NDA / NA
National Defence Academy / Naval Academy

MATHEMATICS II

QUESTION PAPER 2023

Time : 2 : 30 Hours **Total Marks: : 300**

Instructions

1. *This Test Booklet contains **120** items (questions). Each item is printed in **English**. Each item comprises four responses (answers's). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.*
2. *You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.*
3. ***All** items carry equal marks.*
4. *Before you proceed to mark in the Answer Sheet the response to the various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions.*
5. ***Penalty for wrong answers :***
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i) *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.*
   (ii) *If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii) *If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.*

1. If $z\bar{z} = |z + \bar{z}|$, where $z = x + iy$, $i = \sqrt{-1}$, then the locus of z is a pair of :
   (a) straight lines
   (b) rectangular hyperbolas
   (c) parabolas
   (d) circles

2. If 1 ! + 3! + 5! + 7! + ,… + 199! is divided by 24, what is the remainder?
   (a) 3 (b) 6
   (c) 7 (d) 9

3. What is the value of $\sqrt{12+5i} + \sqrt{12-5i}$, where $i = \sqrt{-1}$?
   (a) 24 (b) 25
   (c) $5\sqrt{2}$ (d) $5(\sqrt{2}-1)$

4. If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then what is the value of det (I + AA′), where I is the 3×3 identity matrix?
   (a) 15 (b) 6
   (c) 0 (d) –1

5. If A, B and C are square matrices of order 3 and det(BC) = 2 det(A), then what is the value of det($2A^{-1}$ BC)?
   (a) 16 (b) 8
   (c) 4 (d) 2

6. If the n^{th} term of a sequence is $\frac{2n+5}{7}$, then what is the sum of its first 140 terms?
   (a) 2840 (b) 2780
   (c) 2920 (d) 5700

7. Let A be a skew-symmetric matrix of order 3. What is the value of det($4A^4$) – det ($3A^3$) + det($2A^2$) – det(A)+ det(-I) where I is the identity matrix of order 3 ?
   (a) –1 (b) 0
   (c) 1 (d) 2

8. If $A = \begin{bmatrix} 0 & 3 & 4 \\ -3 & 0 & 5 \\ -4 & -5 & 0 \end{bmatrix}$, then which one of the following statements is correct?
   (a) A^2 is symmetric matrix with det(A^2) = 0.
   (b) A^2 is symmetric matrix with det(A^2) ≠ 0.
   (c) A^2 is skew-symmetric matrix with det(A^2) = 0.
   (d) A^2 is skew-symmetric matrix with det(A^2) ≠ 0.

9. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then which of the following statements are correct?
   1. A^n will always be singular for any positive integer n.
   2. A^n will always be a diagonal matrix for any positive integer n.

3. A^n will always be a symmetric matrix for any positive integer n.

Select the correct answer using the code given below:

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

10. If $(a + b), 2b, (b + c)$ are in HP, then which one of the following is correct?
(a) a, b and c are in AP
(b) a – b, b – c and c – a are in AP
(c) a, b and c are in GP
(d) a – b, b – c and c – a are in GP

11. Let t_1, t_2, t_3 … be in GP. What is $(t_1 t_3 \ldots t_{21})^{\frac{1}{11}}$ equal to?
(a) t_{10} (b) t_{10}^2
(c) t_{11} (d) t_{11}^2

12. Which one of the following is a square root of $-\sqrt{-1}$?
(a) $1 + i$ (b) $\frac{1-i}{\sqrt{2}}$
(c) $\frac{1+i}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}i$

13. What is the maximum number of points of intersection of 10 circles?
(a) 45 (b) 60
(c) 90 (d) 120

14. A set S contains (2n + 1) elements. There are 4096 subsets of S which contain at most n elements. What is n equal to?
(a) 5 (b) 6
(c) 7 (d) 8

15. If $\begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e,$
then what is the value of e?
(a) –1 (b) 0
(c) 1 (d) 2

16. If all elements of a third order determinant are equal to 1 or –1, then the value of the determinant is :
(a) 0 only
(b) an even number but not necessarily 0
(c) an odd number
(d) 0, 1 or –1

17. If $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, then what is the value of det|adj(adjA)|?
(a) 5 (b) 25
(c) 125 (d) 625

18. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then what is the $23A^3 - 19A^2 - 4A$ equal to?
(a) Null matrix of order 3
(b) Identity matrix of order 3
(c) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
(d) $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

19. The value of the determinant of a matrix A of order 3 is 3. If C is the matrix of cofactors of the matrix A, then what is the value of determinant of C^2?
(a) 3 (b) 9
(c) 81 (d) 729

20. If $A_k = \begin{bmatrix} k-1 & k \\ k-2 & k+1 \end{bmatrix}$, then what is $\det(A_1) + \det(A_2) + \det(A_3) + \ldots + \det(A_{100})$ equal to?
(a) 100 (b) 1000
(c) 9900 (d) 10000

21. The Cartesian product A × A has 16 elements among which are (0, 2) and (1, 3). Which of the following statements is/are correct?
1. It is possible to determine set A.
2. A × A contains the element (3, 2).

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

22. Let A = {1, 2, 3, …, 20}. Define a relation R from A to A by $R = \{(x, y) : 4x - 3y = 1\}$, where $x, y \in A$. Which of the following statements is/are correct?
1. The domain of R is {1, 4, 7, 10, 13, 16}.
2. The range of R is {1, 5, 9, 13, 17}.
3. The range of R is equal to codomain of R.

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) 1 and 2 (d) 2 and 3

23. Consider the following statements:
1. The relation f defined by $f(x) = \begin{cases} x^3, & 0 \le x \le 2 \\ 4x, & 2 \le x \le 8 \end{cases}$ is a function.
2. The relation g defined by $g(x) = \begin{cases} x^3, & 0 \le x \le 4 \\ 3x, & 4 \le x \le 8 \end{cases}$ is a function.

Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

24. Consider the following statements:
1. A = (A ∪ B) ∪ (A – B)
2. A ∪ (B – A) = (A ∪ B)
3. B = (A ∪ B) – (A – B)

Which of the statements given above are correct?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

25. A function satisfies $f(x-y) = \frac{f(x)}{f(y)}$, where $f(y) \neq 0$. If $f(1) = 0.5$, then what is $f(2) + f(3) + f(4) + f(5) + f(6)$ equal to?
(a) $\frac{15}{32}$ (b) $\frac{17}{32}$
(c) $\frac{29}{64}$ (d) $\frac{31}{64}$

26. What is $2\cot\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$ equal to?
(a) –1 (b) 1
(c) $3+\sqrt{5}$ (d) $3-\sqrt{5}$

27. If $\sec^{-1} p - \operatorname{cosec}^{-1} q = 0$, where $p > 0$, $q > 0$; then what is the value of $p^{-2} + q^{-2}$?
(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) $\frac{1}{2\sqrt{2}}$

28. What is $1+\sin^2\left[\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)\right]$ equal to?
(a) $\frac{25}{17}$ (b) $\frac{8}{17}$
(c) $\frac{9}{17}$ (d) $\frac{47}{17}$

29. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, $0<\theta<\frac{\pi}{2}$; then what is the value of $8\sin^2\left(\theta+\frac{\pi}{4}\right)$?
(a) 16 (b) 2
(c) 1 (d) $\frac{1}{2}$

30. If $\tan\alpha = \frac{1}{7}$, $\sin\beta = \frac{1}{\sqrt{10}}$; $0<\alpha, \beta<\frac{\pi}{2}$, then what is the value of $\cos(\alpha+2\beta)$?
(a) $-\frac{1}{2}$ (b) $-\frac{1}{\sqrt{2}}$
(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

Consider the following for the next two (02) items that follow:

Consider the equation $(1-x)^4 + (5-x)^4 = 82$.

31. What is the number of real roots of the equation?
(a) 0 (b) 2
(c) 4 (d) 8

32. What is the sum of all the roots of the equation?
(a) 24 (b) 12
(c) 10 (d) 6

Consider the following for the next three (03) items that follow:

Consider equation –I : $z^3 + 2z^2 + 2z + 1 = 0$ and equation – II : $z^{1985} + z^{100} + 1 = 0$.

33. What are the roots of equation–I?
(a) 1, ω, ω² (b) –1, ω, ω²
(c) 1, –ω, ω² (d) –1, –ω, –ω²

34. Which one of the following is a root of equation-II?
(a) –1 (b) –ω
(c) –ω² (d) ω

35. What is the number of common roots of equation-I and equation-II?
(a) 0 (b) 1
(c) 2 (d) 3

Consider the following for the next two (02) items that follow:

A quadratic equation is given by $(a+b)x^2 - (a+b+c)x + k = 0$, where a, b, c are real.

36. If $k = \frac{c}{2}, (c \neq 0)$, then the roots of the equation are:
(a) Real and equal (b) Real and unequal
(c) Real iff a > c (d) Complex but not real

37. If k = c, then the roots of the equation are:
(a) $\frac{a+c}{a+b}$ and $\frac{b}{a+b}$
(b) $\frac{a+c}{a+b}$ and $-\frac{b}{a+b}$
(c) 1 and $\frac{c}{a+b}$
(d) –1 and $-\frac{c}{a+b}$

Consider the following for the next three (03) items that follow:

Let $(1+x)^n = 1 + T_1x + T_2x^2 + T_3x^3 + \ldots + T_nx^n$.

38. What is $T_1 + 2T_2 + 3T_3 + \ldots + nT_n$ equal to?
(a) 0 (b) 1
(c) 2^n (d) $n2^{n-1}$

39. What is $1 - T_1 + 2T_2 - 3T_3 + \ldots + (-1)^n nT_n$ equal to?
(a) 0 (b) -2^{n-1}
(c) $n2^{n-1}$ (d) 1

40. What is $T_1 + T_2 + T_3 + \ldots + T_n$ equal to?
(a) 2^n (b) $2^n - 1$
(c) 2^{n-1} (d) $2^n + 1$

Consider the following for the next two (02) items that follow:

Let $f(x) = x^2 - 1$ and gof(x) = $x - \sqrt{x} + 1$.

41. Which one of the following is a possible expression for $g(x)$?
   (a) $\sqrt{x+1} - \sqrt[4]{x+1}$ (b) $\sqrt{x+1} - \sqrt[4]{x+1} + 1$
   (c) $\sqrt{x+1} + \sqrt[4]{x+1}$ (d) $x + 1 - \sqrt{x+1} + 1$

42. What is g(15) equal to?
   (a) 1 (b) 2
   (c) 3 (d) 4

Consider the following for the next two (02) items that follow:

Let a function f be defined on R – [0] and $2f(x) + f\left(\frac{1}{x}\right) = x + 3$.

43. What is $f(0.5)$ equal to?
   (a) $\frac{1}{2}$ (b) $\frac{2}{3}$
   (c) 1 (d) 2

44. If f is differentiable, then what is f′(0.5) equal to?
   (a) $\frac{1}{4}$ (b) $\frac{2}{3}$
   (c) 2 (d) 4

Consider the following for the next (02) items that follow:

A function is defined by

$$f(x) = \begin{vmatrix} x+1 & 2 & 3 \\ 2 & x+4 & 6 \\ 3 & 6 & x+9 \end{vmatrix}$$

45. The function is decreasing on:
   (a) $\left[-\frac{28}{3}, 0\right]$ (b) $\left[0, \frac{28}{3}\right]$
   (c) $\left[0, \frac{50}{3}\right]$ (d) $\left[0, \frac{56}{3}\right]$

46. The function attains local minimum value at:
   (a) $x = -\frac{28}{3}$ (b) $x = -1$
   (c) $x = 0$ (d) $x = \frac{28}{3}$

Consider the following for the next (02) items that follow:

Given that $4x^2 + y^2 = 9$.

47. What is the maximum value of y?
   (a) $\frac{3}{2}$ (b) 3
   (c) 4 (d) 6

48. What is the maximum value of xy?
   (a) $\frac{9}{4}$ (b) $\frac{3}{2}$
   (c) $\frac{4}{9}$ (d) $\frac{2}{3}$

Consider the following for the next (02) items that follow:

A function is defined by $f(x) = \pi + \sin^2 x$.

49. What is the range of the function?
   (a) [0, 1] (b) $[\pi, \pi+1]$
   (c) $[\pi-1, \pi+1]$ (d) $[\pi-1, \pi-1]$

50. What is the period of the function?
   (a) 2π (b) π
   (c) $\frac{\pi}{2}$ (d) The function is non-periodic

Consider the following for the next (02) items that follow:

A parabola passes through (1, 2) and satisfies the differential equation $\frac{dy}{dx} = \frac{2y}{x}, x > 0, y > 0$.

51. What is the directrix of the parabola?
   (a) $y = -\frac{1}{8}$ (b) $y = \frac{1}{8}$
   (c) $x = -\frac{1}{8}$ (d) $x = \frac{1}{8}$

52. What is the length of latus rectum of the parabola?
   (a) 1 (b) $\frac{1}{2}$
   (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

Consider the following for the next (02) items that follow:

Let $f(x) = \frac{a^{x-1} + b^{x-1}}{2}$ and $g(x) = x - 1$.

