 34 | c | Height and Distance | Trigonometry |
| 35 | c | Triangle | Trigonometry |
| 36 | c | Triangle | Trigonometry |
| 37 | c | Arithmetic and Geometric Progression | Trigonometry |
| 38 | d | Minimum Value | Trigonometry |
| 39 | c | Triangle property | Trigonometry |

| Q No | Answer Key | Topic | Chapter |
|------|-----------|-------|---------|
| 40 | c | Area of triangle | Trigonometry |
| 41 | b | Extreme values | Continuity and Differentiability |
| 42 | a | Extreme values | Continuity and Differentiability |
| 43 | c | Factorial | Permutations and Combinations |
| 44 | d | Factorial | Permutations and Combinations |
| 45 | c | Triangle | Trigonometry |
| 46 | a | Triangle | Trigonometry |
| 47 | a | Trignometric Relation | Trigonometry |
| 48 | d | Trignometric Relation | Trigonometry |
| 49 | b | Harmonic Mean | Sequence and Series |
| 50 | a | Geometric Mean | Sequence and Series |
| 51 | b | Expansion of determinant | Determinants |
| 52 | c | Properties of determinants | Determinants |
| 53 | d | Trigonometric expressions | Trigonometry |
| 54 | b | Trigonometric expressions | Trigonometry |
| 55 | c | Equation of a line | Straight lines |
| 56 | d | Equation of a line | Straight lines |
| 57 | b | Ellipse | Conic Section |
| 58 | d | Ellipse | Conic Section |
| 59 | a | Circle | Conic Section |
| 60 | b | Circle | Conic Section |
| 61 | a | Sphere | 3D Geometry |
| 62 | b | Sphere | 3D Geometry |
| 63 | a | Direction ratios | Three Diimensional Geomtery |
| 64 | d | Direction ratios | Three Diimensional Geomtery |
| 65 | b | Product of two vectors | Vector Algebra |
| 66 | a | Product of two vectors | Vector Algebra |
| 67 | b | Direction cosines | 3D Geometry |
| 68 | a | Direction cosines | 3D Geometry |
| 69 | b | Line | 3D Geometry |
| 70 | c | Line | 3D Geometry |
| 71 | b | Definite integral | Calculus |
| 72 | c | Definite integral | Calculus |
| 73 | d | Differentiation | Calculus |
| 74 | a | Order and degree | Differential equations |
| 75 | b | Order and degree | Differential equations |
| 76 | b | Evaluation of limits | Limits |
| 77 | b | Evaluation of limits | Limits |
| 78 | a | Evaluation of limits | Limits |
| 79 | b | Trigonometric functions | Trigonometry |
| 80 | d | Trigonometric Functions | Trigonometry |
| 81 | Bonus | Definite Integral | Calculus |

| Q No | Answer Key | Topic | Chapter |
|------|-----------|-------|---------|
| 82 | d | Definite Integral | Calculus |
| 83 | b | Definite Integral | Calculus |
| 84 | a | Definite Integral | Calculus |
| 85 | c | Definite Integral | Calculus |
| 86 | a | Differentiation | Calculus |
| 87 | c | Differentiation | Calculus |
| 88 | d | Differentiation | Calculus |
| 89 | b | Continuity | Calculus |
| 90 | c | Continuity | Calculus |
| 91 | a | Increasing-decreasing functions | Calculus |
| 92 | a | Differentiation | Calculus |
| 93 | a | General Equation of a line | Straight Lines |
| 94 | b | Differentiation | Calculus |
| 95 | a | Variable separable | Differential Equations |
| 96 | d | Operations on functions | Functions |
| 97 | b | Range | Functions |
| 98 | a | Value of a function | Functions |
| 99 | d | Evaluation of limits | Limits |
| 100 | c | Evaluation of limits | Limits |
| 101 | c | Mean and variance | Statistics |
| 102 | c | Independent events | Probability |
| 103 | b | Independent events | Probability |
| 104 | b | Independent events | Probability |
| 105 | c | Binomial distribution | Probability |
| 106 | c | Regression | Statistics |
| 107 | d | Angles | Trigonometry |
| 108 | a | Mean | Statistics |
| 109 | a | Regression | Statistics |
| 110 | a | Mode | Statistics |
| 111 | c | Median | Statistics |
| 112 | d | Median | Statistics |
| 113 | d | Mean, median, mode | Statistics |
| 114 | d | Geometric mean | Sequence and Series |
| 115 | b | Harmonic mean | Sequence and Series |
| 116 | c | Derivative | Continuity and Differentiability |
| 117 | b | Total Probability | Probability |
| 118 | c | Probability | Probability |
| 119 | c | Probability | Probability |
| 120 | b | Probability | Probability |

# ANSWERS WITH EXPLANATION

**1. Option (a) is correct.**

*Explanations:* We have,

$$\left|\frac{1-\omega}{\omega+\omega^2}\right| = \left|\frac{1-\omega}{-1}\right| = |-1+\omega|$$

$$= \left|-1+\left(\frac{-1+1\sqrt{3}}{2}\right)\right|$$

$$= \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{3}$$

**2. Option (d) is correct.**

*Explanations:* For number to be divisible by 6, the number should be divisible by 2 and 3 both. Now, number is divisible by 2 if units place digit is 0, 2, or 4:

Also, sum of all digits = 0 + 1 + 2 + 3 + 4 + 5 = 15

**Case I:** If units digit is 0; then no. of ways = 5! = 120

**Case II:** If units digit is either 2 or 4, then no. of ways = 2 × 4! × 4 = 192

So, total number of 6 digit number formed = 120 + 192 = 312

**3. Option (Bonus) is correct.**

*Explanations:* To covert 1011 decimal number, we have,

| Divisible by 2 | Quotient | Remainder | Binary Bit |
|---|---|---|---|
| 1011 ÷ 2 | 505 | 1 | 1 |
| 505 ÷ 2 | 252 | 1 | 1 |
| 252 ÷ 2 | 126 | 0 | 0 |
| 126 ÷ 2 | 63 | 0 | 0 |
| 63 ÷ 2 | 31 | 1 | 1 |
| 31 ÷ 2 | 15 | 1 | 1 |
| 15 ÷ 2 | 7 | 1 | 1 |
| 7 ÷ 2 | 3 | 1 | 1 |
| 3 ÷ 2 | 1 | 1 | 1 |
| 1 ÷ 2 | 0 | 1 | 1 |

1011 = (1111110011)

**4. Option (b) is correct.**

*Explanations:* $|2 \text{ adj } (3A)| = 2^3 |\text{adj } (3A)|$   (i)

