NDA/NA
National Defence Academy/ Naval Academy

MATHEMATICS

QUESTION PAPER 2024 (II)

Time Allowed: 2 hrs 30 min **Total Marks: 300**

Instructions
1. *This Test Booklet contains **120** items (questions). Each item is printed in **English**. Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.*
2. *You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.*
3. ***All** items carry equal marks.*
4. *Before you proceed to mark in the Answer Sheet the response to the various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions.*
5. ***Penalty for wrong answers:***
THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i) *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.*
   (ii) *If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii) *If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.*

1. Let X be a matrix of order 3×3, Y be a matrix of order 2×3 and Z be a matrix of order 3×2. Which of the following statements are correct?
(I) $(ZY)X$ is defined and is a square matrix of order 3.
(II) $Y(XZ)$ is defined and is a square matrix of order 2.
(III) $X(YZ)$ is not defined.
Select the answer using the code given below.
(a) I and II only **(b)** II and III only
(c) I and III only **(d)** I, II and III

2. Consider the following statements:
(I) The set of all irrational numbers between $\sqrt{12}$ and $\sqrt{15}$ is an infinite set.
(II) The set of all odd integers less than 1,000 is a finite set.
Which of the statements given above is/are correct?
(a) I only **(b)** II only
(c) Both I and II **(d)** Neither I nor II

3. How many 4-digit numbers are there having all digits as odd?
(a) 625 **(b)** 400 **(c)** 196 **(d)** 120

4. If $\omega \neq 1$ is a cube root of unity, then what is $(1+\omega-\omega^2)^{100}+(1-\omega+\omega^2)^{100}$ equal to?
(a) $2^{100}\omega^2$ **(b)** $2^{100}\omega$ **(c)** 2^{100} **(d)** -2^{100}

5. Let A and B be two square matrices of same order. If AB is a null matrix, then which one of the following is correct?
(a) Both A and B are null matrices.
(b) Either A or B is a null matrix.
(c) B is a null matrix if A is a non-singular matirx.
(d) Both A and B are singular matrices.

6. In the expansion of $(1+x)^p(1+x)^q$, if the coefficient of x^3 is 35, then what is the value of $(p+q)$?
(a) 5 **(b)** 6 **(c)** 7 **(d)** 8

7. If p times the p^{th} term of an AP is equal to q times the q^{th} term $(p \neq q)$, then what is the $(p+q)^{th}$ term equal to?
(a) 0 **(b)** $p+q$ **(c)** pq **(d)** $pq(p+q)$

8. Let $p = ln(x)$, $q = ln(x^3)$ and $r = ln(x^5)$, where $x > 1$. Which of the following statements is/are correct?
(I) p, q and r are in A.P.
(II) p, q and r can never be in GP.
Select the answer using the code given below.
(a) I only **(b)** II only
(c) Both I and II **(d)** Neither I nor II

9. If
$$Z = \frac{1}{3}\begin{bmatrix} i & 2i & 1 \\ 2i & 3i & 2 \\ 3 & 1 & 3 \end{bmatrix} = x + iy;\ i = \sqrt{-1}.$$
then what is modulus of Z equal to?
(a) 1 **(b)** $\sqrt{2}$ **(c)** 2 **(d)** $\sqrt{3}$

10. What is the value of the sum

$$\sum_{n=1}^{20}(i^{n-1}+i^{n}+i^{n+1}),$$

where $i = \sqrt{-1}$?

(a) $-2i$ **(b)** 0 **(c)** 1 **(d)** $2i$

11. Let $x > 1, y > 1$ and $z > 1$ be in GP. Then $\frac{1}{1+\ln x}$, $\frac{1}{1+\ln y}$ and $\frac{1}{1+\ln z}$ are

(a) in AP.
(b) in GP.
(c) in HP.
(d) neither in AP nor in GP nor in HP.

12. If $\omega = -\frac{1}{2}+i\frac{\sqrt{3}}{2}$ then what is

$$\begin{bmatrix} 1+\omega & 1+\omega^2 & \omega+\omega^2 \\ 1 & \omega & \omega^2 \\ \frac{1}{\omega} & \frac{1}{\omega^2} & 1 \end{bmatrix}$$

equal to?

(a) 0 **(b)** ω
(c) ω^2 **(d)** $1-\omega^2$

13. If the sum of the first n terms of a series is $n(2n + 1)$, then what is the n^{th} term?

(a) $4n - 1$ **(b)** $4n$ **(c)** $4n + 1$ **(d)** $4n + 3$

14. In how many ways can the letters of the word INDIA be permutated such that in each combination, vowels should occupy odd positions?

(a) 3 **(b)** 6 **(c)** 9 **(d)** 12

15. The letters of the word EQUATION are arranged in such a way that all vowels as well as consonants are together. How many such arrangements are there?

(a) 240 **(b)** 720 **(c)** 1,440 **(d)** 1,620

16. If n is a root of the equation $x^2 + px + m = 0$ and m is a root of the equation $x^2 + px + n = 0$, where $m \neq n$, then what is the value of $p + m + n$?

(a) -1 **(b)** 0 **(c)** 1 **(d)** 2

17. In how many ways can a student choose $(n - 2)$ courses out of n courses that are compulsory $(n > 4)$?

(a) $(n-3)(n-4)$ **(b)** $(n-1)(n-2)$
(c) $\frac{(n-3)(n-4)}{2}$ **(d)** $\frac{(n-2)(n-3)}{2}$

18. If $D_n = \begin{bmatrix} n & 20 & 30 \\ n^2 & 40 & 50 \\ n^3 & 60 & 70 \end{bmatrix}$,

then what is the value of $\sum_{n=1}^{4} D_n$?

(a) $-10{,}000$ **(b)** -10
(c) 10 **(d)** 10,000

19. Consider the following in respect of the matrices

$$P = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \text{ and } Q = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

(I) PQ is null matrix.
(II) QP is an identity matrix of order 3.
(III) PQ = QP.

Which of the above is/are correct?

(a) I only **(b)** II only
(c) I and III **(d)** II and III

20. If P is a skew-symmetric matrix of order 3, then what is det(P) equal to?

(a) -1 **(b)** 0 **(c)** 1 **(d)** 3

21. If $4\sin^{-1} x + \cos^{-1} x = \pi$, then what is $\sin^{-1} x + 4\cos^{-1} x$ equal to?

(a) $\frac{\pi}{2}$ **(b)** π **(c)** $\frac{3\pi}{2}$ **(d)** 2π

22. What is $\cot^2(\sec^{-1} 2) + \tan^2(\operatorname{cosec}^{-1} 3)$ equal to?

(a) $\frac{11}{12}$ **(b)** $\frac{11}{24}$ **(c)** $\frac{7}{24}$ **(d)** $\frac{1}{24}$

23. In a triangle ABC,

$$\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$$

What is the area of the triangle if $a = 6$ cm?

(a) $9\sqrt{3}$ square cm **(b)** 12 square cm
(c) $18\sqrt{3}$ square cm **(d)** 24 square cm

24. The roots of the equation $7x^2 - 6x + 1 = 0$ are $\tan\alpha$ and $\tan\beta$, where 2α and 2β are the angles of a triangle. Which one of the following is correct?

(a) The triangle is equilateral.
(b) The triangle is isosceles but not right-angled.
(c) The triangle is right-angled.
(d) The triangle is right-angled isosceles.

25. In a triangle ABC, $\angle A = 75°$ and $\angle B = 45°$. What is $2a - b$ equal to?
(a) c (b) $\sqrt{2}c$ (c) $2c$ (d) $2\sqrt{2}c$

26. What is the number of solutions of the equation $\cot 2x \cdot \cot 3x = 1$ for $0 < x < \pi$?
(a) Only one (b) Only two
(c) Only five (d) More than five

27. What is the general solution of $\cos^{100} x - \sin^{100} x = 1$?
(a) $n\pi$ (b) $(2n+1)\pi$
(c) $2n\pi$ (d) $\frac{(2n+1)\pi}{2}$
where n is an integer.

28. In a triangle ABC.
$$\tan A + \tan B + \tan C = k.$$
What is the value of cotA cotB cotC?
(a) $0.5\,k$ (b) $\frac{1}{k}$ (c) $\frac{3}{k}$ (d) $\frac{1}{k^3}$

29. What is $\sin 12° \sin 48°$ equal to?
(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{\sqrt{5}+1}{4}$
(c) $\frac{\sqrt{5}-1}{8}$ (d) $\frac{\sqrt{5}+1}{8}$

30. What is $\frac{\cos 17° - \sin 17°}{\cos 17° + \sin 17°}$ equal to?
(a) tan34° (b) cot34°
(c) tan62° (d) cot62

31. Consider the following numbers:
(I) tan22.5°
(II) cot22.5°
(III) tan22.5° – cot22.5°
How many of the above are irrational numbers?
(a) None (b) Only one
(c) Only two (d) All three

32. If
$$\frac{x}{\cos\theta} = \frac{y}{\cos\left(\frac{2\pi}{3} - \theta\right)} = \frac{z}{\cos\left(\frac{2\pi}{3} + \theta\right)},$$
then what is $x + y + z$ equal to?
(a) –1 (b) 0 (c) 1 (d) 3

33. If $p \tan(\theta - 30°) = q \tan(\theta + 120°)$, then what is $(p+q)/(p-q)$ equal to?
(a) $\sin 2\theta$ (b) $\cos 2\theta$
(c) $2\sin 2\theta$ (d) $2\cos 2\theta$

34. Let P and Q be two non-void relations on a set A. Which of the following statements are correct?
I. P and Q are reflexive $\Rightarrow P \cap Q$ is reflexive.
II. P and Q are symmetric $\Rightarrow P \cup Q$ is symmetric.
III. P and Q are transitive $\Rightarrow P \cap Q$ is transitive.
Select the answer using the code given below.
(a) I and II only (b) II and III only
(c) I and III only (d) I, II and III

35. If A and B are two non-empty sets having 10 elements in common, then how many elements do $A \times B$ and $B \times A$ have in common?
(a) 10 (b) 20 (c) 40 (d) 100

36. What is the remainder when $7^n - 6n$ is divided by 36 for $n = 100$?
(a) 0 (b) 1 (c) 2 (d) 6

37. What is the maximum number of possible points of intersection of four straight lines and a circle (intersection is between lines as well as circle and lines)?
(a) 6 (b) 10 (c) 14 (d) 16

38. In an AP, the ratio of the sum of the first p terms to the sum of the first q terms is $p^2 : q^2$. Which one of the following is correct?
(a) The first term is equal to the common difference.
(b) The first term is equal to twice the common difference.
(c) The common difference is equal to twice the first term.
(d) The first term is equal to square of the common difference.

39. What is the number of real roots of the equation $(x-1)^2 + (x-3)^2 + (x-5)^2 = 0$?
(a) None (b) Only one
(c) Only two (d) Three

40. In a class of 240 students, 180 passed in English, 130 passed in Hindi and 150 passed in Sanskrit. Further, 60 passed in only one subject, 110 passed in only two subjects and 10 passed in none of the subjects. How many passed in all three subjects?
(a) 60 (b) 55 (c) 40 (d) 35

Direction: *Consider the following for the next **two** items that follow:*

Let Z_1 and Z_2 be any two complex numbers such that $Z_1^2 + Z_2^2 + Z_1Z_2 = 0$.

41. What is the value of $\left|\frac{Z_1}{Z_2}\right|$?
(a) 1 (b) 2 (c) 3 (d) 4

42. What is the value of

$$\frac{1}{2}+\text{Re}\left(\frac{Z_1}{Z_2}\right)?$$

(a) –1 **(b)** 0 **(c)** 1 **(d)** 2

Direction: *Consider the following for the next **two** items that follow:*

The product of five consecutive terms of an AP is 2,29,635. The first, second and fifth terms are in GP.

43. What is the common difference?

(a) 3 **(b)** 4 **(c)** 5 **(d)** 6

44. What is the sum of all five terms?

(a) 60 **(b)** 65 **(c)** 75 **(d)** 80

Direction: *Consider the following for the two (02) items that follow:*

Let $(8+3\sqrt{7})^{20}$ = U + V and $(8-3\sqrt{7})^{20}$ = W, where U is an integer and 0 < V < 1.

45. What is V + W equal to?

(a) 8 **(b)** 4 **(c)** 2 **(d)** 1

46. What is the value of (U + V)W?

(a) $\frac{1}{2}$ **(b)** 1 **(c)** $\frac{3}{2}$ **(d)** 2

Direction: *Consider the following for the next **two** items that follow:*

The roots of the quadratic equation $a^2(b^2-c^2)x^2+b^2(c^2-a^2)x+c^2(a^2-b^2)=0$ are equal $(a^2 \neq b^2 \neq c^2)$.

47. Which one of the following statements is correct?

(a) a^2, b^2, c^2 are in AP.

(b) a^2, b^2, c^2 are in GP.

(c) a^2, b^2, c^2 are in HP.

(d) a^2, b^2, c^2 are neither in AP nor in GP nor in HP.