53. What is $\lim_{x \to 1} \frac{f(x) - 1}{g(x)}$ equal to?
   (a) $\frac{\ln(ab)}{4}$ (b) $\frac{\ln(ab)}{2}$
   (c) ln (ab) (d) 2ln (ab)

54. What is $\lim_{x \to 1} f(x)^{\frac{1}{g(x)}}$ equal to?
   (a) $\sqrt{ab}$ (b) ab
   (c) 2ab (d) $\frac{\sqrt{ab}}{2}$

Consider the following for the next (02) items that follow:

Let $f(x) = \sqrt{2-x} + \sqrt{2+x}$.

55. What is the domain of the function?
   (a) (–2, 2) (b) [–2, 2]
   (c) R –(–2, 2) (d) R – [–2, 2]

56. What is the greatest value of the function?
   (a) $\sqrt{3}$ (b) $\sqrt{6}$
   (c) $\sqrt{8}$ (d) 4

Consider the following for the next (02) items that follow:

Let $f(x) = |x|$ and $g(x) = [x] - 1$, where [.] is the greatest integer function.

Let $h(x) = \dfrac{f(g(x))}{g(f(x))}$.

57. What is $\lim_{x\to 0^+} h(x)$ equal to?
(a) –2 (b) –1
(c) 0 (d) 1

58. What is $\lim_{x\to 0-} h(x)$ equal to?
(a) –2 (b) –1
(c) 0 (d) 2

Consider the following for the next (02) items that follow:

Let f(x) = $\begin{cases} \dfrac{x-3}{|x-3|}+a; & x<3 \\ a-b; & x=3 \\ \dfrac{x-3}{|x-3|}+b; & x>3 \end{cases}$ and

$f(x)$ be continuous at $x = 3$.

59. What is the value of a?
(a) –1 (b) 1
(c) 2 (d) 3

60. What is the value of b?
(a) –1 (b) 1
(c) 2 (d) 3

Consider the following for the next (02) items that follow:

Let I = $\int_{-2\pi}^{2\pi} \dfrac{\sin^4 x + \cos^4 x}{1+3^x}dx$

61. What is $\int_0^{\pi}(\sin^4 x + \cos^4 x)dx$ equal to?
(a) $\frac{3\pi}{8}$ (b) $\frac{3\pi}{4}$
(c) $\frac{3\pi}{2}$ (d) 3π

62. What is I equal to?
(a) 0 (b) $\frac{3\pi}{4}$
(c) $\frac{3\pi}{2}$ (d) 3π

Consider the following for the next (02) items that follow:

Let $f(x) = \begin{cases} ax(x+1)+b, & x<1 \\ x-1, & 1\le x\le 2 \end{cases}$

63. If the function $f(x)$ is differentiable at $x = 1$, then what is the value of $(a + b)$?
(a) $-\frac{1}{3}$ (b) –1
(c) 0 (d) 1

64. What is $\lim_{x\to 0} f(x)$ equal to?
(a) $-\frac{1}{3}$ (b) $-\frac{2}{3}$
(c) 0 (d) 1

65. If $f(x) = |\ln|x||$ where $0 < x < 1$, then what is $f'(0.5)$ equal to?
(a) –2 (b) –1
(c) 0 (d) 2

66. If $f'(x) = \cos(\ln x)$ and $y = f\left(\frac{2x-3}{x}\right)$, then what is $\frac{dy}{dx}$ equal to?
(a) $\cos\left(\ln\left(\frac{2x-3}{x}\right)\right)$ (b) $-\frac{3}{x^2}\sin\left(\ln\left(\frac{2x-3}{x}\right)\right)$
(c) $\frac{3}{x^2}\cos\left(\ln\left(\frac{2x-3}{x}\right)\right)$ (d) $-\frac{3}{x^2}\cos\left(\ln\left(\frac{2x-3}{x}\right)\right)$

67. What is $\int_0^{8\pi}|\sin x|dx$ equal to?
(a) 2 (b) 4
(c) 8 (d) 16

68. What is the area between the curve $f(x) = x|x|$ and x-axis for $x \in [-1, 1]$?
(a) $\frac{2}{3}$ (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

69. What are the order and the degree respectively of the differential equation $x^2\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right)^4 + \sin x = 0$?
(a) 3, 4 (b) 1, 4
(c) 2, 2 (d) 3, 2

70. What is the differential equation of all parabolas of the type $y^2 = 4a(x - b)$?
(a) $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ (b) $\frac{d^2y}{dx^2} + x^2\left(\frac{dy}{dx}\right)^2 = 0$
(c) $y^2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$ (d) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

Consider the following for the next two (02) items that follow:

Let a_1, a_2, a_3 … be in AP such that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{25} + a_{30} + a_{34} = 300$.

71. What is $a_1 + a_5 - a_{10} - a_{15} - a_{20} - a_{25} + a_{30} + a_{34}$ equal to?
(a) 0 (b) 25
(c) 125 (d) 250

72. What is $\sum_{n=1}^{34} a_n$ equal to?
(a) 900 (b) 1025
(c) 1200 (d) 1275

Consider the following for the next two (02) items that follow:

Let p = $\cos\left(\frac{\pi}{5}\right)\cos\left(\frac{2\pi}{5}\right)$ and q = $\cos\left(\frac{4\pi}{5}\right)\cos\left(\frac{8\pi}{5}\right)$.

73. What is the value of p + q?

(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$

(c) 0 (d) $\frac{1}{2}$

74. What is the value of pq?

(a) $-\frac{1}{16}$ (b) $-\frac{1}{4}$

(c) $\frac{1}{4}$ (d) $\frac{1}{16}$

Consider the following for the next two (02) items that follow:

Let $p=\frac{1}{3}-\frac{\tan 3x}{\tan x}$ and $q = 1 - 3\tan^2 x$, $0<x<\pi, x\neq\frac{\pi}{2}$.

75. What is pq equal to?

(a) 1 (b) 2

(c) $\frac{8}{3}$ (d) $-\frac{8}{3}$

76. For how many values of x does $\frac{1}{p}$ become zero?

(a) No value (b) Only one value

(c) Only two values (d) Only three values

Consider the following for the next two (02) items that follow:

Let $\sin x + \sin y = \sqrt{3}(\cos y - \cos x)$; $x+y=\frac{\pi}{2}$, $0<x,y<\frac{\pi}{2}$

.

77. What is a value of sin 3x + sin 3y?

(a) –1 (b) 0

(c) 1 (d) 3

78. What is the value of $\cos^3 x + \cos^3 y$?

(a) $\frac{3\sqrt{3}}{8}$ (b) $\frac{3\sqrt{6}}{8}$

(c) $\frac{3\sqrt{6}}{4}$ (d) 1

Consider the following for the next two (02) items that follow:

The angles A, B and C of a triangle ABC are in the ratio 3 : 5 : 4.

79. What is the value of $a+b+\sqrt{2}c$ equal to?

(a) 3a (b) 2b

(c) 3b (d) 2c

80. What is the ratio of $a^2 : b^2 : c^2$?

(a) $2:2+\sqrt{3}:3$ (b) $2:2-\sqrt{3}:2$

(c) $2:2+\sqrt{3}:2$ (d) $2:2-\sqrt{3}:3$

81. What is the equation of directrix of parabola $y^2 = 4bx$, where $b < 0$ and $b^2 + b - 2 = 0$?

(a) $x + 1 = 0$ (b) $x - 2 = 0$

(c) $x - 1 = 0$ (d) $x + 2 = 0$

82. The points $(-a, -b)$, $(0, 0)$, (a, b) and (a^2, ab) are:

(a) lying on the same circle

(b) vertices of a square

(c) vertices of a parallelogram that is not a square

(d) collinear

83. Given that $16p^2 + 49q^2 - 4r^2 - 56pq = 0$. Which one of the following is a point on a pair of straight lines $(px + qy + r)(px + qy - r) = 0$?

(a) $\left(2,\frac{7}{2}\right)$ (b) $\left(2,-\frac{7}{2}\right)$

(c) (4, –7) (d) (4, 7)

84. If $3x + y - 5 = 0$ is the equation of a chord of the circle $x^2 + y^2 - 25 = 0$, then what are the coordinates of the mid-point of the chord?

(a) $\left(\frac{3}{4},\frac{1}{4}\right)$ (b) $\left(\frac{3}{2},\frac{1}{2}\right)$

(c) $\left(\frac{3}{4},-\frac{1}{4}\right)$ (d) $\left(\frac{3}{2},-\frac{1}{2}\right)$

85. Consider the following in respect of the equation $\frac{x^2}{24-k}+\frac{y^2}{k-16}=2$.

1. The equation represents an ellipse if $k = 19$.
2. The equation represents a hyperbola if $k = 12$.
3. The equation represents a circle if $k = 20$.

How many of the statements given above are correct?

(a) Only one (b) Only two

(c) All three (d) None

86. Consider the following statements in respect of hyperbola $\frac{x^2}{\cos^2\theta}-\frac{y^2}{\sin^2\theta}=1$.

1. The two foci are independent of θ.
2. The eccentricity is sec θ.
3. The distance between the two foci is 2 units.

How many of the statements given above are correct?

(a) Only one (b) Only two

(c) All three (d) None

87. Consider the following in respect of the circle $4x^2 + 4y^2 - 4ax - 4ay + a^2 = 0$:

1. The circle touches both the axes.
2. The diameter of the circle is 2a.
3. The centre of the circle lies on the line $x + y = a$.

How many of the statements given above are correct?

(a) Only one (b) Only two.

(c) All three (d) None

88. For what values of k is the line $(k-3)x - (5-k^2)y + k^2 - 7k + 6 = 0$ parallel to the line $x + y = 1$?

(a) –1, 1 (b) –1, 2

(c) 1, –2 (d) 2, –2

89. The line $x + y = 4$ cuts the line joining P(–1, 1) and Q(5, 7) at R. What is PR : RQ equal to?

(a) 1 : 1 (b) 1 : 2

(c) 2 : 1 (d) 1 : 3

90. What is the sum of the intercepts of the line whose perpendicular distance from origin is 4 units and the angle which the normal makes with positive direction of x-axis is 15°?
(a) 8 (b) $4\sqrt{6}$
(c) $8\sqrt{6}$ (d) 16

91. What is the length of projection of the vector $\hat{i}+2\hat{j}+3\hat{k}$ on the vector $2\hat{i}+3\hat{j}-2\hat{k}$?
(a) $\frac{1}{\sqrt{17}}$ (b) $\frac{2}{\sqrt{17}}$
(c) $\frac{3}{\sqrt{17}}$ (d) $\frac{2}{\sqrt{14}}$

92. If $\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}\cdot\vec{b}\right)^2 = 144$ and $|\vec{b}| = 4$, then what is the value of $|\vec{a}|$?
(a) 3 (b) 4
(c) 5 (d) 6

93. If θ is the angle between vector $\vec{a}$ and $\vec{b}$ such that $\vec{a}\cdot\vec{b}\geq 0$, then which one of the following is correct?
(a) $0\leq\theta\leq\pi$ (b) $\frac{\pi}{2}\leq\theta\leq\pi$
(c) $0\leq\theta\leq\frac{\pi}{2}$ (d) $0<\theta<\frac{\pi}{2}$

94. The vectors $60\hat{i}+3\hat{j}$, $40\hat{i}-8\hat{j}$ and $\beta\hat{i}-52\hat{j}$ are collinear if:
(a) $\beta = 20$ (b) $\beta = 40$
(c) $\beta = -40$ (d) $\beta = 26$

95. Consider the following in respect of the vectors $\vec{a}=(0,1,1)$ and $\vec{b}=(1,0,1)$:
1. The number of unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$ is only one.
2. The angle between the vectors is $\frac{\pi}{3}$.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

96. If L is the line with direction ratios < 3, –2, 6 > and passing through (1, –1, 1), then what are the coordinates of the points on L whose distance from (1, –1, 1) is 2 units?
(a) $\left(-\frac{11}{7},\frac{13}{7},\frac{19}{7}\right)$ and $\left(\frac{1}{7},\frac{3}{7},\frac{5}{7}\right)$
(b) $\left(\frac{19}{7},-\frac{11}{7},\frac{13}{7}\right)$ and $\left(-\frac{1}{7},\frac{3}{7},-\frac{5}{7}\right)$
(c) $\left(\frac{13}{7},\frac{11}{7},\frac{19}{7}\right)$ and $\left(-\frac{1}{7},-\frac{3}{7},\frac{5}{7}\right)$
(d) $\left(\frac{13}{7},-\frac{11}{7},\frac{19}{7}\right)$ and $\left(\frac{1}{7},-\frac{3}{7},-\frac{5}{7}\right)$

97. Which one of the planes is parallel to the line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$?
(a) $2x + 2y + z - 1 = 0$
(b) $2x - y - 2z + 5 = 0$
(c) $2x + 2y - 2z + 1 = 0$
(d) $x - 2y + z - 1 = 0$

98. What is the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$?
(a) 0° (b) 30°
(c) 60° (d) 90°

99. What is the equation of the sphere concentric with the sphere $x^2 + y^2 + z^2 - 2x - 6y - 8z - 5 = 0$ and which passes through the origin?
(a) $x^2+y^2+z^2-2x-8z=0$
(b) $x^2+y^2+z^2-2x-6y=0$
(c) $x^2+y^2+z^2-6y-8z=0$
(d) $x^2+y^2+z^2-2x-6y-8z=0$

100. A point P lies on the line joining A(1, 2, 3) and B(2, 10, 1). If z-coordinate of P is 7, what is the sum of other two coordinates?
(a) –15 (b) –13
(c) –11 (d) –9

101. The sum of deviations of a n numbers from 10 and 20 are p and q respectively. If $(p - q)^2 = 10000$, then what is the value of n?
(a) 10 (b) 20
(c) 50 (d) 100

102. If $\bar{X}=20$ is the mean of 10 observations $x_1, x_2, \ldots, x_{10}$; then what is the value of $\sum_{i=1}^{10}\left(\frac{3x_i-4}{5}\right)$?
(a) 0 (b) 12
(c) 112 (d) 1012

103. If the mean and the sum of squares of 10 observations are 40 and 16160 respectively, then what is the standard deviation?
(a) 16 (b) 6
(c) 5 (d) 4

104. Three dice are thrown. What is the probability of getting a sum which is a perfect square?
(a) $\frac{17}{108}$ (b) $\frac{5}{108}$
(c) $\frac{19}{108}$ (d) $\frac{23}{108}$

105. A, B, C and D are mutually exclusive and exhaustive events.
If 2P(A) = 3P(B)= 4P(C)= 5P(d), then what is 77P(A) equal to?
(a) 12 (b) 15
(c) 20 (d) 30

106. Two distinct natural numbers from 1 to 9 are picked at random. What is the probability that their product has 1 in its unit place?