Now, $|3A| = 3^3 |A| = 3^3.4 = 3^3.2^2$

$|\text{adj } (3A)| = |3A|^{3-1} = |3A|^2 = |3^3.2^2|^2 = 3^6.2^4$

from (i), we have

$|2 \text{ adj } (3A)| = 2^3.2^4.3^6 = 2^7.3^6 = 2^\alpha.3^\beta$

$\Rightarrow \quad \alpha = 7 \text{ and } \beta = 6$

$\therefore \alpha + \beta = 7 + 6 = 13$

**5. Option (d) is correct.**

*Explanations:* We have,

$x^2 - x + 1$

$x = \dfrac{1 \pm \sqrt{3}i}{2} \Rightarrow x = -\omega \text{ or } -\omega^2$

So, $\alpha = -\omega$ and $\beta = -\omega^2$

$$\left|\frac{\alpha^{1\omega} + \beta^{1\omega}}{\alpha^{1\omega} - \beta^{1\omega}}\right| = \left|\frac{\omega^{1\omega} + \omega^{2\omega}}{\omega^{1\omega} - \omega^{2\omega}}\right|$$

$$= \left|\frac{1+\omega^{1\omega}}{1-\omega^{1\omega}}\right| = \left|\frac{1+\omega}{1-\omega}\right|$$

$$\left|\frac{\pm^{100} + \beta^{100}}{\pm^{100} - \beta^{100}}\right| = \left|\frac{(-\omega)^{100} + (-\omega^2)^{100}}{(-\omega)^{100} - (-\omega^2)^{100}}\right|$$

$$= \left|\frac{\omega^{100} + (1+\omega^{100})}{\omega^{100} + (1-\omega^{100})}\right| = \left|\frac{1+\omega^{100}}{1-\omega^{100}}\right| = \left|\frac{1+\omega^{3\times33}\omega}{1-\omega^{3\times33}\omega}\right|$$

$$= \left|\frac{1+\omega}{1-\omega}\right| = \left|\frac{1+\left(\dfrac{-1-\sqrt{3}i}{2}\right)}{1+\left(\dfrac{-1+\sqrt{3}i}{2}\right)}\right|$$

$$= \left|\frac{1+\sqrt{3}i}{3+\sqrt{3}i}\right| = \frac{\sqrt{1+3}}{\sqrt{9+3}} = \frac{1}{\sqrt{3}}$$

**6. Option (b) is correct.**

*Explanations:* When A and B be symmetric matrices then (AB – BA) is skew symmetric.

**7. Option (d) is correct.**

*Explanations:*

$$\because \begin{bmatrix} 3 & 5 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} K \\ 2K \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} K \\ 2K \end{bmatrix}$$

$$\begin{bmatrix} 13K \\ 13K \end{bmatrix} = \begin{bmatrix} 13K \\ 13K \end{bmatrix}$$

$$\begin{bmatrix} 3 & 5 \\ 7 & 3 \end{bmatrix} \neq \begin{bmatrix} 7 & 3 \\ 3 & 5 \end{bmatrix}$$

So, both statements are wrong.

**8. Option (b) is correct.**

*Explanations:* We have,

$$x + 2y + z = 4$$
$$2x + 4y + 2z = 8$$
$$\Rightarrow 2(x + 2y + z) = 8$$
$$\Rightarrow x + 2y + z = 4$$
and $3x + 6y + 3z = 10$
$$\Rightarrow 3(x + 2y + z) = 10$$
$$\Rightarrow x + 2y + z = \frac{10}{3}$$

So, the linear equations have infinity many solutions.

**9. Option (b) is correct.**

*Explanations:* We know that if $X_1$ and $X_2$ are solution of system of equations $AX = B$, $B = 0$ then $aX_1 + bX_2$ is also solution iff $a + b = 1$

**10. Option (b) is correct.**

*Explanations:* $\begin{vmatrix} 0 & x - a & x - b \\ 0 & 0 & x - c \\ x + b & x + c & 1 \end{vmatrix} = 0$

$$\Rightarrow 0 - (x - a)(0 - (x - c)(x + b)) + (x - b)(0 - 0) = 0$$
$$\Rightarrow (x - a)(x + b)(x - c) = 0$$
$$\Rightarrow x = a, x = -b \text{ or } x = c$$
Sum of roots $= a - b + c$

**11. Option (d) is correct.**

*Explanations:* $2 - i\sqrt{3}$ is a root of $x^2 + ax + b$.

So, $2 + i\sqrt{3}$ is also the root of $x^2 + ax + b$.

Sum of roots $= 4$
$$-a = 4 \Rightarrow a = -4$$
Product of roots $= 4 + 3 = 7$
$$\Rightarrow b = 7$$
So, $a + b = -4 + 7 = 3$

**12. Option (b) is correct.**

*Explanations:* We have,

$$z = \frac{1 + i\sqrt{3}}{1 - i\sqrt{3}} \times \frac{1 + i\sqrt{3}}{1 + i\sqrt{3}} = \frac{1 - 3 + 2\sqrt{3}\,i}{1 + 3}$$

$$= \frac{-2 + 2\sqrt{3}i}{4} = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$$

Now, $\tan\theta = \left( \dfrac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}} \right) = -\sqrt{3}$

$$\Rightarrow \theta = \tan^{-1}\left(-\sqrt{3}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

**13. Option (b) is correct.**

*Explanations:* We have,
$$2b = a + c \qquad \text{(i)} \quad (a, b, c \text{ in AP})$$

Let $\Delta = \begin{vmatrix} x + 1 & x + 2 & x + 3 \\ x + 2 & x + 3 & x + 4 \\ x + a & x + b & x + c \end{vmatrix}$

$$= \begin{vmatrix} x + 1 & 1 & 2 \\ x + 2 & 1 & 2 \\ x + a & b - a & c - a \end{vmatrix} = \begin{vmatrix} x + 1 & 1 & 2 \\ 1 & 0 & 0 \\ x + a & b - a & c - a \end{vmatrix}$$

$$(x + 1)(0 - 0) - 1(c - a - 0) + 2(b - a - 0)$$
$$= a - c + 2b - 2a$$
$$= -a - c + a + c \qquad \text{[Using (i)]}$$
$$= 0$$

**14. Option (c) is correct.**

*Explanations:* Since, $\log_x a$, $a^x$, $\log_b x$ are in G.P

$$\therefore \left(a^x\right)^2 = \left(\log_x a\right)\left(\log_b x\right)$$

$$\Rightarrow a^{2x} = \frac{\log a}{\log x} \frac{\log x}{\log b} = \log_b a$$

Taking log both sides, we get
$2x \log_a = \log\left(\log_b^a\right)$

$$x = \frac{1}{2}\log_a\left(\log_b^a\right)$$

**15. Option (a) is correct.**

*Explanations:* $2^{1/c}, 2^{b/ac}, 2^{1/a}$ are in G.P
$2^{2b/ac} = 2^{1/c}.2^{1/a} = 2^{2b/ac} = 2^{1/c + 1/a}$

$$= \frac{2b}{ac} = \frac{1}{c} + \frac{1}{a} = 2b = a + c$$

Hence, $a, b, c$ are in A.P

**16. Option (b) is correct.**

*Explanations:* We have,

$$a_n = 0 = \frac{5}{2} + (n - 1)\left(\frac{-7}{12}\right)$$

$$\Rightarrow n - 1 = \frac{30}{7} \Rightarrow n = \frac{37}{7}$$

So, largest negative term will be for integer $n$
$= 6$

**17. Option (a) is correct.**

*Explanations:* We have,

$f(x) = x^2 - 4x + x$ has real roots

$D > 0 = (4)^2 - 4k, 1 > 0 = 16 - 4k > 0$

$k < 4$          (i)

Now, roots of above equation are lying in the internal (0, 5).