48. Which one of the following is a root of the equation?

(a) $\frac{b^2(c^2-a^2)}{a^2(c^2-b^2)}$ **(b)** $\frac{b^2(c^2-a^2)}{a^2(b^2-c^2)}$

(c) $\frac{b^2(c^2-a^2)}{2a^2(c^2-b^2)}$ **(d)** $\frac{b^2(c^2-a^2)}{2a^2(b^2-c^2)}$

Direction: *Consider the following for the **two** items that follow:*

Let $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$

49. What is A(adj A) equal to?

(a) $\begin{bmatrix}5 & 0 & 0\\ 0 & 5 & 0\\ 0 & 0 & 5\end{bmatrix}$ **(b)** $\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}$

(c) $\begin{bmatrix}\frac{1}{2} & 0 & 0\\ 0 & \frac{1}{2} & 0\\ 0 & 0 & \frac{1}{2}\end{bmatrix}$ **(d)** $\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

50. What is A^{-1} equal to?

(a) $\begin{bmatrix}1 & -1 & 0\\ -2 & 3 & -4\\ -2 & 3 & -3\end{bmatrix}$ **(b)** $\begin{bmatrix}\frac{1}{2} & \frac{-1}{2} & 0\\ -1 & \frac{3}{2} & -2\\ -1 & \frac{3}{2} & \frac{-3}{2}\end{bmatrix}$

(c) $\begin{bmatrix}2 & -2 & 0\\ -4 & 6 & -8\\ -4 & 6 & -6\end{bmatrix}$ **(d)** $\begin{bmatrix}\frac{1}{5} & \frac{-1}{5} & 0\\ \frac{-2}{5} & \frac{3}{5} & \frac{-4}{5}\\ \frac{-2}{5} & \frac{3}{5} & \frac{-3}{5}\end{bmatrix}$

51. What is 3α + 2β equal to if $(2\hat{i}+6\hat{j}+27\hat{k}) \times (\hat{i}+\alpha\hat{j}+\beta\hat{k})$ is a null vector?

(a) 36 **(b)** 33 **(c)** 30 **(d)** 27

52. For what value of the angle between the vectors $\vec{a}$ and $\vec{b}$ is the quantity $|\vec{a}\times\vec{b}|+\sqrt{3}|\vec{a}.\vec{b}|$ maximum?

(a) 0° **(b)** 30° **(c)** 45° **(d)** 60°

53. Let θ be the angle between two unit vectors $\vec{a}$ and $\vec{b}$, If $\vec{a}+2\vec{b}$ is perpendicular to $5\vec{a}-4\vec{b}$, then what is cosθ + cos2θ equal to?

(a) 0 **(b)** $\frac{1}{2}$ **(c)** 1 **(d)** $\frac{\sqrt{3}+1}{2}$

54. Let ABCDEF be a regular hexagon. If $\vec{AD}=m\vec{BC}$ and $\vec{CF}=n\vec{AB}$, then what is mn equal to?

(a) –4 **(b)** –2 **(c)** 2 **(d)** 4

55. The vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are of the same length. If taken pairwise, they form equal angles. If $\vec{a}=\hat{i}+\hat{j}$ and $\vec{b}=\hat{j}+\hat{k}$, then what can $\vec{c}$ be equal to?

I. $\hat{i}+\hat{k}$ **II.** $\dfrac{-\hat{i}+4\hat{j}-\hat{k}}{3}$

Select the correct answer using the code given below.

(a) I only **(b)** II only
(c) Both I and II **(d)** Neither I nor II

56. The diagonals of a quadrilateral ABCD are along the lines $x - 2y = 1$ and $4x + 2y = 3$. The quadrilateral ABCD may be a

(a) rectangle. **(b)** cyclic quadrilateral.
(c) parallelogram. **(d)** rhombus.

57. The foci of the ellipse $4x^2 + 9y^2 = 1$ are at Q and R. If P(x, y) is any point on the ellipse, then what is PQ + PR equal to?

(a) 2 **(b)** 1 **(c)** $\frac{2}{3}$ **(d)** $\frac{1}{3}$

58. If P(2, 4), Q(8, 12), R(10, 14) and S(x, y) are vertices of a parallelogram, then what is $(x + y)$ equal to?

(a) 8 **(b)** 10 **(c)** 12 **(d)** 14

59. The equation of a circle is $(x^2 - 4x + 3) + (y^2 - 6y + 8) = 0$. Which of the following statements are correct?

I. The end points of a diameter of the circle are at (1, 2) and (3, 4).
II. The end points of a diameter of the circle are at (1, 4) and (3, 2).
III. The end points of a diameter of the circle are at (2, 4) and (4, 2).

Select the answer using the code given below.

(a) I and II only **(b)** II and III only
(c) I and III only **(d)** I, II and III

60. Consider the points P($4k$, $4k$) and Q($4k$, $-4k$) lying on the parabola $y^2 = 4kx$. If the vertex is A, then what is ∠PAQ equal to?

(a) 60° **(b)** 90° **(c)** 120° **(d)** 135°

Direction: *Consider the following for the next* ***two*** *items that follow:*

A triangle ABC is inscribed in the circle $x^2 + y^2 = 100$. B and C have coordinates (6,8) and (–8, 6), respectively.

61. What is ∠BAC equal to?

(a) $\frac{\pi}{2}$ **(b)** $\frac{\pi}{3}$ or $\frac{2\pi}{3}$
(c) $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ **(d)** $\frac{\pi}{6}$ or $\frac{5\pi}{6}$

62. What are the coordinates of A?

(a) (–6, 8) **(b)** (–6, –8)
(c) $(5\sqrt{2}, 5\sqrt{2})$
(d) Cannot be determined due to insufficient data

Direction: *Consider the following for the next* ***two*** *items that follow:*

ABCD is an isosceles trapezium and AB is parallel to DC. Let A(2, 3), B(4, 3), C(5, 1) be the vertices.

63. What are the coordinates of vertex D?

(a) (2, 1) **(b)** (1, 2) **(c)** (1, 1) **(d)** (3, 1)

64. What is the point of intersection of the diagonals of the trapezium?

(a) $\left(3, \frac{7}{2}\right)$ **(b)** $\left(3, \frac{7}{3}\right)$
(c) $\left(\frac{7}{2}, 2\right)$ **(d)** $\left(\frac{5}{2}, 2\right)$

Direction: *Consider the following for the two (02) items that follow:*

Let $2x^2 + 2y^2 + 2z^2 + 3x + 3y + 3z - 6 = 0$ be a sphere.

65. What is the diameter of the sphere?

(a) $\frac{5\sqrt{3}}{4}$ **(b)** $\frac{5\sqrt{3}}{2}$ **(c)** $\frac{3\sqrt{5}}{4}$ **(d)** $\frac{3\sqrt{5}}{2}$

66. The centre of the sphere lies on the plane

(a) $2x + 2y + 2z - 3 = 0$
(b) $4x + 4y + 4z - 3 = 0$
(c) $4x + 8y + 8z - 15 = 0$
(d) $4x + 8y + 8z + 15 = 0$

Direction: *Consider the following for the next* ***two (02)*** *items that follow:*

Let S be the line of intersection of two planes $x + y + z = 1$ and $2x + 3y - 4z = 8$.

67. Which of the following are the direction ratios of S?

(a) (–7, –6, 1) **(b)** (–7, 6, 1)
(c) (–6, 5, 1) **(d)** (6, 5, 1)

68. If (l, m, n) are direction cosines of S, then what is the value of $43(l^2 - m^2 - n^2)$?

(a) 6 **(b)** 5 **(c)** 4 **(d)** 1

Direction: *Consider the following for the next* ***two (02)*** *items that follow:*

Let L: $x + y + z + 4 = 0 = 2x - y - z + 8$ be a line and P: $x + 2y + 3z + 1 = 0$ be a plane.

69. What are the direction ratios of the line?

(a) (2, 1, –1) **(b)** (0, –1, 2)
(c) (0, 1, –1) (d) (2, 3, –3)

70. What is the point of intersection of L and P?

(a) (4, 3, –3) **(b)** (4, –3, 3)
(c) (–4, –3, –3) **(d)** (–4, –3, 3)

71. Let $z = [y]$ and $y = [x] - x$, where [] is the greatest integer function. If x is not an integer but positive, then what is the value of z?
(a) –1 **(b)** 0 **(c)** 1 **(d)** 2

72. If $f(x) = 4x + 1$ and $g(x) = kx + 2$ such that $f.g(x) = g.f(x)$, then what is the value of k?
(a) 7 **(b)** 5 **(c)** 4 **(d)** 3

73. What is the minimum value of the function $f(x) = \log_{10}(x^2 + 2x + 11)$?
(a) 0 **(b)** 1 **(c)** 2 **(d)** 10

74. Which one of the following is correct regarding $\lim_{x\to 3} \frac{|x-3|}{x-3}$?
(a) Limit exists and is equal to 1.
(b) Limit exists and is equal to 0.
(c) Limit exists and is equal to –1.
(d) Limit does not exist.

75. What is the maximum value of $a\cos x + b\sin x + c$?
(a) $\sqrt{a^2+b^2+c}$ **(b)** $\sqrt{a^2+b^2}+c$
(c) $\sqrt{a^2+b^2}-c$ **(d)** $\sqrt{a^2+b^2}$

76. If $f(2x) = 4x^2 + 1$, then for how many real values of x will $f(2x)$ be the GM of $f(x)$ and $f(4x)$?
(a) Four **(b)** Two **(c)** One **(d)** None

77. If $f(x) = [x]^2 - 30[x] + 221 = 0$, where $[x]$ is the greatest integer function, then what is the sum of all integer solutions?
(a) 13 **(b)** 17 **(c)** 27 **(d)** 30

78. If $f(x) = 9x - 8\sqrt{x}$ such that $g(x) = f(x) - 1$, then which one of the following is correct?
(a) $g(x) = 0$ has no real roots.
(b) $g(x) = 0$ has only one real root which is an integer.
(c) $g(x) = 0$ has two real roots which are integers.
(d) $g(x) = 0$ has only one real root which is not an integer.

79. What is $\lim_{x\to\frac{\pi}{2}}$ $(\sec\theta - \tan\theta)$ equal to?
(a) –1 **(b)** 0 **(c)** $\frac{1}{2}$ **(d)** 1

80. Let $f(x)\,f(y) = f(xy)$ for all real x, y. If $f(2) = 4$, then what is the value of $f\left(\frac{1}{2}\right)$?
(a) $\frac{1}{4}$ **(b)** $\frac{1}{2}$ **(c)** 1 **(d)** 4

Direction: *Consider the following for the next* ***two*** *items that follow:*
Let $fog(x) = \cos^2\sqrt{x}$ and $gof(x) = |\cos x|$

81. Which one of the following is $f(x)$?
(a) $\cos x$ **(b)** $\cos x^2$
(c) $\cos^2 x$ **(d)** $\cos|x|$

82. Which one of the following is $g(x)$?
(a) $\sqrt{x}$ **(b)** $|x|$ **(c)** x^2 **(d)** $x|x|$

Direction: *Consider the following for the* ***two*** *items that follow:*
Let $f(x) = [x]^2 - [x^2]$.

83. What is $f(0.999) + f(1.001)$ equal to?
(a) –1 **(b)** 0 (c) 1 **(d)** 2

84. Consider the following statements :
I. $f(x)$ is continuous at $x = 0$.
II. $f(x)$ is continuous at $x = 1$.
Which of the statements given above is/are correct?
(a) I only **(b)** II only
(c) Both I and II **(d)** Neither I nor II

Direction: *Consider the following for the next* ***two*** *items that follow:*
Let $f(x) = \cos 2x + x$ on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

85. What is the greatest value of f(x)?
(a) $\frac{\sqrt{3}}{2}-\frac{\pi}{12}$ **(b)** $\frac{\sqrt{3}}{2}+\frac{\pi}{12}$
(c) $\frac{\sqrt{3}}{2}+\frac{\pi}{9}$ **(d)** $\frac{\sqrt{3}}{2}+\frac{\pi}{6}$

86. What is the least value of $f(x)$?
(a) $-\left(1+\frac{\pi}{2}\right)$ **(b)** $-\left(\frac{1}{2}+\frac{\pi}{2}\right)$
(c) $-\left(1+\frac{\pi}{4}\right)$ **(d)** $-2\left(\frac{1}{2}-\frac{\pi}{4}\right)$

Direction: *Consider the following for the next* ***two*** *items that follow:*
The area bounded by the parabola $y^2 = kx$ and the line $x = k$, where $k > 0$, is $\frac{4}{3}$ square units.

87. What is the value of k ?
(a) $\frac{1}{2}$ **(b)** 1 **(c)** $\sqrt{2}$ **(d)** 2

88. What is the area of the parabola bounded by the latus rectum?
(a) $\frac{1}{6}$ square unit **(b)** $\frac{2}{3}$ square unit
(c) 1 square unit **(d)** $\frac{4}{3}$ square units

Direction: *Consider the following for the next* ***two (02)*** *items that follow:*
Let $ydx + (x - y^3)\,dy = 0$ be a differential equation.

89. What are the order and degree, respectively, of the differential equation?

(a) 1 and 1 **(b)** 1 and 2
(c) 2 and 1 **(d)** 1 and 3

90. What is the solution of the differential equation?

(a) $y^4 + 2x = c$ **(b)** $y^4 + 3x = c$
(c) $2xy^4 + x = c$ **(d)** $4xy - y^4 = c$

Direction: *Consider the following for the next* ***two (02)*** *items that follow:*

Let $f(x) = |x^2 - x - 2|$.

91. What is $\int_0^2 f(x)dx$ equal to?

(a) 0 **(b)** 1 **(c)** $\frac{5}{3}$ **(d)** $\frac{10}{3}$

92. What is $\int_1^3 f(x)dx$ equal to?

(a) 2 **(b)** 3 **(c)** 4 **(d)** 5

Direction: *Consider the following for the next* ***two (02)*** *items that follow:*

Let $f(t) = \ln\left(t + \sqrt{1+t^2}\right)$ and $g(t) = \tan(f(t))$.