(a) $\frac{1}{81}$ (b) $\frac{1}{72}$

(c) $\frac{1}{18}$ (d) $\frac{1}{36}$

107. Two dice are thrown. What is the probability that difference of numbers on them is 2 or 3?

(a) $\frac{7}{36}$ (b) $\frac{7}{18}$

(c) $\frac{5}{18}$ (d) $\frac{11}{36}$

108. What is the mean of the numbers 1,2,3, ..., 10 with frequencies ${}^9C_0, {}^9C_1, {}^9C_2, ..., {}^9C_9$ respectively?

(a) 1.1×2^8 (b) 1.2×7^4

(c) 5.5 (d) 0.55

109. The probability that a person recovers from a disease is 0·8. What is the probability that exactly 2 persons out of 5 will recover from the disease?

(a) 0.00512 (b) 0.02048

(c) 0.2048 (d) 0.0512

110. Suppose that there is a chance for a newly constructed building to collapse, whether the design is faulty or not. The chance that the design is faulty is 10%. The chance that the building collapses is 95% if the design is faulty, otherwise it is 45%. If it is seen that the building has collapsed, then what is the probability that it is due to faulty design?

(a) 0.10 (b) 0.19

(c) 0.45 (d) 0.95

111. If r is the coefficient of correlation between x and y, then what is the correlation coefficient between $(3x + 4)$ and $(-3y + 3)$?

(a) $-r$ (b) r

(c) $\sqrt{3}r$ (d) $-\sqrt{3}r$

112. A fair coin is tossed 6 times. What is the probability of getting a result in the 6^{th} toss which is different from those obtained in the first five tosses?

(a) $\frac{7}{16}$ (b) $\frac{1}{16}$

(c) $\frac{1}{32}$ (d) $\frac{1}{64}$

113. If H is the Harmonic Mean of three numbers ${}^{10}C_4, {}^{10}C_5$, and ${}^{10}C_6$, then what is the value of $\frac{270}{H}$?

(a) 1 (b) $\frac{14}{17}$

(c) $\frac{17}{14}$ (d) $\frac{1}{31}$

114. In a class, there are n students including the students P and Q. What is the probability that P and Q sit together if seats are assigned randomly?

(a) $\frac{1}{n}$ (b) $\frac{2}{n}$

(c) $\frac{4}{n}$ (d) $\frac{1}{2n}$

115. In a Binomial distribution $B(n, p)$, $n = 6$ and $9P(X = 4) = P(X = 2)$. What is p equal to?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) $\frac{4}{5}$

Consider the following for the next five (05) items that follow:

Three boys P, Q, R and three girls S, T, U are to be arranged in a row for a group photograph.

116. What is the probability that all three boys sit together?

(a) $\frac{1}{5}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) $\frac{1}{12}$

117. What is the probability that boys and girls sit alternatively?

(a) $\frac{4}{5}$ (b) $\frac{1}{10}$

(c) $\frac{5}{6}$ (d) $\frac{1}{7}$

118. What is the probability that no two girls sit together?

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{1}{18}$ (d) $\frac{1}{5}$

119. What is the probability that P and Q take the two end positions?

(a) $\frac{1}{15}$ (b) $\frac{7}{15}$

(c) $\frac{14}{15}$ (d) $\frac{11}{45}$

120. What is the probability that Q and U sit together?

(a) $\frac{2}{3}$ (b) $\frac{1}{4}$

(c) $\frac{5}{6}$ (d) $\frac{1}{3}$

Answers			
Q No	**Answer Key**	**Topic Name**	**Chapter Name**
1	(d)	Geometrical Representation	Complex Number
2	(c)	Factorial	Permutation and Combination
3	(c)	Square Roots	Complex Number
4	(a)	Values of Determinant	Determinants
5	(a)	Inverse of Matrices	Determinants
6	(c)	Special Series	Sequence and Series
7	(a)	Values of Determinant	Determinants
8	(a)	Values of Determinant	Determinants
9	(b)	Values of Determinant	Determinant
10	(c)	H.P.	Sequence and Series
11	(c)	G.P.	Sequence and Series
12	(b)	Values if i	Complex Number
13	(c)	Special Series	Sequence and Series
14	(b)	Relation of Determinants	Binomial Theorem
15	(b)	Values of Determinant	Determinants
16	(b)	Cofactor	Determinants
17	(d)	Adjoint	Determinants
18	(a)	Product of Matrices	Matrices
19	(c)	Adroit	Determinants
20	(d)	Values of Determinants	Determinants
21	(c)	Cartesian Product	Relations & Function
22	(b)	Range	Relations & Function
23	(a)	Function	Relations & Function
24	(c)	Complement of Set	Sets
25	(a)	Values of Function	Relations & Function
26	(c)	Identities	Trigonometry
27	(a)	Identities	Inverse Trigonometry
28	(a)	Identities	Inverse Trigonometry
29	(c)	Trigonometric Equations	Trigonometry
30	(c)	Identities	Trigonometry
31	(b)	Nature of Roots	Quadratic Equation
32	(b)	Sum of Roots	Quadratic Equation
33	(b)	Cube Roots of Unity	Complex Number
34	(d)	Cube Roots of Unity	Complex Number
35	(c)	Cube Roots of Unity	Complex Number
36	(b)	Nature of Roots	Quadratic Equations
37	(c)	Nature of Roots	Quadratic Equations
38	(d)	Relation of Coefficients	Binomial Theorem
39	(d)	Relation of Coefficients	Binomial Theorem

40	(b)	Relation of Coefficients	Binomial Theorem
41	(b)	Composite Function	Relation & Function
42	(c)	Values of Function	Relation & Function
43	(b)	Values of Function	Relation & Function
44	(c)	Values of Function	Differentiation
45	(a)	Increasing and Decreasing	Application of Derivatives
46	(c)	Maxima & Minima	Application of Derivatives
47	(b)	Maximum value of function	Application of Derivatives
48	(a)	Maximum and Minimum	Application of Derivatives
49	(b)	Range	Trigonometry
50	(b)	Period	Trigonometry
51	(a)	Solution of Different Equates	Differential Equations
52	(b)	Parabola	Conic section
53	(b)	Limit	Limit & Derivatives
54	(a)	Limit	Limit & Derivatives
55	(b)	Domain	Relation & Function
56	(c)	Greatest Value of Function	Application of Derivatives
57	(b)	Limit	Limit & Derivatives
58	(a)	Limit	Limit & Derivatives
59	(d)	Continuity	Continuity and Differentiability
60	(b)	Continuity	Continuity and Differentiability
61	(b)	Values of Definite Integral	Definite Integral
62	(d)	Properties of Definite Integral	Definite Integral
63	(a)	Differentiability	Continuity and Differentiability
64	(b)	Limit	Limit and Derivative
65	(a)	Value of Differentiation	Differentiation
66	(c)	Differentiation	Differentiation
67	(d)	Properties of Definite Integrals	Application of Integral
68	(a)	Area Bounded by a Curve	Definite Integral
69	(d)	Degree and Order	Differential Equation
70	(d)	Formation of Differential Equation	Differential Equation
71	(a)	A.P.	Sequence and Series
72	(d)	A.P.	Sequence and Series
73	(c)	Identities	Trigonometry
74	(a)	Identities	Trigonometry
75	(d)	Identities	Trigonometry
76	(c)	Identities	Trigonometry
77	(b)	Identities	Trigonometry
78	(b)	Values	Trigonometry
79	(c)	Properties of triangle	Trigonometry
80	(a)	Properties of triangle	Trigonometry

81	(a)	Parabola	Conic Section
82	(d)	Collinear	3D
83	(b)	Pair of Straight Lines	Straight Lines
84	(b)	Circle	Conic Section
85	(c)	Ellipse	Conic Section
86	(c)	Hyperbola	Conic Section
87	(b)	Circle	Conic Section
88	(b)	Parallel Lines	Straight Lines
89	(b)	Section Formula	coordinate geometry
90	(c)	Equation of Straight Lines	Straight Lines
91	(b)	Projection	Vector
92	(a)	Cross Product	Vector
93	(c)	Dot Product	Vector
94	(c)	Collinear	Vector
95	(b)	Cross Product	Vector
96	(d)	Straight Lines	3D
97	(d)	Plane	3D
98	(d)	Angle Between Line	3D
99	(d)	Sphere	3D
100	(a)	Straight Lines	3D
101	(a)	Mean	Statistics
102	(c)	Mean	Statistics
103	(d)	Variance and Standard Deviation	Statistics
104	(a)	Basic Probability	Probability
105	(d)	Basic Probability	Probability
106	(c)	Basic Probability	Probability
107	(b)	Basic Probability	Probability
108	(c)	Relation Between Coefficient	Binomial Theorem
109	(d)	Binomial Distribution	Probability
110	(b)	Bayes Theorem	Probability
111	(a)	Coefficient of Correlation	Correlation and Regression
112	(c)	Multiplication	Probability
113	(c)	Harmonic Mean	Statistics
114	(b)	Basic Probability	Probability
115	(a)	Binominal Distribution	Probability
116	(a)	Basic Probability	Probability
117	(b)	Basic Probability	Probability
118	(d)	Basic Probability	Probability
119	(a)	Basic Probability	Probability
120	(d)	Basic Probability	Probability

ANSWERS WITH EXPLANATION

1. **Option (d) is correct.**
Given that $z = x + iy$
$\therefore \bar{z} = x - iy$
Now, $z\bar{z} = |z + \bar{z}|$
$\Rightarrow (x + iy)(x - iy) = |x + iy + x - iy|$
$\Rightarrow x^2 + y^2 = \pm 2x$
$\Rightarrow x^2 \pm 2x + y^2 = 0$
$\Rightarrow x^2 \pm 2x + 1 + y^2 = 1$
$\Rightarrow (x \pm 1)^2 + y^2 = 1$
Represent the equation of circles

2. **Option (c) is correct.**
$1! + 3! + 5! + 7! + \ldots + 199!$
$= 1 + 6 + 5.24 + 7.6.5.24 + \ldots.$
$= 1 + 6 + 24\,[5 + 7.6.5 + \ldots]$
$= 7 + 24\,[5 + 7.6.5 + \ldots.]$
When divided by 24 we get the remainder 7.