$f(0) > 0 = k > 0$        (ii)

and $f(5) > 0 = 25 - 20 + k > 0 = k > -5$    (iii)

from (i), (ii), and (iii) we have,

$k = (0, 4)$

Possible integral values of l are 1, 2 and 3 i.e. 3 is number.

**18. Option (b) is correct.**

*Explanations:* We have

$a = x, S_n = 0$

$\Rightarrow \dfrac{n}{2}\left[2a + (n-1)d\right] = 0 \Rightarrow 2x + (n-1)d = 0$

$\Rightarrow d = \left(\dfrac{-2x}{n-1}\right)$

$= \dfrac{m+n}{2}\left[2x + (m+n-1)d\right] - 0$

$= \dfrac{m+n}{2}\left[2x + md - 2x\right]$

$= \left(\dfrac{m+n}{2}\right)m\left(\dfrac{-2x}{n-1}\right)$

$= \dfrac{mx(m+n)}{1-n}$

**19. Option (b) is correct.**

*Explanations:*

(1) as $5! = 120$

and $5! + 1 = 121$ has 1 at unit place.

so, $25! + 1$ also has 1 at units place.

$25! + 1$ is not divisible by 26.

(2) $6! = 720$

$6! + 1 = 721$, which is divisible by 7.

So, only (2) is true.

**20. Option (c) is correct.**

*Explanations:* Let $x = x + iy$

then $\dfrac{z-1}{z+1} = \dfrac{x+iy-1}{x+iy+1}$

$= \dfrac{(x-1)+iy}{(x+1)+iy} \times \dfrac{(x+1)-iy}{(x+1)-iy}$

$= \dfrac{x^2 + x + ixy - x - 1 + iy + ixy + iy - i^2 y^2}{(x+1)^2 - i^2 y^2}$

$= \dfrac{x^2 + y^2 - 1 + 2iy}{x^2 + 1 + 2x + y^2}$     $(\because i^2 = -1)$

If $\dfrac{z-1}{z+1}$ is purely imaginary number, then

$\mathrm{Re}\left(\dfrac{z-1}{z+1}\right) = 0$

$\Rightarrow x^2 + y^2 = 0$

$\Rightarrow x^2 + y^2 = 1 \Rightarrow |z|^2 = 1$ or $|z| = 1$

Thus the value of $|z| = 1$

**21. Option (b) is correct.**

*Explanations:* We have, $|x - 4| + |x - 7| = 15$

There are two cases arise.

**Case I:** When $x < 4$

$-x + 4 - x + 7 = 15 \Rightarrow n = -2$

**Case II:** When $x \geq 7$

So, only 2 Solution possible.

**22. Option (d) is correct.**

*Explanations:* $f(x)$ is onto

$3x + 5 = 0 \Rightarrow x = -\dfrac{5}{3}$

So, $A = \{x = R - (-5/3)\}$

Let, $y = 2x + 3/3x + 5 \Rightarrow 3xy + 5y = 2x + 3$

$= x = 3 - \dfrac{5y}{3y} - 2$

$3y - 2 = 0 = y = \dfrac{2}{3}$

$B = \{y = R - (2 - 3)\}$

**23. Option (a) is correct.**

*Explanations:* We have,

$\alpha + \beta = 0$        (i)

$\alpha^2 + \beta^2 = 2$

$(\alpha + \beta)^2 - 2\alpha\beta = 2$

$2\alpha\beta = -2$       [Using (i)]

$\alpha\beta = -1$

Now, $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta = 2 - 2(-1) = 4$

$\alpha - \beta = +2$       (ii)

Solving (i) and (ii), we get

$\alpha = 1$ and $\alpha + \beta = +1$

So, only (1) is sufficient to find $x$.

**24. Option (d) is correct.**

*Explanations:* We have,

$T_{5+1} = 5600$

$8_{C5}(x^{-8/3})^{8-5} = (x^2 \log_{10} x)^5 = 5600$

$56 . x^{-8} . x^{10} (\log_{10} x)^5 = 5600$

$x^2 (\log_{10} x)^5 = (10)^2 . (\log_{10} 10)^5$

So, $x = 10$

**25. Option (c) is correct.**

*Explanations:* We have,

$(3x - y)^4(x + 3y)^4 = [(3x - y)(x + 3y)]^4$
$\qquad = (3x^2 + 9xy - xy - 3y^2)^4$
$\qquad = (3x^2 + 8xy - 3y^2)^4$

Here, $r = 3$ and $n = 4$

Required number of terms $= {}^{n+r-1}C_{r-1}$
$\qquad = {}^{4+3-1}C_{3-1}$
$\qquad = {}^6C_2 = 15$

**26. Option (a) is correct.**

*Explanations:* Let $P = 1 - 3d, q = a - d, r = a + d$

Then,

$\quad P + S = 8$
$\quad a - 3d + a + 3d = 8 \Rightarrow a = 4$

Also,

$\quad qr = 15$
$\qquad = a^2 - d^2 = 15$
$\qquad = d^2 = 16 - 15$
$\quad d = +1$

If $d = +1$ and $a = 4$, then

Largest number $= 7$ and smallest number $= 1$

Required difference $= 7 - 1 = 6$

**27. Option (c) is correct.**

*Explanations:*

Both statements are true.

**28. Option (d) is correct.**

*Explanations:*

Selection of 2 parallel lines from $m$ lines $= {}^mC_2$
Selection of 2 parallel lines from $n$ lines $= {}^nC_2$
No. of parallelograms formed $= {}^mC_2 \cdot {}^nC_2$
$\quad = 60 = {}^mC_2 \cdot {}^nC_2$
$\quad = {}^5C_2 \times {}^4C_2 = {}^mC_2 \cdot {}^nC_2$
$\therefore m = 5$ and $n = 4$
So, $m + n = 5 + 4 = 9$

**29. Option (c) is correct.**

*Explanations:* No. of permutations of the word PERMUTATIONS = 12!/2! (T accurs twice)

No. of permutations of the word COMBINA-TIONS = 12!/2! 2! 2!

(As 0, I, M occurs twice)

$y = 12!/2! . 1/4$
$\quad = 4y = x$

**30. Option (b) is correct.**

*Explanations:* Total 5 digit numbers that can be formed using 0, 1, 2, 4 and 5 without repetition $= 4 \times 4! = 96$

No. of 5 digit numbers greater that 50000 $= 1 \times 4! = 24$

(Ten thousand should be filled by 5 only)

Required percentage $= 24/96 \times 100 = 25\%$

**(31-32.)**

We have

$\quad \sin^2\beta = \sin\alpha \cos\alpha$ \qquad\qquad (i)
and $2\tan\gamma = \sin\alpha + \cos\alpha$ \qquad (ii)

**31. Option (a) is correct.**

*Explanations:*

Now, $\cos 2\beta = 1 - 2\sin^2\beta$
$\qquad = 1 - 2\sin\alpha \cos\alpha$
$\qquad = (\sin\alpha - \cos\alpha)^2$

**32. Option (b) is correct.**

*Explanations:*

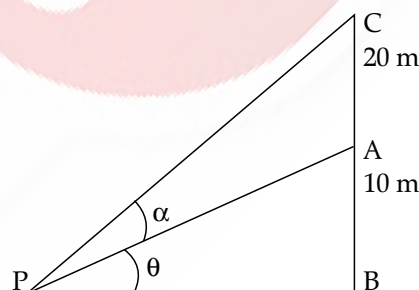$$\cos 2\gamma = \frac{1 - \tan^2\gamma}{1 + \tan^2\gamma}$$

$$\Rightarrow \sec 2\gamma = \frac{1 + \tan^2\gamma}{1 - \tan^2\gamma} = \frac{1 + \left(\dfrac{\sin\alpha + \cos\alpha}{2}\right)^2}{1 - \left(\dfrac{\sin\alpha + \cos\alpha}{2}\right)^2}$$

$$= \frac{5 + 2\sin\alpha\cos\alpha}{3 - 2\sin\alpha\cos\alpha}$$

$$\Rightarrow \sec 2\gamma = \frac{5 + \sin 2\alpha}{3 - \sin 2\alpha}$$

**(33-34).**

From the given question, figure should be as follows:



Let AB be the pillar and $\alpha$ be the angle formed by flagstaff.