93. Consider the following statements:

I. $f(t)$ is an odd function.
II. $g(t)$ is an odd function.

Which of the statements given above is/are correct?

(a) I only **(b)** II only
(c) Both I and II **(d)** Neither I nor II

94. What is $\int_{-\pi}^{\pi} g(t)dt$ equal to?

(a) –1 **(b)** 0 **(c)** $\frac{1}{2}$ **(d)** 1

Direction: *Consider the following for the next* ***two*** *items that follow:*

Let $f : (-1, 1) \rightarrow R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $h(x) = f(2f(x) + 2)$ and $g(x) = (h(x))^2$.

95. What is $h'(0)$ equal to?

(a) –2 **(b)** –1 **(c)** 0 **(d)** 2

96. What is $g'(0)$ equal to?

(a) –4 **(b)** –2 **(c)** 0 **(d)** 4

Direction: *Consider the following for the next* ***two (02)*** *items that follow:*

Let $I = \int_0^{\frac{\pi}{2}} \frac{f(x)}{g(x)}dx$, where $f(x) = \sin x$ and $g(x) = \sin x + \cos x + 1$.

97. What is $\int_0^{\frac{\pi}{2}} \frac{d(x)}{g(x)}$ equal to?

(a) $\frac{ln2}{2}$ **(b)** $\frac{ln2}{4}$ **(c)** $ln2$ **(d)** $2ln2$

98. What is I equal to?

(a) $\frac{\pi}{4} + \ln 2$ **(b)** $\frac{\pi}{4} - \ln 2$
(c) $\frac{\pi}{4} - \frac{\ln 2}{2}$ **(d)** $\frac{\pi}{4} + \frac{\ln 2}{2}$

Direction: *Consider the following for the next* ***two*** *items that follow:*

Let

$$2\int \frac{x^2 - 1}{\sqrt{x^2 + 1}}dx = U(x)\,V(x) - 3ln\,\{U(x) + V(x)\} + c$$

99. What is $|U^2(x) - V^2(x)|$ equal to?

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3

100. What is $U(x)\,V(x)$ equal to?

(a) $\sqrt{x^2 + x^4}$ **(b)** $\sqrt{x + x^3}$
(c) $\frac{\sqrt{x^2 + x^4}}{2}$ **(d)** $2\sqrt{x^2 + x^4}$

101. Let $x - 3y + 4 = 0$ and $2x - 7y + 8 = 0$ be two lines of regression computed from some bivariate data. If b_{yx} and b_{xy} are regression coefficients of lines of regression of y on x and x on y, respectively, then what is the value of $b_{xy} + 7b_{yx}$?

(a) –2 **(b)** 1 **(c)** 2 **(d)** 5

102. The mean of n observations 1, 4, 9, 16, ⋯, n^2 is 130. What is the value of n?

(a) 18 **(b)** 19 **(c)** 20 **(d)** 21

103. Three distinct natural numbers are chosen at random from 1 to 10. What is the probability that they are consecutive?

(a) $\frac{1}{12}$ **(b)** $\frac{3}{40}$ **(c)** $\frac{1}{15}$ **(d)** $\frac{7}{120}$

104. A, B and C are three mutually exclusive and exhaustive events associated with a random experiment. If 3P (B) = 4 P (A) and 3P(C) = 2P (B), then what is P(A) equal to?

(a) $\frac{7}{29}$ **(b)** $\frac{8}{29}$ **(c)** $\frac{9}{29}$ **(d)** $\frac{10}{29}$

105. A die has two faces with number 4, three faces with number 5 and one face with number 6. If the die is rolled once, then what is the probability of getting 4 or 5?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{5}{6}$ (d) $\frac{1}{2}$

106. A box contains 2 black, 4 yellow and 6 white balls. Three balls are drawn in succession with replacement. What is the probability that all three are of the same colour?

(a) $\frac{1}{6}$ (b) $\frac{1}{36}$ (c) $\frac{1}{12}$ (d) $\frac{5}{12}$

107. A can hit a target 5 times in 6 shots, B can hit 4 times in 5 shots and C can hit 3 times in 4 shots. What is the probability that A and C may hit but B may lose?

(a) $\frac{1}{8}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

108. The letters of the word ZOOLOGY are arranged in all possible ways. What is the probability that the consonants and vowels occur alternatively?

(a) $\frac{6}{35}$ (b) $\frac{3}{35}$ (c) $\frac{2}{35}$ (d) $\frac{1}{35}$

109. A natural number x is chosen at random from the first 100 natural numbers. What is the probability that $x^2 + x > 50$?

(a) $\frac{93}{100}$ (b) $\frac{47}{50}$ (c) $\frac{24}{25}$ (d) $\frac{23}{25}$

110. What is the mean deviation of the first 10 natural numbers?

(a) 2 (b) 2.5 (c) 3 (d) 3.5

111. Let $\sum_{i=1}^{9} x_i^2 = 855$. If M is the mean and σ is the standard deviation of $x_1, x_2, ..., x_9$, then what is the value of $M^2 + \sigma^2$?

(a) 100 (b) 95 (c) 90 (d) 85

112. The mean of the series x_1, x_2, x_n is $\bar{x}$. If x_n is replaced by k, then what is the new mean?

(a) $\bar{x} - x_n + k$

(b) $\frac{n\bar{x} - \bar{x} + k}{n}$

(c) $\frac{\bar{x} - x_n - k}{n}$

(d) $\frac{n\bar{x} - x_n + k}{n}$

113. A fair coin is tossed till two heads occur in succession. What is the probability that the number of tosses required is less than 6?

(a) $\frac{5}{64}$ (b) $\frac{15}{32}$ (c) $\frac{31}{64}$ (d) $\frac{19}{32}$

114. Urn A contains 2 white and 2 black balls, while urn B contains 3 white and 2 black balls. One ball is transferred from urn A to urn B, and then a ball is drawn out of urn B. What is the probability that the ball is white?

(a) $\frac{11}{20}$ (b) $\frac{7}{12}$ (c) $\frac{3}{5}$ (d) 1

115. For two events A and B, $P(A) = P(A|B) = 0.25$ and $P(B|A) = 0.5$. Which of the following are correct?

I. A and B are independent.
II. $P(A^c \cup B^c) = 0.875$.
III. $P(A^c \cap B^c) = 0.375$.

Select the answer using the code given below.

(a) I and II only (b) II and III only
(c) I and III only (d) I, II and III

116. Two perfect dice are thrown. What is the probability that the sum of the numbers on the faces is neither 9 nor 10?

(a) $\frac{1}{36}$ (b) $\frac{5}{36}$ (c) $\frac{7}{36}$ (d) $\frac{29}{36}$

117. The occurrence of a disease in an industry is such that the workers have 20% chance of suffering from it. What is the probability that out of 6 workers chosen at random, 4 or more will suffer from the disease?

(a) $\frac{53}{3125}$ (b) $\frac{63}{3125}$ (c) $\frac{73}{3125}$ (d) $\frac{83}{3125}$

118. Three perfect dice are rolled. Under the condition that no two show the same face, what is the probability that one of the faces shown is an ace (one)?

(a) $\frac{5}{9}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

119. Three perfect dice D_1, D_2 and D_3 are rolled. Let x, y and z represent the numbers on D_1, D_2 and D_3, respectively. What is the number of possible outcomes such that $x < y < z$?

(a) 20 (b) 18 (c) 14 (d) 10

120. In a binomial distribution, if the mean is 6 and the standard deviation is $\sqrt{2}$, then what are the values of the parameters n and p, respectively?

(a) 18 and $\frac{1}{3}$ (b) 9 and $\frac{1}{3}$

(c) 18 and $\frac{2}{3}$ (d) 9 and $\frac{2}{3}$

Answer Key

Q. No	Answer Key	Topic's Name	Chapter's Name
1	d	Multiplication of matrices	Matrices and Determinants
2	a	Types of Sets	Sets
3	a	Arrangement	Permutations and Combinations
4	d	Cube roots of Unity	Complex Numbers
5	c	Singular matrix	Matrices and Determinants
6	c	General and middle term	Binomial theorem
7	a	AP	Sequence and Series
8	c	GP	Sequence and Series
9	b	Modulus	Complex Numbers
10	b	Values of 'Y'	Complex Number
11	c	HP	Sequence and Series
12	a	Values of Determinant	Matrices and Determinants
13	a	Sum of n^{th} Term	Sequence and Series
14	b	Permutation	Permutation and Combination
15	c	Permutation	Permutation and Combination
16	c	Quadratic Equation	Quadratic equations
17	d	Combination	Permutation and Combination
18	a	Value of Determinant	Matrices and Determinant
19	c	Multiplication of Matrices	Matrices and Determinant
20	b	Skew Symmetric	Matrices and Determinant
21	c	Properties	Inverse Trigonometry
22	b	Properties	Inverse Trigonometry
23	a	Cosine formula	Trigonometry
24	c	Identities	Trigonometry
25	b	Sine Formula	Trigonometry
26	c	Trigonometric equations	Trigonometry
27	a	Trigonometric equations	Trigonometry
28	b	Identities	Trigonometry
29	c	Identities	Trigonometry
30	d	Identities	Trigonometry
31	c	Identities	Trigonometry
32	b	Identities	Trigonometry
33	d	Identities	Trigonometry
34	d	Type of Relation	Relation and Function
35	d	Cartesian Product	Binomial and Functions
36	b	Expansion	Binomial Theorem
37	c	Combination	Permutation and Combination
38	c	Sum of A.P	Sequence and Series
39	a	Quadratic Equation	Quadratic Equations
40	a	Cardinal Number	Sets
41	a	Modulus	Complex Number

Q. No	Answer Key	Topic's Name	Chapter's Name
42	b	Operation	Complex Number
43	d	AP	Sequence and series
44	c	AP	Sequence and Series
45	d	Expansion	Binomial Theorem
46	b	Expansion	Binomial Theorem
47	c	HP	Sequence and Series
48	c	HP	Sequence and Series
49	d	Adjoint Matrices	Matrices and Determinant
50	a	Adjoint Matrices	Matrices and Determinant
51	a	Cross Product	Vectors
52	b	Cross Product	Vectors
53	a	Dot Product	Vectors
54	a	Parallel Vectors	Vectors
55	c	Angle Between two Vectors	Vectors
56	d	Slope of Lines	Straight Lines
57	b	Ellipse	Conic Section
58	b	Rectangular coordinates	Straight Line
59	a	Circle	Conic Section
60	b	Parabola	Conic Section
61	c	Circle	Conic Section
62	d	Circle	Conic Section
63	d	Mid-point	Coordinate Geometry
64	c	Mid-point	Coordinate Geometry
65	b	Sphere	3-D Geometry
66	d	Sphere	3-D Geometry
67	b	Planes	3-D Geometry
68	a	Planes	3-D Geometry
69	c	Lines and planes	3-D Geometry
70	d	Lines and planes	3-D Geometry
71	a	Types of Functions	Relation and Function
72	a	Composite function	Relation and Function
73	b	Maxima and Minima	Application of Derivatives
74	d	Limit	Limit & Derivatives
75	b	Maxima and Minima	Application of Derivatives
76	c	Function	Relation and Function
77	d	Function	Relation and Function
78	b	Function	Relation and Function
79	b	Limits	Limit and Derivatives
80	a	Function	Relations and Function
81	c	Composite Function	Relation and Functions
82	a	Composite Function	Relation and Functions
83	b	Continuity	Continuity and Differentiability
84	b	Continuity	Continuity and Differentiability
85	b	Maxima and minima	Application of Derivative

Q. No	Answer Key	Topic's Name	Chapter's Name
86	**a**	Maxima and Minima	Application of Derivative
87	**b**	Area of Region	Application of Integral
88	**a**	Area of Region	Application of Integral
89	**a**	Order and degree	Differential Equation
90	**d**	Linear Differential Equation	Differential Equation
91	**d**	Properties of Definite Integral	Integrals
92	**b**	Properties of Definite Integral	Integrals
93	**c**	Properties of Definite Integral	Integrals
94	**b**	Properties of Definite Integral	Integrals
95	**d**	Chain Rule	Differentiation
96	**a**	Chain Rule	Differentiation
97	**c**	Definite Integral	Integrals
98	**c**	Properties of Definite Integral	Integral
99	**b**	Indefinite Integral	Integral
100	**a**	Indefinite Integral	Integrals
101	**d**	Lines of Regression	Statistics
102	**b**	Mean	Statistics
103	**c**	Basic probability	Probability
104	**c**	Basic probability	Probability
105	**c**	Basic probability	Probability
106	**a**	Multiplication Theoren	Probability
107	**a**	Independent Event	Probability
108	**a**	Basic Probability	Probability
109	**b**	Basic probability	Probability
110	**b**	Mean deviation	Statistics
111	**b**	Variance	Statistics
112	**d**	Mean	Statistics
113	**b**	Multiplication Theorem	Probability
114	**b**	Conditional Probability	Probability
115	**d**	Conditional Probability	Probability
116	**d**	Basic Probability	Probability
117	**a**	Probability Distribution	Probability
118	**d**	Multiplication Theorem	Probability
119	**a**	Possible Outcomes	Probability
120	**d**	Binomial Distribution	Probability

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MATHEMATICS

SOLVED PAPER 2024 (II)

ANSWERS WITH EXPLANATION

1. Option (d) is correct.

Since, $A_{m\times n}.B_{n\times p} = (AB)_{m\times p}$

Hence, $[Z_{3\times 2}.Y_{2\times 3}].X_{3\times 3} = [ZYX]_{3\times 3}$,

$Y_{2\times 3}.[X_{3\times 3}.Z_{3\times 2}] = [YXZ]_{2\times 2}$

and $X_{3\times 3}.[Y_{2\times 3}.Z_{3\times 2}] = X_{3\times 3}[YZ]_{2\times 2}$

$\because$ No. of columns in X $\neq$ No. of rows in (YZ)

Hence, X(YZ) is not defined.