3. **Option (c) is correct.**
Let $x = \sqrt{12+5i} + \sqrt{12-5i}$
$\Rightarrow x^2 = 12 + 5i + 12 - 5i + 2\sqrt{144+25}$
$= 24 + 26 = 50$
$x = 5\sqrt{2}$

4. **Option (a) is correct.**
$$A = \begin{bmatrix}1\\2\\3\end{bmatrix} \Rightarrow A' = [1\ 2\ 3]$$
$$AA' = \begin{bmatrix}1\\2\\3\end{bmatrix}[\,1\ 2\ 3] = \begin{bmatrix}1 & 2 & 3\\2 & 4 & 6\\3 & 6 & 9\end{bmatrix}$$
$$I + AA' = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} + \begin{bmatrix}1 & 2 & 3\\2 & 4 & 6\\3 & 6 & 9\end{bmatrix}$$
$$= \begin{bmatrix}2 & 2 & 3\\2 & 5 & 6\\3 & 6 & 10\end{bmatrix}$$
$$\text{Now, } |I + AA'| = \begin{vmatrix}2 & 2 & 3\\2 & 5 & 6\\3 & 6 & 10\end{vmatrix}$$
$= 2(50 - 36) - 2(20 - 18) + 3(12 - 15)$
$= 28 - 4 - 9 = 15$

5. **Option (a) is correct.**
Given that $|BC| = 2|A|$ ...(i)
Now, $|2A^{-1}BC| = |2A^{-1}|\,|BC|$
$[\because |AB| = |A|\,|B|]$ from (i)
$= 2^3\,|A^{-1}|\,.\,2|A|$
$[|2A| = 2^n|A|$, where order of A is $n]$
$= 2^4 \dfrac{1}{|A|}.|A| \qquad \left[\because |A^{-1}| = \dfrac{1}{|A|}\right]$
$= 16$

6. **Option (c) is correct.**
Given that $a_n = \dfrac{2n+5}{7} = \dfrac{2}{7}n + \dfrac{5}{7}$
$\therefore S = \dfrac{2}{7}\Sigma n + \dfrac{5}{7}\Sigma 1$
$= \dfrac{2}{7}\dfrac{n(n+1)}{2} + \dfrac{5}{7}n$
$= \dfrac{n}{7}(n+1+5) = \dfrac{n}{7}(n+6)$
Now, $S_{140} = \dfrac{140}{7}(140+6) = 20 \times 146 = 2920$

7. **Option (a) is correct.**
Given that A is skew symmetric matrix of order 3.
$\therefore A^T = -A \Rightarrow |A^T| = |-A| = (-1^3)|A|$
$\Rightarrow |A| = -|A| \Rightarrow |A| = 0 \qquad [\because |A^T| = |A|]$
Now, $|4A^4| - |3A^3| + |2A^2| - |A| + |-I|$
$= 4^3\,|A|^4 - 3^3\,|A|^3 + 2^3|A|^2 - |A| + (-1)^3\,|I|$
$= -1$

8. **Option (a) is correct.**

$\because A = \begin{bmatrix} 0 & 3 & 4 \\ -3 & 0 & 5 \\ -4 & -5 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 3 & 4 \\ -3 & 0 & 5 \\ -4 & -5 & 0 \end{bmatrix}\begin{bmatrix} 0 & 3 & 4 \\ -3 & 0 & 5 \\ -4 & -5 & 0 \end{bmatrix}$

$= \begin{bmatrix} -25 & -20 & 15 \\ -20 & -34 & -12 \\ 15 & -12 & -41 \end{bmatrix}$

$\therefore [A^2]^T = \begin{bmatrix} -2 & -20 & 15 \\ -20 & -34 & -12 \\ 15 & -12 & -41 \end{bmatrix}$

$\therefore |A^2| = \begin{vmatrix} -25 & -20 & 15 \\ -20 & -34 & -12 \\ 15 & -12 & -41 \end{vmatrix}$

$= -25\,(1394 - 144) + 20\,(820 + 180) + 15\,(240 + 510)$

$= -31250 + 20000 + 11250 = 0$

9. Option (b) is correct.

$\therefore A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$= \begin{bmatrix} 2^2 & 0 & 0 \\ 0 & 3^2 & 0 \\ 0 & 0 & 4^2 \end{bmatrix}$

$\therefore A^n = \begin{bmatrix} 2^n & 0 & 0 \\ 0 & 3^n & 0 \\ 0 & 0 & 4^n \end{bmatrix}$ and $|A^n| \neq 0$

Hence statements 2 and 3 are true.

10. Option (c) is correct.

Given that $(a + b), 2b, (b + c)$ are H.P.

$\therefore \frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ are in A.P.

$\Rightarrow \frac{2}{2b} = \frac{1}{a+b} + \frac{1}{b+c}$

$\Rightarrow \frac{1}{b} = \frac{a+2b+c}{ab+ac+b^2+bc}$

$\Rightarrow ab + ac + b^2 + bc = ab + 2b^2 + bc$

$\Rightarrow ac = b^2$

Hence a, b, c are in G.P.

11. Option (c) is correct.

Lt $t_1 = a, t_2 = ar, t_3 = ar^2$ ....

$\therefore (t_1 t_3 ....t_{21})^{\frac{1}{11}} = (a.ar^2.ar^4 ...at^{20})^{\frac{1}{11}}$

$= \left(a^{11}.r^{2+4+...20}\right)^{\frac{1}{11}}$

$= (a^{11}.\, r^{110})^{1/11}$

$= ar.^{10} = t_{11}$

12. Option (b) is correct.

$\therefore \left(\frac{1-i}{\sqrt{2}}\right)^2 = \frac{1+i^2-2i}{2} = \frac{1-1-2i}{2} = -i = -\sqrt{-1}$

13. Option (c) is correct.

Maximum number of points of intersection of two circles = 2.

Maximum number of points of intersection of three circles = 2 + (2 × 2) = 6

Maximum number of points of intersection of four circles = 6 + (3 × 2) = 12

Maximum number of points of intersection of five circles = 12 + (4 × 2) = 20

Let S = 2 + 6 + 12 + 20 + ...+ a_9

S = 2 + 6 + 12 + 20 + ...+ a_9

– – – – – – –

0 = 2 + 4 + 6 + 8 + ... a_9

a_9 = 2 + 4 + 6 + 8 + ... upto 9 terms

= 9 (4 + (9 – 1) 2) = 90

Hence, maximum number of points of intersection of 10 circles = 90.

Method-2:

$\because$ No. of point of intersection = $^{10}C_2 \times 2$

$= \frac{10\times 9}{2\times 1} \times 2 = 90$

14. Option (b) is correct.

Given that n(S) = $2n + 1$

Number of subsets of S which contain at must n elements

$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n$

$4096 = 2^{(2n+1)} = 2^{2n}$

$\Rightarrow 2^{12} = 2^{2n} \Rightarrow 2n = 12$

$\Rightarrow n = 6$

15. Option (b) is correct.

$\begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix}$

$= (x^2 + 3x)\,[-6x^2 - x^2 + 16] - (x - 1)\,[3x^2 + 3x - x^2 + 7x - 12] + (x + 3)\,[x^2 + 5x + 4 + 2x^2 - 6x]$

$= (x^2 + 3x)\,(-5x^2 + 16) - (x - 1)\,[2x^2 + 10x - 12] + (x + 3)\,[3x^2 - x + 4]$

$= -5x^4 + 16x^2 - 15x^3 + 18x - 2x^3 - 10x^2 + 12x$

$+ 2x^2 + 10x - 12 + 3x^3 - x^2 + 4x + 9x^2 - 3x + 12$
$= -5x^4 - 14x^3 + 16x^2 + 41x$
$= ax^4 + bx^3 + cx^2 + dx + e$
$\Rightarrow e = 0$

Method-2:

$$\begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix} = ax^4+bx^3+cx^3+dx+e$$

Put $x = 0$

$$e = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} = 0$$

[$\because$ determinant value of odd order skew-symmetric is zero]

16. Option (b) is correct.

$$\because \begin{vmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 1(-1-1) + 1(1-1) + 1(-1-1)$$

$= -2 + 0 - 2 = -4$

Since, values of co factor of each element are 0, 2, –2.

So, values of determinants is an even but not necessarily 0.

17. Option (d) is correct.

$$|A| = \begin{vmatrix} 2 & -1 & 0 \\ -1 & 3 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 1(6-1) = 5$$

Now, $|\text{adj}(\text{adj } A)| = |\text{adj } A|^{n-1}$

[$\because |\text{adj } A| = |A|^{n-1}$]

$$= \left[|A|^{(n-1)}\right]^{n-1} = |A|^{(n-1)^2}$$

$\because$ Order of matrix A is 3

$\therefore |\text{adj}(\text{adj } A)| = |A|^{(3-1)^2} = 5^4 = 625$

18. Option (a) is correct.

$$\therefore A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \Rightarrow A^2 = A^3 = A$$

Now, $23A^3 - 19A^2 - 4A = 23A - 19A - 4A = 0$

Null matrix of order 3.

19. Option (c) is correct.

Given that $|A| = 3$ and adj $A = C'$

Now, $|C^2| = |C|^2 \ |C'|^2 = |\text{adj } A|^2$

$= (|A|^{3-1})^2 = |A|^4 = 3^4 = 81$

20. Option (d) is correct.

$$|A_k| = \begin{vmatrix} k-1 & k \\ k-2 & k+1 \end{vmatrix} = k^2 - 1 - k^2 + 2k$$

$= 2k - 1$

$|A_1| + |A_2| + |A_3| + ... + |A_{100}|$

$$= \sum_{k=1}^{100} (2k-1)$$

$$= \frac{2.100(100+1)}{2} - 100$$

$= 100\,(101 - 1) = 10000$

21. Option (c) is correct.

Given that n (A × A) = 16 and (0, 2) and (1, 3) ∈ A × A

$\Rightarrow A = \{0, 1, 2, 3\}$

Also (3, 2) ∈ A × A as 3, 2 ∈ A

Hence both statements are true,

22. Option (b) is correct.

Given that $R = \{(x, y) : 4x - 3y = 1\}$

$$4x - 3y = 1 \Rightarrow y = \frac{1}{3}(4x-1)$$

$\therefore$ Domain = {1, 4, 7, 10, 13}

Range = {1, 5, 9, 13, 17}

Co domain = A

So, statement 2 is correct.

23. Option (a) is correct.

$$f(x) = \begin{cases} x^3, & 0 \le x \le 2 \\ 4x. & 2 \le x \le 8 \end{cases}$$

$\because f(2) = (2)^3 = 8$ for $0 \le x \le 2$

$f(2) = 4\,(2) = 8$ for $2 \le x \le 8$

So, $f(x)$ is function

$$g(x) = \begin{cases} x^2, & 0 \le x \le 4 \\ 3x & 4 \le x \le 8 \end{cases}$$

$\because g(4) = (4)^2 = 16$ for $0 \le x \le 4$

$g(4) = 3\,(4) = 12$ for $4 \le x \le 8$

$\therefore g : \{(4, 16), (4, 12)\}$

So, $g(x)$ is not function.

Hence, only statement 1 is correct.

24. Option (c) is correct.

$\because (A \cup B) \cup (A - B) = (A \cup B) \cup (A \cap B')$

$= A \cup [B \cup (A \cap B')] = A \cup (A \cup B)$

$= A \cup B \neq A$

So, statement 1 is wrong.

$A \cup (B - A) = A \cup (B \cap A')$

$= (A \cup B) \cap (A \cup A') = A \cup B$

So, statement 2 is correct.

$(A \cup B) - (A - B) = (A \cup B) \cap (A \cap B')'$

$= (A \cup B) \cap (A' \cup B)$

$= (A \cap A') \cup B = B$

So, statement 3 is correct.

25. Option (a) is correct.

Given that $f(x-y) = \dfrac{f(x)}{f(y)}$ and $f(1) = 0.5$

$$f(1) = f(2-1) = \frac{f(2)}{f(1)} = f(2) = [f(1)]^2$$

$$\therefore f(2) = (0.5)^2 = \frac{1}{4}$$

$$f(2) = f(3-1) = \frac{f(3)}{f(1)}$$

$$f(3) = f(2).f(1) = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$$

$$\therefore f(4) = f(3).f(1) = \frac{1}{8}\times\frac{1}{2} = \frac{1}{16}$$

$$f(5) = f(4).f(1) = \frac{1}{16}\times\frac{1}{2} = \frac{1}{32}$$

$$f(6) = f(5).f(1) = \frac{1}{32}\times\frac{1}{2} = \frac{1}{64}$$

$\therefore f(2) + f(3) + f(4) + f(5) + f(6)$

$$= \frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64} = \frac{16+8+4+2}{64}$$

$$= \frac{30}{64} = \frac{15}{32}$$

26. Option (c) is correct.

Let $\dfrac{1}{2}\cos^{-1}\dfrac{\sqrt{5}}{3} = \theta \Rightarrow \cos^{-1}\dfrac{\sqrt{5}}{3} = 2\theta$

$$\Rightarrow \cos 2\theta = \frac{\sqrt{5}}{3} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\Rightarrow \sqrt{5}+\sqrt{5}\tan^2\theta = 3-3\tan^2\theta$$

$$\Rightarrow \tan^2\theta = \frac{3-\sqrt{5}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{(3-\sqrt{5})^2}{9-5}$$

$$\Rightarrow \tan^2\theta = \frac{(3-\sqrt{5})^2}{4}$$

$$\Rightarrow \tan\theta = \frac{3-\sqrt{5}}{2} \Rightarrow \cot\theta = \frac{2}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$\cot\theta = \frac{3+\sqrt{5}}{2}$$

$$\therefore 2\cot\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right) = 2\left(\frac{3+\sqrt{5}}{2}\right)$$

$$= 3+\sqrt{5}$$

27. Option (a) is correct.

Given that $\sec^{-1}p - \text{cosec}^{-1}q = 0$

$$\Rightarrow \sec^{-1}p = \text{cosec}^{-1}q \Rightarrow \cos^{-1}\frac{1}{p} = \sin^{-1}\frac{1}{q}$$

$$\Rightarrow \sin^{-1}\sqrt{1-\frac{1}{p^2}} = \sin^{-1}\frac{1}{q}$$

$$\Rightarrow \sqrt{1-\frac{1}{p^2}} = \frac{1}{q}$$

$$\Rightarrow 1-\frac{1}{p^2} = \frac{1}{q^2}$$

$$\Rightarrow \frac{1}{p^2}+\frac{1}{q^2} = 1 \Rightarrow p^{-2}+q^{-2} = 1$$

28. Option (a) is correct.

$$1+\sin^2\left(\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)\right)$$

$$= 1+\sin^2\left(\sin^{-1}\sqrt{1-\frac{9}{17}}\right)$$

$$= 1+\sin^2\left(\sin^{-1}\sqrt{\frac{8}{17}}\right)$$

$$= 1+\left(\sqrt{\frac{8}{17}}\right)^2 = 1+\frac{8}{17} = \frac{25}{17}$$

29. Option (c) is correct.

$\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$

$$\Rightarrow \frac{\sin(\pi\cos\theta)}{\cos(\pi\cos\theta)} = \frac{\cos(\pi\sin\theta)}{\sin(\pi\sin\theta)}$$