**33. Option (a) is correct.**

*Explanations:* It is given that,

$$\tan\theta = \frac{AB}{AP} = \frac{10}{x}$$

$$\tan(\theta + \alpha) = \frac{30}{x}$$

$$\Rightarrow \frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha} = \frac{30}{x}$$

$$\Rightarrow \frac{\dfrac{10}{x} + \dfrac{1}{2}}{1 - \left(\dfrac{10}{x}\right)\left(\dfrac{1}{2}\right)} = \frac{30}{x}$$

$\Rightarrow \dfrac{20 + x}{2x - 10} = \dfrac{30}{x}$

$\Rightarrow 20x + x^2 - 60x + 300 = 0$

$\Rightarrow x^2 - 40x + 300 = 0$

$\Rightarrow (x - 30)(x - 10) = 0$

$\Rightarrow x = 30 \, or \, x = 10$

Ratio of two values of $x = 1 : 3$

And $x \neq 20$ m

So, only (1) is correct.

**34. Option (c) is correct.**

*Explanations:*

Now, $\tan\theta = \dfrac{10}{30}$ or $\tan\theta = \dfrac{10}{10}$

$\tan\theta = \dfrac{1}{3}$ or $1$

**(35-36).**

Let A, B, C be the angle of $\Delta$ABC

Now,

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = k$

$\sin A = ak$, $\sin B = bk$ and $\sin c = ck$

It is given that,

$a + b + c = 6 \times \left( \dfrac{\sin A + \sin B + \sin C}{3} \right)$

$\Rightarrow 2k = 1 \Rightarrow k = \dfrac{1}{2}$

So, $\dfrac{\sin A}{a} = k \Rightarrow \sin A = BC \dfrac{1}{2} = \dfrac{\sqrt{3}}{2} \Rightarrow A = \dfrac{\pi}{3}$

$\dfrac{\sin B}{b} = k \Rightarrow \sin B = AC \dfrac{1}{2} = \dfrac{1}{2} \Rightarrow B = \dfrac{\pi}{6}$

$C = \pi - \left( \dfrac{\pi}{3} + \dfrac{\pi}{6} \right) = \dfrac{\pi}{2}$

**35. Option (c) is correct.**

*Explanations:* Perimeter of triangle

$= \sqrt{3} + 1 + 2 = 3 + \sqrt{3}$

**36. Option (c) is correct.**

*Explanations:* $C = \pi/3 = 1/2 \, (\pi/2 + \pi/6)$

$C = 1/2 \, (A + B)$

A, C, B are in A.P

Both (1) and (2) are true.

**37. Option (c) is correct.**

*Explanations:* We have,

$x = \dfrac{\sin^2 A + \sin A + 1}{\sin A}$

$= \sin A + 1 + \dfrac{1}{\sin A}$

Now, $\sin A + \dfrac{1}{\sin A} \geq 2 \quad (\because \text{AM} > \text{GM})$

$\Rightarrow \sin A + \dfrac{1}{\sin A} + 1 \geq 3$

$\Rightarrow x \geq 3$

Minimum value of $x = 3$

**38. Option (d) is correct.**

*Explanations:* Now, $x = 3$

$\sin^2 A + \sin A + 1 = 3 \sin A$

$\sin A - 2 \sin A + 1 = 0$

$(\sin A - 1)^2 = 0$

$\sin A = 1 = A = \pi/2$

**39. Option (c) is correct.**

*Explanations:* We know that

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = c^2 + a^2 - 2ca \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

Adding above equations, we get

$a^2 + b^2 + c^2 = 2a^2 + 2b^2 + 2c^2$

$= 2bc \cos A = 2ca \cos B - 2ab \cos C$

$a^2 + b^2 + c^2 = 2ab \cos C + 2bc \cos A + 2ac \cos B$

Now, it is given that,

$a^2 + b^2 + c^2 = ac + \sqrt{3} \, bc$

$2ab \cos C + 2bc \cos A + 2ac \cos B = ac + \sqrt{3} \, bc$

On comparing, we get

ABC is right angled triangle.

**40. Option (c) is correct.**

*Explanations:*

Now, area of ABC $= 1/2 \times AC \times BC$

$= 1/2 \times 4\sqrt{3} \times 4$

$\left( \dfrac{1}{2} = \dfrac{BC}{8} = BC = 4 \text{ and } \dfrac{\sqrt{3}}{2} = \dfrac{AC}{8} = AC = 4\sqrt{3} \right)$

$= 8\sqrt{3}$

**(41-42).**

We have,

$f(x) = |x - 2| + |3 - x| + |4 - x|$

$f(x) = |x - 2| + |3 - x| + |4 - x|$

$\Rightarrow f(x) = \begin{cases} -x + 2 + 3 - x + 4 - x, x \in (-\infty, 2) \\ x + 2 + 3 - x + 4 - x, x \in [2, 3) \\ x + 2 + 3 - x + 4 - x, x \in [3, 4) \\ x + 2 + 3 - x + 4 - x, x \geq 4 \end{cases}$

$\Rightarrow f(x) = \begin{cases} 9 - 3x, x < 2 \\ 5 - x, x \in [2, 3) \\ x - 1, x \in [3, 4) \\ 3x - g, x \geq 4 \end{cases}$

$$\Rightarrow f^1(x) = \begin{cases} -3, & x < 2 \\ -1, & x \in (2,3) \\ 1, & x \in (3,4) \\ 3, & x \geq 4 \end{cases}$$

**41. Option (b) is correct.**
*Explanations:* Since sign changes from negative to positive a $x = 3$
$f(x)$ is minimum at $x = 3$

**42. Option (a) is correct.**
*Explanations:* Minimum value of $f(x) = f(3)$
$= |3 - 2| + |3 - 3| + |4 - 3|$
$= 1 + 0 + 1 = 2$

**43. Option (c) is correct.**
*Explanations:* Given, $s = 0! + 1! + 2! + \ldots + 100!$
From 41 onwards every terms has $4 \times 2$, which is divisible by 8.
Remaining sum $= 0! + 1! + 2! + 3!$
$= 1 + 1 + 2 + 6 = 10$
Now, remainder when 10 is divisible by 8 is 2 so, required remainder $= 2$