2. Option (a) is correct.

$\because$ Between any two irrational numbers, an infinite number of irrational numbers are there.

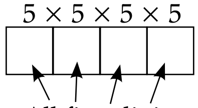
$\therefore$ The set of all irrational numbers between $\sqrt{12}$ and $\sqrt{15}$ is an infinite set.

Set of all odd integers less than 1,000 is {...–5, –3, 1, 3 ... 999} which is an infinite set.

3. Option (a) is correct.

Odd digits can be 1, 3, 5, 7 and 9.

$5 \times 5 \times 5 \times 5$



All five digits

$\because$ All five odd digits can be filled at every place.

Hence, total required four-digit numbers are $5 \times 5 \times 5 \times 5 = 625$

4. Option (d) is correct.

$(1 + \omega - \omega^2)^{100} + (1 - \omega + \omega^2)^{100}$

$= (-\omega^2 - \omega^2)^{100} + (-\omega - \omega)^{100}$ $\{\because 1 + \omega + \omega^2 = 0\}$

$= 2^{100}(\omega^{200} + \omega^{100})$

$= 2^{100}(\omega^2 + \omega)$ $\{\because \omega^3 = 1\}$

$= -2^{100}$

5. Option (c) is correct.

Given AB = O

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

and $B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$.

Now, $AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} ap+br & aq+bs \\ cp+dr & cq+ds \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Comparing both sides, we get

$ap + br = 0$ ... (i)

$aq + bs = 0$ ... (ii)

$cp + dr = 0$ ... (iii)

$cq + ds = 0$ ... (iv)

From Eqs. (i) and (iii), we get:

$(ad - bc)p = 0$ and $(ad - bc)r = 0$

From Eqs. (ii) and (iv), we get:

$(ad - bc)\,q = 0$ and $(ad - bc)s = 0$

Now, if A is non-singular

$\Rightarrow ad - bc \neq 0$

So $p = q = r = s = 0$.

$\therefore$ B is null matrix.

6. Option (c) is correct.

$(1 + x)^p (1 + x)^q = (1 + x)^{p+q}$

$\because$ Coefficient of x^3 in the expansion = 35

$\Rightarrow {}^{(p+q)}C_3 = 35 = {}^7C_3$

$\Rightarrow p + q = 7$

7. Option (a) is correct.

Let 'a' and 'd' be the first term and common difference of AP.

$(p + q)^{th}$ term $= T_{p+q}$

$= a + (p + q - 1)$ ...(i)

Given $p.T_p = q.T_q$

$\Rightarrow p.[a + (p - 1)\,d] = q.[a + (q - 1)\,d]$

$\Rightarrow (p - q)\,[a + (p + q - 1)d] = 0$

$\Rightarrow [a + (p + q - 1)\,d] = 0$ $\{\because p \neq q\}$

$T_{p+q} = 0$ {from (i)}

8. Option (c) is correct.

Given $p = lnx$, $q = lnx^3 = 3lnx$

$r = lnx^5 = 5lnx$

Clearly, $q - p = r - q = 2lnx$

$\Rightarrow p, q, r$ are in AP

Also, $\because \frac{q}{p} \neq \frac{r}{q}$ $\therefore$ p, q, r can never be in GP.

9. Option (b) is correct.

Given $Z = \frac{1}{3}\begin{bmatrix} i & 2i & 1 \\ 2i & 3i & 2 \\ 3 & 1 & 3 \end{bmatrix}$

$\Rightarrow Z = \frac{1}{3}[i(9i-2)-2i(6i-6)+1(2i-9i)]$

$\Rightarrow Z = \frac{1}{3}[3+3i] = 1 + i$

$|Z| = \sqrt{1^2+1^2} = \sqrt{2}$

10. Option (b) is correct.

Let $I = \sum_{n=1}^{20}(i^{n-1}+i^n+i^{n+1})$

$\Rightarrow I = \sum_{n=1}^{20} i^n\left(\frac{1}{i}+1+i\right)$

$= \sum_{n=1}^{20} i^n(1)$

$= i^1 + i^2 + i^3 + i^4 + \ldots i^{20}$

$= 5.(i + i^2 + i^3 + i^4)$

$= 0 \quad \{\because i + i^2 + i^3 + i^4 = 0\}$

11. Option (c) is correct.

$\because$ x, y and z are in GP.

$\Rightarrow y^2 = xz$

$\Rightarrow 2lny = lnx + lnz$

lnx, $ln\, y$ and lnz are in AP.

$(1 + lnx)$, $(1 + lny)$ and $(1 + lnz)$ are in AP.

$\frac{1}{(1+lnx)}$, $\frac{1}{(1+lny)}$ and $\frac{1}{(1+lnz)}$ are in HP.

12. Option (a) is correct.

Let $I = \begin{bmatrix} 1+\omega & 1+\omega^2 & \omega+\omega^2 \\ 1 & \omega & \omega^2 \\ \frac{1}{\omega} & \frac{1}{\omega^2} & 1 \end{bmatrix}$

$C_1 \rightarrow C_1 + C_2 + C_3$, we get

$I = \begin{bmatrix} 2(1+\omega+\omega^2) & 1+\omega^2 & \omega+\omega^2 \\ (1+\omega+\omega^2) & \omega & \omega^2 \\ \frac{(1+\omega+\omega^2)}{\omega^2} & \frac{1}{\omega^2} & 1 \end{bmatrix}$

$I = \begin{bmatrix} 0 & 1+\omega^2 & \omega+\omega^2 \\ 0 & \omega & \omega^2 \\ 0 & \frac{1}{\omega^2} & 1 \end{bmatrix} = 0$

13. Option (a) is correct.

Given $S_n = n(2n + 1)$, $S_{n-1} = (n-1)(2n-1)$

$n^{th}\ term = S_n - S_{n-1}$

$= n(2n + 1) - (n-1)(2n-1)$

$= 4n - 1$

14. Option (b) is correct.

Word 'INDIA' contains three vowels (I, I and A) and two consonants (N and D).

Three vowels (I, I and A) can be arranged at odd position in $\frac{3!}{2!}$ = 3 ways and 2 consonants (N and D) can be arranged in 2! ways = 2 ways.

odd		**odd**		**odd**

Thus, required numbers of ways = (3) (2) = 6.

15. Option (c) is correct.

Word 'EQUATION' contains five vowels (A, E, I, O and U) and three consonants (Q, T and N).

N	Q	T	A	E	I	O	U

Consonants Vowels

Number of ways = (3!) . (5!)

A	E	I	O	U	N	Q	T

Vowels Consonants

Numbers of ways = (3!) . (5!)

Total required number of ways

=(3!) . (5!) + (3!) . (5!)

= 1440

16. Option (c) is correct.

$\because$ n is root of $x^2 + px + m = 0$.

$\therefore \quad n^2 + pn + m = 0 \qquad$ ...(i)

$\because$ m is root of the equation $x^2 + px + n = 0$.

$\therefore \quad m^2 + pm + n = 0 \qquad$ ...(ii)

Now Eq. (i) – Eq. (ii), we get

$(n^2 - m^2) + p(n - m) - (n - m) = 0$

$\Rightarrow (n - m)[(n + m) + p - 1] = 0$

$\Rightarrow n + m + p - 1 = 0 \qquad \{n \neq m\}$

$\Rightarrow n + m + p = 1$

17. Option (d) is correct.

Two courses are compulsory.

$\therefore$ Total courses is $n-2$ and total remaining courses that can be chosen from (n – 2) courses is ($n-4$).

$\therefore$ Required number of ways $= {}^{(n-2)}C_{(n-4)}$

$= \frac{(n-2)(n-3)}{2}$

18. Option (a) is correct.

$$D_n = \begin{bmatrix} n & 20 & 30 \\ n^2 & 40 & 50 \\ n^3 & 60 & 70 \end{bmatrix}$$

$$= (20)(10)\begin{bmatrix} n & 1 & 3 \\ n^2 & 2 & 5 \\ n^3 & 3 & 7 \end{bmatrix}$$

$$D_n = 200\,(-n + 2n^2 - n^3)$$

Now, $$\sum_{n=1}^{4} D_n = 200\sum_{n=1}^{4}(-n + 2n^2 - n^3)$$

$$= 200\left[\frac{-(4)(5)}{2} + 2\frac{(4)(5)(9)}{6} - \left(\frac{(4)(5)}{2}\right)^2\right]$$

$$= 200\,(-50) = -10{,}000$$

19. Option (c) is correct.

Given $$P = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix},$$

$$Q = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$\Rightarrow \quad PQ = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and $$QP = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad PQ = QP$$

20. Option (b) is correct.

Determinant of any skew symmetric matrix with odd order is always zero.

21. Option (c) is correct.

Given $$4\sin^{-1}x + \cos^{-1}x = \pi$$

$$\Rightarrow 4\left(\frac{\pi}{2} - \cos^{-1}x\right) + \left(\frac{\pi}{2} - \sin^{-1}x\right) = \pi$$

$$\Rightarrow 2\pi - 4\cos^{-1}x + \frac{\pi}{2} - \sin^{-1}x = \pi$$

$$\Rightarrow \quad 4\cos^{-1}x + \sin^{-1}x = \frac{3\pi}{2}$$

22. Option (b) is correct.

Let $$I = \cot^2(\sec^{-1}2) + \tan^2(\text{cosec}^{-1}3)$$

$$\Rightarrow \quad I = \cot^2(60°) + \tan^2\left(\cot^{-1}\sqrt{9-1}\right)$$

$$\Rightarrow \quad I = \frac{1}{3} + \tan^2\left(\tan^{-1}\frac{1}{\sqrt{8}}\right)$$

$$= \frac{1}{3} + \frac{1}{8} = \frac{11}{24}$$

23. Option (a) is correct.

Given $$\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$$

$$\Rightarrow \frac{a}{\left(\frac{b^2+c^2-a^2}{2bc}\right)} = \frac{b}{\left(\frac{a^2+c^2-b^2}{2ac}\right)} = \frac{c}{\left(\frac{a^2+b^2-c^2}{2ab}\right)}$$

$$\Rightarrow \quad b^2 + c^2 - a^2 = c^2 + a^2 - b^2 = a^2 + b^2 - c^2$$

$$\Rightarrow \quad a^2 = b^2 = c^2$$

$$\Rightarrow \quad a = b = c$$

Hence, ΔABC is an equilateral triangle.

So area of $\Delta ABC = \frac{\sqrt{3}}{4}\cdot a^2$ (Here, $a = 6$)

$$= \frac{\sqrt{3}}{4}\cdot(6)^2 = 9\sqrt{3}$$

24. Option (c) is correct.

$\tan\alpha$ and $\tan\beta$ are roots of $7x^2 - 6x + 1 = 0$.

Hence, $$\tan\alpha + \tan\beta = \frac{6}{7} \quad ...(i)$$

$$\tan\alpha \,.\, \tan\beta = \frac{1}{7} \quad ...(ii)$$

$$\because \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta} = \frac{\left(\frac{6}{7}\right)}{1 - \frac{1}{7}} = 1$$

$$\Rightarrow \quad \alpha + \beta = 45°$$

$$\Rightarrow \quad 2\alpha + 2\beta = 90°$$

It means third angle of triangle is 90°.

Hence, triangle is right-angled triangle.

Since, $(\tan\alpha - \tan\beta)$

$$= \sqrt{(\tan\alpha + \tan\beta)^2 - 4\tan\alpha.\tan\beta} = \frac{2\sqrt{2}}{7}$$

$$\because \quad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha.\tan\beta}$$

$$= \frac{1}{2\sqrt{2}}$$

and $$\tan 2(\alpha-\beta) = \frac{2\tan(\alpha-\beta)}{1 - \tan^2(\alpha-\beta)}$$

$$= \left(\frac{4\sqrt{2}}{7}\right)$$

$\Rightarrow \quad 2\alpha \neq 2\beta$

$\Rightarrow$ Hence, triangle is not an isosceles triangle.

25. Option (b) is correct.

$\because \angle A = 75°, \angle B = 45°$, and $\angle C = 60°$

and $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{a}{\sin 75°} = \frac{b}{\sin 45°} = \frac{c}{\sin 60°}$$

Hence, $$a = \frac{\sin 75°}{\sin 60°}.c = \frac{(\sqrt{3}+1)c}{\sqrt{6}}$$

and $$b = \frac{\sin 45°}{\sin 60°}.c = \frac{2c}{\sqrt{6}}$$

$$\therefore \quad 2a - b = \frac{2\sqrt{3}c}{\sqrt{6}} = \sqrt{2}c$$

26. Option (c) is correct.

Given $\cot 2x . \cot 3x = 1$

$\cos 2x . \cos 3x - \sin 2x . \sin 3x = 0$

$\cos(2x+3x) = \cos 5x = 0$

$0 < x < \pi$

Hence, $$5x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$$

$$x = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10}$$

Thus, only five solutions are possible.