$\Rightarrow \cos(\pi\sin\theta) . \cos(\pi\cos\theta) - \sin(\pi\cos\theta)\sin(\pi\sin\theta) = 0$

$$\Rightarrow \cos[\pi(\sin\theta+\cos\theta)] = \cos\frac{\pi}{2}$$

$$\Rightarrow \pi(\sin\theta+\cos\theta) = \frac{\pi}{2}$$

$$\Rightarrow \sin\theta+\cos\theta = \frac{1}{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta+\frac{1}{\sqrt{2}}\cos\theta = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \sin\left(\frac{\pi}{4}+\theta\right) = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow 8\sin^2\left(\frac{\pi}{4}+\theta\right) = 1$$

30. Option (c) is correct.

$$\tan\alpha = \frac{1}{7} \Rightarrow \sin\alpha = \frac{1}{5\sqrt{2}} \text{ and } \cos\alpha = \frac{7}{5\sqrt{2}}$$

$\sin\beta = \frac{1}{\sqrt{10}} \Rightarrow \cos\beta = \sqrt{1-\sin^2\beta} = \sqrt{1-\frac{1}{10}} = \frac{3}{\sqrt{10}}$

$\sin 2\beta = 2\sin\beta.\cos\beta = 2.\frac{3}{10} = \frac{3}{5}$

$\cos 2\beta = 1 - 2\sin^2\beta = 1 - \frac{2}{10} = \frac{4}{5}$

Now, $\cos(\alpha + 2\beta)$

$= \cos\alpha.\cos 2\beta - \sin\alpha.\sin 2\beta$

$= \frac{7}{5\sqrt{2}} - \frac{1}{5\sqrt{2}}.\frac{3}{5}$

$= \frac{28-3}{5\sqrt{2}.5} = \frac{25}{25\sqrt{2}} = \frac{1}{\sqrt{2}}$

31. Option (b) is correct.

$(1-x)^4 + (5-x)^4 = 82 = (3)^4 + (1)^4$

Case-I : $(1-x) = \pm 3$ and $5 - x = \pm 1$

$\Rightarrow x = 4$

Case-II : $(1-x) = \pm 1$ and $5 - x = \pm 3$

$\Rightarrow x = 2$

Hence, two real roots.

32. Option (b) is correct.

$(1-x)^4 + (5-x)^4 = 82$

$\Rightarrow 1 - 4x + 6x^2 - 4x^3 + x^4 + 625 - 500x + 150x^2 - 20x^3 + x^4 = 82$

$\Rightarrow 2x^4 - 24x^3 + 156x^2 - 504x + 544 = 0$

$\therefore$ Sum of all the roots of the equation

$= \frac{-b}{a} = \frac{+24}{2} = 12$

33. Option (b) is correct.

$z^3 + 2z^2 + 2z + 1 = 0$

$\Rightarrow z^3 + z^2 + z^2 + z + z + 1 = 0$

$\Rightarrow (z + 1)(z^2 + z + 1) = 0$

If $z + 1 = 0 \Rightarrow z = -1$

or $(z^2 + z + 1) = 0 \Rightarrow z = \omega, \omega^2$

34. Option (d) is correct.

$\because Z^{1985} + Z^{100} + 1 = 0$

Put $z = \omega$

$\omega^{1985} + \omega^{100} + 1 = 0$

$\Rightarrow \omega^{3\times661+2} + \omega^{3\times33+1} + 1 = 0$

$\Rightarrow \omega^2 + \omega + 1 = 0$ (satisfy)

35. Option (c) is correct.

Statement-1: $z^3 + 2z^2 + 2z + 1 = 0$

$\Rightarrow (z + 1)(z^2 + z + 1) = 0$

$\Rightarrow z = -1, \omega, \omega^2$

Statement-2:

$\because \omega, \omega^2$ are satisfy the equation $z^{1985} + z^{100} + 1 = 0$

$\therefore$ Common roots w, w^2

Two common root.

36. Option (b) is correct.

$\because (a + b)x^2 - (a + b + c)x + k = 0$

$D = (a + b + c)^2 - 4(a + b)k$

put $k = \frac{c}{2}$

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 2ac - 2bc$

$= a^2 + b^2 + c^2 + 2ab$

$= (a + b)^2 + c^2 \geq 0$

$\therefore$ Roots are real and unequal.

37. Option (c) is correct.

$D\ (a + b + c)^2 - 4(a + b)$ k

put $k = c$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 4ac - 4bc$

$= (a + b - c)^2$

$x = \frac{(a+b+c) \pm (a+b-c)}{2(a+b)}$

$x = \frac{a+b+c+a+b-c}{2(a+b)}$ or $\frac{a+b+c-a-b+c}{2(a+b)}$

$= 1$ or $\frac{c}{a+b}$

38. Option (d) is correct.

$(1 + x)^n = 1 + T_1x + T_2x^2 + T_3x^3 + ... + T_nx^n$

Differentiative w.r.t. to x

$x(1 + x)^{n-1} = T_1 + 2T_2x + 3T_3x^2 + ... + nT_nx^n$

Put $x = 1$, we get

$n(1 + 1)^{n-1} = T_1 + 2T_2 + 3T_3 + ... + nT_n$

$= n2^{n-1}$

39. Option (d) is correct.

$(1 + x)^n = 1 + T_1x + T_2x^2 + T_3x^3 + ... + T_nx^n$

Differentiate w.r.t. to x

$n(1 + x)^{n-1} = T_1 + 2T_2x + 3T_3x^2 + ... + nT_nx^{n-1}$

Put $n = -1$ we get

$0 = T_1 - 2T_2 + 3T_3 + ... + (-1)^{n-1} nT_n$

Now $1 - T_1 + 2T_2 - 3T_3 + ... + (-1)^n nT_n$

$= 1 - [T_1 - 2T_2 + 3T_3 - ... + (-1)^n . nT_n]$

$= 1 - 0 = 1$

40. Option (b) is correct.

$(1 + x)^n = 1 + T_1x + T_2x^2 + T_3x^3 + ... + T_nx^n$

Put $x = 1$, we get

$2^n = 1 + T_1 + T_2 + T_3 .... + T_n$

$\Rightarrow 2^n - 1 = T_1 + T_2 + T_3 .... + T_n$

41. Option (b) is correct.

Let $g(x) = \sqrt{x+1} - \sqrt[4]{x+1} + 1$

$\therefore gof(x) = g(x^2 - 1)$

$= \sqrt{x^2 - 1 + 1} - \sqrt[4]{x^2 - 1 + 1} + 1$

$= x - \sqrt{x} + 1$

42. Option (c) is correct.

$\because g(x) = \sqrt{x+1} - \sqrt[4]{x+1} + 1$

$g(15) = \sqrt{16} - \sqrt[4]{16} + 1$

$= 4 - 2 + 1 = 3$

43. Option (b) is correct.

$\because 2f(x) + f\left(\dfrac{1}{x}\right) = x + 3,$ ...(i)

Replace x by $\dfrac{1}{x}$

$2f\left(\dfrac{1}{x}\right) + f(x) = \dfrac{1}{x} + 3$ ...(ii)

On 2 (i) – (ii), we get

$3f(x) = 2x - \dfrac{1}{x} + 3$

$f(x) = \dfrac{2}{3}x - \dfrac{1}{3x} + 1$

$f(0.5) = \dfrac{2}{3} \times \dfrac{1}{2} - \dfrac{2}{3} + 1$

$= \dfrac{1}{3} - \dfrac{2}{3} + 1 = \dfrac{1-2+3}{3} = \dfrac{2}{3}$

44. Option (c) is correct.

$f(x) = \dfrac{2}{3}x - \dfrac{1}{3x} + 1$

$f'(x) = \dfrac{2}{3} + \dfrac{1}{3x^2}$

$f'(0.5) = \dfrac{2}{3} + \dfrac{4}{3} = \dfrac{6}{3} = 2$

45. Option (a) is correct.

$$f(x) = \begin{vmatrix} x+1 & 2 & 3 \\ 2 & x+4 & 6 \\ 3 & 6 & x+9 \end{vmatrix}$$

$= (x + 1) [(x + 4) (x + 9) - 36] - 2 [2(x+9) - 18] + 3 [12 - 3(x + 4)]$

$= (x + 1) (x^2 + 13x) - 4x - 9x$

$f(x) = (x + 1) (x^2 + 13x) - 13x$

$= x^3 + 14x^2$

$f'(x) = 3x^2 + 28x = 0 \Rightarrow x = 0, \dfrac{-28}{3}$

Sign of HCM

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$\dfrac{-28}{3}$ 0

$\therefore f(x)$ is decreasing on $\left(\dfrac{-28}{3}, 0\right)$

46. Option (c) is correct.

$f(x) = x^3 + 14x^2$

$f'(x) = 3x^2 + 28x = 0 \Rightarrow x = 0, \dfrac{-28}{3}$

$f''(x) = 6x + 28$

Put $x = 0$

$f''(x) = 28 > 0$

$\therefore f(x)$ is minimum at $x = 0$

47. Option (b) is correct.

$\because 4x^2 + y^2 = 9$

$4x^2 = 9 - y^2 \Rightarrow x = \dfrac{1}{2}\sqrt{9 - y^2}$

$\therefore 9 - y^2 \geq 0 \Rightarrow y^2 \leq 9$

$\Rightarrow -3 \leq y \leq 3$

Hence maximum value of y is 3.

48. Option (a) is correct.

$4x^2 + y^2 = 9 \Rightarrow y^2 = 9 - 4x^2$

$\Rightarrow y = \sqrt{9 - 4x^2}$

$f(x) = xy = x\sqrt{9 - 4x^2}$

$f'(x) = \sqrt{9 - 4x^2} - \dfrac{1}{2\sqrt{9 - 4x^2}} \times 8x$

$= \dfrac{9 - 8x^2}{\sqrt{9 - 4x^2}} = 0 \Rightarrow x = \dfrac{3}{2\sqrt{2}}$

$$f''(x) = \frac{-16x\sqrt{9-4x^2} - (9-8x^2) \times \dfrac{-8x}{2\sqrt{9-4x^2}}}{(9-4x^2)}$$

$$f''\left(\frac{3}{2\sqrt{2}}\right) = \frac{-16 \times \dfrac{3}{2\sqrt{2}}\sqrt{9 - \dfrac{9}{2}}}{9 - \dfrac{9}{2}} < 0$$

$\therefore f(x)$ is maximum at $x = \dfrac{3}{2\sqrt{2}}$

Maximum values $= x\sqrt{9 - 4x^2} = \dfrac{3}{2\sqrt{2}}\sqrt{9 - \dfrac{9}{2}}$

$= \dfrac{3}{2\sqrt{2}} \times \dfrac{3}{\sqrt{2}} = \dfrac{9}{4}.$

49. Option (b) is correct.

$f(x) = \pi + \sin^2 x$

$\because -1 \leq \sin x \leq 1$

$\Rightarrow 0 \leq \sin^2 x \leq 1 \Rightarrow 0 + \pi \leq \pi + \sin^2 x \leq 1 + \pi$

$\Rightarrow \pi \leq f(x) \leq \pi + 1$

Range $= [\pi, \pi + 1]$

50. Option (b) is correct.

$f(x) = \pi + \sin^2 x$

$f(\pi + x) = \pi + \sin^2 (\pi + x) = \pi + \sin^2 x$

$\therefore$ Period of $f(x)$ is π

51. Option (a) is correct.

$$\frac{dy}{dx}=\frac{2y}{x}\Rightarrow\int\frac{1}{y}dy=2\int\frac{1}{x}dx$$

$\Rightarrow \log y = 2\log x + \log c$

$$\Rightarrow \log y=\log\left(\frac{x^2}{c}\right)\Rightarrow x^2=cy$$

$\because$ It passes through (1, 2)

$\therefore 1 = 2c = c = \frac{1}{2}$

$\therefore$ Equation of parabola is

$x^2=\frac{1}{2}y$

$\Rightarrow 4a=\frac{1}{2}\Rightarrow a=\frac{1}{8}$

Equation of directrix

$y=\frac{-1}{8}$

52. Option (b) is correct.

$\because$ Equation of parabola is $x^2=\frac{1}{2}y$

$\therefore$ Length of latus rectum $=4a=\frac{1}{2}$

53. Option (b) is correct.

$$\lim_{x\to 1}\frac{f(x)-1}{g(x)}=\lim_{x\to 1}\frac{\frac{a^{x-1}+b^{x-1}}{2}-1}{x-1}$$