**44. Option (d) is correct.**
*Explanations:* Similarly from 5! onwards every terms has 10, which is divisible by 60
Remainder $= 0! + 1! + 2! + 3! + 4!$
$= 1 + 1 + 2 + 6 + 24 = 34$

**45. Option (c) is correct.**
*Explanations:* We have
PN = PM    (Tangents from an external point)
PN = PM = $n$
Similarly, QL = QN = $n + 2$
and, RM = RL = $n + 4$
So, sides of triangle are,
PQ = $2n + 2$, QR = $2n + 6$, PR = $2n + 4$
Now, cos P = 1/3

$$\Rightarrow \frac{(PQ)^2 + (PR)^2 - (QR)^2}{2\,PQ \cdot PR} = \frac{1}{3}$$

$$\Rightarrow \frac{(2n+2)^2 + (2n+4)^2 - (2n+6)^2}{2 \cdot (2n+2)^2\,(2n+4)} = \frac{1}{3}$$

$$\Rightarrow \frac{4\left((n+1)^2 + (n+2)^2 - (n+3)^2\right)}{4(n+1)(2n+4)} = \frac{1}{3}$$

$$\Rightarrow \frac{n^2 + 1 + 2n + n^2 + 4 + 4n - n^2 - n - 6n}{2n^2 + 6n + 4} = \frac{1}{3}$$

$n = 8$, or $n = -2$

**46. Option (a) is correct.**
*Explanations:* Length of smallest side
$= 2n + 2 = 18$

**47. Option (a) is correct.**
*Explanations:* We have,

$$\sin x + \cos x + \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} + \frac{1}{\cos x} + \frac{1}{\sin x} = 7$$

$$\sin x + \cos x + \frac{1}{\sin x . \cos x} + \frac{\sin x + \cos x}{\sin x . \cos x} = 7$$

$$\Rightarrow (\sin x + \cos x)\left(1 + \frac{2}{\sin x}\right) = 7 - \frac{2}{\sin 2x}$$

Squaring both sides, we get,

$$\Rightarrow (1 + \sin 2x)\left(1 + \frac{2}{\sin x}\right)^2 = 7 - \frac{2}{\sin 2x}$$

$$\Rightarrow \sin^2 2x - 44 \sin 2x + 36 = 0$$

**48. Option (d) is correct.**
*Explanations:* $\sin^2 2x - 44 \sin 2x + 36 = 0$
$a = 22$, $b = 8$ and $c = 7$
So, $a - b + 2c = 22 - 8 + 14 = 28$

**49. Option (b) is correct.**
*Explanations:* Let $a$ and $b$ are the roots of the given equation

$$\therefore \alpha + \beta = \frac{4 + 2\sqrt{3}}{3 + 2\sqrt{2}} \text{ and } \alpha\beta = \frac{8 + 4\sqrt{3}}{3 + 2\sqrt{2}}$$

$$\text{HM of } \alpha \text{ and } \beta = \frac{2\alpha\beta}{\alpha + \beta}$$

$$= \frac{2 \cdot (8 + 4\sqrt{3})}{4 + 2\sqrt{3}} \times \frac{4 - 2\sqrt{3}}{4 - 2\sqrt{3}}$$

$$= \frac{2(32 - 16\sqrt{3} + 16\sqrt{3} - 24)}{16 - 12} = \frac{16}{4} = 4$$

**50. Option (a) is correct.**
*Explanations:* GM of $\alpha$ and $\beta = \sqrt{\alpha\beta}$

$$= \sqrt{\frac{8 + 4\sqrt{3}}{3 + 2\sqrt{2}}} = \sqrt{\frac{2(4 + 2\sqrt{3})}{3 + 2\sqrt{2}}}$$

$$= \sqrt{2}\left(\sqrt{\frac{(\sqrt{3} + 1)^2}{(\sqrt{2} + 1)^2}}\right)$$

$$= \sqrt{2}\left(\frac{\sqrt{3} + 1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1}\right)$$

$$= \sqrt{2}\left(\sqrt{6} - \sqrt{3} + \sqrt{2} - 1\right)$$

**51. Option (b) is correct.**
*Explanations:*

$$\therefore \begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & b & a\alpha+b \\ 0 & c & b\alpha+c \\ a\alpha^2+2b\alpha+c & b\alpha+c & 0 \end{vmatrix} = 0$$

$= 0 - 0 + (a\alpha^2 + 2b\alpha + c)(b^2\alpha + bc - ac\alpha - bc)$

$= 0$

$= b^2\alpha + ac\alpha = 0$ or $b^2 - ac = 0 = b^2 = ac$

So, $a, b, c$ are in G.P

**52. Option (c) is correct.**

*Explanations:* $(7, 4, 2, \alpha) = 0$

$7\alpha^2 + 8\alpha + 2 = 0$

So, $\alpha$ is root of the equation, $7x^2 + 8x + 2 = 0$

**53. Option (d) is correct.**

*Explanations:* We have,

$m(0) = \cot\theta^2 + n^2\tan^2\theta + 2n$

$\quad = (\cot\theta + n\tan\theta)^2$

$m(\theta) > 0$

Now, $\dfrac{\cot\theta + n\tan\theta}{2} \geq \sqrt{n}$

$\quad = (\cot\theta + n\tan\theta)^2 > 4n$

$\therefore$ Minimum value of $m(\theta) = 4n$

**54. Option (b) is correct.**

*Explanations:* $(\cot\theta + n\tan\theta)^2 - 4x = 0$

$\Rightarrow (\cot\theta - n\tan\theta)^2 - 4x = 0$

$\Rightarrow \cot\theta = n\tan\theta$

$\Rightarrow x = \cot^2\theta$

**55. Option (c) is correct.**

*Explanations:* Equation of line of the quadrilateral is, $x = 0, y = 0, x + y = 1$ and $6x + y = 3$

Point of intersection of these lines are

$A\left(\dfrac{1}{2}, 0\right), B(0,0), C(0,1), D\left(\dfrac{2}{5}, \dfrac{3}{5}\right)$

So, equation of diagonal passes through B is ,

$BD = y - 0 = \dfrac{3/5}{2/5}(x - 0)$

$2y = 3x \Rightarrow 3x - 2y = 0$

**56. Option (d) is correct.**

*Explanations:* Equation of diagonal AC is

$y - 0 = \dfrac{1-0}{0-1/2}\left(x - \dfrac{1}{2}\right)$

$\Rightarrow y = -2\left(x - \dfrac{1}{2}\right) \Rightarrow y = -2x + 1 \Rightarrow 2x + y - 1 = 0$

**57. Option (b) is correct.**

*Explanations:* The given ellipse is,

$\dfrac{x^2}{1} + \dfrac{y^2}{\left(\dfrac{1}{2}\right)^2} = 1$

As we know, sum of distances of any point P from two is,

PE + PE = $2a$ = 2

**58. Option (d) is correct.**

*Explanations:* Equation of latus return of ellipse is $x = 2\sqrt{3}/2$

So, points 1, 2 and 3 will be on it.