27. Option (a) is correct.

$\cos^{100}x - \sin^{100}x = 1$ is only possible when $\cos^2 x = 1$ or $\sin^2 x = 0$

because $\cos^2 x \in [0, 1] \Rightarrow \cos^{100}x \in [0, 1]$

and $\sin^2 x \in [0, 1] \Rightarrow \sin^{100}x \in [0, 1]$

Now, $\cos^2 x = 1$

$\Rightarrow \quad x = n\pi \pm 0°$

$\Rightarrow \quad x = n\pi$

28. Option (b) is correct.

In ΔABC, $A + B + C = \pi$

$\Rightarrow \tan(A + B + C) = \tan\pi = 0$

$$\Rightarrow \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A . \tan B - \tan B . \tan C - \tan C . \tan A} = 0$$

$\Rightarrow \tan A . \tan B . \tan C = \tan A + \tan B + \tan C$

$$\Rightarrow \cot A . \cot B . \cot C = \frac{1}{\tan A + \tan B + \tan C} = \frac{1}{k}$$

29. Option (c) is correct.

$$\sin 12°.\sin 48° = \frac{2\sin 12°\sin 48°}{2}$$

$$= \frac{1}{2}[\cos(36°) - \cos(60°)]$$

$$= \frac{1}{2}\left[\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right] = \frac{\sqrt{5}-1}{8}$$

30. Option (d) is correct.

$$\frac{\cos 17° - \sin 17°}{\cos 17° + \sin 17°} = \frac{\cos 17°\left(1 - \frac{\sin 17°}{\cos 17°}\right)}{\cos 17°\left(1 + \frac{\sin 17°}{\cos 17°}\right)}$$

$$= \frac{\tan 45° - \tan 17°}{1 + \tan 45° . \tan 17°}$$

$$= \tan 28° = \cot 62°$$

31. Option (c) is correct.

(I) Let $\tan 22.5° = \tan\frac{45°}{2} = t$

Now, $$\tan 45° = \tan\left(2 \times \frac{45°}{2}\right)$$

$$= \frac{2\tan\left(\frac{45°}{2}\right)}{1 - \tan^2\left(\frac{45°}{2}\right)}$$

$$\Rightarrow \quad 1 = \frac{2t}{1-t^2} \Rightarrow t^2 + 2t - 1 = 0$$

$$\Rightarrow \quad t = \frac{-2 \pm \sqrt{4+4}}{2} = -1 \pm \sqrt{2}$$

$$\Rightarrow \quad \tan\left(\frac{45°}{2}\right) = -1 \pm \sqrt{2}$$

$$\Rightarrow \quad \tan\left(\frac{45°}{2}\right) = -1 + (\sqrt{2})$$

{$\because$ In first quadrant tan is positive}

This is an irrational number.

(II) $$\cot 22.5° = \frac{1}{\tan 22.5°} = \frac{1}{-1+\sqrt{2}} = 1 + \sqrt{2}$$

This is also an irrational number.

(III) $\tan 22.5° - \cot 22.5° = -1 + \sqrt{2} - (1 + \sqrt{2})$

$= -2$

This is not an irrational number.

32. Option (b) is correct.

Given $$\frac{x}{\cos\theta} = \frac{y}{\cos\left(\frac{2\pi}{3} - \theta\right)}$$

$$= \frac{z}{\cos\left(\frac{2\pi}{3}+\theta\right)} = k(\text{say})$$

$$\Rightarrow \quad x = k\cos\theta$$

$$y = k\cos\left(\frac{2\pi}{3}-\theta\right)$$

$$z = k\cos\left(\frac{2\pi}{3}+\theta\right)$$

Now, $x + y + z = k$

$$\left[\cos\theta + \cos\left(\frac{2\pi}{3}-\theta\right) + \cos\left(\frac{2\pi}{3}+\theta\right)\right]$$

$$= k\left[\cos\theta + 2\cos\frac{2\pi}{3}\cos\theta\right]$$

$$= k\left[\cos\theta + 2\left(-\frac{1}{2}\right)\cos\theta\right]$$

$$= k[\cos\theta - \cos\theta] = 0$$

$$\Rightarrow \quad x + y + z = 0$$

33. Option (d) is correct.

Given $p\tan(\theta - 30°) = q\tan(\theta + 120°)$

$$\Rightarrow \quad \frac{p}{q} = \frac{\tan(\theta+120°)}{\tan(\theta-30°)}$$

$$= \frac{\sin(\theta+120°)\cos(\theta-30°)}{\cos(\theta+120°)\sin(\theta-30°)}$$

$$\frac{p+q}{p-q}$$

$$= \frac{\sin(\theta+120°)\cos(\theta-30°)+\cos(\theta+120°)\sin(\theta-30°)}{\sin(\theta+120°)\cos(\theta-30°)-\cos(\theta+120°)\sin(\theta-30°)}$$

$$\Rightarrow \quad \frac{p+q}{p-q} = \frac{\sin[(\theta+120°)+(\theta-30°)]}{\sin[(\theta+120°)-(\theta-30°)]}$$

$$= \frac{\sin(90°+2\theta)}{\sin(150°)}$$

$$\Rightarrow \quad \frac{p+q}{p-q} = \frac{\sin(90°+2\theta)}{\left(\frac{1}{2}\right)} = 2\cos 2\theta$$

34. Option (d) is correct.

(I) Given that P and Q are reflexive relations on A.

$\Rightarrow$ For any $a \in A$, $(a, a) \in P$ and $(a, a) \in Q$

$\Rightarrow (a, a) \in P \cap Q$

$\Rightarrow P \cap Q$ is reflexive relation. So statement (I) is true.

(II) Given that P and Q are symmetric relation

Let $(a, b) \in (P \cup Q) \Rightarrow (a, b) \in P$ or $(a, b) \in Q$.

$\Rightarrow (b, a) \in P$ or $(b, a) \in Q$

$\Rightarrow (b, a) \in (P \cup Q)$

$\Rightarrow P \cup Q$ is symmetric relation.

So statement (II) is true.

(III) Given that P and Q are transitive

Let $(a, b) \in (P \cap Q)$ and $(b, c) \in (P \cap Q)$

$\Rightarrow \{(a, b) \in P, (b, c) \in P\}$ and $\{(a, b) \in Q, (b, c) \in Q\}$

$\Rightarrow (a, c) \in P$ and $(a, c) \in Q$

{$\because$ P and Q are transitive}

$\Rightarrow (a, c) \in P \cap Q$

So $P \cap Q$ is a transitive relation. So statement (III) is also true.

35. Option (d) is correct.

Given $\quad A \cap B = 10$

Now, $(A \times B) \cap (B \times A) = 10^2 = 100$

36. Option (b) is correct.

Let $\quad p(n) = 7^n - 6n$

$$= (6+1)^n - 6n$$

$$= 1 + 6n + {}^nC_2 6^2 + {}^nC_3 6^3 + \ldots + 6^n - 6n)$$

$$= 1 + 6^2({}^nC_2 + {}^nC_3 \times 6 + \ldots + 6^{n-2})$$

$$= 1 + 36({}^{100}C_2 + {}^{100}C_2 \times 6 + \ldots + 6^{98})$$

{$\because n = 100$}

$= 1 + 36k$, where $k = {}^{100}C_2 + {}^{100}C_2 \times 6 + \ldots 6^{98}$

$\therefore$ When $7^n - 6n$ is divided by 36, the remainder will be 1.

37. Option (c) is correct.

Since there are 4 lines, the maximum number of intersection points are ${}^4C_2 = 6$.

And maximum number of intersection of circle with 4 lines are $4 \times 2 = 8$.

$\therefore$ Required intersection points $= 6 + 8 = 14$

38. Option (c) is correct.

Given $\quad \frac{S_p}{S_q} = \frac{p^2}{q^2}$

$$\Rightarrow \quad \frac{\frac{p}{2}[2a+(p-1)d]}{\frac{q}{2}[2a+(q-1)d]} = \frac{p^2}{q^2}$$

$$\Rightarrow \quad \frac{2a+(p-1)d}{2a+(q-1)d} = \frac{p}{q}$$

$$\Rightarrow \quad 2aq + q(p-1)d = 2ap + p(q-1)d$$

$$\Rightarrow \quad 2a(q-p) = d(q-p)$$

$$\Rightarrow \quad 2a = d \text{ or } d = 2a$$

$\therefore$ The common difference is equal to twice of the first term.

39. Option (a) is correct.

Given equation is

$$(x-1)^2 + (x-3)^2 + (x-5)^2 = 0$$

$\Rightarrow x^2 - 2x + 1 + x^2 - 6x + 9 + x^2 - 10x + 25 = 0$

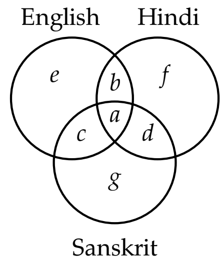
$\Rightarrow 3x^2 - 18x + 35 = 0$

Now, discriminant D $= (-18)^2 - 4 \times 3 \times 35$

$= -96 < 0$

$\therefore$ The given equation does not have any real roots.

40. Option (a) is correct.



Total student = 240

$\because$ 10 students failed in every subject.

Hence, remaining total = 230

According to question,

$a + b + c + d + e + f + g = 230$ ...(i)

and $b + c + d = 110$ ....(ii)

and $e + f + g = 60$ ...(iii)

Eq. (i) – Eqs. [(ii) + (iii)], we get

$\Rightarrow a = 230 - (110 + 60) = 60$

For Questions 41 and 42

$\because Z_1^2 + Z_2^2 + Z_1Z_2 = 0$

$\Rightarrow Z_1 = \omega$ and $Z_2 = \omega^2$

41. Option (a) is correct.

Now, $\left|\frac{Z_1}{Z_2}\right| = \left|\frac{\omega}{\omega^2}\right| = \left|\frac{1}{\omega}\right| = 1$

42. Option (b) is correct.

$$\frac{1}{2} + \text{Re}\left(\frac{Z_1}{Z_2}\right) = \frac{1}{2} + \text{Re}\left(\frac{\omega}{\omega^2}\right)$$

$$= \frac{1}{2} + \text{Re}(\omega^2)$$

$$= \frac{1}{2} + \text{Re}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$$

$$= \frac{1}{2} + \left(-\frac{1}{2}\right) = 0$$

43. Option (d) is correct.

Let the required terms of AP are

$a - 2d, a - d, a, a + d, a + 2d$

$\therefore (a - 2d)(a - d)a(a + d)(a + 2d) = 2,29,635$

$\Rightarrow a(a^2 - d^2)(a^2 - 4d^2) = 2,29,635$ ...(i)

$\because a - 2d, a - d$ and $a + 2d$ are in GP.

$\Rightarrow (a-d)^2 = a^2 - 4d^2$

$\Rightarrow 5d^2 - 2ad = 0$

$\Rightarrow d(5d - 2a) = 0$

$\Rightarrow 5d = 2a$ ...(ii)

Solving equation (i) and (ii), we get

$d = 6$

44. Option (c) is correct.

Putting the value of d in equation (ii), we get

$5 \times 6 = 2a$

$\Rightarrow a = 15$

$\therefore$ The terms are 3, 9, 15, 21 and 27.

So required sum = 3 + 9 + 15 + 21 + 27 = 75

For questions 45 and 46:

Given $\left(8+3\sqrt{7}\right)^{20} = \text{U} + \text{V}$ ...(i)

and $\left(8-3\sqrt{7}\right)^{20} = \text{W}$

Here, $0 < \text{W} < 1$

Now, adding Eqs. (i) and (ii), we get

$$\text{U} + \text{V} + \text{W} = \left(8+3\sqrt{7}\right)^{20} + \left(8-3\sqrt{7}\right)^{20}$$

$$= 2\left[{}^{20}C_0 8^{20} + {}^{20}C_2 8^{18}.\left(3\sqrt{7}\right)^2 + \ldots + \left(3\sqrt{7}\right)^{20}\right]$$

It is an even number.

Also, $0 < \text{V} < 1$, $0 < \text{W} < 1$ and U is an integer.

$\therefore$ V + W is an integer $\Rightarrow$ V + W = 1

45. Option (d) is correct.

V + W = 1

46. Option (b) is correct.

$$(\text{U} + \text{V})\text{W} = \left(8+3\sqrt{7}\right)^{20}\left(8-3\sqrt{7}\right)^{20}$$

$$= (64 - 63)^{20} = 1^{20} = 1$$

For questions 47 and 48:

The given equation is

$a^2(b^2 - c^2)x^2 + b^2(c^2 - a^2)x + c^2(a^2 - b^2) = 0$...(i)

$\because$ Equation has equal roots:

$\Rightarrow D = 0$

$\therefore [b^2(c^2 - a^2)]^2 - 4a^2c^2(b^2 - c^2)(a^2 - b^2) = 0$

$\Rightarrow b^4(c^2 - a^2)^2 - 4a^2c^2(b^2 - c^2)(a^2 - b^2) = 0$

Solving above equation, we get

$b^4(c^2 + a^2)^2 = 4a^4c^4$

$\Rightarrow b^2(a^2 + c^2) = 2a^2c^2$

$\Rightarrow \frac{2}{b^2} = \frac{1}{a^2} + \frac{1}{c^2}$

$\therefore a^2, b^2$ and c^2 are in HP.