Let $x - 1 = h$ when $x \to 1$ then $h \to 0$

$$=\lim_{h\to 0}\frac{a^h+b^h-2}{2h}$$

$$=\lim_{h\to 0}\frac{(a^h-1)}{2h}+\frac{(b^h-1)}{2h}$$

$$=\frac{1}{2}.\log_e a+\frac{1}{2}\log_e b=\frac{1}{2}\log_e ab$$

$$=\frac{1}{2}\ln ab$$

54. Option (a) is correct.

$$\lim_{x\to 1}f(x)\frac{1}{g(x)}=\lim_{x\to 1}\left(\frac{a^{x-1}+b^{x-1}}{2}\right)^{\frac{1}{x-1}}$$

Let $x - 1 = h$, when $x \to 1$ then $h \to 0$

$$\lim_{h\to 0}\left(\frac{a^h+b^h}{2}\right)^{\frac{1}{h}}$$

$\therefore$ It is form of , ∞

$$\therefore \lim_{h\to 0} e^{\frac{1}{h}\left[\frac{a^h+b^h}{2}-1\right]}$$

We know that $\lim_{x\to 0} f(x)g(x)$ (form 1^∞)

$$=\lim_{h\to 0} e^{\frac{a^h+b^h-2}{2}}=e^{\frac{1}{2}\ln ab}$$

$$=e^{\ln\sqrt{ab}}=\sqrt{ab}.$$

55. Option (b) is correct.

$\because f(x)=\sqrt{2-x}+\sqrt{2+x}$

for domain

$2 - x \geq 0$ and $2 + x \geq 0$

$\Rightarrow -x \geq -2$ and $x \geq -2$

$\Rightarrow x \leq 2$ and $x \geq -2$

$\therefore -2 \leq x \leq 2$

Domain = [–2, 2]

56. Option (c) is correct.

$f(x)=\sqrt{2-x}+\sqrt{2+x}$

$$f'(x)=\frac{-1}{2\sqrt{2-x}}+\frac{1}{2\sqrt{2+x}}=0\Rightarrow x=0$$

$\therefore$ Greatest value is

$f(0)=\sqrt{2}+\sqrt{2}=2\sqrt{2}=\sqrt{8}$

57. Option (b) is correct.

$$h(x)=\frac{f(g)x))}{g(f(x))}=\frac{|[x]-1|}{[|x|]-1}$$

$$\lim_{x\to 0^+}h(x)=\lim_{h\to 0}h(0+h)$$

$$\lim_{h\to 0}\frac{|[0+h]-1|}{[|0+h|-1]}=\frac{|0-1|}{0-1}=-1$$

58. Option (a) is correct.

$$\lim_{x\to 0^-}h(x)=\lim_{h\to 0}h(0-h)$$

$$=\lim_{h\to 0}\frac{|[0-h]-1|}{[|0-h|-1]}=\frac{|-1-1|}{0-1}=-2$$

59. Option (d) is correct.

$$f(x)=\begin{cases}a-1 & ; \quad x<3\\ a-b & ; \quad x=3\\ 1+b & ; \quad x>3\end{cases}$$

$$\because |x-3|=\begin{cases}x-3 & x\geq 3\\ -(x-3) & x<3\end{cases}$$

$\because f(x)$ is continuous at $x = 3$

$\therefore a - 1 = a - b = 1 + b$

$a - 1 = a - b \Rightarrow b = 1$

$a - b = 1 + b \Rightarrow a = 1 + 2b = 3.$

60. Option (b) is correct.

$$f(x)=\begin{cases} a-1 & ; \quad x<3 \\ a-b & ; \quad x=3 \\ 1+b & ; \quad x>3 \end{cases}$$

$$\because |x-3| = \begin{cases} x-3 & x \geq 3 \\ -(x-3) & x<3 \end{cases}$$

$\because f(x)$ is continuous at $x = 3$

$\therefore a - 1 = a - b = 1 + b$

$a - 1 = a - b \Rightarrow b = 1$

$a - b = 1 + b \Rightarrow a = 1 + 2b = 3.$

61. Option (b) is correct.

$$\int_0^{\pi} (\sin^4 x + \cos^4 x) dx$$

$$= \int_0^{\pi} [(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x . \cos^2 x] dx$$

$$= \int_0^{\pi} \left(1 - 2\frac{\sin^2 2x}{4}\right) dx$$

$$= \int_0^{\pi} \left[1 - \frac{1}{2}\frac{(1-\cos 4x)}{2}\right] dx$$

$$= \left[x - \frac{1}{4}\left(x - \frac{\sin 4x}{4}\right)\right]_0^{\pi}$$

$$= \left[\pi - \frac{1}{4}(\pi - 0)\right] - 0$$

$$= \pi - \frac{\pi}{4} = \frac{3\pi}{4}.$$

62. Option (d) is correct.

$$I = \int_{-2\pi}^{2\pi} \frac{\sin^4 x + \cos^4 x}{1+3^x} dx \quad \text{...(i)}$$

Let $x = -t \Rightarrow dx = -dt$

when $x = -2\pi \Rightarrow t = 2\pi$

$n = 2\pi \Rightarrow t = -2\pi$

$$= -\int_{-2\pi}^{2\pi} \frac{\sin^4 t + \cos^4 t}{1+3^{-t}} dt$$

$$= -\int_{-2\pi}^{2\pi} \frac{\sin^4 t + \cos^4 t}{1+\frac{1}{3^x}} dx$$

$$= \int_{-2\pi}^{2\pi} \frac{3^x(\sin^4 x + \cos^4 x)}{3^x + 1} dx \quad \text{...(ii)}$$

Adding (i) and (ii) we get

$$I = \int_{-2\pi}^{2\pi} (\sin^4 x + \cos^4 x) dx$$

$$= 2\int_0^{2\pi} (\sin^4 x + \cos^4 x) dx \quad [\because \text{ even function}]$$

$\because f(2\pi - x) = f(x)$

$$\therefore I = 4\int_0^{\pi} (\sin^4 x + \cos^4 x) dx$$

$$= 4 \times \frac{3\pi}{4} = 3\pi$$

63. Option (a) is correct.

$\because f(x)$ is differentiable at $x = 1$

L.H.D. $= f'(1^-) = [a(x + 1) + ax]_{x=1} = 3a$

R.H.D. $= f'(1^+) = 1$

$$\therefore 3a = 1 \Rightarrow a = \frac{1}{3}$$

$f(x)$ is continuous also.

$$\therefore \text{L.H.L.} = \lim_{x \to 1^-} a(x^2 + x) + b = 2a + b$$

$$\text{R.H.L.} = \lim_{x \to 1^+} x - 1 = 0$$

$$\therefore 2a + b = 0 \Rightarrow \frac{2}{3} + b = 0$$

$$b = \frac{-2}{3}$$

$$\text{Now, } a + b = \frac{1}{3} - \frac{2}{3} = \frac{-1}{3}$$

64. Option (b) is correct.

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} a(x^2 + x) + b$$

$$= b = -\frac{2}{3}$$

65. Option (a) is correct.

$f(x) = |\ln |x|| = -\ln x \,;\, 0 < x < 1$

$$f'(x) = -\frac{1}{x}$$

$f'(0.5) = -2$

66. Option (c) is correct.

$$y = f\left(\frac{2x-3}{x}\right)$$

$$\frac{dy}{dx} = f'\left(\frac{2x-3}{x}\right) . \frac{2(x)-(2x-3)}{x^2}$$

$$= \cos\left(\ln\left(\frac{2x-3}{x}\right)\right) . \frac{2x-2x+3}{x^2}$$

$$= \frac{3}{x^2}\cos\left(\ln\left(\frac{2x-3}{x}\right)\right)$$

67. Option (d) is correct.

$$I = \int_0^{8\pi} |\sin x| dx$$

$$= 4\int_0^{2\pi} |\sin x|\, dx$$

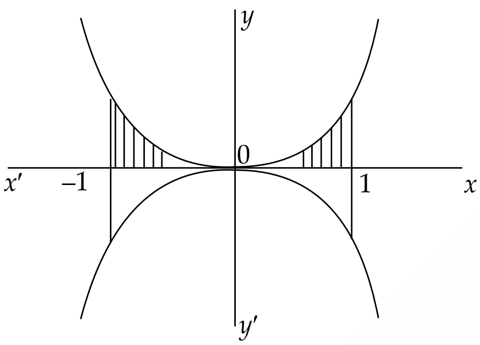
$$= 4\left[\int_0^{\pi} \sin x\, dx + \int_{\pi}^{2\pi} -\sin x\, dx\right]$$

$$= 4[-\cos x]_0^{\pi} - 4[-\cos x]_{\pi}^{2\pi}$$

$= 4\,[1 - (-1)] - 4\,[-1-1]$
$= 8 + 8 = 16$

68. Option (a) is correct.

$$f(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$



$$\text{Area} = 2\int_0^1 x^2 dx = 2\left[\frac{x^3}{3}\right]_0^1 = \frac{2}{3}$$

69. Option (d) is correct.

Order = 3
Degree = 2

70. Option (d) is correct.

$y^2 = 4a(x - b)$

$$2y\frac{dy}{dx} = 4a \Rightarrow y\frac{dy}{dx} = 29$$

$$\frac{dy}{dx}.\frac{dy}{dx} + y\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

71. Option (a) is correct.

Let $a_1 = a, a_2 = a + d, a_3 = a + 2d$
$a_4 = a + 3d$... so, on
$\because a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{25} + a_{30} + a_{34} = 300$
$a + a + 4d + a + 9d + a + 14d + a + 19d + a$
$+ 24d + a + 29d + a + 33d = 300$
$8a + 132d = 300$
$\Rightarrow 2a + 33d = 75$ ...(i)
Now, $a_1 + a_5 - a_{10} - a_{15} - a_{20} - a_{25} + a_{30} + a_{34}$
$= a + a + 4d - a - 9d - a - 14d$
$-a - 19d - a - 24d + a + 29d + a + 33d = 0$

72. Option (d) is correct.

$$\sum_{n=1}^{34} a_n = a_1 + a_2 + ... + a_{34}$$

$= 34a + d(1 + 2 + 3 + ... + 33)$

$$= 34a + d.\frac{33(33+1)}{2}$$

$= 34a + d.33.17$
$= 17(2a + 33d)$
$= 17 \times 75 = 1275$

73. Option (c) is correct.

$$p + q = \cos\frac{\pi}{5}.\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5}.\cos\frac{8\pi}{5}$$

$$= \cos\frac{\pi}{5}.\cos\frac{2\pi}{5} + \cos\left(\pi - \frac{\pi}{5}\right).\cos\left(2\pi - \frac{\pi}{5}\right)$$

$$= \cos\frac{\pi}{5}.\cos\frac{2\pi}{5} - \cos\frac{\pi}{5}.\cos\frac{2\pi}{5}$$

$= 0$

74. Option (a) is correct.

$$p.q = \cos\frac{\pi}{5}.\cos\frac{2\pi}{5}.\cos\frac{4\pi}{5}.\cos\frac{8\pi}{5}$$

$$= \cos\frac{\pi}{5}.\cos\frac{2\pi}{5}.\cos\left(\pi - \frac{\pi}{5}\right).\cos\left(2\pi - \frac{2\pi}{5}\right)$$

$$= -\cos^2\frac{\pi}{5}\cos^2\frac{2\pi}{5}$$

$$= \frac{-4\sin^2\frac{\pi}{5}}{4\sin^2\frac{\pi}{5}}\cos^2\frac{\pi}{5}\cos^2\frac{2\pi}{5}$$

$$= \frac{-1}{4.4\sin^2\frac{\pi}{5}}.4.\sin^2\frac{2\pi}{5}.\cos^2\frac{2\pi}{5}$$

$$= \frac{-1}{16\sin^2\frac{\pi}{5}}.\sin^2\frac{4\pi}{5}$$

$$= \frac{-1}{16\sin^2\frac{\pi}{5}}.\sin^2\left(\pi - \frac{\pi}{5}\right)$$

$$= \frac{-1}{16\sin^2\frac{\pi}{5}} \times \sin^2\frac{\pi}{5} = -\frac{1}{16}.$$

75. Option (d) is correct.

$$p.q. = \left(\frac{1}{3} - \frac{\tan 3x}{\tan x}\right)(1 - 3\tan^2 x)$$

$$= \left[\frac{1}{3} - \frac{3\tan x - \tan^3 x}{\tan x(1 - 3\tan^2 x)}\right][1 - 3\tan^2 x]$$

$$= \left[\frac{1}{3} - \frac{3-\tan^2 x}{1-3\tan^2 x}\right][1-3\tan^2 x]$$

$$= \frac{1-3\tan^2 x - 9 + 3\tan^2 x}{3(1-3\tan^2 x)}.(1-3\tan^2 x)$$

$$= \frac{-8}{3}.$$

76. Option (c) is correct.

$$p = \frac{1}{3} - \frac{\tan 3x}{\tan x}$$

$$= \frac{1}{3} - \frac{3\tan x - \tan 3x}{(1-3\tan^2 x)\tan x}$$

$$= \frac{1}{3} - \frac{3-\tan^2 x}{1-3\tan^2 x}$$

$$= \frac{1-3\tan^2 x - 9 + 3\tan^2 x}{3(1-3\tan^2 x)}$$

$$= \frac{-8}{3(1-3\tan^2 x)}$$

$$\frac{1}{p} = \frac{3(1-3\tan^2 x)}{-8} = 0$$

$\Rightarrow 1 - 3\tan^2 x = 0$

$$\Rightarrow \tan^2 x = \frac{1}{3} \Rightarrow \tan x = \frac{\pm 1}{\sqrt{3}}$$

$$x = \frac{\pi}{6}, \left(\pi - \frac{\pi}{6}\right)$$

Two solutions.