**59. Option (a) is correct.**

*Explanations:* Given equation of circle is

$(x - a)^2 + y^2 = a^2$

Now, $y = x$ intersect it 2 parts

Point of intersection of line and circle is, (0, 0) and $(a, a)$

So, required area $= \displaystyle\int_0^a \sqrt{a^2 - (x-a)^2}\,dx - \int_0^a x\,dx$

$= \left( \dfrac{x-a}{2}\sqrt{2ax - x^2} + \dfrac{a^2}{2}\sin^{-1}\left(\dfrac{x-a}{a}\right) - \left(\dfrac{x^2}{2}\right) \right)\Big|_0^a$

$= \dfrac{a^2}{2}\sin^{-1}(0) - \dfrac{a^2}{2}\sin^{-1}(-1) - \dfrac{a^2}{2}$

$= \dfrac{a^2}{2}\times 0 + \dfrac{a^2}{2}\times\dfrac{\pi}{2} - \dfrac{a^2}{2}$

$= \dfrac{a^2}{4}(\pi - 2)$

**60. Option (b) is correct.**

*Explanations:* Area of major segment

= $\pi r^2$ – Area of minor segment

$= \pi a^2 - \dfrac{\pi a^2}{4} - \dfrac{a^2}{2}$

$= \dfrac{3\pi a^2}{4} + \dfrac{a^2}{2} = \dfrac{a^2}{4}(3\pi + 2)$

**61. Option (a) is correct.**

*Explanations:* End points of diameter are A(1, –1, 2) and B(2, 1, –1)

Centre of sphere = $\left(\dfrac{3}{2}, 0, \dfrac{1}{2}\right)$

and radius = $\sqrt{\left(\dfrac{3}{2}-1\right)^2 + (0+1)^2 + \left(\dfrac{1}{2}-2\right)^2}$

$= \sqrt{\dfrac{1}{4} + 1 + \dfrac{9}{4}} = \sqrt{\dfrac{14}{4}} = \sqrt{\dfrac{7}{2}}$

Equation of space is

$\Rightarrow x^2 + y^2 + z^2 + \dfrac{9}{4} + \dfrac{1}{4} - 3x - z = \dfrac{7}{2}$

$\Rightarrow x^2 + y^2 + z^2 + 9/4 + 1/4 - 3x - z = 7/2$

$\Rightarrow x^2 + y^2 + z^2 - 3x - z - 1 = 0$

So, from given equation of sphere we have

$2a = -3, 2v = 0$ and $2w = -1$

$\Rightarrow a = -3/2, v = 0, w = -1/2$

So, $u + v + w = 4/2 = -2$

**62. Option (b) is correct.**

*Explanations:*

$PA^2 + PB^2 = AB^2$

$= (2-1)^2 + (1+1)^2 + (-1-2)^2$

$= 1 + 4 + 9 = 14$

**63. Option (a) is correct.**

*Explanations:* dr1 $(2, -1, 2)$ and $(k, 3, 5)$ indicated at $\pi/4$

$\therefore \cos\dfrac{\pi}{4} = \dfrac{2k - 3 + 10}{\sqrt{4 + 1 + 4}\sqrt{k^2 + 9 + 25}}$

$\Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{2k + 7}{3\sqrt{k^2 + 34}}$

$\Rightarrow 9\,(k^2 + 34) = 2(2k + 7)^2$

$\Rightarrow 9k^2 + 306 = 2(4k^2 + 49 + 28k)$

$\Rightarrow 9k^2 + 306 - 8k^2 - 98 - 56k = 0$

$\Rightarrow k^2 - 56k + 209 = 0$

$\Rightarrow k^2 - 52k - 4k + 208 = 0$

$\Rightarrow (k - 52)(k - 4) = 0 \Rightarrow k = 52$ or $k = 4$

**64. Option (d) is correct.**

*Explanations:* Let the drs of line perpendicular to given lines be $(a, b, c)$

Then, $2a - b + 2c = 0$

and $4a + 3b + 5c = 0$

$\Rightarrow \dfrac{a}{-5-6} = \dfrac{b}{8-10} = \dfrac{c}{6-4}$

$\Rightarrow \dfrac{a}{-11} = \dfrac{b}{-2} = \dfrac{c}{10}$

So, $(11, 2, -10)$ as the required drs.

**65. Option (b) is correct.**

*Explanations:* Let $\vec{b} = ai = bj = ck$

Then, $\vec{a} \cdot \vec{b} = 27$

$\Rightarrow 3a + 3b + 3c = 27 \Rightarrow a + b + c = 9$     (i)

Also,

$\Rightarrow \begin{vmatrix} i & j & k \\ 3 & 3 & 3 \\ a & b & c \end{vmatrix} = 9(j - k)$

$\Rightarrow i(3c - 3b) - j(3c - 3a) + k(3b - 3a) = 9(j - k)$

$\Rightarrow 3c - 3b = 0, 3c - 3A = 9, 3b - 3a = -9$

$\Rightarrow c = b, a - c = 3, a - b = 3 \Rightarrow c = a - 3, b = a - 3$

From (i), we have

$a + a - 3 + a - 3 = 9$

$\Rightarrow 3a = 15 \Rightarrow a = 5$

$b = 5 - 3 = 2 = c$

So, $b = 5i + 2j + 2k$

**66. Option (a) is correct.**

*Explanations:* Now, $a + b = 8i + 5j + 5k$

$c = j - k$

$(a+b).c = |a + b| |c| \cos\theta,$

where $\theta$ is the required angle

$\Rightarrow 0 = \sqrt{8^2 + 5^2 + 5^2}\, \sqrt{1 + 1}\, \cos\theta$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \dfrac{\pi}{2}$

**67. Option (b) is correct.**

*Explanations:* We have, $a = 4i - 8j + k$

$\therefore \cos\alpha = \dfrac{4}{\sqrt{4^2 + 8^2 + 1^2}} = \dfrac{4}{9}$

**68. Option (a) is correct.**

*Explanations:* Also, $\cos\beta = 8/9$ and $\cos\gamma = 1/9$

Now, $\cos 2\beta + \cos 2\gamma = 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$

$= 2\left(\dfrac{64}{81} + \dfrac{1}{81}\right) - 2 = \dfrac{-32}{81}$

**69. Option (b) is correct.**

*Explanations:* We have, A $= (1, -1, 0)$ and B $= (0, 1, 1)$

Equation of line AB is,

$\dfrac{x - 1}{0 - 1} = \dfrac{y + 1}{1 + 1} = \dfrac{z - 0}{1 - 0}$

$\dfrac{1 - x}{1} = \dfrac{y + 1}{2} = \dfrac{z}{1}$

Now, only (2) and (3) satisfy this equation.