47. Option (c) is correct.

48. Option (c) is correct.

Let α be the required root.

$$\therefore \quad \text{Sum of roots} = \frac{-B}{A} = \frac{-b^2(c^2-a^2)}{a^2(b^2-c^2)}$$

$$\Rightarrow \quad \alpha + \alpha = \frac{-b^2(c^2-a^2)}{a^2(b^2-c^2)}$$

$$\Rightarrow \quad \alpha = \frac{b^2(c^2-a^2)}{2a^2(c^2-b^2)}$$

For questions 49 and 50:

$$\text{Given A} = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

Now, $|A| = 3(-3+4) - 2(-3+4) + 0 = 3 - 2$
$= 1$

$$\because \quad A(\text{adj}A) = |A|I = I$$

49. Option (d) is correct.

50. Option (a) is correct.

$$\text{Given} \quad \text{adj(A)} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$$\text{Now,} \quad A^{-1} = \frac{1}{|A|}(\text{Adj(A)})$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

51. Option (a) is correct.

$$\because \left(2\hat{i} + 6\hat{j} + 27\hat{k}\right) \times \left(\hat{i} + \alpha\hat{j} + \beta\hat{k}\right) = 0$$

$$\Rightarrow \quad \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \alpha & \beta \end{bmatrix} = 0$$

$$\Rightarrow \hat{i}(6\beta - 27\alpha) - \hat{j}(2\beta - 27) + \hat{k}(2\alpha - 6) = 0$$

Comparing both sides, we get

$$6\beta - 27\alpha = 0$$

$$\Rightarrow \quad 2\beta = 9\alpha$$

$$2\beta - 27 = 0$$

$$\Rightarrow \quad \beta = \frac{27}{2}$$

$$\text{and} \quad 2\alpha - 6 = 0$$

$$\Rightarrow \quad \alpha = 3$$

$$\text{Now,} \quad 3\alpha + 2\beta = 3 \times 3 + 2 \times \frac{27}{2}$$

$$9 + 27 = 36$$

52. Option (b) is correct.

$$\text{Let} \quad p = \left|\vec{a} \times \vec{b}\right| + \sqrt{3}\left|\vec{a} \cdot \vec{b}\right|$$

$$= \left|\vec{a}\right| . \left|\vec{b}\right| |\sin\theta| + \sqrt{3}\left|\vec{a}\right|\left|\vec{b}\right| |\cos\theta|$$

$$= \left|\vec{a}\right|\left|\vec{b}\right|\left[|\sin\theta| + \sqrt{3}|\cos\theta|\right]$$

p will be max if $\left(\sin\theta + \sqrt{3}\cos\theta\right)$ is maximum.

Now, for its maxima,

$$\frac{d}{d\theta}\left(\sin\theta + \sqrt{3}\cos\theta\right) = 0$$

$$\Rightarrow \cos\theta - \sqrt{3}\sin\theta = 0$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = 30°$$

53. Option (a) is correct.

$\because \left(\vec{a} + 2\vec{b}\right)$ and $\left(5\vec{a} - 4\vec{b}\right)$ are perpendicular vectors.

$$\Rightarrow \quad \left(\vec{a} + 2\vec{b}\right) . \left(5\vec{a} - 4\vec{b}\right) = 0$$

$$\Rightarrow \quad 5\left|\vec{a}\right|^2 - 4\vec{a} . \vec{b} + 10\vec{a} . \vec{b} - 8\left(\vec{b}\right)^2 = 0$$

$$\Rightarrow \quad 5 \times 1 + 6\vec{a} . \vec{b} - 8 = 0$$

($\because \vec{a}$ and $\vec{b}$ are unit vectors.)

$$\Rightarrow \quad 6\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta = 3$$

$$\Rightarrow \quad 1.1\cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

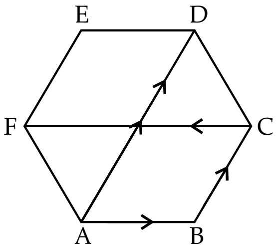
$$\therefore \quad \cos 2\theta = \cos^2\theta - 1$$

$$= 2 \times \frac{1}{4} - 1$$

$$= -\frac{1}{2}$$

$$\text{Now,} \quad \cos\theta + \cos 2\theta = \frac{1}{2} - \frac{1}{2} = 0$$

54. Option (a) is correct.



$\because \quad \vec{AB} = 2\vec{FC}$

$\Rightarrow \quad \vec{AB} = -2\vec{CF}$

$\therefore \quad n = -\frac{1}{2}$

Also, $\quad \vec{AD} = 2\vec{BC}$

$\therefore \quad m = 2$

Now, $\quad mn = 2\left(-\frac{1}{2}\right) = -1$

55. Option (c) is correct.

$\vec{a} = \hat{i}+\hat{j}, \vec{b} = \hat{j}+\hat{k}$

Also, $\vec{a}$, $\vec{b}$ and $\vec{c}$ has same length.

$$\Rightarrow \quad |\vec{a}| = |\vec{b}| = |\vec{c}|$$

$$= \sqrt{1+1} = \sqrt{2}$$

Let θ be the angle between the vectors.

$$\therefore \quad \cos\theta = \frac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|} = \frac{0+1+0}{\sqrt{2}\times\sqrt{2}} = \frac{1}{2}$$

(I) Let $\quad \vec{c} = \hat{i}+\hat{k}$

$$|\vec{c}| = \sqrt{2}$$

and $\quad \cos\theta = \frac{\vec{a}\,.\,\vec{c}}{|\vec{a}||\vec{c}|} = \frac{1+0+0}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$

All the conditions are satisfied, so it can be vector $|\vec{c}|$.

(II) Let $\quad \vec{c} = \frac{-\hat{i}+4\hat{j}-\hat{k}}{3}$

$$\Rightarrow \quad |\vec{c}| = \frac{1}{3}\sqrt{1+16+1} = \sqrt{2}$$

and $\quad \cos\theta = \frac{\vec{a}\,.\,\vec{c}}{|\vec{a}||\vec{c}|} = \frac{\frac{-1}{3}+\frac{4}{3}}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$

All the conditions are satisfied, so it can be vector $|\vec{c}|$.

56. Option (d) is correct.

Slope of diagonal along the line $x - 2y = 1$

$$m_1 = \frac{1}{2}$$

Slope of the diagonal along the line

$$4x + 2y = 3$$

$$\Rightarrow \quad m_2 = -2$$

Now, $\quad m_1m_2 = \frac{1}{2}(-2) = -1$

∴ Diagonals are perpendicular.

⇒ The quadrilateral ABCD is a rhombus.

57. Option (b) is correct.

Given $\quad 4x^2 + 9y^2 = 1$

Here, $\quad a = \frac{1}{2}$ and $b = \frac{1}{3}$

$$e = \sqrt{1-\frac{b^2}{a^2}} = \frac{\sqrt{5}}{3}$$

$$\text{foci} = (\pm ae, 0)$$

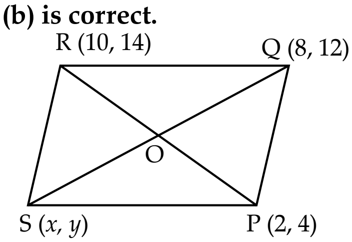
$$= \left(\pm\frac{1}{2}\frac{\sqrt{5}}{3}, 0\right)$$

$$= \left(\pm\frac{\sqrt{5}}{6}, 0\right)$$

∴ P(x, y) is any point on the ellipse.

$$\therefore \quad PQ + PR = 2a = 2\times\frac{1}{2} = 1$$

58. Option (b) is correct.



∵ O is midpoint of P and R

$$\Rightarrow \quad O \equiv \left(\frac{10+2}{2}, \frac{14+4}{2}\right) = (6, 9)$$

Also, O is midpoint of S and Q.

$$\therefore \quad O \equiv \left(\frac{x+8}{2}, \frac{y+12}{2}\right)$$

$$\therefore \quad (6, 9) = \left(\frac{x+B}{2}, \frac{y+12}{2}\right)$$

Comparing both sides, we get

$$\frac{x+8}{2} = 6$$

$\Rightarrow \quad x = 4$

and $\quad \dfrac{y+12}{2} = 9$

$\Rightarrow \quad y = 6$

Now, $\quad x + y = 4 + 6 = 10$

59. Option (a) is correct.

Given equation of circle is

$(x^2 - 4x + 3) + (y^2 - 6y + 8) = 0$

$(x - 3)(x - 1) + (y - 4)(y - 2) = 0$

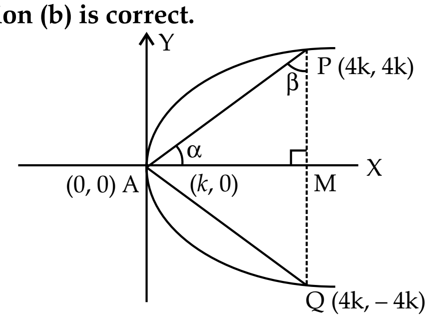
So the possible ends of the diameters are (3, 4), (3, 2), (1, 4) and (1, 2).

Also, $\quad$ Radius $= \sqrt{2}$

and $\quad$ center $= (2, 3)$

So the required pair of ends of diameters are (I) (1, 2) and (3, 4) and (II) (1, 4) and (3, 2).

60. Option (b) is correct.



Let $\quad \angle PAM = \alpha$

and $\quad \angle APM = \beta$

From figure, it is clear that

$\angle PAQ = \angle PAM + \angle QAM$

$= \alpha + \alpha = 2\alpha$

In right-angled triangle AMP,

$AM = PM = 4k$

$\Rightarrow \quad \alpha = \beta$

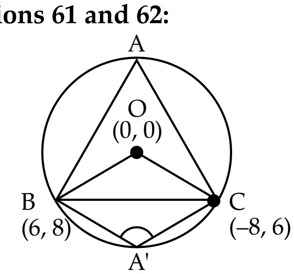
$\therefore \quad \alpha + \beta + 90° = 180°$

$\Rightarrow \quad 2\alpha = 90°$

$\Rightarrow \quad \alpha = 45°$

Hence, $\angle PAQ = 2\alpha = 90°$

For questions 61 and 62:



61. Option (c) is correct.

$$BC = \sqrt{(6+8)^2 + (8-6)^2}$$

$$= \sqrt{200} = 10\sqrt{2}$$

Let O is centre of circle,

$x^2 + y^2 = 100,$

Radius = OB = OC = 10 units

$OB^2 + OC^2 = \sqrt{100+100}$

$= \sqrt{200} = 10\sqrt{2}$

In ΔBOC, $OB^2 + OC^2 = BC^2 = (10\sqrt{2})^2$

$= 200$

So $\angle BOC = 90°$ (by converse of pythagoras theorem)

$\therefore \angle BAC = 45°$ or $180° - 45° = \dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$

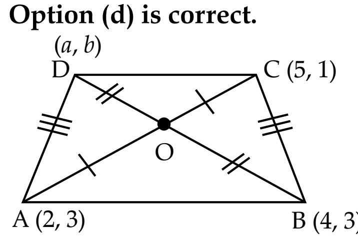
(Angle subtended by a chord at centre is double the angle at any point subtended by it on the remaining circle.)

62. Option (d) is correct.

Point A can be taken at any point on the circumference of circle.

So coordinates of A cannot be determined due to insufficient data.

63. Option (d) is correct.



In isosceles trapezium, diagonals bisect each other.

O is midpoint of AC and BD.

$$\frac{5+2}{2} = \frac{4+a}{2}, \frac{3+b}{2} = \frac{3+1}{2}$$

$a = 3, b = 1$

$D(a, b) = (3, 1)$

64. Option (c) is correct.

Since diagonals of isosceles trapezium bisect each other, then intersecting point of diagonal is O.

Coordinates of O is $\left(\dfrac{2+5}{2}, \dfrac{3+1}{2}\right) = \left(\dfrac{7}{2}, 2\right)$.

For questions 65 and 66:

Sphere: $2x^2 + 2y^2 + 2z^2 + 3x + 3y + 3z - 6 = 0$

$\Rightarrow x^2 + y^2 + z^2 + \dfrac{3}{2}x + \dfrac{3}{2}y + \dfrac{3}{2}z - 3 = 0$

65. Option (b) is correct.

$$\text{Radius} = \sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^2 + 3}$$

$$= \sqrt{\frac{27}{16}+3} = \sqrt{\frac{75}{16}}$$

$$\text{Radius} = 5\frac{\sqrt{3}}{4}$$

$$\text{Diameter} = 5\frac{\sqrt{3}}{2}$$

66. Option (d) is correct.

Centre of sphere is $C\left(\frac{-3}{4}, \frac{-3}{4}, \frac{-3}{4}\right)$.

$4x + 8y + 8z + 15 = 0$, plane satisfy the centre of sphere $-3-6-6+15 = 0 \Rightarrow 0 = 0$

For questions 67 and Q 68:

Two planes are $x + y + z = 1$ and $2x + 3y - 4z = 8$.

67. Option (b) is correct.

Let a, b, c are direction ratios of lines of intersection of planes. Then the lines is perpendicular to normal of both planes.

$$a + b + c = 0 \quad ...(i)$$

$$2a + 3b - 4c = 0 \quad ...(ii)$$

$$\frac{a}{-4-3} = \frac{b}{2+4} = \frac{c}{3-2} = \lambda$$

$$a = -7\lambda, b = 6\lambda, c = \lambda$$

Direction ratios are (–7, 6, 1).