77. Option (b) is correct.

$$\sin x + \sin y = \sqrt{3}(\cos y - \cos x)$$

$$\Rightarrow 2\sin\frac{(x+y)}{2}.\cos\frac{(x-y)}{2}$$

$$= \sqrt{3}.2\sin\frac{(x+y)}{2}\sin\frac{(x-y)}{2}$$

$$= \frac{1}{\sqrt{3}} = \tan\frac{(x-y)}{2}$$

$$\Rightarrow \frac{x-y}{2} = \frac{\pi}{6}$$

$$\Rightarrow x - y = \frac{\pi}{3} \quad \text{...(i)}$$

$$x + y = \frac{\pi}{2}$$

$$x - y = \frac{\pi}{3}$$

$$2x = \frac{5\pi}{6}$$

$$x = \frac{5\pi}{12}$$

$$y = \frac{\pi}{12}$$

$$\sin 3x + \sin 3y = \sin\frac{5\pi}{4} + \sin\frac{\pi}{4}$$

$$= \sin\left(\pi + \frac{\pi}{4}\right) + \sin\frac{\pi}{4} = -\sin\frac{\pi}{4} + \sin\frac{\pi}{4}$$

$= 0$

78. Option (b) is correct.

$$\cos x = \cos\frac{5\pi}{12} = \cos 75° = \cos(45° + 30°)$$

$= \cos 45°.\cos 30° - \sin 45°.\sin 30°$

$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\cos y = \cos\frac{\pi}{12} = \cos 15° = \cos(45°-30°)$$

$= \cos 45°.\cos 30° + \sin 45°.\sin 30°$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Now, $\cos^3 x + \cos^3 y$

$$= \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^3 + \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^3$$

$$= \frac{3\sqrt{3} - 9 + 3\sqrt{3} - 1 + 3\sqrt{3} + 9 + 3\sqrt{3} + 1}{16\sqrt{2}}$$

$$= \frac{12\sqrt{3}}{16\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{6}}{32}$$

$$= \frac{3\sqrt{6}}{8}$$

79. Option (c) is correct.

Let angles of triangle be $3x$, $5x$ and $4x$

$3x + 5x + 4x = 180°$

$$\Rightarrow 12x = 180° \Rightarrow x = \frac{180}{12} = 15°.$$

So, angles are 45°, 75° and 60°

from sine rule

$$\frac{a}{\sin 45°} = \frac{b}{\sin 75°} = \frac{c}{\sin 60°} = k$$

$a = k\sin 45°$

$b = k\sin 75°$

$c = k \sin 60°$

Now, $a+b+\sqrt{2}c$

$= k [\sin 45° + \sin 75° + \sqrt{2} \sin 60°]$

$= k [2 \sin 60°. \cos 15° + \sqrt{2} \sin 60°]$

$= k \sin 60° [2 \cos 15° + \sqrt{2}]$

$= k \sin 60°[2\cos 15° + \sqrt{2}]$

$= k \sin 60°[2\cos(45°-30°) + \sqrt{2}]$

$= k \sin 60°\left[2\left(.\frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2}\right) + \sqrt{2}\right]$

$= k \sin 60°\left[\frac{\sqrt{3}+1}{\sqrt{2}} + \sqrt{2}\right]$

$= k.\frac{\sqrt{3}}{2}\left[\frac{\sqrt{3}+3}{\sqrt{2}}\right] = 3k\frac{(\sqrt{3}+1)}{2\sqrt{2}}$

$= 3k \sin 75° = 3b$

80. Option (a) is correct.

$a^2 : b^2 : c^2 = \sin^2 45° : \sin^2 75° : \sin^2 60°$

$= \left(\frac{1}{\sqrt{2}}\right)^2 : \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^2 : \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{2} : \frac{3+1+2\sqrt{3}}{8} : \frac{3}{4}$

$= \frac{1}{2} : \frac{2+\sqrt{3}}{4} : \frac{3}{4}$

$= 2 : 2+\sqrt{3} : 3$

81. Option (a) is correct.

$b^2 + b - 2 = 0 \Rightarrow b^2 + 2b - b - 2 = 0$

$\Rightarrow b(b+2) - 1(b+2) = 0 \Rightarrow (b-1)(b+2) = 0$

$b = 1, -2$, but $b < 0$

$\therefore b = 1$

Now equation of parabola

$y^2 = 4bx = 4x$

Equation of direction is

$x = -1 \Rightarrow x + 1 = 0$

82. Option (d) is correct.

$$\begin{vmatrix} 0 & 0 & 1 \\ a & b & 1 \\ -a & -b & 1 \end{vmatrix} = 1(-ab+ab) = 0$$

$$\begin{vmatrix} 0 & 0 & 1 \\ a & b & 1 \\ a^2 & ab & 1 \end{vmatrix} = 1(a^2b - a^2b) = 0$$

$\therefore$ points are collinear.

83. Option (b) is correct.

$16p^2 + 49q^2 - 4r^2 - 56pq = 0$

$\Rightarrow (4p - 7q)^2 = 4r^2$

$\Rightarrow 4p - 7q = \pm 2r$

$\Rightarrow 2p - \frac{7}{2}q = \pm r$

$\Rightarrow \left(2p - \frac{7}{2}q + r\right)\left(2p - \frac{7}{2}q - r\right) = 0$

$\therefore x = 2, y = \frac{-7}{2}$

Hence given pair of straight line passes through a point $\left(2, \frac{-7}{2}\right)$

84. Option (b) is correct.

$3x + y - 5 = 0 \Rightarrow y = 5 - 3x$

$x^2 + y^2 - 25 = 0$

$\Rightarrow x^2 + (5-3x)^2 - 25 = 0$

$\Rightarrow x^2 + 25 + 9x^2 - 30x - 25 = 0$

$\Rightarrow 10x^2 - 30x = 0$

$\Rightarrow x = 0, 3 \Rightarrow y = 5, -4$

Point of intersections are (0, 5), (3, –4)

Mid-point $= \left(\frac{3+0}{2}; \frac{5-4}{2}\right) = \left(\frac{3}{2}, \frac{1}{2}\right)$

85. Option (c) is correct.

$\frac{x^2}{24-k} + \frac{y^2}{k-16} = 2$ ...(i)

Put $k = 19$ in equation (i)

$\frac{x^2}{5} + \frac{y^2}{3} = 2$

Represent the equation of ellipse

Put $k = 12$ in equation (i)

$\frac{x^2}{24-12} + \frac{y^2}{12-16} = 2$

$\Rightarrow \frac{x^2}{12} - \frac{y^2}{4} = 2$

Represent equation of hyperbola

Putting $k = 20$ in equation (i)

$\frac{x^2}{24-20} + \frac{y^2}{20-16} = 2$

$\Rightarrow \frac{x^2}{4} + \frac{y^2}{4} = 2$

$\Rightarrow x^2 + y^2 = 8$

Represent equation of circle.

86. Option (c) is correct.

Equation of hyperbola

$\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$

$a^2 = \cos^2\theta$ and $b^2 = \sin^2\theta$

$c^2 = a^2 + b^2 = \sin^2\theta + \cos^2\theta = 1$

$c = \pm 1$

foci $= (\pm c, 0) = (\pm 1, 0)$

Independent of θ

$c^2 = 1 \Rightarrow a^2e^2 = 1$

$$e^2 = \frac{1}{a^2} = \frac{1}{\cos^2\theta} = \sec^2\theta$$

$e = \sec\theta$

Distance between two foci = 1 + 1 = 2

87. Option (b) is correct.

$4x^2 + 4y^2 - 4ax - 4ay + a^2 = 0$

$$\Rightarrow x^2 + y^2 - ax - ay + \frac{a^2}{4} = 0$$

$$\text{Centre} = \left(\frac{a}{2}, \frac{a}{2}\right)$$

$$\text{Radium} = \sqrt{\frac{a^2}{4} + \frac{a^2}{4} - \frac{a^2}{4}} = \frac{a}{2}$$

$\therefore$ The circle touches both the axes.

$\therefore$ Statement (1) is correct

$$\text{Diameter of the circle} = 2d = 2 \times \frac{a}{2} = 9$$

Statement (2) is wrong.

$$x + y = a \Rightarrow \frac{a}{2} + \frac{a}{2} = a$$

$\Rightarrow a = a$

Statement (3) is correct.

88. Option (b) is correct.

Given that both equations are parallel

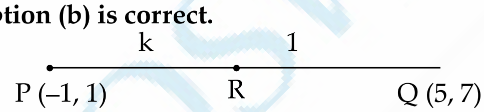
$$\frac{k-3}{1} = \frac{-(5-k^2)}{1}$$

$\Rightarrow k - 3 = -5 + k^2$

$\Rightarrow k^2 - k - 2 = 0 \Rightarrow (k + 1)(k - 2) = 0$

$k = -1, 2$

89. Option (b) is correct.



$$R(x, y) = \left(\frac{5k-1}{k+1}, \frac{7k+1}{k+1}\right)$$

$\because$ R lies on $x + y = 4$

$$\frac{5k-1}{k+1} + \frac{7k+1}{k+1} = 4$$

$\Rightarrow 12k = 4k + 4$

$$\Rightarrow 8k = 4 \Rightarrow k = \frac{1}{2} = 1:2$$

90. Option (c) is correct.

Equation of st. line

$x \cos 15° + y \sin 15° = 4$

$$x\frac{(\sqrt{3}+1)}{2\sqrt{2}} + y\frac{(\sqrt{3}-1)}{2\sqrt{2}} = 4$$

$$x\text{-intercept} = \frac{8\sqrt{2}}{\sqrt{3}+1}$$

$$\text{y-intercept} = \frac{8\sqrt{2}}{\sqrt{3}-1}$$

$$\text{Sum of intercept} = \frac{8\sqrt{2}}{\sqrt{3}+1} + \frac{8\sqrt{2}}{\sqrt{3}-1}$$

$$= \frac{8\sqrt{2}(\sqrt{3}-1+\sqrt{3}+1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{8\sqrt{2}(2\sqrt{3})}{2} = 8\sqrt{6}$$

91. Option (b) is correct.

$$\text{Projection} = \frac{(\hat{i}+2\hat{j}+3\hat{k}).(2\hat{i}+3\hat{j}-2\hat{k})}{|2\hat{i}+3\hat{j}-2\hat{k}|}$$

$$= \left|\frac{2+6-6}{\sqrt{4+9+4}}\right| = \frac{2}{\sqrt{17}}$$

92. Option (a) is correct.

$$(\vec{a} \times \vec{b})^2 + (\vec{a}.\vec{b})^2 = 144$$

$$\Rightarrow (|\vec{a}||\vec{b}|^2 [\sin^2\theta + \cos^2\theta] = 144$$

$$\Rightarrow |\vec{a}|^2|\vec{b}|^2 [\sin^2\theta + \cos^2\theta] = 144$$

$$\Rightarrow |\vec{a}|^2 .16 = 144 \Rightarrow |\vec{a}|^2 = 9$$

$$\Rightarrow |\vec{a}| = 3$$

93. Option (c) is correct.

$$\because \vec{a}.\vec{b} \geq 0 \Rightarrow |\vec{a}||\vec{b}|\cos\theta \geq 0$$

$$\Rightarrow \cos\theta \geq 0 \Rightarrow 0 \leq \theta \leq \frac{\pi}{2}$$

94. Option (c) is correct.

Let $\overline{OA} = 60\hat{i} + 3\hat{j}, \overline{OB} = 40\hat{i} - 8\hat{j}$ and

$\overline{OC} = \beta\hat{i} - 52\hat{j}$

$$\overline{AB} = (40\hat{i} - 8\hat{j}) - (60\hat{i} + 3\hat{j})$$

$$= 20\hat{i} - 11\hat{j}$$

$$\overline{BC} = (\beta\hat{i} - 52\hat{j}) - (40\hat{i} - 8\hat{j})$$

$$= (\beta - 40)\hat{i} - 44\hat{j}$$

$\because$ Given vectors are collinear

$\therefore \frac{-20}{\beta-40} = \frac{-11}{-44}$

$\Rightarrow -80 = \beta - 40 \Rightarrow \beta = -40$

95. Option (b) is correct.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i}+\hat{j}-\hat{k}$$

Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

$$= \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{\hat{i}+\hat{j}-\hat{k}}{\pm\sqrt{3}}$$

Two vectors

So, statement 1 is wrong

$$\text{Now, } \cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{0+0+1}{\sqrt{2}\,\sqrt{2}} = \frac{1}{2}$$

$$\theta = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$$

So, statement 2 is correct.