**70. Option (c) is correct.**

*Explanations:*

We have, A $= \hat{i} + \hat{j} + 0.\hat{k}$ and B $= oi + j + k$

AB $= (0 - 1)i + (1+1)i + (1 - 0) = i + 2j + k$

$\left|\overline{AB}\right| = \sqrt{1^2 + 2^2 + 1} = \sqrt{6}$

**(71-73).**

We have,

$f(x) = Pe^x + Qe^{2x} + Re^{3x}$

It is given that, $f(0) = 6$

$P + Q + R = 6$     (i)

$\displaystyle\int_0^{\ln 2} f(x)\, dx = 11$

$\Rightarrow \left(Pe^x + \dfrac{Qe^{2x}}{2} + \dfrac{Re^{3x}}{3}\right)_0^{\ln 2} = 11$

**71. Option (b) is correct.**

**72. Option (c) is correct.**

**73. Option (d) is correct.**

$f(0) = P + 2Q + 3R = 1 + 4 + 9 = 14$

**(74-76).**

We have,

$y^2 = 2c\,x + 2c$

$= y^4 + 4y^2(y)^2 x^2 - 4y^3(y)x - 4y^3.(y)^3 = 0$

**74. Option (a) is correct.**

**75. Option (b) is correct.**

**76. Option (b) is correct.**

*Explanations:* We have,

$$f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$$

$$\therefore f(0) = \begin{vmatrix} \cos 0 & 0 & 1 \\ 2\sin 0 & 0 & 0 \\ \tan 0 & 0 & 1 \end{vmatrix} = 0$$

**77. Option (b) is correct.**

*Explanations:*

$$\lim_{x \to 0} \frac{f(x)}{x} = \lim_{x \to 0} \begin{vmatrix} \cos x & x & 1 \\ 2\sin x/x & x & 2 \\ \tan x & x & 1 \end{vmatrix} = 0$$

**78. Option (a) is correct.**

*Explanations:*

$$\lim_{x \to 0} \frac{f(x)}{x^2} = \lim_{x \to 0} \begin{vmatrix} \cos x & 1 & 1 \\ 2\sin x/x & 1 & 2 \\ \tan x & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix} = -1$$

**79. Option (b) is correct.**

*Explanations:* We have,

$f(x) = \sin[\pi^2]x + \cos[-\pi^2]x$

$= \sin 9x + \cos(-10x)$

$= \sin 9x + \cos 10x$

$= 1 + (-1) = 0$

**80. Option (d) is correct.**

*Explanations:*

$$f\left(\frac{\pi}{4}\right) = \sin\frac{9\pi}{4} + \cos\frac{5\pi}{2}$$

$$= \sin\left(2\pi + \frac{\pi}{4}\right) + 0 = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

**81. Option (Bonus) is correct.**

*Explanations:* Since $I_1 = I_2 = \dfrac{\pi^2}{2\sqrt{2}}$

So, 81 and 83 is bonus.

**82. Option (d) is correct.**

*Explanations:* $8I^2 = 8\left[\dfrac{\pi^2}{2\sqrt{2}}\right]^2 = \dfrac{8\pi^4}{8} = \pi^4$

**83. Option (b) is correct.**

**84. Option (a) is correct.**

*Explanations:* Now, $a < 0 < b$

$l = b - (-a) = a + b$

**85. Option (c) is correct.**

*Explanations:* Now, $a < b < 0$

$l = -(-b) + (-a) = b - a$

**86. Option (a) is correct.**

*Explanations:* $f'(0.5) = -1/0.5 = -2$

**87. Option (c) is correct.**

*Explanations:* $F'(2) = 1/2$

**88. Option (d) is correct.**

*Explanations:*

$$\begin{cases} ln(-ln\,x) & , x < 0.1 \\ -ln(-ln\,x) & , x \in (0.1,\,1) \\ -ln(-ln\,x) & , x \in (1,\,2) \end{cases}$$

$$\therefore \frac{d}{dx}\left(fof(x)\right) = \frac{-1}{ln\,x} \cdot \frac{1}{x} = \frac{-1}{x\,ln\,x}$$

**89. Option (b) is correct.**

*Explanations:* $f(x)$ is continuous

$7 = P + q$       (i)

$10 = 2P + q$    (ii)

Solving (i) and (ii) we get

$P = 3$

**90. Option (c) is correct.**

*Explanations:* Also, $p = 3,\ q = 4$

**91. Option (a) is correct.**

*Explanations:* Only (1) is true

**92. Option (a) is correct.**

*Explanations:*

$$\Rightarrow \frac{du}{dx} = 2\sin x \cos x = \sin 2x$$

$$\text{and } \frac{dv}{dx} = 2\cos x(-\sin x) = -\sin 2x$$

$$\therefore \frac{du}{dv} = \frac{du}{dx}\Big/\frac{dv}{dx} = \frac{\sin 2x}{-\sin 2x} = -1$$

**93. Option (a) is correct**

Let the equation of line segments of ABC are given A = (2, 2), B(2, 2 m) and C(0, 0)

Since area of ABC = 3

$= |1/2(0 + 2(2m - 0) + 2(0 - 2)| = 3$

$= 4m - 4 = \pm 6$

$m = 5/2$ or $m = -1/2$

$\because m < 0$

$\therefore m = -1/2$

**94. Option (b) is correct.**
*Explanations:*

$$x° = \frac{\pi x}{180} \text{ radians}$$

$$\therefore \frac{d}{dx}(\operatorname{cosec} x°) = \frac{d}{dx}\left(\operatorname{cosec} \frac{\pi x}{180}\right)$$

$$= \frac{-\pi}{180}\operatorname{cosec} x° \cdot \cot x°$$

**95. Option (a) is correct.**
*Explanations:*

$$\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{dy}{dx} - x\right) = 0$$

$$\Rightarrow \frac{dy}{dx} = 0 \ or \ \frac{dy}{dx} = x$$

$$\Rightarrow y = C(\text{constant}) \ or \ y = \frac{x^2}{2} + c$$

**96. Option (d) is correct.**
*Explanations:* $f(x) = x^2 + 2$, $g(x) = 2x - 3$
$f(1) = 1 + 2 = 3$ and $g(1) = -1$
$(fg)(1) = 3$

**97. Option (b) is correct.**
*Explanations:* We have

$$\Rightarrow f(x) = \begin{pmatrix} 0 & , x < 0 \\ 2x & , x \geq 0 \end{pmatrix}$$

Range of $f(x) = [0, \infty)$

**98. Option (a) is correct.**
*Explanations:*

$$f(x) = x(4x^2 - 3)$$
$$f(\sin\theta) = \sin\theta(4\sin^2\theta - 3)$$
$$= 4\sin^3\theta - 3\sin\theta = -\sin 3\theta$$

**99. Option (d) is correct.**
*Explanations:*

$$\lim_{x \to 5} \frac{5 - x}{|x - 5|}$$

LHL = 1 and RHL = –1
So, limit at $x = 5$ does not exist.