68. Option (a) is correct.

Direction ratios of S are $l = \frac{-7}{\sqrt{86}}$,

$$m = \frac{6}{\sqrt{86}}, n = \frac{1}{\sqrt{86}}$$

$$\therefore 43\,(l^2-m^2-n^2) = 43\left(\frac{49}{86}-\frac{36}{86}-\frac{1}{86}\right)$$

$$= 43\times\frac{12}{86} = 6$$

For questions 69 and Q 70:

$L = x + y + z + 4 = 0 = 2x - y - z + 8$

and $P = x + 2y + 3z + 1 = 0$ is a plane.

69. Option (c) is correct.

Let direction ratios of line (a, b, c)

Since line is obtained by intersection of planes $x + y + z + 4 = 0$ and $2x - y - z + 8 = 0$

So $a + b + c = 0 \quad ...(i)$

$$2a - b - c = 0 \quad ...(ii)$$

$$\frac{a}{-1+1} = \frac{b}{2+1} = \frac{c}{-1-2} = \lambda$$

(solving (i) and (ii))

$$a = 0, b = 3\lambda \text{ and } c = -3\lambda$$

Direction ratios of line is (0, 3, –3) or (0, 1, –1).

70. Option (d) is correct.

To find point of intersection of L and P,

The point must satisfy both line L and plane P,

Option (d) (–4, –3, 3) will satisfy both line L and plane P.

So option (d) is the correct answer.

71. Option (a) is correct.

Since, $\{x\}+[x] = x$

$\Rightarrow \quad x - [x] = \{x\}$

$0 \le x - [x] < 1 \; (0 \le \{x\} < 1)$

$-1 \le [x] - x \le 0$

But x is positive and non-integer; then

$-1 < [x]-x < 0$

$-1 < y < 0$

$\therefore \quad [y] = -1$

72. Option (a) is correct.

$f(x) = 4x + 1, g(x) = kx + 2$

$fog(x) = 4(kx + 2) + 1 = 4kx + 9$

$go\,f(x) = k(4x + 1) + 2 = 4kx + k + 2$

$fog(x) = go\,f(x)$

$\Rightarrow \quad 9 = k + 2$

$\Rightarrow \quad k = 7$

73. Option (b) is correct.

$f(x) = \log_{10}(x^2 + 2x + 11)$ for minimum

$f'(x) = 0$

$$\frac{2x+2}{x^2+2x+11} = 0 \qquad (\text{Since } x^2 + 2x + 11 > 0)$$

At $x = -1$,

we get minumum value of $f(x)$.

$f(-1) = \log_{10}10 = 1$

74. Option (d) is correct.

$$\lim_{x\to3}\frac{|x-3|}{x-3}$$

$$\text{LHL} = \lim_{x\to3^-}\frac{|x-3|}{x-3} = \frac{-(x-3)}{x-3} = -1$$

$$\text{RHL} = \lim_{x\to3^+}\frac{|x-3|}{x-3} = \frac{(x-3)}{x-3} = +1$$

LHL ≠ RHL; limit does not exist.

75. Option (b) is correct.

Since, maximum value of $a\cos x + b\sin x$ is $\sqrt{a^2+b^2}$

So maximum value of $a\cos x + b\sin x + c$ is $\sqrt{a^2+b^2}+c$.

76. Option (c) is correct.

$f(2x) = 4x^2 + 1 \Rightarrow f(x) = x^2 + 1$

$f(4x) = 16x^2 + 1$

$$(4x^2 + 1)^2 = (x^2 + 1) \times (16x^2 + 1)$$

[$f(2x)$ is GM of $f(x)$ & $f(4x)$]

$\Rightarrow \quad x^2 = 0$

$x = 0$

$\therefore$ One real value of x is possible.

77. Option (d) is correct.

$f(x) = [x]^2 - 30[x] + 221 = 0$

$$[x] = \frac{30 \pm \sqrt{900-884}}{2} = \frac{30 \pm 4}{2}$$

$$[x] = \frac{30+4}{2} \text{ or } \frac{30-4}{2}$$

$[x] = 17$ or 13

Integral value of x are 17 and 13.

$\therefore$ Sum of integral solution of $f(x) = 30$.

78. Option (b) is correct.

Given $f(x) = 9x - 8\sqrt{x}$, $g(x) = 9x - 8\sqrt{x} - 1$

Let $x = t^2$

$g(t^2) = 9t^2 - 8t - 1 = 0$

$$t = \frac{8 \pm 10}{18}$$

$$\Rightarrow \quad t = \frac{18}{18} \text{ or } \frac{-1}{9}$$

$$t = 1 \text{ or } \frac{-1}{9}$$

$$\Rightarrow \quad x = 1, \frac{1}{81}$$

$\therefore$ $g(x) = 0$ has only one real root which is an integer.

79. Option (b) is correct.

$$\lim_{\theta \to \frac{\pi}{2}} (\sec\theta - \tan\theta)$$

$$\lim_{\theta \to \frac{\pi}{2}} (\sec\theta - \tan\theta) = \lim_{\theta \to \frac{\pi}{2}} \frac{1-\sin\theta}{\cos\theta} \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{\theta \to \frac{\pi}{2}} \left(\frac{-\cos\theta}{-\sin\theta}\right) = 0 \quad \text{(using L' hospital rule)}$$

80. Option (a) is correct.

$f(x)f(y) = f(xy)$

Let $f(x) = x^n$ and $f(y) = y^n$.

$f(xy) = (xy)^n$

$f(2) = 4 \Rightarrow 4 = (2)^n \Rightarrow n = 2$

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

For questions 81 and 82:

$$fog\,(x) = \cos^2\sqrt{x} \text{ and } gof(x) = |\cos x|$$

81. Option (c) is correct.

If we take option (c),

$f(x) = \cos^2 x$; then $g(x) = \sqrt{x}$

$fog(x) = \cos^2\sqrt{x}$ and $gof(x) = \sqrt{\cos^2 x}$

$= |\cos x|$

$\therefore f(x) = \cos^2 x$

82. Option (a) is correct.

If $f(x) = \cos^2 x$ then $g(x) = \sqrt{x}$

For questions 83 and 84:

$$f(x) = [x]^2 - [x^2]$$

83. Option (b) is correct.

$f(0.999) = [0.999]^2 - [(0.999)^2]$

$= 0 -$ [value lies b/w 0 and 1]

$= 0 - 0 = 0$

$f(1.001) = [1.001]^2 - [(1.001)^2]$

$= 1 - 1 = 0$

$f(0.999) + f(1.001) = 0 - 0 = 0$

84. Option (b) is correct.

$f(x) = [x]^2 - [x^2]$

At $x = 0$

$$\text{LHL} = \lim_{h \to 0} [0-h]^2 - [(0-h)^2]$$

$= (-1)^2 - 0 = 1$

$$\text{RHL} = \lim_{h \to 0} [0+h]^2 - [(0+h)^2]$$

$= 0 - 0 = 0$

LHL $\neq$ RHL; $f(x)$ is not continuous at $x = 0$

At $x = 1$

$$\text{LHL} = \lim_{h \to 0} [1-h]^2 - [(1-h)^2]$$

$= 1 - 1 = 0$

$$\text{RHL} = \lim_{h \to 0} [1+h]^2 - [(1+h)^2]$$

$= 1 - 1 = 0$

$f(x)$ is continuous at $x = 1$; LHL = RHL.

For questions 85 and 86:

$$f(x) = \cos 2x + x \text{ on } \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

85. Option (b) is correct.

$f'(x) = -2\sin 2x + 1$

For maxima and minima, $f'(x) = 0$

$$\sin 2x = \frac{1}{2}$$

$$x = \frac{\pi}{12} \text{ or } \frac{5\pi}{12} \text{ (not in interval)}$$

So maximum value of $f(x)$ occurs at

$$x = \frac{\pi}{12}$$

$$f\left(\frac{\pi}{12}\right) = \cos\frac{\pi}{6}+\frac{\pi}{12} = \frac{\sqrt{3}}{2}+\frac{\pi}{12}$$

86. Option (a) is correct.

$$f\left(\frac{-\pi}{2}\right) = -1-\frac{\pi}{2}$$

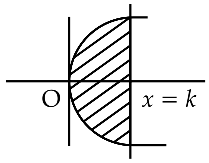
$$f\left(\frac{\pi}{2}\right) = -1+\frac{\pi}{2}$$

So minimum value of f(x) $-\left(1+\frac{\pi}{2}\right)$

For questions 87 and 88:

The area bounded by the parabola $y^2 = kx$ and the line $x = k$, where $k > 0$, is $\frac{4}{3}$ square units.

87. Option (b) is correct.



Area bounded $= \frac{4}{3}$ sq. units

$$= 2\int_0^k y\,dx$$

$$\Rightarrow \quad \frac{4}{3} = 2\int_0^k \sqrt{k}\sqrt{x}\,dx$$

$$\Rightarrow \quad 1 = k^2$$

$\Rightarrow \quad k = \pm 1$ (since k>0)

$$\Rightarrow \quad k = 1$$

88. Option (a) is correct.

So equation of parabola is $y^2 = x$

$$4a = 1, a = \frac{1}{4}$$

Area of parabola $y^2 = x$ and latus rectum at $x = \frac{1}{4}$ is

$$= 2\int_0^{\frac{1}{4}} \sqrt{x}\,dx = 2\times\frac{2}{3}\left(\frac{1}{4}\right)^{\frac{3}{2}}$$

$$= \frac{4}{3}\times\frac{1}{8} = \frac{1}{6} \text{ sq. units}$$

For questions 89 and 90:

Let $ydx + (x - y^3)\,dy = 0$ be a differential equation.

89. Option (a) is correct.

$$ydx + (x-y^3)dy = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{y^3-x}$$

degree $= 1$

90. Option (d) is correct.

$$y\frac{dx}{dy}+x = y^3$$

$\Rightarrow \frac{dx}{dy}+x\frac{1}{y} = y^2$ (linear differential equation)

$$\text{I.F.} = e^{\int\frac{1}{y}dy} = y$$

$$x(\text{I.F}) = \int(\text{I.F.})\,y^2dy$$

$$\Rightarrow \quad xy = \int y^3dy$$

$$\Rightarrow 4xy - y^4 = c$$

For questions 91 and 92:

$f(x) = |x^2 - x - 2|$.

91. Option (d) is correct.

$f(x) = |x^2 - x - 2|$
$= \{x^2 - x - 2;$
$x \in (-\infty, -1)$
$\cup (2,\infty) - (x^2 - x - 2); x \in [-1, 2]$

Let $\quad \text{I} = -\int_0^2 (x^2 - x - 2)\text{d}x$

$$= -\left[\frac{x^3}{3}-\frac{x^2}{2}-2x\right]_0^2$$

$$= -\left(\frac{8}{3}-\frac{4}{2}-4\right)+0 = -\left(\frac{8}{3}-6\right)$$

$$= -\left(\frac{8-18}{3}\right) = \frac{10}{3}$$

92. Option (b) is correct.

$$f(x) = |x^2-x-2|$$

$$= \begin{cases} x^2-x-2; \; x\in(-\infty,-1)\cup(2,\infty) \\ -(x^2-x-2); \; x\in[-1,\,2] \end{cases}$$

Let $\quad \text{I} = \int_1^3 f(x)dx$

$$= -\int_1^2 (x^2-x-2)dx + \int_2^3 (x^2-x-2)dx$$

$$= -\left[\frac{x^3}{3}-\frac{x^2}{2}-2x\right]_1^2 + \left[\frac{x^3}{3}-\frac{x^2}{2}-2x\right]_2^3$$

$$= -\left[\frac{8}{3}-\frac{4}{2}-4\right]+\left[\frac{1}{3}-\frac{1}{2}-2\right]$$

$$+\left[\frac{27}{3}-\frac{9}{2}-6\right]-\left[\frac{8}{3}-\frac{4}{2}-4\right]$$

$$= -\left[\frac{16-12-24}{6}\right]+\left[\frac{2-3-12}{6}\right] + \left[\frac{54-27-36}{6}\right]-\left[\frac{16-12-24}{6}\right]$$

$$= \frac{20}{6}-\frac{13}{6}-\frac{9}{6}+\frac{20}{6} = \frac{20-13-9+20}{6}$$

$$= \frac{18}{6} = 3$$

93. Option (c) is correct.

Let $f(t) = \ln\left(t+\sqrt{1+t^2}\right)$

$$f(-t) = \ln\left(\sqrt{1+t^2}-t\right)$$

$$= \ln\left(\frac{1}{t+\sqrt{1+t^2}}\right)$$

$$= -\ln\left(t+\sqrt{1+t^2}\right) = -f(t)$$

$\therefore$ $f(t)$ is an odd function.

$\because$ $g(t) = \tan f(t)$

$\therefore$ $g(-t) = \tan f(-t)$

$= -\tan(f(t)) = -g(t)$

So $g(t)$ is also an odd function.

Hence, both statements I and II are correct.