96. Option (d) is correct.

Equation of st. line is

$$\frac{x-1}{3} = \frac{y+1}{-2} = \frac{z-1}{6} = k \text{ (say)}$$

$x = 3k + 1, y = -2k - 1, z = 6k + 1$

$\therefore$ Required point is $(3k + 1, -2k - 1, 6k + 1)$

$$\sqrt{(3k+1-1)^2+(-2k-1+1)^2+(6k+1-1)^2} = 2$$

$$\Rightarrow \sqrt{9k^2+4k^2+36k^2} = 2$$

$$\Rightarrow \sqrt{49k^2} = 2 \Rightarrow 7k = \pm 2$$

$$\Rightarrow k = \pm\frac{2}{7}$$

$\therefore$ Required points are

$$\left(\frac{13}{7}, \frac{-11}{7}, \frac{19}{7}\right) \text{ and } \left(\frac{1}{7}, \frac{-3}{7}, \frac{-5}{7}\right)$$

97. Option (d) is correct.

D.r. of line are <3, 4, 5>

D.r. of plane $x - 2y + z - 1$ are <1, –2, 1>

$3.1 - 4.2 + 5.1 = 0$

So, given line is parallel to plane

$x - 2y + z - 1 = 0$

98. Option (d) is correct.

$$2x = 3y = -z \Rightarrow \frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$$

$$\text{and } 6x = -y = -4z \Rightarrow \frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$$

$$\therefore \cos\theta = \frac{6-24+18}{\sqrt{9+4+36}\sqrt{4+144+9}} = 0$$

$\therefore \theta = 90°$

99. Option (d) is correct.

$x^2 - 2x + y^2 - 6y + z^2 - 8z = 5$

$\Rightarrow (x-1)^2 + (y-3)^2 + (z-4)^2 = 31$

Equation of sphere concentric with given sphere is

$(x-1)^2 + (y-3)^2 + (z-4)^2 = k$

$\because$ It passes through (0, 0, 0)

$\therefore 1 + 9 + 16 = k \Rightarrow k = 26$

$\therefore$ Equation of circle is

$x^2 + y^2 + z^2 - 2x - 6y - 8z = 0$

Shortcut: Equation of sphere concentric with given sphere and passes through origin is same as given sphere but constant term is zero

So equation of sphere is

$x^2 + y^2 + z^2 - 2x - 6y - 8z = 0$

100. Option (a) is correct.

Equation of st. line is

$$\frac{x-1}{2-1} = \frac{y-2}{10-2} = \frac{z-3}{1-3}$$

$$\Rightarrow \frac{x-1}{1} = \frac{y-2}{8} = \frac{z-3}{-2} = k$$

$\therefore p(k + 1, 8k + 2, -2k + 3)$

Given that z-coordinate is 7

$\therefore -2k + 3 = 7 \Rightarrow k = -2$

$\therefore$ Sum of other two coordinate

$= k + 1 + 8k + 2 = 9k + 3 = -18 + 3 = -15$

101. Option (a) is correct.

Given that $\Sigma (x_i - 10) = p$

$\Sigma x_i - 10\,\Sigma 1 = p$

$\Rightarrow \Sigma x_i - 10x = p \Rightarrow \Sigma x_i = 10x + p$ ...(i)

Similarly, $\Sigma x_i = 20x + q$ ...(ii)

from (i) and (ii)

$10x + p = 20x + q \Rightarrow p - q = 10x$

$\because (p - q)^2 = 10000$

$\Rightarrow (10x)^2 = 10000$

$\Rightarrow 100\,x^2 = 10000$

$\Rightarrow x^2 = 100 \Rightarrow x = 10$

102. Option (c) is correct.

$$\because \bar{x} = 20 \Rightarrow \frac{1}{10}\sum_{i=1}^{10} x_i = 20$$

$$\Rightarrow \sum_{i=1}^{10} x_i = 200$$

$$\sum_{i=1}^{10}\frac{3x_i - 4}{5} = \frac{3}{5}\sum_{i=1}^{10} x_i - \frac{4}{5}\sum_{i=1}^{10} 1$$

$= \frac{3}{5} \times 200 - \frac{4}{5} \times 10$

$= 120 - 8 = 112$

103. Option (d) is correct.

Given that, $n = 10, \bar{x} = 40, \sum_{j=1}^{10} x_i^2 = 16160$

$$\text{S.D.} = \sqrt{\frac{1}{n}\sum_{j=1}^{n} x_i^2 - (\bar{x})^2}$$

$$= \sqrt{\frac{1}{10} \times 16160 - (40)^2}$$

$= \sqrt{1616 - 1600} = \sqrt{16} = 4$

104. Option (a) is correct.

$n(S) = 6 \times 6 \times 6 = 216$

Sum of numbers lies between 3 to 18

perfect square = 4, 9, 16

Sum is 4 = (1, 1, 2) $\rightarrow$ 3 ways

Sum is 9 = (1, 2, 6) $\rightarrow$ 6 ways

= (2, 2, 5) $\rightarrow$ 3 ways

= (3, 1, 5) $\rightarrow$ 6 ways

= (1, 4, 4) $\rightarrow$ 3 ways

= (3, 2, 4) $\rightarrow$ 6 ways

= (5, 3, 3) $\rightarrow$ 1 way

= (4, 6, 6) $\rightarrow$ 3 ways

= (5, 5, 6) $\rightarrow$ 1 way

Total favourable events = 34

$\therefore p = \frac{34}{216} = \frac{17}{108}$

105. Option (d) is correct.

$\because$ A, B, C and D are mutually exclusive and exhaustive

and 2P (A) = 3P(B) = 4P(C) = 5P(D)

$\therefore$ L.C.M. of 2, 3, 4, 5 = 60

$$\frac{P(A)}{30} = \frac{P(B)}{20} = \frac{P(C)}{15} = \frac{P(D)}{12} = k$$

P(A) = $30k$, P(B) = $20k$, P(C) = $15k$ and P(D) = $12k$

$\therefore$ P(A) + P(B) + P(C) + P(D) = 1

$30k + 20k + 15k + 12k = 1$

$77k = 1;$

$= k = \frac{1}{77}$

Now, 77 P(A) = $77 \times 30.k = 77 \times 30 \times \frac{1}{77} = 30$

106. Option (c) is correct.

$n(S) = {}^9C_2 = \frac{9 \cdot 8}{2 \cdot 1} = 36$

Product has 1 in its unit place (3, 7), (7, 3)

$\therefore P = \frac{n(E)}{n(S)} = \frac{2}{36} = \frac{1}{18}$

107. Option (b) is correct.

$n(S) = 6 \times 6 = 36$

Difference of numbers is 2 or 3.

= (1, 3), (1, 4), (2, 4), (2, 5), (3, 5), (3, 6), (4, 6), (6, 4), (6, 3), (5, 3), (5, 2), (2, 4), (4, 1), (6, 1)

$n(E) = 14$

$P = \frac{n(E)}{n(S)} = \frac{14}{36} = \frac{7}{18}$

108. Option (c) is correct.

$\because (1+x)^9 = {}^9C_0 + {}^9C_1 x + {}^9C_2 x^2 + ... + {}^9C_9 x^9$

$\Rightarrow x(1+x)^9 = {}^9C_0 x + {}^9C_1 x + {}^9C_2 x^3 + ... + {}^9C_9 x^{10}$

Differentiate w.r.t to x

$(1+x)^9 + 9x(1+x)^8 = {}^9C_0 + 2\,{}^9C_1 x + 3\,{}^9C_2 x^2$

$+ .... 10\,{}^9C_9\, x^9$

Put $x = 1$, we get

$2^9 + 9.2^8 = {}^9C_0 + 2\,{}^9C_1 + 3\,{}^9C_2 + 10\,{}^9C_9$

$= 11.2^8$

We know that

${}^9C_0 + {}^9C_1 + {}^9C_2 + ... + {}^9C_9 = 2^9$

$$\therefore \text{Mean} = \frac{1.\,{}^9C_0 + 2.\,{}^9C_1 + 3.\,{}^9C_2 + ... + 10\,{}^9C_9}{{}^9C_0 + {}^9C_2 + .. + {}^9C_9}$$

$= \frac{11.2^8}{2^9} = \frac{11}{2} = 5.5$

109. Option (d) is correct.

$p = 0.8 \Rightarrow q = 1 - 0.8 = 0.2$ and $n = 5$

$P(x = 2) = {}^5C_2 (0.8)^2 (0.2)^3$

$= \frac{5.4}{2} \times 0.64 \times 0.008 = 0.0512$

110. Option (b) is correct.

F = building has collapsed

E_1 = design is faulty

E_2 = design is not faulty

$$P\left(\frac{E_1}{F}\right) = \frac{P\left(\frac{F}{E_1}\right).P(E_1)}{P\left(\frac{F}{E_1}\right).P(E_1) + P\left(\frac{F}{E_2}\right).P(E_2)}$$

$$= \frac{\frac{95}{100} \times \frac{10}{100}}{\frac{95}{100} \times \frac{10}{100} + \frac{45}{100} \times \frac{90}{100}}$$

$$= \frac{950}{950+4050} = \frac{950}{5000}$$

$= 0.19$

111. Option (a) is correct.

$$\because r = \frac{\text{cov}.(x,y)}{\sqrt{\text{var}(x).\text{var}(y)}}$$

and cov $(x, y) = \Sigma (x - x')(y - y')$

cov $(3x + 4, -3y + 3)$

$= \Sigma(3x - 3x')(-3y + 3y')$

$= \Sigma - 9(x - x')(y - y'') = -9$ cov (x, y)

var $(3x + y) = (3)^2$ var (x)

var $(-3y + 3) = (-3)^2$ var (y)

$$\therefore r(3x + 4, -3y + 3) = \frac{-9\,\text{cov}(x,y)}{\sqrt{9\,\text{var}(x)}\sqrt{9\,\text{var}(y)}}$$

$$= \frac{-9\,\text{cov}(x,y)}{9\sqrt{\text{var}(x)}\sqrt{\text{var}(y)}}$$

$= -r$

112. Option (c) is correct.

Required probability

= P (TTTTTH) + P (HHHHHT)

$$= \frac{1}{64} + \frac{1}{64} = \frac{2}{64} = \frac{1}{32}$$

113. Option (c) is correct.

$$H = \frac{3}{\frac{1}{^{10}C_4} + \frac{1}{^{10}C_5} + \frac{1}{^{10}C_6}}$$

$$= \frac{3}{\frac{4.3.2.1}{10.9.8.7} + \frac{5.4.3.2.1}{10.5.8.7.6} + \frac{4.3.2.1}{10.9.8.7}}$$

$$= \frac{3}{\frac{1}{210} + \frac{1}{252} + \frac{1}{210}}$$

$$= \frac{3}{\frac{6+5+6}{1260}} = \frac{3\times1260}{17}$$

$$\therefore \frac{270}{H} = 270\times\frac{17}{3\times1260} = \frac{17}{14}$$

114. Option (b) is correct.

Total arrangement $= n!$

Number of arrangement when P and Q or together $= (n - 1)!\,2!$

$$P = \frac{(n-1)!2!}{n!} = \frac{2}{n}$$

115. Option (a) is correct.

$\because 9P(x = 4) = P(x = 2)$

$9.\,^6C_4\,p^4q^2 = {}^6C_2\,p^2q^4$

$9p^2 = q^2$ $\quad (\because {}^6C_4 = {}^6C_2)$

$3p = q$

$$\because p + q = 1 \Rightarrow p + 3p = 1 \Rightarrow p = \frac{1}{4}$$

116. Option (a) is correct.

Total arrangement of P, Q, R, S, T and U = 6!

Number of arrangement when P, Q and R together = 4! 3!

$$\therefore \text{Required probability} = \frac{4!3!}{6!}$$

$$= \frac{1}{5}$$

117. Option (b) is correct.

Total arrangement = 6!

Number of ways when boys and girls sit alternatively

= 2!.3!.3! = 2.3!.3!

$$p = \frac{2.3!3!}{6!} = \frac{1}{10}$$

118. Option (d) is correct.

Total arrangement = 6!

No. of ways of arrangement that no two girls sit together.

P.Q.R.

$^4C_3$ 3! 3!

$$p = \frac{^4C_3 3!3!}{6!} = \frac{1}{5}$$

119. Option (a) is correct.

Total arrangement = 6!

No. of ways of arrangement that P and Q take the two end positions = 4! 2!

$$\text{Required probability} = \frac{4!2!}{6!} = \frac{1}{15}$$

120. Option (d) is correct.

Total arrangement = 6!

No. of ways of arrangement that Q and U sit together = 5! 2!

$$\text{Required probability} = \frac{5!2!}{6!} = \frac{1}{3}.$$