**100. Option (c) is correct.**
*Explanations:* We have

$$\lim_{x \to 1}\frac{x^9 - 1}{x^3 - 1} = \lim x - 1\left(\frac{x^9 - 1}{x - 1} \times \frac{x - 1}{x^3 - 1}\right)$$

$$= \frac{9 \cdot (1)^8}{3 \cdot (1)^2} = 3$$

**101. Option (c) is correct.**
*Explanations:* Let the other two observations be $x$ and $y$.

$$\therefore \text{ Mean} = \frac{11 + 16 + 20 + x + y}{5}$$

$$= 14 \times 5 = 47 + x + y$$
$$= x + y = 23 \tag{i}$$

Now, variance $= \frac{1}{5}[(11 - 14)^2 + (16 - 14)^2$
$$+ (20 - 14)^2 + (x - 14) + (y - 14)^2]$$
$$= 13.2 \times 5 = 9 + 4 + 36 + (x - 14)^2 + (y - 14)^2$$
$$= x^2 + y^2 - 28(x + y) + 2x.196 + 49 = 66$$
$$= x^2 + y^2 - 28(23) = -375$$
$$= x^2 + y^2 = 269 \tag{ii}$$
$x = 13, y = 10$

**102. Option (c) is correct.**
*Explanations:* Since A and B are independent events
$$0.8 = (1 - 0.7) + (1 - k) - (1 - 0.7)(1 - k)$$
$$0.8 = 0.3 + 1 - k - 0.3(1 - k)$$
$$0.5 = 1 - k - 0.3 + 0.3\,k$$
$$0.1k = 0.2 = k = 2/7$$

**103. Option (b) is correct**
*Explanations:* P (getting head) $= \frac{1}{4}$

P(getting tail) $= 1 - \frac{1}{4} = \frac{3}{4}$

Now, required probability

$$\left(\frac{3}{4}\right)^4 = \frac{81}{1024}$$

**104. Option (b) is correct**
*Explanations:* We have

$$p(H) = \frac{3}{4} \text{ and } p(T) = \frac{1}{4}$$

Required probability $= 4 \times (3/4)^3 \ 1/4 = 27/64$

**105. Option (c) is correct.**
*Explanations:* We have, $n = 100$, $P = \frac{4}{5}$, $q = \frac{1}{5}$

For random variable $x$

$$V(x) = npq = 80 \times \frac{1}{5} = 16$$

Now, $y = 100 - x$
and, var $(y) = v(100 - x) = 0 + (-1)^2 v(x) = 16$

**106. Option (c) is correct.**
*Explanations:* We have
$$x + 4y + 1 = 0 \tag{i}$$
$$4x + 9y + 7 = 0 \tag{ii}$$

From (i), $bxy = -4$
From (ii), $byx = -1$
    $r^2 = 4$ which is not possible as $0 < r^2 < 1$
$4x + 9y + 7 = 0$ is a line of regression $x$ on $y$.
$4x = -(9y + 7)$
$x = -(9y + 7)/4 = -(-27 + 7)/4 = 5$

**107. Option (d) is correct.**

*Explanations:* Sum of all angles = 360°
$= p + q + r + s = 360°$
$= \dfrac{k}{9} + \dfrac{k}{3} + \dfrac{k}{2} + \dfrac{k}{6} = 360^o$

So, $4p - q = 144 - 108 = 36°$

**108. Option (a) is correct**

*Explanations:*

$m$ = mean of 8 lowest observations

$= 1 + 1 + 1 + 2 + 3 + 3 + 4 + \dfrac{4}{8}$

$\Rightarrow m = \dfrac{19}{8}$

Now, M = Mean of highest 4 observation

$M = 6 + 6 + 5 + \dfrac{4}{4} = \dfrac{21}{4}$

$\therefore \quad 2m + M = \dfrac{19}{4} + \dfrac{21}{4} = \dfrac{40}{4} = 10$

**109. Option (a) is correct**

*Explanations:* We have,
$x = -1 + 3 = 2, y = 1 + 2 = 3$
$xy = -1 + 6 = 5, x^2 = 1 + 9 = 10$

Now, $x = \dfrac{2}{2} = 1$ and $y = \dfrac{3}{2}$

So, line of regression $y$ on $x$ is,
$y - y = b_{yx}(x - x)$

$y - \dfrac{3}{2} = \dfrac{1}{4}(x - 1) = x - 4y + 5 = 0$

**110. Option (a) is correct**

*Explanations:* Mode = 1

**111. Option (c) is correct**

*Explanations:*

| $x$ | $f$ | $cf$ |
|-----|-----|------|
| 1 | 4 | 4 |
| 2 | 6 | 10 |
| 3 | 9 | 19 |
| 5 | 7 | 26 |
|   | 26 |   |

Now, N/2 = 13
Median = 3 as $cf$ just greater than lies for $x = 3$.

**112. Option (d) is correct**

*Explanations:* Arranging observations of first five and last five observations, we get
Set-I: –1, 1, 3, 4, 8
Set-II: 9, 11, 12, 17, 19
M = Median of set I = 3
N = Median of set II = 12
$4M - N = 4 \times 3 - 12 = 12 - 12 = 0$

**113. Option (d) is correct**

*Explanations:* Let $P = \dfrac{k}{5}$, $Q = \dfrac{k}{4}$ and $R = 2k$

$P + Q/2P + 0.7R = \dfrac{\left(\dfrac{9k}{20}\right)}{\left(\dfrac{9k}{5}\right)} = \dfrac{1}{4}$

**114. Option (d) is correct**

*Explanations:* G.M. of $1, 2, 2^2 ..... 2^{n-1}$

$= \sqrt[n]{1.2.2^2(.......)2^3....2^{n-1}}$

$= \sqrt[n]{2^{1+2+3+...+n-1}}$

$= \sqrt[n]{2^{(n-1)n/2}}$

$= G = 2^{(n-1)/2}$
Now, $1 + 2\log_2 G = 1 + 2(n - 1/2)$
$= 1 + n - 1 = n$

**115. Option (b) is correct**

*Explanations:* HM $= \dfrac{n}{1 + \dfrac{1}{2} = \dfrac{1}{2^2} + .... \dfrac{1}{2^{n-1}}}$

$\Rightarrow \dfrac{n}{H} = 1 + \dfrac{1}{2} = \dfrac{1}{2^2} + .... \dfrac{1}{2^{n-1}}$

$= \dfrac{1\left(1 - \dfrac{1}{2^n}\right)}{1 - \dfrac{1}{2}} = 2\left(1 - \dfrac{1}{2^n}\right)$

$\Rightarrow \dfrac{n}{H} - 2 - \dfrac{1}{2^{n-1}}$

**116. Option (c) is correct**

*Explanations:* Let $P = (2x_i - a)^2$
On differentiating, we get
$\dfrac{dp}{da} = 8x_i - 4a$

$\dfrac{dp}{da} = 0$

$8x_i = 4a$
$a = 2x_i$

$a = \dfrac{2x_i}{n} = 2Q$

117. **Option (Bonus) is correct**
   *Explanations:*
   Let E1 : Two balls are white
   E2 : Two balls are black
   E3 : One ball is white & other is black
   A: A white ball is from by B
   Probability

   $$= \frac{^3C_2}{^5C_2} + \frac{4}{7} + \frac{^2C_2}{^5C_2} + \frac{2}{7} + \frac{^3C_1 \times ^2C_1}{^5C_2} \times \frac{3}{7} = \frac{33}{70}$$

118. **Option (c) is correct**
   *Explanations:* Total number of ways
   $= 6 \times 6 \times 6$
   Out of these, multiple of 3 shows $2 \times 2 \times 2$
   times

   Required probability $= \dfrac{8}{216} = \dfrac{1}{27}$

119. **Option (c) is correct**
   *Explanations:* There are 31 days in december
   i.e., $31 = \dfrac{28}{7} + 3$
   Now, on these 3 days, one can be sunday
   Required probability $= \dfrac{3}{7}$

120. **Option (b) is correct**
   *Explanations:* $n + \dfrac{50}{n} < 50$

   So, $n$ can be 2, 3, ......, 48
   Favourable cases $= 47$
   Total cases $= 50$

   Required probability $= \dfrac{47}{50}$