94. Option (b) is correct.

When $g(t)$ is an odd function, then $\int_{-\pi}^{\pi} g(t)dt$ $= 0$

95. Option (d) is correct.

Let f: $(-1, 1) \to$ R be a differentiable function with $f(0) = -1$ and $f'(0) = 1$

$$h(x) = f(2f(x) + 2)$$

$$h'(x) = f'(2f(x) + 2)2f'(x)$$

$$h'(0) = f'(2f(0) + 2).2f'(0)$$

$$= f'(-2 + 2).2(1)$$

$$= f'(0).2 = (1).2 = 2$$

96. Option (a) is correct.

$$g(x) = (h(x))^2$$

$$g'(x) = 2(h(x)).h'(x)$$

$$g'(0) = 2(h(0)).h'(0)$$

$$= 2f(2f(0) + 2).2$$

$$= 2f(0).2 = (-2).2 = -4$$

97. Option (c) is correct.

Given that $g(x) = \sin x + \cos x + 1$

$$\int_0^{\frac{\pi}{2}} \frac{dx}{g(x)} = \int_0^{\frac{\pi}{2}} \frac{1}{\sin x+\cos x+1}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}+\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}+1}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}}{2\left(1+\tan\frac{x}{2}\right)}dx$$

Let $1+\tan\frac{x}{2} = t$, $\frac{1}{2}\sec^2\frac{x}{2}\,dx = dt$

When $x = \frac{\pi}{2}$,

then $t = 2$,

and when $x = 0$,

then $t = 1$.

$$= \int_1^2 \frac{1}{t}dt = [\ln t]_1^2$$

$$= \ln 2 - \ln 1$$

$$= \ln 2$$

98. Option (c) is correct.

$$I = \int_0^{\frac{\pi}{2}} \frac{f(x)}{g(x)}dx = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x+\cos x+1}dx \quad ...(i)$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)+1}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x+\cos x+1}dx \quad ...(ii)$$

Adding equation (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x+\cos x}{\sin x+\cos x+1}dx$$

$$= \int_0^{\frac{\pi}{2}} \left(1-\frac{1}{\sin x+\cos x+1}\right)dx$$

$$= \int_0^{\frac{\pi}{2}} 1dx - \int_0^{\frac{\pi}{2}} \frac{1}{\sin x+\cos x+1}dx$$

$$= \int_0^{\frac{\pi}{2}} 1dx - \ln 2 \quad \text{(From solution 97)}$$

$$= \frac{\pi}{2} - \ln 2$$

$$\therefore \quad I = \frac{\pi}{4} - \frac{\text{In}2}{2}$$

For questions 99 and 100:

$$I = 2\int \frac{x^2-1}{\sqrt{x^2+1}}dx = 2\int\left(\frac{x^2+1}{\sqrt{x^2+1}} - \frac{2}{\sqrt{x^2+1}}\right)dx$$

$$= 2\int\left(\sqrt{x^2+1} - \frac{2}{\sqrt{x^2+1}}\right)dx$$

$$= 2\left(\frac{x}{2}\sqrt{x^2+1} + \frac{1}{2}\ln(x+\sqrt{x^2+1}\right) - 2\ln\left(x+\sqrt{x^2+1}\right) + C$$

$$= x\sqrt{x^2+1} - 3\ln(x+\sqrt{x^2+1}) + c$$

$$= U(x).V(x) - 3\ln\{U(x)+V(x)\} + C$$

So, $U(x) = x$ and $V(x) = \sqrt{x^2+1}$

99. Option (b) is correct.

Now, $|U^2(x) - V^2(x)|$

$= |x^2 - (x^2+1)| = |x^2 - x^2 - 1|$

$= |-1| = 1$

100. Option (a) is correct.

$U(x).V(x) = x\sqrt{x^2+1} = \sqrt{x^4+x^2}$

101. Option (d) is correct.

Lines of regression of x on y

$x - 3y + 4 = 0$

$\Rightarrow \quad x = 3y - 4$

$\Rightarrow \quad b_{xy} = 3$

Line of regression of y on x

$2x - 7y + 8 = 0$

$\Rightarrow \quad y = \frac{2}{7}x + \frac{8}{7}$

$\Rightarrow \quad b_{yx} = \frac{2}{7}$

Now, $b_{xy} + 7b_{yx} = 3 + 7.\frac{2}{7} = 5$

102. Option (b) is correct.

$$\text{Mean} = \frac{1+4+9+...+n^2}{n}$$

$$130 = \frac{n(n+1)(2n+1)}{6n}$$

$\Rightarrow \quad 780 = 2n^2 + 3n + 1$

$\Rightarrow \quad 2n^2 + 3n - 779 = 0$

$\Rightarrow \quad 2n^2 + 41n - 38n - 779 = 0$

$\Rightarrow \quad n(2n + 41) - 19(2n + 41) = 0$

$\Rightarrow \quad (2n + 41)(n - 19) = 0$

$\Rightarrow \quad x = 19$

($\because n = \frac{-41}{2}$ is not possible.)

103. Option (c) is correct.

$n(S) = {}^{10}C_3$

Favourable outcomes = (1, 2, 3), (2, 3, 4), (3, 4, 5), (4, 5, 6), (5, 6, 7), (6, 7, 8), (7, 8, 9), (8, 9, 10)

$n(E) = 8$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{8}{{}^{10}C_3}$

$= \frac{8\times3\times2\times1}{10\times9\times8} = \frac{1}{15}$

104. Option (c) is correct.

Given that A, B and C are mutually exclusive and exhaustive events:

$\therefore \quad P(A) + P(B) + P(C) = 1 \quad$ ...(i)

Also, $\quad 3P(B) = 4P(A)$

$\Rightarrow \quad P(A) = \frac{3}{4}P(B)$

and $3P(C) = 2P(B)$

$\Rightarrow \quad P(C) = \frac{2}{3}P(B)$

From (i), $\frac{3}{4}P(B) + P(B) + \frac{2}{3}P(B) = 1$

$\Rightarrow \quad \frac{9P(B)+12P(B)+8P(B)}{12} = 1$

$\Rightarrow \quad \frac{29P(B)}{12} = 1$

$\Rightarrow \quad P(A) = \frac{3}{4}\times\frac{12}{29} = \frac{9}{29}$

105. Option (c) is correct.

Number on faces on die are 4, 4, 5, 5, 5 and 6.

$\therefore \quad n(S) = 6$

P (getting 4 or 5) $= \frac{5}{6}$

106. Option (a) is correct.

Black = 2, yellow = 4 and white = 6

Total balls = 2 + 4 + 6 = 12

P (all three are of the same colour).

$$= \frac{2^3}{12^3} + \frac{4^3}{12^3} + \frac{6^3}{12^3}$$

$$= \frac{1}{6}$$

107. Option (a) is correct.

Here, $\quad P(A) = \frac{5}{6}, P(B) = \frac{4}{5}, P(C) = \frac{3}{4}$

and $\quad P(\overline{B}) = 1 - \frac{4}{5} = \frac{1}{5}$

$\therefore \quad P(A \cap \overline{B} \cap C) = P(A).P(B).P(C)$

$$= \frac{5}{6}\times\frac{1}{5}\times\frac{3}{4} = \frac{1}{8}$$

108. Option (a) is correct.

Number of words formed from letters of word

ZOOLOGY $= \frac{7!}{3!}$

Vowels = O, O, O

Consonants = ZLGY

$\therefore$ Number of words in which consonants and vowels occur alternatively = Z*L*G*Y = 4! × 3!

↓ ↓ ↓

Vowels

Therefore, required probability

$$= 4! \times 3! \times \frac{3!}{7!} = \frac{6}{35}$$

109. Option (b) is correct.

$n(S) = 100$

Numbers from 7 to 100 satisfy $x^2 + x > 50$

$\therefore \quad n(E) = 100 - 6 = 94$

$$P(E) = \frac{n(E)}{n(S)} = \frac{94}{100} = \frac{47}{50}$$

110. Option (b) is correct.

Mean of 10 natural numbers

$$\bar{x} = \frac{1+2+3+...+10}{10}$$

$$= \frac{10\times 11}{2\times 10} = 5.5$$

Mean deviation

$$= \frac{|1-5.5|+|2-5.5|+|3-5.5|+...+|10-5.5|}{10}$$

$$= \frac{4.5 + 3.5 + 2.5 + 1.5 + 0.5 + 0.5 + 1.5 + 2.5 + 3.5 + 4.5}{10} = 2.5$$

111. Option (b) is correct.

Given that $\sum_{i=i}^{9} x_i^2 = 855$

$$\text{Variance } \sigma = \sum_{i=1}^{9} x_i^2 - (\text{mean})^2$$

$$= \frac{855}{9} - M^2$$

$$\Rightarrow \quad \sigma + M^2 = \frac{855}{9} = 95$$

112. Option (d) is correct.

Given that $\bar{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$

$\Rightarrow x_1 + x_2 + x_3 + ... + x_n = n\bar{x}$

Mean when x_n is replaced by k

$$\text{New mean} = \frac{x_1 + x_2 + x_3 + ... + x_{n-1} + k}{n}$$

$$= \frac{n\bar{x} - x_n + k}{n}$$

113. Option (b) is correct.

$\because$ Fair coin is tossed till two heads occur in succession.

$\therefore$ P (the number of tosses required is less than 6)

= P(HH) + P(THH) + P(TTHH) + P(TTTHH)

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^5$$

$$= \frac{\left(\frac{1}{2}\right)^2\left[1-\left(\frac{1}{2}\right)^4\right]}{1-\frac{1}{2}} = \frac{\frac{1}{4}\left[1-\frac{1}{16}\right]}{\frac{1}{2}}$$

$$= \frac{1}{4}\times\frac{15}{16}\times\frac{2}{1} = \frac{15}{32}$$

114. Option (b) is correct.

White = 2 White = 3

Black = 2 Black = 2

A B

E_1 = One white ball transferred from A to B; then urn A contains white = 1 and b = 2 and urn B contains white = 4 and black = 2

E_2 = One black ball transferred from A to B. then urn A contains white = 2 and black = 1 and urn B contains white = 3 and black = 3

F = One white ball is drawn from urn B.

$$P(F) = P(E_1).P\left(\frac{F}{E_1}\right) + P(E_2).P\left(\frac{F}{E_2}\right)$$

$$= \frac{2}{4}\times\frac{4}{6}+\frac{2}{4}\times\frac{3}{6} = \frac{8}{24}+\frac{6}{24}$$

$$= \frac{14}{24} = \frac{7}{12}$$

115. Option (d) is correct.

Given that $P(A) = P\left(\frac{A}{B}\right) = 0.25$

and $P\left(\frac{B}{A}\right) = 0.5$

I. $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow P(A \cap B) = P(A)\, P(B|A)$

$\Rightarrow P(A \cap B) = 0.25 \times 0.5 = 0.125$

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow P(B) = \frac{P(A \cap B)}{P\left(\frac{A}{B}\right)}$

$\Rightarrow P(B) = \frac{0.125}{0.25} = 0.5$

Now, $P(A).P(B) = 0.25 \times 0.5 = 0.125 = P(A \cap B)$

$\therefore$ A and B are independent.

II. $P(\overline{A} \cup \overline{B}) = 1 - P(A \cap B)$

$= 1 - 0.125 = 0.875$

III. $P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)$

$= 1 - [0.25 + 0.5 - 0.125]$

$= 1 - 0.625 = 0.375$

So all statements I, II and III are correct.

116. Option (d) is correct.

$n(S) = 6 \times 6 = 36$

Sum 9 = (3, 6), (4, 5), (5, 4), (6, 3)

Sum 10 = (4, 6), (5, 5), (6, 4)

P (sum 9 or 10) $= \frac{7}{36}$

$\therefore$ P (sum neither 9 nor 10) $= 1 - \frac{7}{36} = \frac{29}{36}$

117. Option (a) is correct.

Here, $n = 6$

$p = P$ (suffering from disease)

$= \frac{20}{100} = \frac{1}{5}$

$q = P$ (not suffering from disease)

$= 1 - \frac{1}{5} = \frac{4}{5}$

$p(x \geq 4) = p(x = 4) + p(x = 5) + p(x = 6)$

$= {}^6C_4 \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^2 + {}^6C_5 \left(\frac{1}{5}\right)^5 \left(\frac{4}{5}\right)^1 + {}^6C_6 \left(\frac{1}{5}\right)^6$

$= \frac{6!}{4!2!} \times \frac{16}{5^6} + \frac{6!}{5!} \times \frac{4}{5^6} + \frac{1}{5^6}$

$= \frac{6.5.4!}{4!2 \times 1} \times \frac{16}{15625} + \frac{6.5!}{5!} \times \frac{4}{15625} + \frac{1}{15625}$

$= \frac{240 + 24 + 1}{15625} = \frac{265}{15625} = \frac{53}{3125}$

118. Option (d) is correct.

$n(S) = 6 \times 5 \times 4$

$n(E) = n$ (one of the faces shown is an ace.)

$= 3(1 \times 5 \times 4) = 3 \times 5 \times 4$

$P(E) = \frac{3 \times 5 \times 4}{6 \times 5 \times 4} = \frac{1}{2}$

119. Option (a) is correct.

Possible outcomes $x < y < z$

= (1, 2, 3) (1, 2, 4), (1, 2, 5), (1, 2, 6) (1, 3, 4) (1, 3, 5) (1, 3, 6), (1, 4, 5), (1, 4, 6), (1, 5, 6), (2, 3, 4),(2, 3, 5) (2, 3, 6), (2, 4, 5), (2, 4, 6), (2, 5, 6), (3, 4, 5), (3, 4, 6) (3, 5, 6), (4, 5, 6)

$\therefore$ Total possible outcomes = 20

120. Option (d) is correct.

Given that mean $\overline{x} = np = 6$ ...(i)

and SD $= \sqrt{npq} = \sqrt{2}$

$npq = 2$ ...(ii)

Dividing (ii) by (i), we get

$\frac{npq}{np} = \frac{2}{6}$

$\Rightarrow q = \frac{1}{3}$

$\therefore p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$

$n \times \frac{2}{3} = 6$ (from Eq. (i))

$n = 9$